

AT-Angle: A Distributed Method for Localization using Angles in Sensor Networks

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Abstract

Determining where a given sensor is physically located is a challenging issue. In this paper, we address the localization problem where, initially, a certain number of sensors called anchors are aware of their positions. Our goal is to localize all sensors with high accuracy, while using a limited number of anchors. So, we focus on localization techniques based on angle of arrival information between neighbor nodes. This paper proposes an original angle-based localization technique, called AT-Angle, which allows to verify two important properties: first, a sensor node can eliminate wrong received information about its position; second, it deduces if its estimated position is closed to its real position. In this last case, the sensor node becomes an estimated anchor and contributes to the positioning of others nodes. Simulations show that AT-Angle achieves good precision for located nodes despite the introduction of position errors and the small number of anchors.

1. Introduction

Recent advancements in micro-electro-mechanical systems and wireless communications has enabled the development of a new kind of networks: *Wireless Sensor Networks*. These networks have been proposed in many fields such that target tracking, intrusion detection, medical applications, climate control and disaster management. Sensors are small devices which are composed by sensing, computing and communication modules. Usually, sensors are equipped with a little battery. The sensing module allows to gather events about their environment, the computing module process these gathered data and finally, sensors can exchange their information via the communication module. To preserve the battery, only a few operations (computations and especially communications) have to be performed. In some

applications the knowledge about sensor localization is required but all sensors cannot be equipped by localization module (e.g. GPS[4]) due to cost and energy constraints. A common example of ad-hoc wireless sensor networks is the aircraft deployment of sensors in a given area. In this network, only a few nodes know their positions thanks to a localization module. These nodes are called *anchors*. A maximum number of remaining nodes have to deduce their positions according to anchor positions. Nevertheless, the number of anchors has to be as small as possible due to cost and energy constraints. The network has to be self-organizing, i.e. it does not depend on global infrastructure. Proposed solutions must take all sensor characteristics into account. Localization schemes can be classified into two categories : *measure-free methods* and *measure-based methods*. The first category contains all methods which just consider anchor positions to compute sensor positions. These methods only assign estimated positions. The second category contains methods which assume that sensors can calculate either distances or angles with their neighbors by using technologies such as ToA (Time of Arrival), TDoA (Time Difference of Arrival), RSSI (Received Signal Strength Indicator) and AoA (Angle of Arrival). This paper focus on approaches where angles can be measured by sensors. The accuracy of technologies AoA, ToA, etc depends on network's environment. Errors which are introduced by these technologies are called *measure errors*. It is the most important drawback for measure-based methods (distances or angles). It is important to note that all previous angle-based methods have no idea about the precision of their computed positions and if its estimated position is close to its real position. This paper presents a new distributed localization technique for wireless sensor networks, called AT-Angle, where sensors can only calculate angles with their neighbors. AT-Angle locates a great number of sensors with accuracy positions and manages introduction (or accumulation) of measure errors. Each node performs the localization technique which

defines a restricted zone containing the node, according to information about anchor positions and angles. To be located, a node computes an estimation of its position which is the gravity center for this zone. AT-Angle verifies **two important properties**: first, a node detects if its estimated position is closed to its real position. In this case, this node becomes an *estimated anchor* and will be used by others nodes to obtain their positions. Second, some wrong localization information (e.g. due to measure errors) can be eliminated regarding to defined sensor zones. These AT-Angle properties allow to obtain good results. Contrary to the existing solutions [2, 6] which derive their schemes under the strong assumption of noisy range measurements (signal, distance, and angle), we do not make any assumption of this kind. However, we introduce the error on the estimated position. In [6], authors propose localization and orientation scheme that is derived under the assumption of noisy angle measurements. But, the problem is how a node can well estimate this error? In our method, a node which estimates its position can determine the position error bound without any additional hardware component. If the error on this position is bounded then the sensor propagates this information to its neighbors. The remainder of the paper is organized as follows : Section 2 introduces basic notions for this problem. Section 3 discusses previous works on sensor localization problem. Section 4 presents our approximation technique. Section 5 discusses simulation results of our method and its comparison to others methods. Finally, in section 6 gives the conclusion of the paper.

2. Model

This paper focuses on static networks. Moreover, it assumes that all sensors have identical reachability radius r . However, it is easy to adapt our methods to sensors having different reachability radii. A wireless sensor networks is represented as a bidirectional graph $G(V, E)$, where V is the set of n nodes representing sensors and E is the set of m edges representing communication links. If two nodes $u, v \in V$ are neighbors, then they are linked, which means distance between u and v is smaller than r .

$$E = \{(u, v) \in V^2 | u \neq v \text{ et } d_{uv} \leq r\}$$

In all figures, white nodes represent sensors which do not know their positions, and white nodes represent anchors. The set of neighbors for a node $u \in V$ is denoted $N(u)$. A priori, some nodes, called *anchors*, have knowledge of their own position with respect to some global coordinate system. Positions can be obtained by a localization system such as GPS [4]. Thanks to these modules, an anchor can locate itself with position error less than 1 meter, but in military application, this accuracy is measured in millimeters. We

assume that all sensors which are anchors have omnidirectional antennas (do not allow to measure AOA) and the others are able to measure angles from incoming signals. The set of anchors is denoted Δ . The set of neighbor anchors for a node u is denoted $N_{\Delta}(u)$ (i.e. $N_{\Delta}(u) = N(u) \cap \Delta$) and the set of non-neighbor anchors is denoted $\overline{N}_{\Delta}(u)$ (i.e. $\overline{N}_{\Delta}(u) = \Delta \setminus N_{\Delta}(u)$). Note that all identical nodes (anchors or other nodes) have the same capabilities (energy, processing, communication, etc). The position of node u is denoted (x_u, y_u) . \mathcal{P} is the set of all possible positions in a network. AT-Family methods define zones containing nodes. The zone of node u , denoted Z_u , is the set of possible positions for u . The exact position of u belongs to Z_u . AT-Angle method assumes that each node can only compute its angle (and does not have a capability to compute distances) to its neighbors when it receives a signal according to one reference axis (north, south, east, west, determined with a compass). All angles are computed according to the same reference axis. The angle formed by two nodes u and v is denoted $\angle u, v$. When a node receives positions of two anchor neighbors, it deduces its position. For example, in figure 1, C does not know its position. A and B send to C their positions (x_1, y_1) , (x_2, y_2) and angles $\angle A, C = \theta_1$, $\angle B, C = \theta_2$. (x_c, y_c) can be calculated with this system : $\frac{y_c - y_i}{x_c - x_i} = \tan(\theta_i)$ for $i \in \{1, 2\}$. Node C becomes an anchor and broadcasts its position.

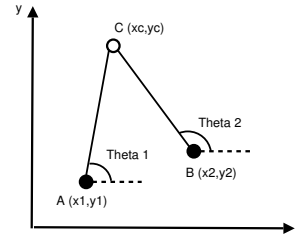


Figure 1. Angle measure

3. Related works

Solutions for the localization problem can be organized in three categories. GPS-free methods, which means that node does not need anchors to locate itself. For example, method in [11] builds a virtual system of coordinates and the node computes its position in this system. Infrastructure-based systems, which need infrastructure like RADAR[1] or Cricket[8]. Robot-based systems. In [7, 3], authors proposed method uses robots to locate nodes. Nodes can be equipped with different capabilities. They can calculate distances or angles with their neighbors when they receive messages from them. The most popular methods in

order to compute distances or angles between two neighbor nodes are RSSI (Received Signal Strength Indicator)[1], ToA/TDoA (Time of arrival / Time difference of arrival) [4, 9] and AoA [5] (Angle of arrival). AoA estimates the angle at which sensors sense the direction from which a signal is received. A simple geometric relationships is used to calculate node positions. AoA sensing requires either antenna array or several ultrasound receivers. Each angle is measured according to an orientation which represents one of axes north, south, east, west. The most popular techniques APS_{AoA} [5] and *Probabilistic* [6] are two angle-based methods. In APS_{AoA} , each anchor floods its position. When a sensor X receives an position of anchor A and regarding to angle with the announcer, it deduces its angle with anchor A . When X knows more than three anchor positions and associated angles, it estimates its position thanks to the triangulation. In *probabilistic* method, positions are estimated through probability distribution functions. It defines, a pseudo-anchor, as a sensor with estimated position probability density function. Anchors and pseudo-anchor send their positions. A sensor which receives these data, measures the angle with the announcer and updates its position distribution and probability density function. Its becomes a pseudo-anchor and broadcasts its updated probability density function. Authors conclude that their technique is better than APS_{AoA} . However, they use an angle error modelisation as being a Gaussian distribution to characterize AoA measurements. Thus, when a sensor sends a signal, it deduces the direction of the line of sight plus or minus an angle error bound. But the efficiency of this modelisation depends on network's environment. When there are obstructions between transmission of any two nodes, this modelisation cannot be used. As APS_{AoA} , AT-Angle does not use any error modelisation. The hypothesis about network's environment and sensor knowledge are the same. Therefore, AT-Angle performances are better compared to the ones of APS_{AoA} which have the same assumptions.

4. AT-Angle Localization Technique

This section proposes an approximation technique called AT-Angle, where nodes are equipped with the AoA-technology in order to calculate angles with respect to their neighbors.

Algorithmic Description Initially, anchors broadcast their positions. Then, each sensor deduces the number of hops (only the smallest numbers are considered) between it and each anchor. When a node receives a position of an anchor, if the node and the anchor are neighbors, then the node deduces that it is inside the disk of radius r and centered on the anchor (later, we will introduce the incertitude on the transmission range and see how this measure error

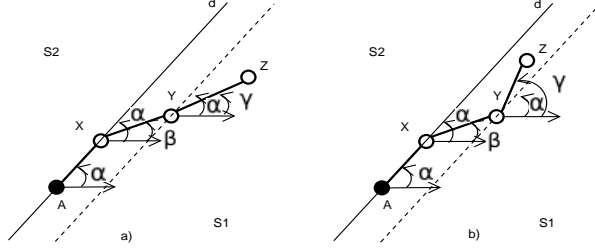


Figure 2. a) Z deduces its side b) Z does not

can influence on the position of sensors). Otherwise, if the node and the anchor are not neighbors, then the node deduces that it is outside the disk of radius r and centered on the anchor, but inside the disk of radius $r \times h$ and centered on the anchor, where h is the number of hops from the node to the anchor. Thus, the intersection of disks defines a restricted zone, denoted Z_u^c , containing the node. Formally, equations (1), (2) (with $\epsilon_a = 0$, this variable will be introduced after) and (3) allow to obtain the restricted zone for each node $u \in V \setminus \Delta$. Let A an anchor, B its neighbor and d the straight line crossing A 's position and B 's position. Then, each node B , which is neighbor of an anchor A , broadcasts the anchor's position and the angle formed between line d and the reference axis (e.g. north, south, east, west, which must be the same for each sensor). All angles are computed according to this axis. In this paper, we consider a plan which will be cut by d into two sides $S1$ and $S2$. Then, a node seeks to deduce if it belongs to this line or to one of the two sides. In the case where a node take a decision then it broadcasts the collected anchor's information (anchor's position and the angle). Otherwise, it does not broadcast anything. Let us consider the example described in figure 2 where A is an anchor and X, Y and Z are neighbor nodes. When the node X receives the position of anchor A , it deduces the line d (crossing X and A) as well as the angle $\angle A, X = \alpha$ which form this line with the horizontal axis. Since X belongs to the line d , it disseminates this information to its neighbors. At the reception of the message from X , Y deduces the angle $\angle X, Y = \beta$ it forms with X . It plots the straight line d thanks to the position of A and α and, according to the values of α and β , it deduces the side of d it belongs to: $S1$ or $S2$. If the condition (5) is verified then Y belongs to $S2$ else Y belongs to $S1$. Then, Y broadcasts the angle α , its side $S1$ and the position of the anchor A . When Z receives this message, it measures the angle $\angle Y, Z = \gamma$ which it forms with Y . According to the values of α and γ and side of Y related to d , Z tries to determine the side to which it belongs. Two cases arise: If the angles

$$Z_{N_{\Delta}(u)} = \bigcap_{a \in N_{\Delta}(u)} \{\forall i \in \mathcal{P}, (x_i, y_i) | (d_{ua} - \epsilon_a)^2 \leq (x_i - x_a)^2 + (y_i - y_a)^2 \leq (d_{ua} + \epsilon_a)^2\} \quad (1)$$

$$Z_{\overline{N_{\Delta}(u)}} = \bigcap_{a \in \overline{N_{\Delta}(u)}} \{\forall i \in \mathcal{P}, (x_i, y_i) | (r - \epsilon_a)^2 < (x_i - x_a)^2 + (y_i - y_a)^2 \leq (\hat{d}_{ua} + \epsilon_a)^2\} \quad (2)$$

$$Z_u^c = Z_{N_{\Delta}(u)} \cap Z_{\overline{N_{\Delta}(u)}} \quad (3)$$

$$S_d(u) = \begin{cases} S_1 & , \exists v \in N(u), v \in S_1 \mid \angle u, v < \alpha_d \text{ or } \angle u, v > \alpha_d + \pi \\ S_2 & , \exists v \in N(u), v \in S_2 \mid \alpha_d < \angle u, v < \alpha_d + \pi \\ S_1 \cap S_2 & , \exists v \in N(u), v \in S_1 \cap S_2 \mid \angle u, v = \alpha_d \text{ or } \angle u, v = \alpha_d + \pi \\ \emptyset & , \text{otherwise} \end{cases} \quad (4)$$

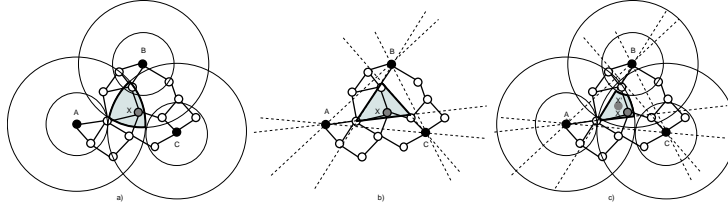


Figure 3. Approximation with: a) disks b) straight lines c) both.

α and γ verify the condition (5) (figure 2a) then Z deduces that it belongs to the same side S_1 as Y . Z broadcasts the position of A , and its side S_1 , and so on... If the angles α and γ does not respect the condition (5) (figure 2b) then Z can deduce nothing. Indeed, without the knowledge of distances, Z does not know if it remains in the same side S_1 or if it passes on the other side S_2 .

$$\alpha < \beta < \alpha + \pi \quad (5)$$

In other words, a node can conclude if it *moves away* from the straight line formed by the position of A and α and thus it stays on the same side of its announcer. Otherwise, a node cannot conclude if it *moves closer* to the straight line and stays on the same side of its announcer. Finally, with the anchor positions and angles, a node computes the zone containing itself. Formally, for each node $u \in V \setminus \Delta$, Z_u is obtained as follows: from equation (4), each node u can determine the side $S_d(u)$ it belongs related to the straight line d obtained by an anchor and one of its neighbors and the angle α_d . Let \mathcal{D} be the set of straight lines deduced relative to anchor positions and angles:

$$\mathcal{D} = \{d(a, \angle a, v) : \forall a \in \Delta, \forall v \in N(a) \cup \Delta \setminus \{a\}\}$$

Let \mathcal{S}_u be the set of sides for u :

$$\mathcal{S}_u = \bigcap_{d \in \mathcal{D}} S_d(u) \quad (6)$$

The zone Z_u^d of u obtained according to straight lines is defined as follows:

$$Z_u^d = \bigcap_{S \in \mathcal{S}_u} \{(x_i, y_i) \in S\} \quad (7)$$

Zone Z_u of u with straight lines and disks is defined as follows:

$$Z_u = Z_u^d \cap Z_u^c \quad (8)$$

Figure 3 represents the estimation technique. The zone outlined with bold lines defines the area containing node X calculating its position. In figure 3a, node X receives positions of anchors A, B, C . It calculates the number of hops with these anchors. X is not a neighbor of A, B, C , so X is outside of disks centered respectively on A, B, C of radius r . But X is inside disks centered on A (resp. B, C) of radius $r \times h_i$ ($i \in \{A, B, C\}$), where h_i represents the number of hops to A (resp. B, C). In figure 3b, X also receives angles formed by anchors with their neighbors and other anchors and X draws straight lines. X deduces sides to which they belong according to the straight lines. Finally, figure 3c represents the calculated zone with disks and straight lines. X computes the gravity center of this zone and estimates its position in X' .

Properties of AT-Angle AT-Angle has two important properties: First, a node knows if its estimated position is close to its real position regarding to its position error

bound : this bound denoted ϵ is the distance between its estimated position (i.e. the gravity center) and the point in its zone, furthest away from the gravity center. Let d_{err} being the distance between its estimated position and its real position which represents the position error. Whatever the real position of the node, it knows that $d_{err} \leq \epsilon$. By using a predefined *threshold*, if $\epsilon \leq \text{threshold}$ then the node has an estimation close to its real position. In this case the node becomes an estimated anchor and broadcasts its position and its ϵ . Thus, when a node applies the approximation technique with an estimated anchor radius, it takes ϵ into account : if an anchor A is not neighbor of node X then radius of disks become $r - \epsilon$ and $r \times h + \epsilon$. If X and A are neighbors then X belong to disk of radius equal to $r + \epsilon$. Therefore, $Z_{N\Delta(u)}$ and $Z_{\overline{N\Delta}(u)}$ become as indicated in (1) and (2) with $\epsilon_a = 0$ for anchors equipped with GPS module. Secondly, according to its zone, a node can detect if some localization information are wrong. Particularly, when a node receives information locating itself outside of its zone. More phenomena can be responsible for this situation: angle measures can be wrong due to the network environment and introduce wrong information; the anchor may be under control of an attacker in a military context and announce a wrong position. Thus, when a sensor receives localization information, it checks that these data are consistent regarding to its defined zone ; otherwise, it deletes them.

5. Simulations

Simulation environment AT-Angle is simulated with OMNET++, a discrete event simulator [10]. Concurrent transmissions are allowed if the transmission areas do not overlap. When a node wants to broadcast a message while another message in its area is in progress, it must wait until that transmission are completed. Simply, the simulator uses CSMA/CA MAC protocol. Message corruption is not considered, so all messages sent during the simulation are delivered. First, a random network topology is generated according to the number of nodes and the number of anchors. Nodes are randomly placed, with an uniform distribution, within a square area. Also, anchors are randomly selected. In order to allow easy comparison between different scenarios, errors on the estimated position are normalized according to the radio range. For example, 50% of position error means a distance of half the range of the radio between the real and estimated positions. Percentage of angle errors are normalized according to π . For example, 10% of angle errors means that the calculated angle can belong to the interval $\text{angle} \pm \pi \times 0.10$. The percentage of angle errors is called δ . All nodes are distributed in a square 100×100 . The connectivity (average number of neighbors) is controlled by specifying the radio range. By default, sce-

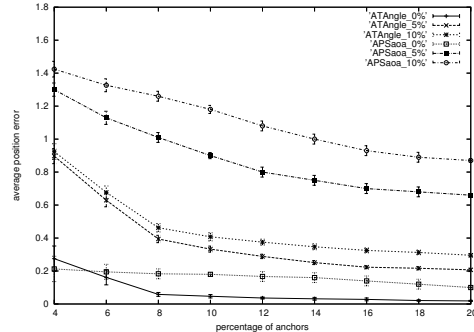


Figure 4. Average error rate for $\delta = \{0, 5, 10\}\%$

narios use networks with 150 nodes and the radio range is set to 14. Thus, sensor density is equal to 9,24. The percentage of anchors varies from 0% to 20% representing density of anchors from 0.12 to 1.23. Different scenarios are used while changing the measure error percentage δ respectively equals to 0%, 5%, 10%. Moreover, a node becomes an estimated anchor if its maximum position error is lower than 15% (i.e. $\epsilon \leq r \times 0.15$). AT-Angle is proposed in order to resolve the localization problem in wireless sensor networks. Therefore, simulations focus on a criterion allowing to evaluate performance of AT-Angle for this problem: the average error rate (i.e. the sum of position errors divided by the number of nodes minus the number of anchor equipped with GPS). This criterion is analysed according to anchor percentage, measure error percentage, and node density. In our analysis, each scenario is performed 100 times. Thus, a relatively small variance is obtained. Graphs represent the means and the confidence intervals for each analysed parameters. Here there is 95% of chance that the real value belong to this interval.

Anchor percentage When AT-Angle does not exactly locate nodes, it assigns estimated positions. Figure 4 illustrates the average position errors according to percentage of anchors equipped GPS. This percentage varies from 4% to 20%. This graph contains six curves representing position average errors with angle error equals to 0%, 5% and 10% for AT-Angle and APS_{aoa} . With percentage of anchors equals to 10% and angle error equals to 0%, position average error is lower than 5% for AT-Angle and 20% for APS_{aoa} . When angle errors are introduced, figure 4 shows that average position error increases. Without surprise, there is a great difference between curves representing the ideal case and others. Nevertheless, this difference decreases (especially between 5% and 10%). This means that AT-Angle manages efficiently the introduction of angle errors and maintains the average position error at the same level in networks containing measure errors.

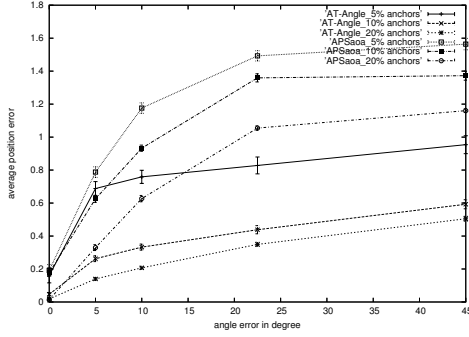


Figure 5. Average error rate with angle errors from 0 to 45

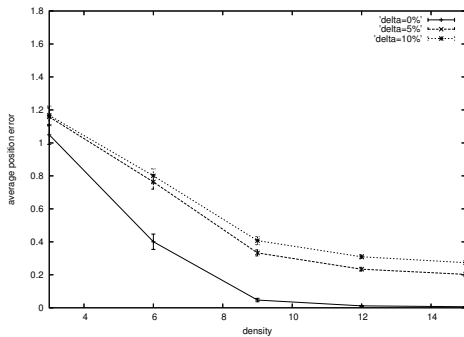


Figure 6. AT-Angle: Average error rate according to density of nodes with $\alpha = 10\%$

Measure error percentage Figure 5 shows impacts of the angle measurement error on average position error in AT-Angle and APS_{aoa} . There are three curves for each method respectively representing the position mean error when the percentage of anchors equipped GPS is equal to 5%, 10% and 20% according to angle errors. On the horizontal axis the standard deviation of the measurement noise is varied from 0 to $\frac{\pi}{4}$. AT-Angle is very efficient because with, for example, a percentage of anchors higher than 10% and angle errors higher than $\frac{\pi}{8}$, APS_{aoa} obtains a average position error higher than 70% while AT-Angle obtains a average position error lower than 60%.

Node density Finally, figure 6 shows impacts of density of nodes on the behavior of average error rate. The more density of nodes increases, the more average error rate decreases. Note that after a density of nodes equals to 12, the behavior of average error rate is not significative. As position constraints (i.e., due to the link between neighbors) increases when sensor density increases, estimated positions converge toward exact positions.

6. Conclusions

This paper proposes a novel approximation technique AT-Angle in order to estimate the position of nodes when they can only calculate angles with their neighbors. No information are known about network's environment. AT-Angle assigns positions (exact or estimated) for all sensors in the networks and proposes two important properties. First, this technique eliminates some wrong propagated information due to measure errors or other. Second, a node knows if its estimated position is closed to its real position and in this case, it becomes an estimated anchor and contributes to the positioning of others nodes. These two properties contributed to performances of obtained simulation results.

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