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A dual model-free control of non-minimum phase systems for generation of stable limit cycles

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***Abstract*— This paper presents a method allowing recent model-free control technique to deal with non-minimum phase systems for stable limit cycles generation. A first controller is designed in order to track parametrized reference trajectories on a subset of the coordinates. In order to stabilize the closed loop system, a second controller is designed using one parameter of reference trajectories as input. The overall system is therefore able to track the desired trajectories while stabilizing the internal dynamics of the system. The proposed method is illustrated through two examples of non-minimum phase systems. Numerical simulations are presented showing the effectiveness of the proposed method as well as its robustness toward external disturbances.**

I. Introduction

Model-free control strategies has been recently proposed in [7], [1] resulting in a breakthrough in nonlinear control. This technique is based on recent results on fast estimation and identification of nonlinear signals [8], [13]. The control scheme is based on local linear approximation of the controlled system dynamics which is valid for a small time window. This approximation is updated in an online fashion thanks to a fast estimator. The control law proposed consists in a PID controller augmented with compensating terms provided by the online estimation of the system dynamics. The overall controller is also called i-PID (intelligent PID) controller. Comparison of such a controller with classical PID controller can be found in [2]. The main advantage of this control strategy is that it doesn't require neither prior knowledge of the system dynamics, nor complex parameters tuning. It is therefore easy to build a controller for an unknown system.

Model-free control has been succesfully applied to many academic control problems as well as various industrial cases. Linear and nonlinear systems are studied in [7], [4], [5] as well as the ball and beam mechanical system for both stabilization and reference trajectories tracking in [5]. Application to switched nonlinear systems, which is a generalization of hybrid systems, is studied in [1]. In [11], a motor throttles are regulated using a model-free controller. Shape Memory Alloys (SMA) are able to modify thier shape when heated. They are used to design compact actuators but their dynamics and therefore their associated control problem remains complex. In [9], [10], model-free control is successfully used to control a SMA based actuator. Water level control in open channels is often subject to unpredictible disturbances of significant magnitudes. In [12], water level control for hydroelectric power plants is achieved using model-free control techniques, providing a single control law for a wide range of flow. In [14], model-free control methodology is applied to a power converter, where stable regulation is achieved for large variation of curent intensity. Modern financial engineering involves tracking control of "risk-free" management. For this application, in [6], [3], model-free control is compared to other existing techniques, showing the superiority of model-free control scheme.

Although this control method is smart, and has been applied to resolve many control problems, it has however some drawbacks. Aside it's dependancy to quality of sensors and sampling frequency on which relies the fast local estimation, model-free control is not currently adapted to control of non-minimum phase systems. Those systems are characterized with unstable internal dynamics. In this paper we focus on stable limit cycles generation for non-minimum phase systems, more precisely for underactuated mechanical systems. A mechanical system with less actuators than degrees of freedom is said to be underactuated and unfortunately, a vast majority of these systems are non-minimum phase. Therefore model-free control techniques cannot be applied as it has initially been proposed. Some efforts have been made for two particular cases of non-minimum phase systems: the ball and beam [5] (where the dynamics of the beam has not been taken into account) and the Planar Vertical Take Off and Landing (PVTOL) aircraft [15].

In order to achieve stable limit cycles on all coordinates of a non-minimum phase underactuated system, we first design a family of parametrized periodic trajectories for a subset of coordinates. Thoses trajectories are then tracked using control inputs thanks to a classical model-free technique. Since the system is non-minimum phase, the internal dynamics of the system is unstable. Therefore stable limit cycles on those coordinates are generated through the control of trajectories' parameters. To achieve this control, we use an other model-free controller using untracked coordinates as output and trajectories parameters as input. The proposed methodology is illustrated through two examples of underactuated mechanical systems with numerical simulations, namely the cart-pole pendulum and the pendubot. Numerical simulations show the effectiveness and the robustness of the proposed control approach towards external distur-

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bances and changes in system dynamics.

The rest of the paper is organised as follow. In Section II the basic model-free control technique is introduced. Section III presents the reference trajectories generation and the dual model-free controller. Applications of the proposed method are illustrated through simulations in Section IV. Finally, conclusion and future work are addressed in the Section V.

II. Model-free control: background

Model-free control design relies on a local linear approximation of the input-output behavior of the system, valid for a short time window. For the sake of simplicity, we present model-free control for a Single-Input Single-Output (SISO) nonlinear systems. However, the method is straightforwardly adaptable to Multiple-Input Multiple-Output (MIMO) systems (cf. [7]).

A. Nonlinear system dynamics

Consider a nonlinear system whith unknown dynamics. The input-output behavior of this system can be expressed in the following general form:

$$E(y, \dot{y}, \ldots, y^{(a)}, u, \dot{u}, \ldots, u^{(b)}) = 0 \tag{1}$$

where y is the system's output and x is its control input. Given that this finite dimensionnal ordinary differential equation is smooth enough, it can be approximated for a short time window by the following simplified model:

$$y^{(\nu)} = F + \alpha u \tag{2}$$

The derivation order ν and the constant parameter $\alpha \in \mathbb{R}$ can be arbitrarily choosen by the designer. In model-free control litterature, ν is generally choosen to be 1 or 2. The non-physical constant α is a design parameter. The term $F \in \mathbb{R}$ captures all the unknown nonlinearities in the input-ouput behavior and can be compensated in the control law. Since equation (2) is valid for a short time window, it must be updated at each sample time. Therefore, the value of F is updated from the measurement of αu and $y^{(\nu)}$ in the following manner:

$$[F(k)]_e = [y^{(\nu)}(k)]_e - \alpha u(k-1) \tag{3}$$

where $[F(k)]_e$ is the estimated value of F at sample instant k which will be used for the computation of the control input $u(k)$. $[y^{(\nu)}(k)]_e$ is the estimated value of the ν-th derivative of the output y at discret time k and $u(k-1)$ is the control input previously computed and applied to the system at discret time $k-1$ (the value of F can be initialized to 0 at $k = 0$)

B. Control law

Given numerical knowledge (i.e. estimation) of F expressed by equation (3), the control input simply cancels the unknown nonlinearities and adds compensating terms corresponding to a closed-loop tracking of a given reference trajectory $y^*(t)$ using a conventionnal PID controller resulting in an *inteligent*-PID (i-PID):

$$u = \frac{1}{\alpha}\left(-F + y^{*(\nu)} + K_p e + K_i \int e + K_d \dot{e}\right) \tag{4}$$

where $y^{*(\nu)}$ is the ν-th derivative of the reference trajectory y^*, K_p, K_i, K_d are the PID gains, $e = y^* - y$ is the output tracking error and $\dot{e}$ is its first derivative. The tuning of the PID gains can be performed using poles placement technique since all nonlinearities are assumed to be canceled. It is no longer necessary to perform complex system identification [4], [5]. If $\nu = 1$ the PID controller reduces to a PI controller since the first derivative of output $\dot{y}$ is taken into account in the estimation of F in (3).

III. Proposed solution: a dual model-free controller

In our case, we are interrested in stable limit cycles generation for under-actuated mechanical systems which are generally nonlinear and non-minimum phase. Again, in order to simplify our presentation, we focus on 1-input 2-degree of freedom mechanical systems which have the minimum dimensions for a system to be underactuated. Dynamics of such systems takes the following lagrangian form [16], [18]:

$$M(q)\ddot{q} + H(q, \dot{q}) + G(q) = Ru \tag{5}$$

where M is a 2×2 symetric positive definite inertia matrix of the system, q is the vector of generalized coordinates. $\dot{q}$, $\ddot{q}$ are respectively their first and second derivatives. H is a vector containing centrifugal and Coriolis forces terms and G is a vector of gravitationnal terms. u is the control input and R is a matrix distributing the effects of u on the generalized coordinates. Using a suitable partition $q = [q_a, q_{na}]$ of the vector of generalized coordinates where q_a is the actuated coordinate and q_{na} is the unactuated one, equation (5) rewrites as:

$$m_{11}(q)\ddot{q}_a + m_{12}(q)\ddot{q}_{na} + h_1(q, \dot{q}) + g_1(q) = u \tag{6}$$

$$m_{21}(q)\ddot{q}_a + m_{22}(q)\ddot{q}_{na} + h_2(q, \dot{q}) + g_2(q) = 0 \tag{7}$$

where $[q_a \; q_{na} \; \dot{q}_a \; \dot{q}_{na}]^t$ is the state vector of the system and:

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, H = \begin{bmatrix} h_1 & h_2 \end{bmatrix}^T, G = \begin{bmatrix} g_1 & g_2 \end{bmatrix}^T$$

A. Basic principle

Our objective is to generate stable limit cycles on both actuated and unactuated coordinates. We first define a family of p-parametrized τ-periodic reference trajectories $q^*_{na}(p, \tau, t)$ for the unactuated coordinate. Those trajectories have the same boundary conditions for all p, allowing the controller to switch from one trajectory to another while the overall trajectory remains smooth. Thanks to the dynamic coupling existing between the

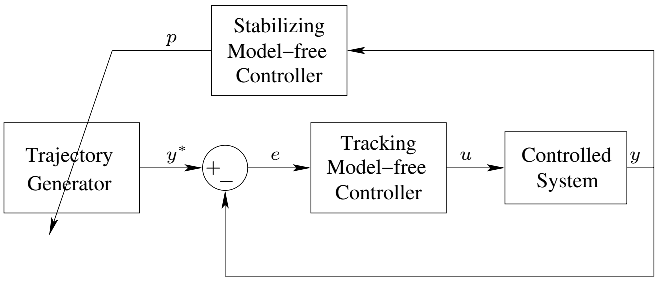


Fig. 1: Schematic view of the dual model-free controller.

actuated and unactuated coordinates, it is possible to control directly the unactuated coordinate using the control input u (i.e. the torque on the actuated coordinate) which allows thoses trajectories to be tracked on the unactuated coordinate q_{na} using the control input u. Indeed, the dynamics (6)-(7) can be rewritten in a form which explicits the relation between unactuated coordinate and control input. First equation (6) is solved for $\ddot{q}_a$ (for clarity reason the dependancy in q and $\dot{q}$ of the terms involved is omitted in the notation):

$$\ddot{q}_a = m_{11}^{-1}\left(-m_{12}\ddot{q}_{na} - h_1 - g_1 + u\right) \tag{8}$$

Injecting this solution in equation (7) leads to:

$$\overline{m}_2\ddot{q}_{na} + \overline{h}_2 + \overline{g}_2 = -m_{21}m_{11}^{-1}u \tag{9}$$

where:

$$\begin{aligned}\overline{m}_2 &= m_{22} - m_{21}m_{11}^{-1}m_{12}\\ \overline{h}_2 &= h_2 - m_{21}m_{11}^{-1}h_1\\ \overline{g}_2 &= g_2 - m_{21}m_{11}^{-1}g_1\end{aligned}$$

A model-free controller can then be designed to perform the tracking of these trajectories on unactuated coordinate using the control input u. In order to stabilize the internal dynamics of the closed-loop system (i.e. the inertia wheel dynamics) and to generate stable limit cycles on both coordinates, a second controller is designed. This second controller therefore takes the reference trajectory parameter p as control input, and use the actuated coordinate as output. At the end of each period (of the periodic reference trajectory), the second controller chooses the right trajectory parameter p in order to stabilize the actuated coordinate. The chosen parameter p fixes the reference trajectories used by the first controller for the duration of the whole next period. The overall control scheme is illustrated in block-diagram of Figure 1.

B. Parametrized reference trajectories generation

The first step of this framework is to generate parametrized reference trajectories $q^*_{na}(p,\tau,t)$ that will be tracked on unactuated coordinate. Those trajectories must fulfil some conditions. First of all, they have to be continuous, derivable and periodic in order to generate limit cycles. That leads us to design oscillatory shaped trajectories which are splitted in half period, where we will use symmetry to generate a whole cycle. The parametrization of these trajectories must allow the controller to update the parameter p (which corresponds to the time at which the trajectory q^*_{na} crosses zero) during tracking while the overall trajectory remains smooth. That leads to some boundary conditions of each half period part. That is for a given period τ and amplitude $2A$:

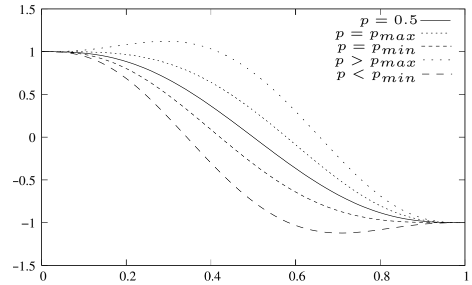


Fig. 2: Normalized reference trajectories during a half-period for unactuated coordinate $q^*_{na}(p,\tau,t)$ for some values of p. Period τ is 2 and amplitude $2A$ is 2.

$$\forall p \in \mathcal{P}, \begin{cases} q^*_{na}(p,\tau,0) = q^*_{na}(p,\tau,\tau) = A\\ q^*_{na}(p,\tau,\frac{\tau}{2}) = -A\\ \dot{q}^*_{na}(p,\tau,0) = \dot{q}^*_{na}(p,\tau,\frac{\tau}{2}) = \dot{q}^*_{na}(p,\tau,\tau) = 0\end{cases} \tag{10}$$

for some domain $\mathcal{P} \subset \mathbb{R}$ (see below). A six-degree polynomial function parametrized with p is chosen such that:

$$\forall p \in \mathcal{P},\ q^*_{na}(p,\tau,t=p) = 0 \tag{11}$$

Figure 2 shows normalized reference trajectories ($\tau = 2$, $A = 1$) for different values of the parameter p during half a period. The domain $\mathcal{P}$ is restricted to interval $[p_{min},\ p_{max}]$ in order to keep an oscillatory shape. Note that $p_{min} = 1 - p_{max}$ due to the symmetry property of half period trajectory parts.

C. Proposed dual model-free controller

The design of the reference trajectories tracking controller is based on using of a model-free controller. The unactuated coordinate nonlinear dynamics (9) is replaced by the local model according to model-free control principle:

$$\ddot{q}_{na} = F_1 + \alpha_1 u \tag{12}$$

where the constant parameter α_1 is a design parameter. F_1 captures the nonlinearities in the unactuated coordinate dynamics and is updated according to equation (3) at each sample time. The reference trajectory tracking controller is obtained based on numericaly computed value of F_1 using an i-PID:

$$u = \frac{1}{\alpha_1}\left(-F_1 + \ddot{q}^*_{na}(p,\tau,t) + K_{p1}e + K_{i1}\int e + K_{d1}\dot{e}\right) \tag{13}$$

with PID gains K_{p1}, K_{i1}, K_{d1}. The unactuated coordinate then follows the desired periodic trajectories $q^*_{na}(p,\tau,t)$.

The parameter p used in the tracking control law (13) is constant over half a period $\forall t \in [k\frac{\tau}{2}\ \ (k+1)\frac{\tau}{2}[$

($k \in \mathbb{N}$) and is updated at the end of each half period at time $(k+1)\frac{\tau}{2}$ by the second controller (16). The unknown nonlinear dynamics of the actuated coordinate is replaced by the local discrete model:

$$\Delta_\tau v_a = F_2 + \alpha_2 p \quad (14)$$

where $\Delta_\tau v_a = \dot{q}_a(k\frac{\tau}{2}) - \dot{q}_a((k-1)\frac{\tau}{2})$ is the variation of actuated articulation velocity $v_a = \dot{q}_a$ measured between half periods and the constant α_2 is a design parameter. The value of F_2 is updated at time $t = k\frac{\tau}{2}$ for $k \in \mathbb{N}$ at the end of each half cycle using the principle of model-free control, that is:

$$[F_2(k\frac{\tau}{2})]_e = [\Delta_\tau \dot{q}_a(k\frac{\tau}{2})]_e - \alpha_2 p((k-1)\frac{\tau}{2}) \quad (15)$$

The notation $[.]_e$ is the estimated value as explained in section II. Note that the actuated coordinate dynamics whithin a half cycle $t =]k\frac{\tau}{2}\ (k+1)\frac{\tau}{2}[$ is not taken into account in this local model since we only aim at limit cycle generation and therefore it is only required that the actuated coordinate trajectory to be periodic. In other words, the aim of the second controller is to bring the actuated coordinate to a fixed desired state $(q_a^{\,d}, \dot{q}_a^{\,d})$ at the end of each half period, ensuring periodicity of the actuated coordinate trajectory and therefore limit cycle generation. The second model-free controller updates the trajectory parameter p according to the following formula:

$$p = \frac{1}{\alpha_2}\left(-F_2 + K_{p2}e_a + K_{i2}\int e_a + K_{d2}\dot{e}_a\right) \quad (16)$$

where K_{p2}, K_{i2}, K_{d2} are PID gains, $e_a = q_a^{\,d} - q_a$ and $\dot{e}_a = \dot{q}_a^{\,d} - \dot{q}_a$. Notice that since the desired state for actuated coordinate $(q_a^{\,d}, \dot{q}_a^{\,d})$ is constant, the $(\Delta_\tau \dot{q}_a)^{\,d}$ term is zero and is then ommited.

IV. Application examples

To illustrate its effectiveness, the proposed control method is applied to two different underactuated mechanical systems. All numerical simulations are performed using MATLAB/SIMULINK software. The dynamic models of the controlled systems are only used in simulation of their behavior, however, they are not used in the controller design. The sampling frequency is set to 150 Hz which is a reasonable value for real-time implementation. In all examples, white noise is added to the measured signals of all coordinates' positions with 0 mean value and 0.0181 standard deviation (in degrees). Velocities are then obtained from numerical derivation of those noisy measurements.

A. Application 1: The cart-pole pendulum

The cart-pole pendulum (cf. Figure 3) is a classical underactuated mechanical system. It consists of a pendulum beam attached to a cart through a passive joint. The cart evolves on a rail and is actuated by a DC motor. The control input u is the horizontal force applied on the cart and the degrees of freedom are the pendulum angular position θ with respect to the vertical and the linear position x of the cart on the rail. Figure 3 shows a schematic view of the cart-pole pendulum system. The nonlinear dynamic model used is the following [17]:

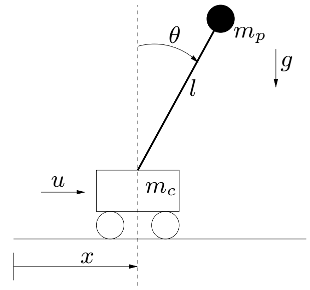


Fig. 3: Schematic view of the cart-pole pendulum.

$$(m_p + m_c)\ddot{x} + m_p l\cos(\theta)\ddot{\theta} - m_c\dot{\theta}^2\sin(\theta) = u \quad (17)$$
$$m_p l\cos(\theta)\ddot{x} + m_p\ddot{\theta} - m_p l g\sin(\theta) = 0 \quad (18)$$

where $m_p = 0.01$ and $m_c = 0.3$ are the respective masses of the pendulum and the cart, $l = 1$ is the beam length and g is the gravity.

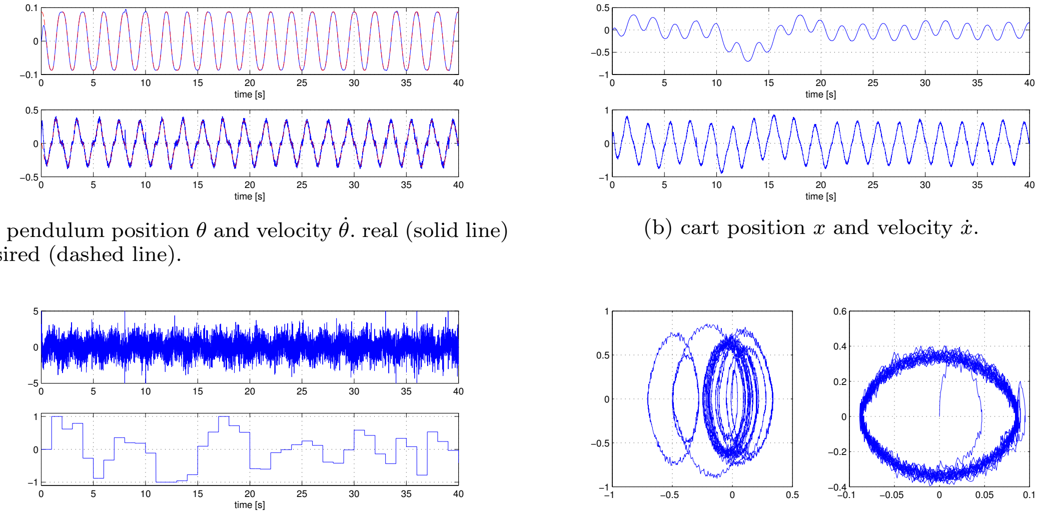
Reference trajectories are generated using the basic principle of section III-B, and to be tracked on θ_1. The amplitude of these trajectories is $2A = 10°$ and their period is $\tau = 2\ s$.

In simulations, the following control design parameters were used: $\alpha_1 = 10$, $\alpha_2 = -0.2$, the first controller gains are choosen as $K_{p1} = 10$, $K_{i1} = 0$ and $K_{d1} = 2$, the second controller gains $K_{p2} = 2$, $K_{i2} = 0$ and $K_{d2} = 6$. Since we don't have prior knowledge on the amplitude of the cart position trajectory oscillations, we cannot set the desired value of the cart position $x^{\,d}$ for the second model-free controller in equation (16). Instead of measuring $x(k\frac{\tau}{2})$, we average the last two measured values of cart position and can then set the desired average value x_{avg}^{d} to 0 and the desired cart velocity $\dot{x}^{\,d}$ to 0 in order to obtain oscillations around zero. An external disturbance is introduced as a torque applied to the pendulum beam at time $t = 10\ s$ with an intensity of 0.2 Nm.

The obtained results of this simulation are depicted in Figure 4. Noise can be seen on phase portraits and velocities of the two coordinates. Despite this noise, the convergence to stable limit cycles can clearly be observed on both phase portraits of the pendulum and the cart. The disturbance can be viewed on the pendulum trajectories as a deviation from the reference trajectories which is rapidly compensated. The effect of this disturbance is clearly stronger on the cart trajectories, but the second controller adapts the reference trajectories accordingly and brings back the cart trajectories to the limit cycle.

B. Application 2: the pendubot

The *pendubot* [18] is an other underactuated mechanical system often proposed in underactuated systems control litterature. Depicted in Figure 5, it consists in



(a) pendulum position θ and velocity $\dot{\theta}$. real (solid line) desired (dashed line).

(b) cart position x and velocity $\dot{x}$.

(c) first controler input u and second controler input p.

(d) phase portraits (x, $\dot{x}$) and (θ, $\dot{\theta}$).

Fig. 4: Simulation results for the cart-pole pendulum: A punctual external disturbance was introduced as a torque applied at time $t = 10\ s$ on the pendulum beam.

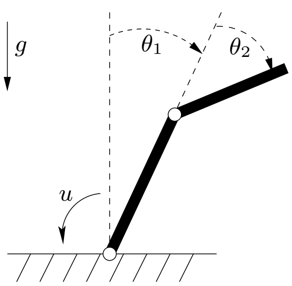


Fig. 5: Schematic view of the pendubot: the first joint θ_1 is actuated, while the second θ_2 is unactuated. Control input u is the torque applied to the first joint.

a 2-degree of freedom planar manipulator where the first joint is actuated, whereas the second one is not actuated. The control input u of the system is the torque applied on the first joint and the degrees of freedom are the angular positions θ_1 and θ_2 as illustrated in Figure 5. In this scenario we are interrested in controlling the absolute angular positions of the two links with respect to the vertical, namely θ_1 and $(\theta_1 + \theta_2)$. The nonlinear dynamic model used for simulation is the following [18]:

$$m_{11}\ddot{\theta}_1 + m_{12}\ddot{\theta}_2 + h_1 + g_1 = u \quad (19)$$

$$m_{21}\ddot{\theta}_1 + m_{22}\ddot{\theta}_2 + h_2 + g_2 = 0 \quad (20)$$

where

$$\begin{aligned}
m_{11} &= m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos(\theta_2)) + I_1 + I_2 \\
m_{12} &= m_2(l_{c2}^2 + l_1 l_{c2}\cos(\theta_2)) + I_2 \\
m_{21} &= m_{12} \\
m_{22} &= m_2 l_{c2}^2 + I_2 \\
h_1 &= -m_2 l_1 l_{c2}\sin(\theta_2)\dot{\theta}_2^2 - 2m_2 l_1 l_{c2}\sin(\theta_2)\dot{\theta}_2\dot{\theta}_1 \\
g_1 &= m_2 l_1 l_{c2}\sin(\theta_2)\dot{\theta}_1^2 \\
h_2 &= (m_1 l_{c1} + m_2 l_1) g \sin(\theta_1) + m_2 l_{c2} g \sin(\theta_1 + \theta_2) \\
g_2 &= m_2 l_{c2} g \sin(\theta_1 + \theta_2)
\end{aligned}$$

with $m_1 = m_2 = 1$ are the masses of the two links, $l_1 = 1$ is the length of first link, $l_{c1} = l_{c2} = 0.5$ are the distances from the joint to their respective centers of mass. $I_1 = I_2 = 0.0833$ are the moments of inertia of the two links and g the acceleration of gravity.

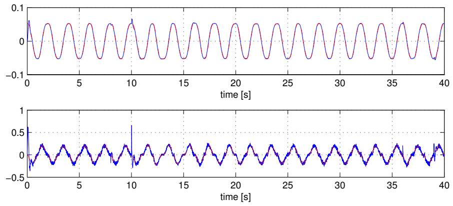
Our objective is to generate stable limit cycles on both absolute angular positions θ_1 and $(\theta_1 + \theta_2)$. Reference trajectories of section III-B are to be tracked on $(\theta_1+\theta_2)$. The amplitude of these trajectories is $2A = 6°$ and their period is $\tau = 2\ s$.

In simulations, the control design parameters used in the controllers are: $\alpha_1 = -10$, $\alpha_2 = 0.1$, the PID gains are $K_{p1} = 30$, $K_{i1} = 0$ and $K_{d1} = 2$, the second controller gains are $K_{p2} = 3$, $K_{i2} = 0$ and $K_{d2} = 7$. For the same reasons as the previous example, we use the average of two consecutives values of $\theta_1(k\frac{\tau}{2})$ in the second controller. We can then set average desired value ${\theta_1}_{avg}^d = 0$ and desired velocity $\dot{\theta}_1^d = 0$. An external disturbance is introduced through a torque applied to the second link at time $t = 10\ s$ with an intensity of $8\ Nm$.

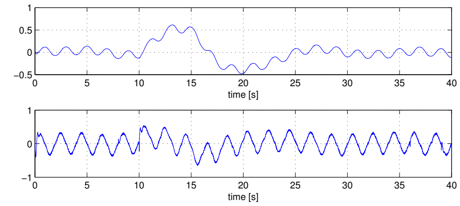
The obtained simulation results are displayed in Figure 6. Limit cycles are obtained on both coordinates (actuated and unactuated) of the system, this can be observed in their respective phase portraits despite the presence of noise. The response of the first controller to the external disturbance limits its effect on the trajectory tracking to small deviation of the output position and velocity. The second controller ajusts the trajectory parameter in order to bring the actuated coordinate position and velocity trajectories oscillations around zero in few periods.

V. Conclusions and future work

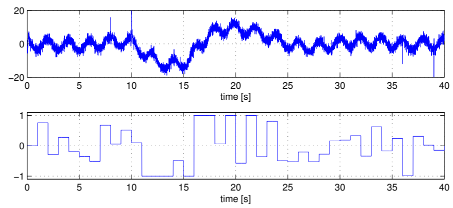
In this paper, a dual model-free controller is proposed to deal with control of non-minimum phase systems for stable limit cycles generation. This method inherits the



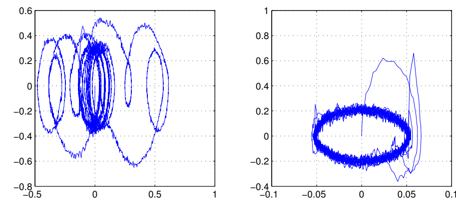
(a) tracked output position $\theta_1 + \theta_2$ and velocity $\dot{\theta}_1 + \dot{\theta}_2$. real (solid line) desired (dashed line).



(b) actuated joint position θ_1 and velocity $\dot{\theta}_1$.



(c) first controler input u and second controler input p.



(d) phase portraits $(\theta_1, \dot{\theta}_1)$ and $(\theta_1 + \theta_2, \dot{\theta}_1 + \dot{\theta}_2)$.

Fig. 6: Simulation results for the pendubot: A punctual external disturbance was introduced as a torque applied at time $t = 10\ s$ on the unactuated joint θ_2.

advantages of model-free control: mathematical modelling of the controlled system is not required and no complex parameters identification is needed. To show the effectiveness of the proposed control method, two applications of underactuated mechanical systems are proposed (the cart-pole pendulum and the pendubot) and illustrated through numerical simulations.

Future work will be focused on the generalization of the proposed method to the stabilization around equilibrium points and regulation and to arbitrary reference trajectories tracking. We are also planning real-time implentation of this control method on real testbeds.

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