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Kinematic calibration of linear-actuated parallel mechanisms from leg observation

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Abstract

In this article, an original algorithm is proposed to achieve the kinematic calibration of parallel mechanisms with linear actuators on the base, using vision as an exteroceptive sensor to perform measurements on the legs of the mechanism. The calibration can be performed without adding proprioceptive sensors or restricting the mechanism’s workspace during the calibration process. The algorithm is implemented for the calibration of the I4 parallel mechanism with experimental results.

1 Introduction

Among the proposed architectures for parallel mechanisms, structures with linear actuators on the base are of great interest: inertial effects are lowered, and the use of modern linear actuators allows us to build mechanisms with high dynamics. Consequently, many recently developed structures belong to this class of mechanisms such as Hexaglide [1], I4 [2], Orthoglide [3]. Their use is highly dependent on their accuracy. Compared to serial mechanisms, they may exhibit a much better repeatability [4], but not necessarily a better accuracy, because of their large number of links and passive joints [5]. A kinematic calibration is thus needed for these structures.

The various approaches proposed to perform the kinematic calibration of parallel mechanisms are based on the minimization of an error function which depends on a geometrical parameter vector defining the mechanism. With error functions expressed using the direct kinematic model, numerical models have generally to be used, leading to stability and convergence problems in the identification process [6]. With methods based on the inverse kinematic model, the full pose of the end-effector has to be measured, which is often complex and tedious to achieve accurately. Vision can be an adequate sensor [7]. The calibration efficiency is however limited for a large mechanism workspace, as the measurement accuracy lowers, and the calibration procedure seems rather difficult to achieve online: the end-effector is generally inaccessible or its observation impossible because of the environment, with for instance lubricant projections in a machine-tool. Alternative methods are based on additional measurements on the passive links [8, 9, 6, 10, 11], or mechanical constraints on some elements of the mechanism [11, 12, 13]. However the former method is not always economically and technologically achievable and the involvement of mechanical constraints implies a significant reduction of the workspace during the calibration, which lowers the quality of the identification.

We recently proposed [14] to get information redundancy by observing with a camera the legs that connect the actuators to the end-effector. No mechanical modification or workspace limitation is then needed, and calibration can be performed online since the end-effector observability is not needed. The kinematics of parallel mechanisms are furthermore intrinsically linked to the kinematics of their legs. In this article, we introduce a calibration method based on the leg observation for mechanisms with linear actuators on the base. For sake of clarity, the algorithm is presented in the context of the I4 parallel mechanism. A more general algorithm may be found in [15].

In the second part, the I4 parallel mechanism is presented with its modeling. In the third part the calibration algorithm is then detailed. Experimental results are given in the fourth part, before concluding on the performance and further developments of the identification method.
tuator axes are considered parallel and coplanar, as
nism are considered valid for the calibration. The ac-
sumptions achieved during the design of the mecha-
Because of the manufacturing tolerances, some as-
parts (figure 1(b)), using two rack-and-pinion sys-
tated parallelograms, link the actuators to a travel-
effector can be translated in three directions, and ro-
in the base: the distance 2
Finally, four parameters are needed to define the joints
on the base: the distance 2H between the two actuator

\[ (\text{OA}_j) \cdot \mathbf{x}_B = \mathbf{q}_i + \mathbf{q}_j \]  

The relative position of the joint centers \( B_j \), \( j \in [1,4] \) is defined by the dimensions \( S \) and \( D \). Seven parameters finally define the mechanism geometry: \( (H, \mathbf{q}_{02}, \mathbf{q}_{03}, \mathbf{q}_{04}, S, D, L) \).
Step 1 Base parameter estimation in the camera frame
\[ \rightarrow (A_0^0)_{RC} \]
Step 2 Parameter estimation in the base frame
\[ \rightarrow (q_{02}, q_{03}, q_{04}, H) \]
Step 3 Estimation of the length of the kinematically equivalent single rods connected to the traveling plate
\[ \rightarrow L \]
Step 4 Estimation of the parameters of the traveling plate
\[ \rightarrow (S, D) \]

Table 1: The calibration method structure

3 Calibration method

To identify the mechanism kinematic parameters described in the former section, we propose a calibration method using a camera to observe the cylindrical elements constituting the articulated parallelograms connected to the traveling plate. One can demonstrate that the observation of a cylindrical element enables us to determine its pose with respect to the camera frame [14], i.e., the position and the orientation of the axis of the element. It must be outlined that the camera location is not considered to be known accurately, which enables a simple experimental procedure.

The method is composed of four steps (table 1). The first one is performed to get information in the camera frame and the kinematic parameters are determined too.

The coordinates of \( A_{j,i} \) in the camera frame can therefore be computed after solving the overdetermined linear system obtained by concatenation of the 6\( N_f \) vector equations (2). As the equations provided by the cross product for each pose are not independent, the solution is obtained by a singular value decomposition. It can be easily proved that two leg orientations are at least necessary to estimate the joint center position: \( N_f \geq 2 \).

3.1 Step 1 - Base parameter estimation in the camera frame

In the first step, the locations of the points \( A_{j,i} \), \( j \in [1, 4] \), are identified in the camera frame. Due to modeling assumptions, this also implies the identification of the actuator axes.

The end-effector is moved to modify the orientation of the legs, while locking an actuator \( j \) in a position \( m \) (figure 3). The observation of the two corresponding cylindrical elements in \( N_f \) different orientations enables one to measure their axes \( \mathbf{u}_{j,k} \) and \( \mathbf{u}'_{j,k} \), \( k \in [1, N_f] \), and the position of points \( M_{j,k} \) and \( M'_{j,k} \) on their axes. The position of the spherical joint centers \( P_{j,m} \) and \( P'_{j,m} \) can hence be computed in the camera frame by expressing their belonging to the axes:

\[
\begin{align*}
P_{j,m} M_{j,k} \times \mathbf{u}_{j,k} &= 0 \\
P'_{j,m} M'_{j,k} \times \mathbf{u}'_{j,k} &= 0
\end{align*}
\]

The position of the joint center \( A_{j,m} \) is then easily determined:
\[
A_{j,m} = \frac{P_{j,m} + P'_{j,m}}{2}
\]

The coordinates of \( A_{j,m} \) in the camera frame can be computed at the same time, and consequently the unit vector \( z_B \) perpendicular to the plane containing the joint centers can be computed at the same time, and consequently the unit vector \( y_B \) too.

3.2 Step 2 - Parameter estimation in the base frame

At the end of the first step, the position of the points \( A_{j,i} \), \( j \in [1, 4] \), and the actuator axes are determined in the camera frame. Expressing these elements in the base frame requires the knowledge of the camera pose.
with respect to the base. Designing a system to impose accurately the camera location would be expensive and its use restricting, so that this location is computed. This computation can be achieved explicitly, introducing the corresponding unknowns, or implicitly by directly searching the kinematic parameters in the base frame. The latter solution is selected since it enables one to suppress the need of a priori knowledge of the transformation $B_{TC}$, which is of great interest as the camera frame is not physically measurable.

To determine the kinematic parameters in the base frame, an error function is expressed using the invariance of the scalar product with Euclidian transformation. Scalar products are expressed both in the base frame and in the successive camera frames.

The position of the joint centers $A_j, j \in [1,4]$ when the actuator encoder values are equal to zero are estimated in the previous step in the camera frame. From these four positions $A_0^j$, $j \in [1,4]$, three independent vectors $A_0^j A_g^j, (j,g) \in [1,4]$ can be computed. One can then constitute a vector set $V$ of independent vectors:

$$V = (A_0^0 A_0^1, A_0^0 A_3^0, A_0^0 A_1^0)$$

with

$$\left\{ \begin{align*}
A_0^0 A_0^1 &= q_{02} x_B \\
A_0^0 A_3^0 &= q_{03} x_B + 2 H y_B \\
A_0^0 A_1^0 &= q_{04} x_B + 2 H y_B
\end{align*} \right.$$  

Consequently the kinematic parameters in the base frame $R_B$ are determined by nonlinear minimization of the error function $F_1$:

$$F_1(q_{02}, q_{03}, q_{04}, H) = \sum_{(p,q), p>q} \| V_p \cdot V_q \|_{R_C} - \| V_p \cdot V_q \|_{R_B} \|^2$$  

with $V_p$ and $V_q$ two elements of $V$ and $\cdot_{R}$ representing the frame $R$ used to express the terms of $V$.

Six independent equations can be expressed from the elements of $V$, which ensures the identifiability of the four kinematic parameters.

At the end of this second step, the four parameters $(q_{02}, q_{03}, q_{04}, H)$ related to the actuators and the location of the points $A_j, j \in [1,4]$ are identified.

### 3.3 Step 3 - Identification of the length of the elements connected to the end-effector

From the first step, one can express the position of the joint center $A_j$ for any end-effector pose. The position of the joint center $B_j$ can hence be written in the camera frame as a function of the length $L$ of the kinematically equivalent single rods connected to the traveling plate:

$$B_{j,k} |_{R_C} = A_{j,k} |_{R_C} + L \eta_{j,k} |_{R_C}, \ k \in [1,N]$$  

with $N$ the number of used end-effector poses, $B_{j,k}$ the position of $B_j, j \in [1,4]$ for the k-th end-effector pose and $\eta_{j,k}$ the unit vector of the axis of the observed element, determined from its image. An error function is introduced by considering like Notash [16] the invariance of the distances between the points $B_j$ between two consecutive end-effector poses $k$ and $k+1$. The invariance of the distances $\|B_1 B_2\|$ and $\|B_3 B_4\|$ can be expressed by:

$$\{ \begin{align*}
\|B_{1,k} + B_{2,k+1}\|_{R_C} &= \|B_{1,k} B_{2,k}\|_{R_C} \\
\|B_{3,k} + B_{4,k+1}\|_{R_C} &= \|B_{3,k} B_{4,k}\|_{R_C}
\end{align*} \} \ k \in [1, N_I - 1]$$

The length $L$ of the elements connected to the end-effector is therefore determined by nonlinear minimization of the error function $F_2$:

$$F_2(L) = \sum_{k=1}^{N_I-1} \left[ \left( \|B_{1,k+1} B_{2,k+1}\|_{R_C} - \|B_{1,k} B_{2,k}\|_{R_C} \right)^2 \\
+ \left( \|B_{3,k+1} B_{4,k+1}\|_{R_C} - \|B_{3,k} B_{4,k}\|_{R_C} \right)^2 \right]$$

At the end of the third step, the length of the kinematically equivalent single rods connected to the traveling plate is identified. It is important to notice that this step is independent from the previous one. No error propagation hence occurs.

### 3.4 Step 4 - Parameter estimation in the end-effector frame

In the fourth step, the parameters related to the traveling plate are identified. The previous step enables one to determine the values of the distances $\|B_1 B_2\|$ and $\|B_3 B_4\|$. These distances only depend on the parameter $S$, which is therefore immediately determined.

To determine the other dimension $D$, the relative displacement of the two traveling plate elements has to be taken into account. The distance between $B_1$ (resp. $B_3$) and $B_2$ (resp. $B_4$) along $Y_B$ is constant and known in the camera frame, with for instance:

$$B_1 B_3 Y_B = - L \eta_{1,k} Y_B + 2(H - D) + L \eta_{3,k} Y_B, \ \forall k$$

The corresponding distance $2D$ is hence also immediately identified since all the other terms in (8) are known at this point.
4 Experimental results

4.1 Experimental setup

The camera is located on the base of the mechanism so that the four legs are observable, as well as a calibration board linked to the end-effector (figures 4 and 5). The camera has a resolution of $1024 \times 768$ pixels, 8-bit encoded, with a 3.6mm lens. The calibration board enables one to perform end-effector pose measurements to evaluate the calibration efficiency.

Five poses are considered for each joint location determination in step 1. Another 20-pose set is used for the steps 2 to 4.

4.2 Experimental results

The determination of the spherical joint positions in the camera frame (step 1) is achieved with an accuracy in the order of 1mm. The identified kinematic parameters are indicated in table 2 with the a priori values. The parameter value modifications are noticeable, with variations in the order of several millimeters.

4.3 Validation tests

End-effector displacement analysis. Observation of the calibration board enables us to measure its pose with respect to the camera. The end-effector displacement between two poses can therefore be estimated from these pose measurements. On the other hand, the end-effector displacement can be estimated using the direct kinematic model, the proprioceptive sensor values recorded during the experiment and a kinematic parameter set, either identified or estimated a priori. In table 3, the mean and root mean squares of the gaps between the vision-based displacement measurements and their estimation are indicated for the parameters identified with the proposed method. The gaps are computed for the 20-pose set used for calibration. The identified parameters enable us to lower significantly the gaps between the measured end-effector displacements and their estimation.

Kinematic constraint. An independent validation test has been also achieved by imposing a kinematic constraint on the end-effector (Figure 6). The end-effector is manually constrained to follow a line materialized by a ruler. The corresponding joint variables are stored. From these joint variable values, poses are computed by means of the direct kinematic model. The physical setup implies that these poses should lie on a straight line. Hence, the straightness of the line is computed for the kinematic parameter sets (a priori and identified) as the root mean square of the distance between the different positions and the line estimated by a least squares criterion.

A 500mm line is measured. The straightness measurement repeatability, experimentally evaluated equal to 0.07mm, is in the order of the improvement induced by the calibration (table 4). The accuracy improvement can hence not be concluded using this second evaluation criterion. This experiment demonstrates the difficulties that might be encountered while using

<table>
<thead>
<tr>
<th>Parameter</th>
<th>L (m)</th>
<th>H (m)</th>
<th>D (m)</th>
<th>S (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A priori</td>
<td>1.0010</td>
<td>0.5000</td>
<td>0.1536</td>
<td>0.1536</td>
</tr>
<tr>
<td>Identified</td>
<td>1.0213</td>
<td>0.4985</td>
<td>0.1602</td>
<td>0.1598</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$q_02$ (m)</th>
<th>$q_03$ (m)</th>
<th>$q_04$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A priori</td>
<td>1.5705</td>
<td>1.2190</td>
<td>2.8700</td>
</tr>
<tr>
<td>Identified</td>
<td>1.5675</td>
<td>1.2190</td>
<td>2.7867</td>
</tr>
</tbody>
</table>

Table 2: A priori and identified kinematic parameters

<table>
<thead>
<tr>
<th>Parameter set</th>
<th>Mean</th>
<th>Root mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>a priori</td>
<td>-0.06mm</td>
<td>0.53mm</td>
</tr>
<tr>
<td>Identified</td>
<td>-3.4E-3mm</td>
<td>0.33 mm</td>
</tr>
</tbody>
</table>

Table 3: Gaps between vision-based end-effector displacement measurements and their estimation computed from a kinematic parameter set.
Table 4: Straightness evaluation with the a priori and identified kinematic parameter sets.

<table>
<thead>
<tr>
<th>Parameter set</th>
<th>Straightness</th>
</tr>
</thead>
<tbody>
<tr>
<td>a priori</td>
<td>0.53mm</td>
</tr>
<tr>
<td>Identified</td>
<td>0.50mm</td>
</tr>
</tbody>
</table>

a kinematic constraint to achieve the calibration of a mechanism with large workspace.

5 Conclusion

Parallel mechanisms with linear actuators on the base are of great interest for applications where high dynamics are required. Like serial mechanisms, they need a kinematic calibration to improve their accuracy. In this article, we proposed an original approach to achieve the calibration process, observing the legs with a camera. The ease of use and the absence of mechanism modifications due to the use of an exteroceptive sensor are combined to the advantage of having information directly bound to the mechanism kinematics, like methods based on the use of redundant proprioceptive sensors. The method has been developed in the context of the calibration of a 14 parallel mechanism, with a noticeable accuracy improvement according to vision-based pose measurement. Future work will now concern the accuracy improvement when considering the dynamics of the end-effector.

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References