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ON THE DESIGN OF MOBILE PARALLEL ROBOTS FOR LARGE WORKSPACE APPLICATIONS

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ABSTRACT
In this paper, several considerations for designing industry oriented robots which combine the mobility of legged robots and advantages of parallel mechanisms are outlined. For designing such optimized robots in terms of simplicity and performance, a topology study is done based on the mobility analysis. Applying some design constraints, potential topologies of such robots are identified. One architecture is chosen for designing a tripod robot. Both inverse and forward kinematic problems of this robot are derived in order to simulate its gait motion. The analysis and simulations show that: integrating some clamping devices and some lockable passive joints, six actuators are enough to build a legged manipulator which can not only perform 6-axis machining but can also walk on a curved supporting media.

INTRODUCTION
Modern industry requires manufacturing systems to be more reconfigurable, flexible and agile to adapt to the increasing competitive climate with sophisticated customers demands [1].

In industrial fields such as automobile assembly lines or semiconductor manufacturing processes, robotic systems have been widely integrated. But when large workspaces are required, the traditional stationary-base robots can not be used.

Therefore, the concept of mobile manipulator, classical serial robot arms mounted on a mobile base, has been considered for the automation of applications like welding, inspection, painting etc [2, 3].

However, most of these solutions will fail when a high precision and/or stiffness are required for applications like drilling or milling. Meanwhile, parallel mechanisms, with great potential to provide high rigidity and motion dynamics, suffer from their inherent limited operational workspace. Several approaches have been proposed recently in order to apply parallel mechanisms to the aeronautic industry, where a large operational workspace is required [4, 5].

The mobility of the base can be provided by linear guide ways, wheeled mobile robots, tracked mobile robots or legged mobile robots. On the one hand, long linear guide ways with high precision and stiffness represent high costs and tedious infrastructure adjustments of workshops. Beside the need of independent control for the mobile base, wheeled and tracked mobile robots, possessing lower degrees of freedom (DOF), have limited mobility, limited obstacle cross-over ability. On the other hand, legged robots, with mechanical structure inherently similar with parallel robots, rarely appear in the workshop because of the lack of efficiency and reliability [6].

An innovative solution which combines the mobility of legged robots and advantages of parallel mechanisms is studied in this paper (Fig. 1 shows an illustration of such robot). Un-
like classical legged robots, the extremity of every leg will be equipped with fixing devices which help to provide solid connections between the robot and the ground. Therefore the walking and machining capacities of such robots will not be seriously constrained by factors like center of gravity, friction between limbs and the ground etc.

In the following sections, the design considerations and process are discussed. A tripod mobile robot which can "walk" on surfaces with moderate curvature and perform as a 6-axis parallel manipulator once it is deployed on the working position is presented. Both inverse and forward kinematic problems of this robot are derived in order to simulate its gait motion.

REDUCED DOF WALKING MANIPULATOR

Legged robots have attracted attention because of their relatively good terrain pass-over capacity [7]. Most of these studies focus on improving the mobility and the reliability of mobile platforms in hazardous environments for exploration purposes. Many prototypes which imitate the limb structures of animals have been built and studied in universities and research centers.

However, few of them have been used to solve industrial problems: both the human-like biped and animal-like quadruped or hexapod have legs with all their joints being actuated. Three actuators are needed for positioning the pinpoint-type foot to a point in the 3D space where no orientation capacity is required [8]. That is why a typical bio-mimetic quadruped has 12 actuators and a hexapod has 18 actuators. When the orientation of a foot needs to be controlled to fit well the terrain, more than five actuators are needed in each leg.

It is difficult to consider using this kind of legged robots for manufacturing applications due to their high material cost and the complexity of their control. Designing a legged robot for manufacturing purpose is very different from designing a legged robot for explorations of hazardous unknown environments [9].

Principles to Reduce the Number of Actuators

Several techniques are discussed below in order to build industry oriented legged robots which are capable to achieve tasks with high stiffness and accuracy.

Sharing actuators for positioning each Limb

The body of a legged robot can be moved to help positioning its limbs\(^1\). Generated by the supporting limbs, the DOFs of the body can be used to position the swinging limb. Instead of actuating each limb independently, sharing actuators for positioning each limb will help to reduce the total number of actuators [10, 11].

Using the same actuators for locomotion and for manipulation

For conventional mobile robots, the locomotion actuation and the manipulation actuation are usually provided by two independent systems. In order to reduce the number of actuators, the
mobility of the locomotion system can also be used for manipulation purpose [12–17]. For example, Roptalmu, a 3-axis drilling robot designed for aeronautic industry applications, is composed by a wheeled mobile platform and a crawler robot. The wheeled mobile platform follows automatically the crawler, and its main goal is to compensate the gravity by exerting an upward vertical force on the crawler (Fig. 2 a). As it is outlined in the joint-and-loop graph (Fig. 2 b), actuators $X$, $Z_1$ are used for locomotion tasks. And actuators $X$, $Y$, $Z_2$ are used for drilling tasks. Using the $X$ axis actuator for both locomotion and machining tasks makes the mechanism of the crawler more efficient.

**Integrating lockers on the passive joints** Legged robots, with closed kinematic chains (KC) formed between the body and the terrain, can be considered as parallel mechanisms. Noticing that the existence of passive joints in the branches of conventional parallel robots helps to build light-weight robot with relatively higher rigidity, passive joints will be introduced in the design of legged robots for this purpose. However, in order to keep the mechanism controllable during locomotion, lockers should be integrated on some of the passive joints. These lockers can eliminate temporarily the passive DOFs when it is necessary [18].

**Docking system** For robots which are supposed to provide high manipulation stiffness and accuracy, solid connections between the robot and the supporting media (tooling, workpiece itself, etc.) are required. The connection force can be provided by a magnetic device, a vacuum device or a mechanical clamping system [19].

**MOBILITY AND TOPOLOGY ANALYSIS**

**Mobility Requirements of A Walking Parallel Robot**

Following the presented techniques, the objective of the structure design is to build a mobile machining center which machines as a parallel machining center and walks as a legged robot. The various working modes of the desired robot can be roughly distinguished as: Machining Head (MH) mode and Branch Extremity (BE) mode.

**MH Mode:** During the MH mode, all the branches of the robot are attached to the base. The robot which is supposed to perform 5-axis tasks can be considered as a classical parallel robot or a hybrid (serial-parallel) robot. When the payload platform (PP) possesses more than five DOFs, then the 5-axis movement for the machining task can be provided by the PP. Otherwise an extra kinematic chain between the PP and machining head, which possesses the rest of the required mobility CS (CS in the Fig. 3 (a) denotes such supplementary connectivity) should be added on the PP.

**BE Mode:** In the BE mode, one branch of the robot is detached from the base in order to reach another supporting point, while the other branches remain attached to the base. Before detaching the branch from the supporting point, passive joints in the swing branch should be locked in order to control the extremity of this branch. Also, as there are less branches connected between the PP and the base, the DOF of the PP might be changed. In the case that the actuators in the branches attached to the base are not sufficient to control the PP, some of the passive joints in these branches also need to be locked in order to reduce the DOF of the PP.

**Mobility Analysis**

A mobility analysis of some KCs with various topologies will lead to mechanisms which are capable to achieve the desired movements. We would like find out what is the proper choice for:

1. the number of branches that should be used in the mechanism
2. the adequate connectivity for every branch
3. the number of actuated DOF in every branch
For reliability considerations, at least four limbs are needed in order to achieve gait motion which is statically stable. The reason is that during a limb swinging phase, at least three other limbs should be in contact with the terrain in order to form a support polygon which covers the Center of Mass (CoM) of Robot. By contrast, for robots which have rigid connection with the supporting media, the number of limbs can be less than four [20–22].

The classical Chebychev-Grubler-Kutzbach mobility formula is used to calculate the mobility of a KC:

\[ C = \lambda (n - j - 1) + \sum_{i=1}^{n} f_i \]  

where \( C \) denotes the mobility of the considered KC, \( \lambda \) is the order of the system (\( \lambda = 3 \) for a planar motion or spherical motion, and \( \lambda = 6 \) for a spatial motion), \( n \) is the number of links, \( j \) is the number of joints, and \( f_i \) is the number of DOFs of the \( i^{th} \) joint. In order to obtain the same performances whatever the swinging branch is, only branches with identical structure will be considered during this analysis. For such KCs connected in a parallel manner to the PP, the mobility of the payload platform can also be written as

\[ M = N(C - 6) + 6 \]  

where \( M \) represents the mobility of the PP, \( N \) the number of branches and \( C \) the connectivity between the PP and the last link of every branch. Being used for describing a movement between two links, the term “mobility” carries the same meaning as “connectivity” in this paper.\(^2\)

### Topology Analysis

Applying Eqn.(2), \( N \) and \( C \) vary independently from one to six, all mobility arrangements of branches are examined according to the following imposed constraints which should be valid in both the MH mode and the BE mode:

a. **Mobility of the PP should be greater than zero:** With all the branches attached to the base, the mobility of the PP, calculated by applying Eqn.(2), should be greater than zero. If it is lower than five, then an extra KC should be added between the PP and the MH. A negative result means that the whole mechanism is either a non-feasible mechanism or an over-constrained one. For the later case, special geometric arrangement between the branches is required [23]. In our application, the robot geometry is changing when the supporting points are changed. That is why it becomes difficult to respect the special geometric requirements which are necessary for forming an over-constrained mechanism.

b. **At least five actuators contribute to the movement of the target link:** Having all the limbs attached to the ground, the legged mobile robot should be capable to achieve 5-axis manipulations which require at least five actuators. This 5-axis movement can be composed by the movement of the PP and/or the movement of a potential extra-added actuated KC mounted between the PP and the MH. Because this extra-added KC will not contribute to the movement of the BE of the swing branch, the total number of actuators in branches (without counting those on the extra-added KC) should be greater than five.

c. **Degree of actuation redundancy should be less than four:** We distinguish between the notions of kinematic redundancy and actuation redundancy in this paper [24]. To the best of our knowledge, there are rarely machine tools with an actuation redundancy greater than three in the literatures [25, 26]. Although the actuation redundancy could be an option for improving the manipulability, we try to avoid actuation redundancy in order to limit both complexity and cost of the robot. For this reason, the solutions with an actuation redundancy greater than four degrees will be eliminated.

Tab.1 shows the possible mobility combinations which are identified according to the former constraints. Three of the configurations are illustrated in Fig. 4:

- Each column in Fig. 4 corresponds to one group’s configuration.
- Row 1 shows the connectivity of the robot during the MH mode.
- Row 2 presents the situation during the BE mode.
- The arrow represents the machining head; the BE of the limb in swing is presented as red dot; the branches with no red dot are fixed on the base.
- The first number in each white circle represents the number of actuators in each branch; it represents the controllable

### TABLE 1: POSSIBLE KC ARRANGEMENT THAT SATISFY THE MOBILITY CONSTRAINTS

<table>
<thead>
<tr>
<th>Group</th>
<th>Nb. of Branches</th>
<th>Connectivity of every Branch</th>
<th>Nb. of Actuators per Branch</th>
<th>Nb. of Actuators in total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>6</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

\(^2\)When “mobility” is used for describing a mechanical system, it represents the number of independent displacement variables of the system, which might be greater than the connectivity between any two links in the system.
connectivity of the KC going from the BE to the PP.
- The second number in the white circle is the total connectivity of each branch.
- The first number in the blue circle is the number of actuators used for controlling the PP.
- The total connectivity between the PP and the base is the second number in the blue circle.

During the BE mode, the branch in swing forms no longer a closed KC with the base. Some of the passive joints in this branch lose their constraints which were exerted by the formerly existed closed KC. In order to control the BE, the free connectivity of this branch should be locked.

Similarly, when the number of actuators is greater than the connectivity of the PP, there is actuation redundancy in the mechanism. And if the former is smaller than latter, then lockers should be used to add constraints.

For instance, the topology in Group 1 of Fig. 4 represents a mechanism with three branches; each branch possesses five DOFs; two of them are actuated. The actuation redundancy happens in the MH mode. The PP of such robot, which has three DOFs, is actuated by six actuators located in the three branches connected to the base. An actuated two DOFs machining head is added on the PP. In the BE mode, the branch with red dot is detached from the base; three DOFs in this detached branch should be locked. The four DOFs of the PP are actuated by four actuators located in the branches attached to the base.

DESCRIPTION AND KINEMATICS OF THE MOBILE TRIPOD MANIPULATOR

Theoretically, every group of KC arrangements listed in Tab. 1 can satisfy the preset mobility constraints. In this section, one of the group in Tab. 1 is chosen in order to validate this new concept of mobile parallel robots.

Generally, the structure with less branches and less actuators is favorable for the following reasons:
- This leads to less overweight, better energy efficiency and less material cost as well.
- During the walking phase, the robot needs to move its limbs one by one to the supporting points of the next operating location. The design of robots with less limbs helps to reduce the machine downtime.

According to the result of topology analysis, the configuration of Group 2 shows that six actuators arranged in three branches are enough to achieve the required tasks in both the MH mode and the BE mode.

Branch’s Structure

Branches involved in Group 2 have six DOFs, two of them being actuated. From a technical point of view, it is favorable to actuate the prismatic joints for tasks with heavy loads. A structure which has two prismatic joints in the KC can be used in the case where two actuators are expected. Fig. 5 (a) shows such mechanism in which a two DOFs planar structure is formed between the three parallel axes (a, b and c). Fig. 5 (b) shows its equivalent serial UPS KC.

Geometry of the Platform

With three branches mounted symmetrically on Payload Platform, a tripod (Fig. 6) can be obtained. When all the branches are attached to the supporting points, such structure can be considered as a 6-3 Stewart platform from a topological point of view. The geometry of the Tripod is described by the geometric parameters (Fig. 6) and the joints variables (Fig. 8).

Branches are symmetrically mounted on the PP, the axes of the last joints of every branch being coplanar. \( \psi_p \), defined as the angles between the axes of the last joints of every branch, equals to \( \frac{2\pi}{3} \). The branch \( i \) is connected to the PP at \( P_{L_i}P_{R_i} \). \( P_{C_i} \) is the middle point of \( P_{L_i}P_{R_i} \), where the virtual serial chain connects to...
the PP, \( r_p \) is the radius of the circle which passes through the connecting points \( P_{L_i}, P_{R_i} \) of all the branches. It defines the size of the PP. \( \theta \) is the angle between \( P_{O}P_{L_i} \) and \( P_{O}P_{R_i} \). Fixed at the center of the circle \( P_{O} \), the frame \( \mathcal{R}_P(P_{O}, u_p, v_p, w_p) \) is attached to the PP with the \( u \)-axis pointing to \( P_L \). The frame \( \mathcal{R}_B(B_{O}, u_b, v_b, w_b) \) is the world frame which is fixed on the supporting media. Frame \( \mathcal{R}_{A_i}(A_{i}, u_a, v_a, w_a)(i=1,2 \text{ or } 3) \) is defined for each limb, with its origin located at the point \( A_i \) of the \( i^{th} \) limb.

**Platform posture variable (x):**

\[
(x_p; y_p; z_p; \alpha_p; \beta_p; \gamma_p)
\]

It describes the posture (position and orientation) of the frame \( \mathcal{R}_P \) with respect to the world frame \( \mathcal{R}_B \). \([x_p, y_p, z_p]^T\) is the position vector of the point \( P_B \) with respect to the world frame. \( \alpha_p, \beta_p \) and \( \gamma_p \) are the rotations about the fixed \( u_b, v_b \) and \( w_b \) axes of the world frame.

**Branch extremity variable (x_{A_i}):**

\[
(x_{A_i}; y_{A_i}; z_{A_i}; \alpha_{A_i}; \beta_{A_i}; \gamma_{A_i})(i=1,2 \text{ or } 3)
\]

These parameters describe the posture of frame \( \mathcal{R}_{A_i} \) with respect to the world frame. \([x_{A_i}, y_{A_i}, z_{A_i}]^T\) is the position vector of points \( A_i \) in the world frame. \( \alpha_{A_i}, \beta_{A_i} \) and \( \gamma_{A_i} \) are the rotations about the fixed \( u_b, v_b \) and \( w_b \) axes of the world frame.

**Actuator variable (q):**

\[
[qL_{L_1}; qR_{L_1}; qL_{L_2}; qR_{L_2}; qL_{L_3}; qR_{L_3}]^T
\]

\( q \) represents the generalized actuation coordinates vector which corresponds to the displacement of the six prismatic joints in the three branches. The subscripts \( L_1, L_2 \) and \( L_3 \) are the indices of the three branches.

**Joints of the virtual serial chains (s_j):**

\[
[qL_{L_1}; qL_{L_2}; qL_{L_3}; qL_{R_1}; qL_{R_2}; qL_{R_3}]^T(i=1,2 \text{ or } 3)
\]

As it is issued in the previous section, the closed KC configuration, which provides the possibility to use two identical linear actuators, can be considered as a virtual UPS serial

![FIGURE 6: GENERAL COORDINATES OF ROBOT](image)

**FIGURE 6: GENERAL COORDINATES OF ROBOT**

KC. \( s_j \) is the joints values of the virtual serial chain of the \( j^{th} \) branch.

**The Tripod Working Modes and Scenario**

**PP Mode:** All the branches of the robot are attached to the supporting media. The tripod which is capable of performing 6-axis tasks can be considered as a 6-3 Stewart platform from a topological point of view.

**BE Mode:** One branch of the robot is detached from the base in order to reach another supporting point, while the other branches remain attached to the base. The passive joints in the swing branch should be locked before detaching the branch from the supporting point. Furthermore, as there are less branches connected between the PP and the base, the actuators in the branches attached to the base are no longer sufficient to control the PP. Some of the passive joints in these branches also need to be locked in order to reduce the DOF of the PP.

These two modes are used to perform machining and locomotion tasks. A working scenario (Fig. 7) which presents one operation cycle from one work location to another can be decomposed into different phases summarized as follows:

**Machining phase** With all the branches attached to the supporting points, the robot works as a parallel manipulator. By using the inverse kinematic models (IKMs), the PP of the manipulator is capable to follow a given trajectory in its workspace. It is important to notice that when the supporting pattern changes, the workspace and force capacity of the robot vary as well. This provides the possibility to reconfigure the robot for various tasks.

**Reconfiguration for limb swinging phase** During this phase, all the limbs of the robot are still attached to the supporting points. Before locking the corresponding lockable joints, the PP is supposed to move to a specific position with a given pose in order to have all the \( b_i \) joints (defined in the next section) in desired positions for locking.

**Limb swinging phase** With the corresponding lockers activated, the extremity of the swinging limb can follow a given 6-axis trajectory.
KINEMATICS AND SIMULATION

Inverse and forward kinematic problems are derived for both the PP mode and the BE mode in order to achieve the simulation. Similar to the classical parallel robots, the PP of the tripod is considered as the end-effector in the PP mode. The robot’s kinematic models are based on the vectors loop equations and numerical methods [27]. In this section, only the BE mode kinematic problem is addressed.

BE Mode Kinematics: BE As End Effector

When one of the limbs is detached from its supporting point, a hybrid mechanism is formed: a 6-DOF parallel mechanism with four actuators plus a 6-DOF mechanism with two actuators mounted on the PP.

In the BE mode, in order to control the branch extremity, the passive joints $q_1, q_2, q_3$ and $q_6$ of the swinging limb will be locked before the limb detaches from its supporting point. Then to control the PP with the four remaining actuators located in the two supporting limbs, the two $q_1$ joints (one in each limb) should be locked before the swinging phase begins.

We introduce variable $b_i$ which denotes the locked joints during the swinging phase of limb $i$:

$$\{q_{Li}; q_{Mi}; q_{Pi}; q_{Qi}; q_{Ri}\}$$

for $((i,j,k) \in \{(1,2,3),(2,1,3),(3,1,2)\}; i: \text{branch in swing}; j, k: \text{branches in stance})$

The pose of the branch extremity of the $i^{th}$ limb are described by the BE coordinate variable $x_{ai}$. So the inverse kinematics problem of the BE mode will be naturally considered as: finding the actuator variable $q$ with the given value of $x_{ai}$. Similar to the forward kinematics problems of a conventional parallel manipulator, the direct relationship between $x_{ai}$ and $q$ are difficult to obtain due to the highly nonlinear equations (polynomial up to 40 degrees in some cases) [28].

To solve this relationship, the problem is formulated in a different way: we consider that the lockers on the lockable joints are not activated, which means the robot works as in PP mode. Then if the pose of the clamping points is changed slightly, the robot will still be capable to keep the platform at the same pose by modifying the values of actuator variable $q$. Consequently, the values of the lockable joints $b_i$ will be changed as well. To compute the values of these passive lockable joints, an inverse kinematics model $BIKMX$ with $x$ as input and $b$ as output is established.

The vector projection approach is used to solve $BIKMX$. As the relations are valid for every branch independently, the subscripts of variables which indicate the index of branches are omitted in the equations.

The signed angle between two intersected unit vectors is computed as follows:

$$\Theta(V_1, V_2, N) = \arctan 2(N \cdot (V_1 \times V_2), V_1 \cdot V_2)$$

where $V_1, V_2$ are two coplanar unit vectors, $N$ is the normal of such plane.

$\Vert V_1, V_2 \Vert$ denotes the direction vector which is parallel to the projection of $V_1$ on the normal of $V_2$.

$$\Vert V_1 - (V_1 \cdot V_2) \times V_2\|_{V_1 - (V_1 \cdot V_2) \times V_2}$$

$BIKMX$ Computation Let $\vec{v}_c$ be the direction vector of $\overrightarrow{BO}$, $\vec{c}$ be the direction vector of $\overrightarrow{P_i P_R}$, $\vec{l}$ denotes the direction vector of $\overrightarrow{AP}$ (Fig. 8). $q_6$ is defined as the angle between plane $P_l P_R A$ and plane $P_0 P_l P_R$. As the two planes intersect at $P_l P_R$, $\vec{c}$ is perpendicular to $\vec{u}$, $q_6$ can be expressed as:

$$q_6 = \frac{\pi}{2} - \Theta(\vec{u}, \vec{v}_c, \vec{c})$$

And $q_5$, being the angle between $P_l P_R$ and $AP$, can be calculated as:

$$q_5 = \frac{\pi}{2} - \Theta(\vec{u}, \vec{l}, \vec{u} \times \vec{l})$$

$q_4$ can be expressed as the distance between $P_l$ and $A$:

$$q_4 = \|\overrightarrow{BP} - \overrightarrow{AP}\|$$
with \( \overrightarrow{B_0P_C} = \overrightarrow{P_0} - \overrightarrow{B_0P_O} \), where \( \overrightarrow{P_0} \) is the 3 \( \times \) 3 rotation matrix from the world frame \( \mathcal{R}_B \) to the frame \( \mathcal{R}_P \): 
\[
\overrightarrow{P_0} = \text{Rot}(\gamma_p)\text{Rot}(\beta_p)\text{Rot}(\alpha_p).
\]

The angle between the projection of \( \overrightarrow{w_a} \) on the plane \( \mathcal{P}_{LR_A} \) and \( \overrightarrow{\ell_c} \) equals \( q_3 \). \( \overrightarrow{\ell_c} \), the direction vector of the \( q_3 \) axis, is always perpendicular to the plane \( \mathcal{P}_{LR_A} \). The direction vector of the projection of \( \overrightarrow{w_a} \) on the plane \( \mathcal{P}_{LR_A} \) can be calculated as \( \perp \overrightarrow{v_3} \overrightarrow{w_a} \).

Therefore, we have
\[
q_3 = \Theta(\overrightarrow{\ell_c}, \perp \overrightarrow{v_3} \overrightarrow{w_a}, \overrightarrow{v_3})
\]

The value of joint \( q_2 \) is the angle between \( \perp \overrightarrow{v_3} \overrightarrow{w_a} \) and the \( w \)-axis of the frame \( \mathcal{R}_A \).
\[
q_2 = \Theta(\perp \overrightarrow{v_3} \overrightarrow{w_a}, \overrightarrow{v_w}, \overrightarrow{u})
\]

Let \( \overrightarrow{w_a} \overrightarrow{v_3} \) be the projection of \( \overrightarrow{v_3} \) on the \( x-y \) plane of the frame \( \mathcal{R}_A \). Then \( q_1 \) can be expressed as the angle between the \( u \)-axis of the frame \( \mathcal{R}_A \) and \( \perp \overrightarrow{v_3} \overrightarrow{w_a} \)
\[
q_1 = \Theta(\overrightarrow{v_u}, \perp \overrightarrow{v_3} \overrightarrow{w_a}, \overrightarrow{v_w})
\]

**XFKMBi Computation** In reality, the value of locked joints will not be changed during the BE mode. So the original branch extremity control problem is transformed as follows: when the supporting points are changed, finding the values of actuator variable \( q \) which allow all the lockable joints to remain to their given values \( b_i \). As we can obtain straightforwardly the actuator variable \( q \) from the platform coordinates \( x \) with the IKM of conventional parallel robots, the problem can be further transformed as: finding \( x \), the very pose of the PP, which allows values of all the activated lockable joints to remain matching the given \( b_i \).

To answer the previous question, a numerical forward kinematics model \( XFKMB_i \) is written as an optimization problem: it consists in finding the \( x \) which minimizes \( ||B_i\text{IKM}(X) - b_i|| \).

**Scenarios Simulation**

The whole cycle of a working scenario is simulated in MatLab. Combining the kinematic models, the scenario presented in Fig. 9 shows the feasibility of the concept of a reduced DOF legged robot with integrated lockable joints to achieve machining and locomotion tasks.

It is worthy to mention that, during all these phases, there is at most one limb detached from the supporting point. Thanks to the clamping devices and lockers, the robot has always solid connection with the supporting media. Unlike most of the tripods that exist in the literature, the limitation of friction between the feet and the ground, the landing impact force and static or dynamic balance [29] issues are not the major concerns as long as the locking components do not fail.

**FIGURE 9: WALKING AND MACHINING SCENARIO**

**Discussions on the Walking Parallel Robots**

The simulation reveals some important issues for the design of a realistic legged mobile robot with lockable joints and clamping devices.

A machining head can be added on the PP with various ori-
entations according to different applications (Fig. 10).

From the topological point of view, the proposed tripod remains the same kinematics structure in each working area. However, when the supporting pattern \( (x_i) \) changes, the robot has no longer the same geometrical parameters. The workspace, the rigidity, the precisions and many other properties of the robot vary as well. In the design process, the optimization of the structure should not only concentrate on the robot’s design parameters but also on the arrangement of the supporting points.

The choice of lockable joints and their values is also one of the key aspects to study but is not addressed in this paper. The simulation of scenario shows that the choice of lockable joints and their values have significant impact on the reachable walking area of the robot. Further study is needed for determining the proper “locking” strategy.

Also, other identified robot topologies in Tab. 1 can be interesting according to different applications. From the simulation, we notice that might be interesting to add kinematic redundancy to the studied tripod. The reason is that, during the walking phase, the extremity of the swing limb is controlled by six actuators. For a given pose of the extremity of the swing limb, the tripod possesses no kinematics redundancy, so cannot be determined a specific behavior for the PP to avoid interferences. One of the possible solution is to add one more limb to the platform, which corresponds to the topology of Group 3 in Fig. 4. The significant advantage of this arrangement is that, during the walking phase, the connectivity between the BE link and the supporting surface is eight, which provides the possibility to follow its own given trajectory and to optimize the trajectory of the PP at the same time.

CONCLUSION

Mobile and flexible manufacturing systems are demanded in many industries. In this paper, several important considerations and approach for designing industry oriented legged robots are presented. Applying some design constraints, several potential topologies of such robots are identified. Based on a mobility analysis, a mobile manipulator which combines the advantages of both parallel robots and legged mobile robots is studied. With the derived kinematic models, a scenario of 6-axis manipulation and 6-axis walking is simulated. From the simulation, we show that: integrating lockers on some passive joints and some clamping devices, six actuators are enough to build a legged manipulator which can not only achieve 6-axis manipulation but also walk on a curved supporting media. Before building a prototype of such walking parallel robot, there are ongoing studies on the arrangement of the supporting points, locking strategies as well as advanced control methods in order to be able to customize the robots configuration according to specific applications.

REFERENCES


