What Can Argumentation Do for Inconsistent Ontology Query Answering?
–Technical Report–

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Abstract. The area of inconsistent ontological knowledge base query answering studies the problem of inferring from an inconsistent ontology. To deal with such a situation, different semantics have been defined in the literature (e.g. AR, IAR, ICR). Argumentation theory can also be used to draw conclusions under inconsistency. Given a set of arguments and attacks between them, one applies a particular semantics (e.g. stable, preferred, grounded) to calculate the sets of accepted arguments and conclusions. However, it is not clear what are the similarities and differences of semantics from ontological knowledge base query answering and semantics from argumentation theory. This paper provides the answer to that question. Namely, we prove that: (1) sceptical acceptance under stable and preferred semantics corresponds to ICR semantics; (2) universal acceptance under stable and preferred semantics corresponds to AR semantics; (3) acceptance under grounded semantics corresponds to IAR semantics. We also prove that the argumentation framework we define satisfies the rationality postulates (e.g. consistency, closure).

1 Introduction

Ontological knowledge base query answering problem has received renewed interest in the knowledge representation community (and especially in the Semantic Web domain where it is known as the ontology based data access problem [17]). It considers a consistent ontological knowledge base (made from facts and rules) and aims to answer if a query is entailed by the knowledge base (KB). Recently, this question was also considered in the case where the KB is *inconsistent* [16, 8]. Maximal consistent subsets of the KB, called *repairs*, are then considered and different *semantics* (based on classical entailment on repairs) are proposed in order to compute the set of accepted formulae.

Argumentation theory is also a well-known method for dealing with inconsistent knowledge [5, 2]. Logic-based argumentation [6] considers constructing arguments from

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inconsistent knowledge bases, identifying attacks between them and selecting acceptable arguments and their conclusions. In order to know which arguments to accept, one applies a particular *argumentation semantics*.

This paper starts from the observation that both inconsistent ontological KB query answering and instantiated argumentation theory deal with the same issue, which is reasoning under inconsistent information. Furthermore, both communities have several mechanisms to select acceptable conclusions and they both call them *semantics*. The *research questions* one could immediately ask are: Is there a link between the semantics used in inconsistent ontological KB query answering and those from argumentation theory? Is it possible to instantiate Dung's ([15]) abstract argumentation theory in a way to implement the existing semantics from ontological KB query answering? If so, which semantics from ontological KB query answering correspond to which semantics from argumentation theory? Does the proposed instantiation of Dung's abstract argumentation theory satisfy the rationality postulates [10]?

There are several benefits from answering those questions. First, it would allow to *import some results* from argumentation theory to ontological query answering and vice versa, and more generally open the way to the Argumentation Web [19]. Second, it might be possible to use these results in order to *explain* to users how repairs are constructed and why a particular conclusion holds in a given semantics by constructing and evaluating arguments in favour of different conclusions [14]. Also, on a more theoretical side, proving a link between argumentation theory and the results in the knowledge representation community would be a step forward in understanding the *expressibility* of Dung's abstract theory for logic based argumentation [21].

The paper is organised as follows. In Section 2 the ontological query answering problem is explained and the logical language used throughout the paper is introduced. The end of this section introduces the existing semantics proposed in the literature to deal with inconsistent knowledge bases. Then, in Section 3, we define the basics of argumentation theory. Section 4 proves the links between the extensions obtained under different argumentation semantics in this instantiated logical argumentation setting and the repairs of the ontological knowledge base. We show the equivalence between the semantics from inconsistent ontological KB query answering area and those defined in argumentation theory in Section 5. Furthermore, the argumentation framework thus defined respects the rationality postulates (Section 6). The paper concludes with Section 7.

2 Ontological Conjunctive Query Answering

The main goal of section is to introduce the syntax and semantics of the $\mathcal{SRC}$language [3, 4], which is used in this paper due to its relevance in the context of the ontological KB query answering.

Note that the goal of the present paper is not to change or criticise the definitions from this area; we simply present the existing work. Our goal is to study the link between the existing work in this area and the existing work in argumentation theory. In the following, we give a general setting knowledge representation language which can then be instantiated according to properties on rules or constraints and yield equivalent languages to those used by [16] and [8].

A knowledge base is a 3-tuple $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ composed of three finite sets of formulae: a set $\mathcal{F}$ of facts, a set $\mathcal{R}$ of rules and a set $\mathcal{N}$ of constraints. Let us formally define what we accept as $\mathcal{F}$, $\mathcal{R}$ and $\mathcal{N}$.

Facts Syntax. Let $\mathbf{C}$ be a set of constants and $\mathbf{P} = P_1 \cup P_2 \ldots \cup P_n$ a set of predicates of the corresponding arity $i = 1, \ldots, n$. Let $\mathbf{V}$ be a countably infinite set of *variables*. We define the set of *terms* by $\mathbf{T} = \mathbf{V} \cup \mathbf{C}$. As usual, given $i \in \{1 \ldots n\}$, $p \in P_i$ and $t_1, \ldots, t_i \in \mathbf{T}$ we call $p(t_1, \ldots, t_i)$ an *atom*. If γ is an atom or a conjunction of atoms, we denote by $var(\gamma)$ the set of variables in γ and by $term(\gamma)$ the set of terms in γ. A *fact* is the existential closure of an atom or an existential closure of a conjunction of atoms. (Note that there is no negation or disjunction in the facts.) As an example, consider $\mathbf{C} = \{Tom\}$, $\mathbf{P} = P_1 \cup P_2$, with $P_1 = \{cat, mouse\}$, $P_2 = \{eats\}$ and $\mathbf{V} = \{x_1, x_2, x_3, \ldots\}$. Then, $cat(Tom)$, $eats(Tom, x_1)$ are examples of atoms and $\gamma = cat(Tom) \wedge mouse(x_1) \wedge eats(Tom, x_1)$ is an example of a conjunction of atoms. It holds that $var(\gamma) = \{x_1\}$ and $term(\gamma) = \{Tom, x_1\}$. As an example of a fact, consider $\exists x_1(cat(Tom) \wedge mouse(x_1) \wedge eats(Tom, x_1))$.

An *interpretation* is a pair $I = (\triangle, .^I)$ where $\triangle$ is the interpretation domain (possibly infinite) and $.^I$, the interpretation function, satisfies:

1. For all $c \in \mathbf{C}$, we have $c^I \in \triangle$,
2. For all i and for all $p \in P_i$, we have $p^I \subseteq \triangle^i$,
3. If $c, c' \in \mathbf{C}$ and $c \neq c'$ then $c^I \neq c'^I$.

Note that the third constraint specifies that constants with different names map to different elements of Δ.

Let γ be an atom or a conjunction of atoms or a fact. We say that γ is *true* under interpretation I iff there is a function ι which maps the terms (variables and constants) of γ into $\triangle$ such that for all constants c, it holds that $\iota(c) = c^I$ and for all atoms $p(t_1, ...t_i)$ appearing in γ, it holds that $(\iota(t_1), ..., \iota(t_i)) \in p^I$. For a set F containing any combination of atoms, conjunctions of atoms and facts, we say that F is *true* under interpretation I iff there is a function ι which maps the terms (variables and constants) of all formulae in F into $\triangle$ such that for all constants c, it holds that $\iota(c) = c^I$ and for all atoms $p(t_1, ...t_i)$ appearing in formulae of F, it holds that $(\iota(t_1), ..., \iota(t_i)) \in p^I$. Note that this means that for example sets $F_1 = \{\exists x(cat(x) \wedge dog(x))\}$ and $F_2 = \{\exists x(cat(x)), \exists x(dog(x))\}$ are true under exactly the same set of interpretations. Namely, in both cases, variable x is mapped to an object of Δ. On the other hand, there are some interpretations under which set $F_3 = \{\exists x_1(cat(x_1)), \exists x_2(dog(x_2))\}$ is true whereas F_1 and F_2 are not.

If γ is true in I we say that I is a model of γ. Let γ' be an atom, a conjunction of atoms or a fact. We say that γ is a logical consequence of γ' (γ' entails γ, denoted $\gamma' \models \gamma$) iff all models of γ are models of γ'. If a set F is true in I we say that I is a model of F. We say that a formula γ is a logical consequence of a set F (denoted $F \models \gamma$) iff all models of F are models of γ. We say that a set G is a logical consequence of set F (denoted $F \models G$) if and only if all models of F are models of G. We say that two sets F and G are logically equivalent (denoted $F \equiv G$) if and only if $F \models G$ and $G \models F$.

Given a set of variables $\mathbf{X}$ and a set of terms $\mathbf{T}$, a *substitution* σ of $\mathbf{X}$ by $\mathbf{T}$ is a mapping from $\mathbf{X}$ to $\mathbf{T}$ (denoted $\sigma : \mathbf{X} \to \mathbf{T}$). Given an atom or a conjunction of atoms γ, $\sigma(\gamma)$ denotes the expression obtained from γ by replacing each occurrence of $x \in \mathbf{X} \cap var(\gamma)$ by $\sigma(x)$. If a fact F is the existential closure of a conjunction γ then we define $\sigma(F)$ as the existential closure of $\sigma(\gamma)$. Finally, let us define *homomorphism*. Let F and F' be atoms, conjunctions of atoms or facts (it is not necessarily the case that F and F' are of the same type, e.g. F can be an atom and F' a conjunction of atoms). Let σ be a substitution such that $\sigma : var(F) \to term(F')$. We say that σ is a homomorphism from F to F' if and only if the set of atoms appearing in $\sigma(F)$ is a subset of the set of atoms appearing in $\sigma(F')$. For example, let $F = cat(x_1)$ and $F' = cat(Tom) \wedge mouse(Jerry)$. Let $\sigma : var(F) \to term(F')$ be a substitution such that $\sigma(x_1) = Tom$. Then, σ is a homomorphism from F to F' since the atoms in $\sigma(F)$ are $\{cat(Tom)\}$ and the atoms in $\sigma(F')$ are $\{cat(Tom), mouse(Jerry)\}$.

Note that it well is known that $F' \models F$ if and only if there is a homomorphism from F to F' [12].

Rules. A rule R is a formula $\forall x_1, \ldots, \forall x_n \; \forall y_1, \ldots, \forall y_m \; (H(x_1, \ldots, x_n, y_1, \ldots, y_m) \to \exists z_1, \ldots \exists z_k \; C(y_1, \ldots, y_m, z_1, \ldots z_k))$ where H, the hypothesis, and C, the conclusion, are atoms or conjunctions of atoms, $n, m, k \in \{0, 1, \ldots\}$, $x_1, \ldots, x_n$ are the variables appearing in H, $y_1, \ldots, y_m$ are the variables appearing in both H and C and $z_1, \ldots, z_k$ the new variables introduced in the conclusion. As two examples of rules, consider $\forall x_1(cat(x_1) \to miaw(x_1))$ or $\forall x_1((mouse(x_1) \to \exists z_1(cat(z_1) \wedge eats(z_1, x_1)))$.

Reasoning consists of applying rules on the set and thus inferring new knowledge. A rule $R = (H, C)$ is *applicable* to set $\mathcal{F}$ if and only if there exists $\mathcal{F}' \subseteq \mathcal{F}$ such that there is a homomorphism σ from the hypothesis of $\mathcal{R}$ to the conjunction of elements of $\mathcal{F}'$. For example, rule $\forall x_1(cat(x_1) \to miaw(x_1))$ is applicable to set $\{cat(Tom)\}$, since there is a homomorphism from $cat(x_1)$ to $cat(Tom)$. If rule R is applicable to set F, the application of R to F according to π produces a set $F \cup \{\pi(C)\}$. In our example, the produced set is $\{cat(Tom), miaw(Tom)\}$. We then say that the new set (which includes the old one and adds the new information to it) is an *immediate derivation* of F by R. This new set is often denoted by $R(F)$. Thus, applying a rule on a set produces a new set.

Let F be a subset of $\mathcal{F}$ and let $\mathcal{R}$ be a set of rules. A set F_n is called an $\mathcal{R}$*-derivation* of F if there is a sequence of sets (called a *derivation sequence*) $(F_0, F_1, \ldots, F_n)$ such that:

– $F_0 \subseteq F$
– F_0 is $\mathcal{R}$-consistent
– for every $i \in \{1, \ldots, n-1\}$, it holds that F_i is an immediate derivation of F_{i-1}
– (no formula in $\mathcal{F}_n$ contains a conjunction and $\mathcal{F}_n$ is an immediate derivation of $\mathcal{F}_{n-1}$) or F_n is obtained from F_{n-1} by conjunction elimination.

Conjunction elimination is the following procedure: while there exists at least one conjunction in at least one formula, take an arbitrary formula φ containing a conjunction. If φ is of the form $\varphi = \psi \wedge \psi'$ then exchange it with two formulae ψ and ψ'. If φ is of the form $\exists x(\psi \wedge \psi')$ then exchange it with two formulae $\exists x(\psi)$ and $\exists x(\psi')$. The idea is just to start with an $\mathcal{R}$-consistent set and apply (some of the) rules. The only technical detail is that the conjunctions are eliminated from the final result. So if the last set in a sequence does not contain conjunctions, nothing is done. Else, we eliminate those conjunctions. This technicality is needed in order to stay as close as possible to the procedures used in the literature in the case when the knowledge base is consistent.

Given a set $\{F_0, \ldots, F_k\} \subseteq \mathcal{F}$ and a set of rules $\mathcal{R}$, the closure of $\{F_0, \ldots, F_k\}$ with respect to $\mathcal{R}$, denoted $\mathtt{Cl}_{\mathcal{R}}(\{F_0, \ldots, F_k\})$, is defined as the smallest set (with respect to $\subseteq$) which contains $\{F_0, \ldots, F_k\}$, and is closed for $\mathcal{R}$-derivation (that is, for every $\mathcal{R}$-derivation F_n of $\{F_0, \ldots, F_k\}$, we have $F_n \subseteq \mathtt{Cl}_{\mathcal{R}}(\{F_0, \ldots, F_k\})$). Finally, we say that a set $\mathcal{F}$ and a set of rules $\mathcal{R}$ *entail* a fact G (and we write $\mathcal{F}, \mathcal{R} \models G$) iff the closure of the facts by all the rules entails F (i.e. if $\mathtt{Cl}_{\mathcal{R}}(\mathcal{F}) \models G$).

As an example, consider a set of facts $\mathcal{F} = \{cat(Tom), small(Tom)\}$ and the rule set $\mathcal{R} = \{R_1 = \forall x_1(cat(x_1) \rightarrow miaw(x_1) \wedge animal(x_1)), R_2 = \forall x_1(miaw(x_1) \wedge small(x_1) \rightarrow cute(x_1))\}$. Then, F_0, F_1, F_2 is a derivation sequence, where $F_0 = \{cat(Tom), small(Tom)\}$, $F_1 = R_1(F_0) = \{cat(Tom), small(Tom), miaw(Tom) \wedge animal(Tom)\}$, $F_2 = \{cat(Tom), small(Tom), miaw(Tom) \wedge animal(Tom), cute(Tom)\}$ and $F_3 = \{cat(Tom), small(Tom), miaw(Tom), animal(Tom), cute(Tom)\}$.

We conclude the presentation on rules in $\mathcal{SRC}$ by a remark on performing union on facts when they are viewed as sets of atoms. In order to preserve semantics the union is done by renaming variables. For example, let us consider a fact $F_1 = \{\exists x cat(x)\}$ and a fact $F_2 = \{\exists x animal(x)\}$. Then the fact $F = F_1 \cup F_2$ is the union of the two fact after variable naming has been performed: $F = \{\exists x_1 cat(x_1), \exists x_2 animal(x_2)\}$.

Constraints. A constraint is a formula $\forall x_1 \ldots \forall\, x_n\ (H(x_1, \ldots, x_n) \rightarrow \bot)$, where H is an atom or a conjunction of atoms and $n \in \{0, 1, 2, \ldots\}$. Equivalently, a constraint can be written as $\neg(\exists x_1, ..., \exists x_n H(x_1, ...x_n))$. As an example of a constraint, consider $\forall x_1(cat(x_1) \wedge dog(x_1) \rightarrow \bot)$. $H(x_1, \ldots, x_n)$ is called the hypothesis of the constraint.

Given a knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$, a set $\{F_1, \ldots, F_k\} \subseteq \mathcal{F}$ is said to be *inconsistent* if and only if there exists a constraint $N \in \mathcal{N}$ such that $\{F_1, \ldots, F_k\} \models H_N$, where H_N denotes the existential closure of the hypothesis of N. A set is consistent if and only if it is not inconsistent. A set $\{F_1, \ldots, F_k\} \subseteq \mathcal{F}$ is $\mathcal{R}$*-inconsistent* if and only if there exists a constraint $N \in \mathcal{N}$ such that $\mathtt{Cl}_{\mathcal{R}}(\{F_1, \ldots, F_k\}) \models H_N$, where H_N denotes the existential closure of the hypothesis of N.

A set of facts is said to be $\mathcal{R}$*-consistent* if and only if it is not $\mathcal{R}$-inconsistent. A knowledge base $(\mathcal{F}, \mathcal{R}, \mathcal{N})$ is said to be *consistent* if and only if $\mathcal{F}$ is $\mathcal{R}$-consistent. A knowledge base is *inconsistent* if and only if it is not consistent.

Example 1. Let us consider the following knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$, with: $\mathcal{F} = \{cat(Tom), bark(Tom)\}$, $\mathcal{R} = \{\forall x_1(cat(x_1) \rightarrow miaw(x_1))\}$, $\mathcal{N} = \{\forall x_1(bark(x_1) \wedge miaw(x_1) \rightarrow \bot)\}$. The only rule in the knowledge base is applicable to the set $\{cat(Tom), bark(Tom)\}$ and its immediate derivation produces the set $\{cat(Tom), bark(Tom), miaw(Tom)\}$. We see that $\mathtt{Cl}_{\mathcal{R}}(\mathcal{F}) \models \exists x_1(bark(x_1) \wedge miaw(x_1))$, thus the KB is inconsistent.

Given a knowledge base, one can ask a conjunctive query in order to know whether something holds or not. Without loss of generality we consider in this paper boolean conjunctive queries (which are facts). As an example of a query, take $\exists x_1 cat(x_1)$. The answer to query α is positive if and only if $\mathcal{F}, \mathcal{R} \models \alpha$.

2.1 Query Answering over Inconsistent Ontological Knowledge Bases

Notice that (like in classical logic), if a knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ is inconsistent, then everything is entailed from it. In other words, every query is true. Thus, the approach we described until now is not robust enough to deal with inconsistent information. However, there are cases when the knowledge base is inconsistent; this phenomenon has attracted particular attention during the recent years [8, 16]. For example, the set $\mathcal{F}$ may be obtained by combining several sets of facts, coming from different agents. In this paper, we study a general case when $\mathcal{K}$ is inconsistent without making any hypotheses about the origin of this inconsistency. Thus, our results can be applied to an inconsistent base independently of how it is obtained.

A common solution [8, 16] is to construct maximal (with respect to set inclusion) consistent subsets of $\mathcal{K}$. Such subsets are called *repairs*. Formally, given a knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$, define:

$$\mathcal{R}epair(\mathcal{K}) = \{\mathcal{F}' \subseteq \mathcal{F} \mid \mathcal{F}' \text{ is maximal for } \subseteq \ \mathcal{R}\text{-consistent set}\}$$

We now mention a very important technical detail. In some papers, a *set* of formulae is identified with the *conjunction* of those formulae. This is not of particular significance when the knowledge base is *consistent*. However, in case of an *inconsistent* knowledge base, this makes a big difference. Consider for example $\mathcal{K}_1 = (\mathcal{F}_1, \mathcal{R}_1, \mathcal{N}_1)$ with $\mathcal{F}_1 = \{dog(Tom), cat(Tom)\}$, $\mathcal{R}_1 = \emptyset$ and $\mathcal{N}_1 = \{\forall x_1(dog(x_1) \wedge cat(x_1) \rightarrow \perp)\}$, compared with $\mathcal{K}_2 = (\mathcal{F}_2, \mathcal{R}_2, \mathcal{N}_2)$ with $\mathcal{F}_2 = \{dog(Tom) \wedge cat(Tom)\}$, $\mathcal{R}_2 = \emptyset$ and $\mathcal{N}_2 = \{\forall x_1(dog(x_1) \wedge cat(x_1) \rightarrow \perp)\}$. In this case, according to the definition of a repair, $\mathcal{K}_1$ would have two repairs and $\mathcal{K}_2$ would have no repairs at all. We could proceed like this, but we find it confusing given the existing literature in this area. This is why, in order to be completely precise, from now on we suppose that $\mathcal{F}$ does not contain conjunctions. Namely, $\mathcal{F}$ is supposed to be a set composed of of atoms and of existential closures of atoms. One could believe that this reduces the expressibility of the language, consider for example $\mathcal{F}_1 = \{\exists x(dog(x)), \exists x(black(x))\}$ as opposed to $\mathcal{F}_2 = \{\exists x(dog(x) \wedge black(x))\}$. Namely, in classical first order logic, $\mathcal{F}_1$ and $\mathcal{F}_2$ do not have the same models. However, in $\mathcal{SRC}$, $\mathcal{F}_1$ and $\mathcal{F}_2$ have the same models (see the definition of an interpretation).

Once the repairs calculated, there are different ways to calculate the set of facts that follow from an inconsistent knowledge base. For example, we may want to accept a query if it is entailed in *all repairs* (AR semantics).

Definition 1. *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base and let α be a query. Then α is **AR-entailed** from $\mathcal{K}$, written $\mathcal{K} \models_{AR} \alpha$ iff for every repair $A' \in \mathcal{R}epair(\mathcal{K})$, it holds that $\mathtt{Cl}_{\mathcal{R}}(A') \models \alpha$.*

Another possibility is to check whether the query is entailed from the *intersection of closed repairs* (ICR semantics).

Definition 2. *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base and let α be a query. Then α is **ICR-entailed** from $\mathcal{K}$, written $\mathcal{K} \models_{ICR} \alpha$ iff $\bigcap_{A' \in \mathcal{R}epair(\mathcal{K})} \mathtt{Cl}_{\mathcal{R}}(A') \models \alpha$.*

Example 2 (Example 1 Cont.). $\mathcal{R}epair(\mathcal{K}) = \{R_1, R_2\}$ with $R_1 = \{cat(Tom)\}$ and $R_2 = \{bark(Tom)\}\}$. $\texttt{Cl}_{\mathcal{R}}(R_1) = \{cat(Tom), miaw(Tom)\}$, $\texttt{Cl}_{\mathcal{R}}(R_2) = \{bark(Tom)\}$. It is *not* the case that $\mathcal{K} \models_{ICR} cat(Tom)$.

Finally, another possibility is to consider the *intersection of all repairs* and then close this intersection under the rules (IAR semantics).

Definition 3. *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base and let α be a query. Then α is **IAR-entailed** from $\mathcal{K}$, written $\mathcal{K} \models_{IAR} \alpha$ iff $\texttt{Cl}_{\mathcal{R}}(\bigcap_{A' \in \mathcal{R}epair(\mathcal{K})}) \models \alpha$.*

The three semantics can yield different results [16, 8], as illustrated by the next two examples.

Example 3. (ICR and IAR different from AR) Consider $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$, with: $\mathcal{F} = \{havecat(Tom), haveMouse(Jerry)\}$, intuitively, we have a cat (called Tom) and a mouse (called Jerry); $\mathcal{R} = \{\forall x_1\ (haveCat(x_1) \rightarrow haveAnimal(x_1)),$
$\forall x_2\ (haveMouse(x_2) \rightarrow haveAnimal(x_2))\}$; $\mathcal{N} = \{\forall x_1 \forall x_2 (haveCat(x_1)$
$\wedge haveMouse(x_2) \rightarrow \bot)\}$, meaning that we cannot have both a cat and a mouse (since the cat would eat the mouse). There are two repairs: $R_1 = \{haveCat(Tom)\}$ and $R_2 = \{haveMouse(Jerry)\}$. $\texttt{Cl}_{\mathcal{R}}(R_1) = \{haveCat(Tom), haveAnimal(Tom)\}$ and $\texttt{Cl}_{\mathcal{R}}(R_2) = \{haveMouse(Jerry), haveAnimal(Jerry)\}$. Consider a query $\alpha = \exists x_1\ haveAnimal(x_1)$ asking whether we have an animal. It holds that $\mathcal{K} \models_{AR} \alpha$ since $\texttt{Cl}_{\mathcal{R}}(R_1) \models \alpha$ and $\texttt{Cl}_{\mathcal{R}} \models \alpha$, but neither $\mathcal{K} \models_{ICR} \alpha$ (since $\texttt{Cl}_{\mathcal{R}}(R_1) \cap \texttt{Cl}_{\mathcal{R}}(R_2) = \emptyset$) nor $\mathcal{K} \models_{IAR} \alpha$ (since $R_1 \cap R_2 = \emptyset$).

Example 4. (AR and ICR different from IAR) Consider $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$, with: $\mathcal{F} = \{cat(Tom), dog(Tom)\}$, $\mathcal{R} = \{\forall x_1 (cat(x_1) \rightarrow animal(x_1)),$
$\forall x_2 (dog(x_2) \rightarrow animal(x_2))\}$, $\mathcal{N} = \{\forall x (cat(x) \wedge dog(x) \rightarrow \bot)\}$.

We have $\mathcal{R}epair(\mathcal{K}) = \{R_1, R_2\}$ with $R_1 = \{cat(Tom)\}$ and $R_2 = \{dog(Tom)\}$. $\texttt{Cl}_{\mathcal{R}}(R_1) = \{cat(Tom), animal(Tom)\}$, $\texttt{Cl}_{\mathcal{R}}(R_2) = \{dog(Tom), animal(Tom)\}$.

It is *not* the case that $\mathcal{K} \models_{IAR} \exists x(animal(x))$ (since $R_1 \bigcap R_2 = \emptyset$). However, $\mathcal{K} \models_{AR} \exists x(animal(x))$. This is due to the fact that $\texttt{Cl}_{\mathcal{R}}(R_1) \models \exists x(animal(x))$ and $\texttt{Cl}_{\mathcal{R}}(R_2) \models \exists x(animal(x))$. Also, we have $\mathcal{K} \models_{ICR} \exists x(animal(x))$ since $\texttt{Cl}_{\mathcal{R}}(R_1) \cap \texttt{Cl}_{\mathcal{R}}(R_2) = \{animal(Tom)\}$.

3 Argumentation over Inconsistent Ontological Knowledge Bases

This section shows that it is possible to define an instantiation of Dung's abstract argumentation theory [15] that can be used to reason with an inconsistent ontological KB.

We first define the notion of an argument. For a set of formulae $\mathcal{G} = \{G_1, \ldots, G_n\}$, notation $\bigwedge G$ is used as an abbreviation for $G_1 \wedge \ldots \wedge G_n$.

Definition 4. *Given a knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$, an argument a is a tuple $a = (F_0, F_1, \ldots, F_n)$ where:*

– $(F_0, \ldots, F_{n-1})$ *is a derivation sequence with respect to $\mathcal{K}$*

– *F_n is an atom, a conjunction of atoms, the existential closure of an atom or the existential closure of a conjunction of atoms such that $F_{n-1} \models F_n$.*

Example 5 (Example 2 Cont.). Consider $a = (\{cat(Tom)\}, \{cat(Tom), miaw(Tom)\}, miaw(Tom))$ and $b = (\{bark(Tom)\}, bark(Tom))$ as two examples of arguments.

This is a straightforward way to define an argument when dealing with $\mathcal{SRC}$language, since this way, an *argument* corresponds to a *derivation*.

To simplify the notation, from now on, we suppose that we are given a fixed knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ and do not explicitly mention $\mathcal{F}$, $\mathcal{R}$ nor $\mathcal{N}$ if not necessary. Let $a = (F_0, ..., F_n)$ be an argument. Then, we denote $\texttt{Supp}(a) = F_0$ and $\texttt{Conc}(a) = F_n$. Let $S \subseteq \mathcal{F}$ a set of facts, $Arg(S)$ is defined as the set of all arguments a such that $\texttt{Supp}(a) \subseteq S$. Note that the set $\texttt{Arg}(S)$ is also dependent on the set of rules and the set of constraints, but for simplicity reasons, we do not write $\texttt{Arg}(S, \mathcal{R}, \mathcal{N})$ when it is clear to which $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ we refer to. Finally, let $\mathcal{E}$ be a set of arguments. The base of $\mathcal{E}$ is defined as the union of the argument supports: $\texttt{Base}(\mathcal{E}) = \bigcup_{a \in \mathcal{E}} \texttt{Supp}(a)$.

Arguments may attack each other, which is captured by a binary attack relation $\texttt{Att} \subseteq \texttt{Arg}(\mathcal{F}) \times \texttt{Arg}(\mathcal{F})$. Recall that the repairs are the subsets of $\mathcal{F}$ while the set $\mathcal{R}$ is always taken as a whole. This means that the authors of the semantics used to deal with an inconsistent ontological KB envisage the set of facts as inconsistent and the set of rules as consistent. When it comes to the attack relation, this means that we only need the so called "assumption attack" since, roughly speaking, all the inconsistency "comes from the facts".

Definition 5. *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base and let a and b be two arguments. The argument a attacks argument b, denoted $(a, b) \in Att$, if and only if there exists $\varphi \in \texttt{Supp}(b)$ such that the set $\{Conc(a), \varphi\}$ is $\mathcal{R}$-inconsistent.*

This attack relation is not symmetric. To see why, consider the following example. Let $\mathcal{F} = \{p(m), q(m), r(m)\}$, $\mathcal{R} = \emptyset$, $\mathcal{N} = \{\forall x_1(p(x_1) \wedge q(x_1) \wedge r(x_1) \rightarrow \bot)\}$. Let $a = (\{p(m), q(m)\}, p(m) \wedge q(m))$, $b = (\{r(m)\}, r(m))$. We have $(a, b) \in \texttt{Att}$ and $(b, a) \notin \texttt{Att}$. Note that using attack relations which are not symmetric is very common in argumentation literature. Moreover, symmetric attack relation have been criticised for violating some desirable properties [1].

Definition 6. *Given a knowledge base $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$, the* corresponding argumentation framework *$\mathcal{AF}_K$ is a pair $(\mathcal{A} = \texttt{Arg}(\mathcal{F}), \texttt{Att})$ where $\mathcal{A}$ is the set of arguments that can be constructed from $\mathcal{F}$ and $\texttt{Att}$ is the corresponding attack relation as specified in Definition 5.*

Let $\mathcal{E} \subseteq \mathcal{A}$ and $a \in \mathcal{A}$. We say that $\mathcal{E}$ is conflict free *iff there exists no arguments $a, b \in \mathcal{E}$ such that $(a, b) \in Att$. $\mathcal{E}$* defends *a iff for every argument $b \in \mathcal{A}$, if we have $(b, a) \in Att$ then there exists $c \in \mathcal{E}$ such that $(c, b) \in Att$.*

$\mathcal{E}$ is admissible *iff it is conflict free and defends all its arguments. $\mathcal{E}$ is a* complete extension *iff $\mathcal{E}$ is an admissible set which contains all the arguments it defends. $\mathcal{E}$ is a* preferred extension *iff it is maximal (with respect to set inclusion) admissible set. $\mathcal{E}$ is a* stable extension *iff it is conflict-free and for all $a \in \mathcal{A} \setminus \mathcal{E}$, there exists an argument $b \in \mathcal{E}$ such that $(b, a) \in Att$.*

$\mathcal{E}$ is a grounded extension *iff $\mathcal{E}$ is a minimal (for set inclusion) complete extension.*

If a semantics returns exactly one extension for every argumentation framework, then it is called a single-extension semantics.

For an argumentation framework $AS = (\mathcal{A}, Att)$ we denote by $Ext_x(AS)$ (or by $Ext_x(\mathcal{A}, Att)$) the set of its extensions with respect to semantics x. We use the abbreviations c, p, s, and g for respectively complete, preferred, stable and grounded semantics.

An argument is sceptically accepted if it is in all extensions, credulously accepted if it is in at least one extension and rejected if it is not in any extension.

Finally, we introduce two definitions allowing us to reason over such an argumentation framework. The output of an argumentation framework is usually defined [10, Definition 12] as the set of conclusions that appear in all the extensions (under a given semantics).

Definition 7 (Output of an argumentation framework). *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base and $\mathcal{AF}_K$ the corresponding argumentation framework. The output of $\mathcal{AF}_K$ under semantics x is defined as:*

$$\texttt{Output}_x(\mathcal{AF}_K) = \bigcap_{\mathcal{E} \in \texttt{Ext}_x(\mathcal{AF}_K)} \texttt{Concs}(\mathcal{E}).$$

In the degenerate case when $\texttt{Ext}_x(\mathcal{AF}_K) = \emptyset$, we define $\texttt{Output}(\mathcal{AF}_K) = \emptyset$ by convention.

Note that the previous definition asks for existence of a conclusion in every extension. This kind of acceptance is usually referred to as *sceptical* acceptance. We say that a query α is sceptically accepted if it is a logical consequence of the output of $\mathcal{AF}_K$:

Definition 8 (Sceptical acceptance of a query). *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base and $\mathcal{AF}_K$ the corresponding argumentation framework. A query α is sceptically accepted under semantics x if and only if $\texttt{Output}_x(\mathcal{AF}_K) \models \alpha$.*

It is possible to make an alternative definition, which uses the notion of universal acceptance instead of sceptical one. According to universal criteria, a query α is accepted if it is a logical consequence of conclusions of every extension:

Definition 9 (Universal acceptance of a query). *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base and $\mathcal{AF}_K$ the corresponding argumentation framework. A query α is universally accepted under semantics x if and only if for every extension $\mathcal{E}_i \in \texttt{Ext}_x(\mathcal{AF}_K)$, it holds that $\texttt{Concs}(\mathcal{E}_i) \models \alpha$.*

In general, universal and sceptical acceptance of a query do not coincide. Take for instance the KB from Example 3, construct the corresponding argumentation framework, and compare the sets of universally and sceptically accepted queries under preferred semantics.

Note that for single-extension semantics (e.g. grounded), the notions of sceptical and universal acceptance coincide. So we simply use word "accepted" in this context.

Definition 10 (Acceptance of a query). *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base, $\mathcal{AF}_K$ the corresponding argumentation framework, x a single-extension semantics and let $\mathcal{E}$ be the unique extension of $\mathcal{AF}_K$. A query α is accepted under semantics x if and only if $\texttt{Concs}(\mathcal{E}) \models \alpha$.*

4 Equivalence between Repairs and Extensions

In this section, we prove two links between the repairs of an ontological KB and the corresponding argumentation framework: Theorem 1 shows that the repairs of the KB correspond exactly to the stable (and preferred, since in this instantiation the stable and the preferred semantics coincide) extensions of the argumentation framework; Theorem 2 proves that the intersection of all the repairs of the KB corresponds to the grounded extension of the argumentation framework.

Theorem 1. *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base, $\mathcal{AF}_K$ the corresponding argumentation framework and $x \in \{s, p\}$*[1]*. Then:*

$$\texttt{Ext}_x(\mathcal{AF}_K) = \{\texttt{Arg}(A') \mid A' \in \mathcal{R}epair(\mathcal{K})\}$$

Proof. The plan of the proof is as follows:

1. We prove that $\{\texttt{Arg}(A') \mid A' \in \mathcal{R}epair(\mathcal{K})\} \subseteq \texttt{Ext}_s(\mathcal{AF}_K)$.
2. We prove that $\texttt{Ext}_p(\mathcal{AF}_K) \subseteq \{\texttt{Arg}(A') \mid A' \in \mathcal{R}epair(\mathcal{K})\}$.
3. Since every stable extension is a preferred one [15], we can proceed as follows. From the first item, we have that $\{\texttt{Arg}(A') \mid A' \in \mathcal{R}epair(\mathcal{K})\} \subseteq \texttt{Ext}_p(\mathcal{AF}_K)$, thus the theorem holds for preferred semantics. From the second item we have that $\texttt{Ext}_s(\mathcal{AF}_K) \subseteq \{\texttt{Arg}(A') \mid A' \in \mathcal{R}epair(\mathcal{K})\}$, thus the theorem holds for stable semantics.

1. We first show $\{\texttt{Arg}(A') \mid A' \in \mathcal{R}epair(\mathcal{K})\} \subseteq \texttt{Ext}_s(\mathcal{AF}_K)$. Let $A' \in \mathcal{R}epair(\mathcal{K})$ and let $\mathcal{E} = \texttt{Arg}(A')$. Let us prove that $\mathcal{E}$ is a stable extension of $(\texttt{Arg}(\mathcal{F}), \texttt{Att})$.
   We first prove that $\mathcal{E}$ is conflict-free. By means of contradiction we suppose the contrary, i.e. let $a, b \in \mathcal{E}$ such that $(a, b) \in \texttt{Att}$. From the definition of attack, there exists $\varphi \in \texttt{Supp}(b)$ such that $\{\texttt{Conc}(a), \varphi\}$ is $\mathcal{R}$-inconsistent. Thus $\texttt{Supp}(a) \cup \{\varphi\}$ is $\mathcal{R}$-inconsistent; consequently A' is $\mathcal{R}$-inconsistent, contradiction. Therefore $\mathcal{E}$ is conflict-free.
   Let us now prove that $\mathcal{E}$ attacks all arguments outside the set. Let $b \in \texttt{Arg}(\mathcal{F}) \setminus \texttt{Arg}(A')$ and let $\varphi \in \texttt{Supp}(b)$, such that $\varphi \notin A'$. Let A'_c be the set obtained from A' by conjunction elimination and let $a = (A', A'_c, \bigwedge A'_c)$. We have $\varphi \notin A'$, so, due to the set inclusion maximality for the repairs, $\{\bigwedge A'_c, \varphi\}$ is $\mathcal{R}$-inconsistent. Therefore, $(a, b) \in \texttt{Att}$. Consequently, $\mathcal{E}$ is a stable extension.
2. We now need to prove that $\texttt{Ext}_p(\mathcal{AF}_K) \subseteq \{\texttt{Arg}(A') \mid A' \in \mathcal{R}epair(\mathcal{K})\}$. Let $\mathcal{E} \in \texttt{Ext}_p(\mathcal{AF}_K)$ and let us prove that there exists a repair A' such that $\mathcal{E} = \texttt{Arg}(A')$. Let $S = \texttt{Base}(\mathcal{E})$. Let us prove that S is $\mathcal{R}$-consistent. Aiming to a contradiction, suppose that S is $\mathcal{R}$-inconsistent. Let $S' \subseteq S$ be such that (1) S' is $\mathcal{R}$-inconsistent and (2) every proper set of S' is $\mathcal{R}$-consistent. Let us denote $S' = \{\varphi_1, \varphi_2, ..., \varphi_n\}$. Let $a \in \mathcal{E}$ be an argument such that $\varphi_n \in \texttt{Supp}(a)$. Let S'_c be the set obtained from $S' \setminus \{\varphi\}$ by conjunction elimination and let $a' = (S' \setminus \{\varphi_n\}, S'_c, \bigwedge S'_c)$. We have that $(a', a) \in \texttt{Att}$. Since $\mathcal{E}$ is conflict free, then $a' \notin \mathcal{E}$. Since $\mathcal{E}$ is an admissible set, there exists $b \in \mathcal{E}$ such that $(b, a') \in \texttt{Att}$. Since b attacks a' then there exists

[1] Recall that s stands for *stable* and p for *preferred* semantics.

$i \in \{1, 2, ..., n-1\}$ such that $\{\mathtt{Conc}(b), \varphi_i\}$ is $\mathcal{R}$-inconsistent. Since $\varphi_i \in \mathtt{Base}(\mathcal{E})$, then there exists $c \in \mathcal{E}$ such that $\varphi_i \in \mathtt{Supp}(c)$. Thus $(b, c) \in \mathtt{Att}$, contradiction. So it must be that S is $\mathcal{R}$-consistent.
Let us now prove that there exists no $S' \subseteq \mathcal{F}$ such that $S \subsetneq S'$ and S' is $\mathcal{R}$-consistent. We use the proof by contradiction. Thus, suppose that S is not a maximal $\mathcal{R}$-consistent subset of $\mathcal{F}$. Then, there exists $S' \in \mathcal{R}epair(\mathcal{K})$, such that $S \subsetneq S'$. We have that $\mathcal{E} \subseteq \mathtt{Arg}(S)$, since $S = \mathtt{Base}(\mathcal{E})$. Denote $\mathcal{E}' = \mathtt{Arg}(S')$. Since $S \subsetneq S'$ then $\mathtt{Arg}(S) \subsetneq \mathcal{E}'$. Thus, $\mathcal{E} \subsetneq \mathcal{E}'$. From the first part of the proof, $\mathcal{E}' \in \mathtt{Ext}_s(\mathcal{AF}_K)$. Consequently, $\mathcal{E}' \in \mathtt{Ext}_p(\mathcal{AF}_K)$. We also know that $\mathcal{E} \in \mathtt{Ext}_p(\mathcal{AF}_K)$. Contradiction, since no preferred set can be a proper subset of another preferred set. Thus, we conclude that $\mathtt{Base}(\mathcal{E}) \in \mathcal{R}epair(\mathcal{K})$.
Let us show that $\mathcal{E} = \mathtt{Arg}(\mathtt{Base}(\mathcal{E}))$. It must be that $\mathcal{E} \subseteq \mathtt{Arg}(S)$. Also, we know (from the first part) that $\mathtt{Arg}(S)$ is a stable and a preferred extension, thus the case $\mathcal{E} \subsetneq \mathtt{Arg}(s)$ is not possible.

3. Now we know that $\{\mathtt{Arg}(A') \mid A' \in \mathcal{R}epair(\mathcal{K})\} \subseteq \mathtt{Ext}_s(\mathcal{AF}_K)$ and $\mathtt{Ext}_p(\mathcal{AF}_K) \subseteq \{\mathtt{Arg}(A') \mid A' \in \mathcal{R}epair(\mathcal{K})\}$. The theorem follows from those two facts, as explained at the beginning of the proof.

To prove Theorem 2, we first prove the following lemma which says that if there are no rejected arguments under preferred semantics, then the grounded extension is equal to the intersection of all preferred extensions. Note that this result holds for every argumentation framework (not only for the one studied in this paper, where arguments are constructed from an ontological knowledge base). Thus, we only suppose that we are given a set and a binary relation on it (called attack relation).

Lemma 1. *Let $AS = (\mathcal{A}, Att)$ be an argumentation framework and* GE *its grounded extension.*

$$\textit{If } \mathcal{A} \subseteq \bigcup_{\mathcal{E}_i \in \mathtt{Ext}_p(AS)} \mathcal{E}_i \quad \textit{then} \quad \mathtt{GE} = \bigcap_{\mathcal{E}_i \in \mathtt{Ext}_p(AS)} \mathcal{E}_i.$$

Proof. Let $\mathtt{Iope} = \cap_{\mathcal{E}_i \in \mathtt{Ext}_p(AS)} \mathcal{E}_i$ denote the intersection of all preferred extensions. It is known [15] the $\mathtt{GE} \subseteq \mathtt{Iope}$. Let us prove that in the case when there are no rejected arguments, it also holds the $\mathtt{Iope} \subseteq \mathtt{GE}$. Let $a \in \mathtt{Iope}$. Let us show that no argument b attacks a. This holds since every argument b is in at least one preferred extension, say $\mathcal{E}_i$, and a is also in $\mathcal{E}_i$ (since a is in all preferred extensions) thus b does not attack a since both a and b are in $\mathcal{E}_i$ and $\mathcal{E}_i$ is a conflict-free set (since it is a preferred extension). All this means that arguments in $\mathtt{Iope}$ are not attacked. Consequently, they must all belong to the grounded extension. In other words, $\mathtt{Iope} \subseteq \mathtt{GE}$.

We can now, using the previous result, prove the link between the intersection of repairs and the grounded extension.

Theorem 2. *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base and $\mathcal{AF}_K$ the corresponding argumentation framework. Denote the grounded extension of $\mathcal{AF}_K$ by* GE*. Then:*

$$\mathtt{GE} = \mathtt{Arg}\Big(\bigcap_{A' \in \mathcal{R}epair(\mathcal{K})} A'\Big).$$

Proof. Denote the intersection of all repairs by $\texttt{Ioar} = \bigcap_{A' \in \mathcal{R}epair(\mathcal{K})} A'$ and the intersection of all preferred extensions by $\texttt{Iope} = \bigcap_{\mathcal{E}_i \in \texttt{Ext}_p(\mathcal{AF}_K)} \mathcal{E}_i$. From Theorem 1, we know that $\texttt{Ext}_x(\mathcal{AF}_K) = \{\texttt{Arg}(A') \mid A' \in \mathcal{R}epair(\mathcal{K})\}$. Consequently,

$$\texttt{Iope} = \bigcap_{A' \in \mathcal{R}epair(\mathcal{K})} \texttt{Arg}(A') \tag{1}$$

Since every argument has an $\mathcal{R}$-consistent support, then its support is in at least one repair. From Theorem 1, that argument is in at least one preferred extension, (i.e. it is not rejected). From Lemma 1,

$$\texttt{Iope} = \texttt{GE} \tag{2}$$

From (1) and (2), we obtain that

$$\texttt{GE} = \bigcap_{A' \in \mathcal{R}epair(\mathcal{K})} \texttt{Arg}(A') \tag{3}$$

Note that for every collection $S_1, \ldots, S_n$ of of sets of formulae, we have $\texttt{Arg}(S_1) \cap \ldots \cap \texttt{Arg}(S_n) = Arg(S_1 \cap \ldots \cap S_n)$. By applying this rule on the set of all repairs, we obtain:

$$\bigcap_{\mathcal{A}' \in \mathcal{R}epair(\mathcal{K})} \texttt{Arg}(A') = \texttt{Arg}(\texttt{Ioar}) \tag{4}$$

From (3) and (4), we obtain $\texttt{GE} = \texttt{Arg}(\texttt{Ioar})$ which ends the proof.

5 Semantics Equivalence

This section presents the main result of the paper. It proves the links between semantics from argumentation theory (stable, preferred, grounded) and semantics from inconsistent ontology KB query answering (ICR, AR, IAR). More precisely, we show that: (1) sceptical acceptance under stable and preferred semantics corresponds to ICR semantics; (2) universal acceptance under stable and preferred semantics corresponds to AR semantics; (3) acceptance under grounded semantics corresponds to IAR semantics. The proof of Theorem 3 is based on Theorem 1 and the proof of Theorem 4 is derived from Theorem 2.

Theorem 3. *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base, let $\mathcal{AF}_K$ be the corresponding argumentation framework and let α be a query. Let $x \in \{s, p\}$ be stable or preferred semantics. Then:*

- $\mathcal{K} \models_{ICR} \alpha$ *iff* α *is sceptically accepted under semantics* x.
- $\mathcal{K} \models_{AR} \alpha$ *iff* α *is universally accepted under semantics* x.

Proof. Theorem 1 implies $Ext_x(Arg(\mathcal{F}), Att) = \{Arg(A') \mid A' \in \mathcal{R}epair(\mathcal{K})\}$. In fact, the restriction of function $\texttt{Arg}$ on $\mathcal{R}epair(\mathcal{K})$ is a bijection between $\mathcal{R}epair(\mathcal{K})$ and $\texttt{Ext}_x(\mathcal{AF}_K)$. Note also that for every query α, for every repair A', we have that $\texttt{Cl}_{\mathcal{R}}(A') \models \alpha$ if and only if $\texttt{Concs}(\texttt{Arg}(A')) \models \alpha$. By using those two facts, the result of the theorem can be obtained as follows:

– For every query α, we have: $\mathcal{K} \models_{ICR} \alpha$ if and only if $\bigcap_{A' \in \mathcal{R}epair(\mathcal{K})} \mathtt{Cl}_{\mathcal{R}}(A') \models \alpha$ if and only if $\bigcap_{\mathcal{E}_i \in \mathtt{Ext}_x(\mathcal{AF}_K)} \mathtt{Concs}(\mathcal{E}_i) \models \alpha$ if and only if $\mathtt{Output}_x(\mathcal{AF}_K) \models \alpha$ if and only if α is sceptically accepted.
– For every query α, we have: $\mathcal{K} \models_{AR} \alpha$ if and only if for every $A' \in \mathcal{R}epair(\mathcal{K})$, $\mathtt{Cl}_{\mathcal{R}}(A') \models \alpha$ if and only if for every $\mathcal{E}_i \in \mathtt{Ext}_x(\mathcal{AF}_K)$, $\mathtt{Concs}(\mathcal{E}_i) \models \alpha$ if and only if α is universally accepted.

Theorem 4. *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base, let $\mathcal{AF}_K$ be the corresponding argumentation framework and let α be a query. Then:*

$$\mathcal{K} \models_{IAR} \alpha \textit{ iff } \alpha \textit{ is accepted under grounded semantics.}$$

Proof. Let us denote the grounded extension of $\mathcal{AF}_K$ by $\mathtt{GE}$ and the intersection of all repairs by $\mathtt{Ioar} = \bigcap_{A' \in \mathcal{R}epair(\mathcal{K})} A'$. From Definition 10, we have:

$$\alpha \text{ is accepted under grounded semantics iff } \mathtt{Concs}(\mathtt{GE}) \models \alpha. \tag{5}$$

From Theorem 2, we have:

$$\mathtt{GE} = \mathtt{Arg}(\mathtt{Ioar}). \tag{6}$$

Note also that for every set of facts $\{F_1, \ldots, F_n\}$ and for every query α, we have that $\mathtt{Cl}_{\mathcal{R}}(\{F_1, \ldots, F_n\}) \models \alpha$ if and only if $\mathtt{Concs}(\mathtt{Arg}(\{F_1, \ldots, F_n\})) \models \alpha$. Thus,

$$\mathtt{Cl}_{\mathcal{R}}(\mathtt{Ioar}) \models \alpha \text{ if and only if } \mathtt{Concs}(\mathtt{Arg}(\mathtt{Ioar})) \models \alpha. \tag{7}$$

From (6) and (7) we have that:

$$\mathtt{Cl}_{\mathcal{R}}(\mathtt{Ioar}) \models \alpha \text{ if and only if } \mathtt{Concs}(\mathtt{GE}) \models \alpha. \tag{8}$$

From Definition 3, one obtains:

$$\mathtt{Cl}_{\mathcal{R}}(\mathtt{Ioar}) \models \alpha \text{ if and only if } \mathcal{K} \models_{IAR} \alpha. \tag{9}$$

The theorem now follows from (5), (8) and (9).

6 Postulates

In this section, we prove that the framework we propose in this paper satisfies the rationality postulates for instantiated argumentation frameworks [10]. We first prove the indirect consistency postulate.

Proposition 1 (Indirect consistency). *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base, $\mathcal{AF}_K$ the corresponding argumentation framework and $x \in \{s, p, g\}$. Then:*

– *for every $\mathcal{E}_i \in \mathtt{Ext}_x(\mathcal{AF}_K)$, $\mathtt{Cl}_{\mathcal{R}}(\mathtt{Concs}(\mathcal{E}_i))$ is a consistent set*
– *$\mathtt{Cl}_{\mathcal{R}}(\mathtt{Output}_x(\mathcal{AF}_K))$ is a consistent set.*

Proof.

– Let $\mathcal{E}_i$ be a stable or a preferred extension of $\mathcal{AF}_K$. From Theorem 1, there exists a repair $A' \in \mathcal{R}epair(\mathcal{K})$ such that $\mathcal{E}_i = \texttt{Arg}(A')$. Note that $\texttt{Concs}(\mathcal{E}_i) = \texttt{Cl}_{\mathcal{R}}(A') \cup \{\alpha \mid \texttt{Cl}_{\mathcal{R}}(A) \models \alpha\}$ (this follows directly from Definition 4). Consequently, the set of $\mathcal{R}$-derivations of $\texttt{Concs}(\mathcal{E}_i)$ and the set of $\mathcal{R}$-derivations of $\texttt{Cl}_{\mathcal{R}}(A')$ coincide. Formally, $\texttt{Cl}_{\mathcal{R}}(\texttt{Cl}_{\mathcal{R}}(A')) = \texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\mathcal{E}_i))$. Since $\texttt{Cl}_{\mathcal{R}}$ is idempotent, this means that $\texttt{Cl}_{\mathcal{R}}(A') = \texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\mathcal{E}_i))$. Since $\texttt{Cl}_{\mathcal{R}}(A')$ is consistent, then $\texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\mathcal{E}_i))$ is consistent.
  Let us now consider the case of grounded semantics. Denote GE the grounded extension of $\mathcal{AF}_K$. We have just seen that for every $\mathcal{E}_i \in \texttt{Ext}_p(\mathcal{AF}_K)$, it holds that $\texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\mathcal{E}_i))$ is a consistent set. Since the grounded extension is a subset of the intersection of all the preferred extensions [15], and since there is at least one preferred extension, say $\mathcal{E}_1$, then $\texttt{GE} \subseteq \mathcal{E}_1$. Since $\texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\mathcal{E}_i))$ is consistent then $\texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\texttt{GE}))$ is also consistent.
– Consider the case of stable or preferred semantics. Let us prove $\texttt{Cl}_{\mathcal{R}}(\texttt{Output}_x(\mathcal{AF}_K))$ is a consistent set. Recall that $\texttt{Output}_x(\mathcal{AF}_K) = \bigcap_{\mathcal{E}_i \in \texttt{Ext}_x(\mathcal{AF}_K)} \texttt{Concs}(\mathcal{E}_i)$. Since every knowledge base has at least one repair then, according to Theorem 1, there is at least one stable or preferred extension $\mathcal{E}_i$. From Definition 7, we have that $\texttt{Output}_x(\mathcal{AF}_K) \subseteq \texttt{Concs}(\mathcal{E}_i)$. $\texttt{Concs}(\mathcal{E}_i)$ is $\mathcal{R}$-consistent thus $\texttt{Output}_x(\mathcal{AF}_K)$ is $\mathcal{R}$-consistent. In other words, $\texttt{Cl}_{\mathcal{R}}(\texttt{Output}_x(\mathcal{AF}_K))$ is consistent.
  Note that in the case of grounded semantics the second part of the proposition follows directly from the first one, since $\texttt{Cl}_{\mathcal{R}}(\texttt{Output}_g(\mathcal{AF}_K)) = \texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\texttt{GE}))$.

Since our instantiation satisfies indirect consistency then it also satisfies direct consistency. This comes from $\mathcal{R}$-consistency definition; namely, if a set is $\mathcal{R}$-consistent, then it is necessarily consistent. Thus, we obtain the following corollary.

Corollary 1 (Direct consistency). *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base, $\mathcal{AF}_K$ the corresponding argumentation framework and $x \in \{s, p, g\}$. Then:*

– *for every $\mathcal{E}_i \in \texttt{Ext}_x(\mathcal{AF}_K)$, $\texttt{Concs}(\mathcal{E}_i)$ is a consistent set*
– $\texttt{Output}_x(\mathcal{AF}_K)$ *is a consistent set.*

We now also prove that the present argumentation formalism also satisfies the closure postulate.

Proposition 2 (Closure). *Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a knowledge base, $\mathcal{AF}_K$ the corresponding argumentation framework and $x \in \{s, p, g\}$. Then:*

– *for every $\mathcal{E}_i \in \texttt{Ext}_x(\mathcal{AF}_K)$, $\texttt{Concs}(\mathcal{E}_i) = \texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\mathcal{E}_i))$.*
– $\texttt{Output}_x(\mathcal{AF}_K) = \texttt{Cl}_{\mathcal{R}}(\texttt{Output}_x(\mathcal{AF}_K))$.

Proof.
– From the definition of $\texttt{Cl}_{\mathcal{R}}$, we see that $\texttt{Concs}(\mathcal{E}_i) \subseteq \texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\mathcal{E}_i))$. Let us prove that $\texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\mathcal{E}_i)) \subseteq \texttt{Concs}(\mathcal{E}_i)$. Suppose that $\alpha \in \texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\mathcal{E}_i))$. This means that there exists $\alpha_1, \ldots, \alpha_k \in \texttt{Concs}(\mathcal{E}_i)$ and that there exists a derivation sequence $F_0, \ldots, F_n$ such that $F_0 = \{\alpha_1, \ldots, \alpha_k\}$ and $\alpha \in F_n$. Note that from Proposition 1, we know that $\{\alpha_1, \ldots, \alpha_k\}$ is $\mathcal{R}$-consistent. Since $\alpha_1, \ldots, \alpha_k \in \texttt{Concs}(\mathcal{E}_i)$ then there exist $a_1, \ldots, a_k \in \mathcal{E}_i$ such that $\texttt{Conc}(a_1) = \alpha_1, \ldots, \texttt{Conc}(a_k) = \alpha_k$. Thus,

there exists an argument a such that $\texttt{Supp}(a) = \texttt{Supp}(a_1) \cup \ldots \cup \texttt{Supp}(a_k)$ and $\texttt{Conc}(a) = \alpha$. Since $\mathcal{E}_i$ is a preferred, a stable or the grounded extension, Theorems 1 and 2 imply that there exists a set of formulae S such that $\mathcal{E}_i = \texttt{Arg}(S)$. Consequently, $\mathcal{E}_i = \texttt{Arg}(\texttt{Base}(\mathcal{E}_i))$. From this observation and since $\texttt{Supp}(a) \subseteq \texttt{Base}(\mathcal{E}_i)$, we conclude that $a \in \mathcal{E}_i$. Thus, $\alpha \in \texttt{Concs}(\mathcal{E}_i)$, which ends the proof.

– In the case of grounded semantics, the result holds directly from the first part of the proposition. The reminder of the proof considers stable or preferred semantics. From the definition of $\texttt{Cl}_{\mathcal{R}}$, $\texttt{Output}_x(\mathcal{AF}_K) \subseteq \texttt{Cl}_{\mathcal{R}}(\texttt{Output}_x(\mathcal{AF}_K))$. So we only need to prove that $\texttt{Cl}_{\mathcal{R}}(\texttt{Output}_x(\mathcal{AF}_K)) \subseteq \texttt{Output}_x(\mathcal{AF}_K)$.
Let $\alpha \in \texttt{Cl}_{\mathcal{R}}(\texttt{Output}_x(\mathcal{AF}_K))$. Then there exist $\alpha_1, \ldots, \alpha_k \in \texttt{Output}_x(\mathcal{AF}_K)$ such that there is a derivation sequence $F_0, \ldots, F_n$ such that $F_0 = \{\alpha_1, \ldots, \alpha_k\}$ and $\alpha \in F_n$. Since $\alpha_1, \ldots, \alpha_k \in \texttt{Output}_x(\mathcal{AF}_K)$ then for every $\mathcal{E}_i \in \texttt{Ext}_x(\mathcal{AF}_K)$, we have $\alpha_1, \ldots, \alpha_k \in \mathcal{E}_i$. Therefore for every $\mathcal{E}_i \in \texttt{Ext}_x(\mathcal{AF}_K)$, $\alpha \in \texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\mathcal{E}_i))$. From the first part of the proof, $\texttt{Cl}_{\mathcal{R}}(\texttt{Concs}(\mathcal{E}_i)) = \texttt{Concs}(\mathcal{E}_i)$. Thus, for every $\mathcal{E}_i \in \texttt{Ext}_x(\mathcal{AF}_K)$, $\alpha \in \texttt{Concs}(\mathcal{E}_i)$. This means that $\alpha \in \texttt{Output}_x(\mathcal{AF}_K)$.

7 Summary and Conclusion

This paper investigates the links between the semantics used in argumentation theory and those from the inconsistent ontological KB query answering.

Contribution of the paper. First, we show that it is possible to instantiate Dung's abstract argumentation theory in a way to deal with inconsistency in an ontological KB. Second, we formally prove the links between the semantics from ontological KB query answering and those from argumentation theory: ICR semantics corresponds to sceptical acceptance under stable or preferred argumentation semantics, AR semantics corresponds to universal acceptance under stable / preferred argumentation semantics and IAR semantics corresponds to acceptance under grounded argumentation semantics. Third, we show that the instantiation we define satisfies the rationality postulates. The fourth contribution of the paper is to make a bridge between the argumentation community and the knowledge representation community in this context, allowing for future exchanges.

Applications of our work. The first possible application of our work is to import some results about semantics and acceptance from argumentation to ontological KB query answering and vice versa. Second, arguments can be used for explanatory purposes. In other words, we can use arguments and counter arguments to graphically represent and explain why different points of view are conflicting or not and why certain argument is (not) in all extensions. However, we suppose that the user understands the notion of logical consequence under first order logic when it comes to *consistent* data. For example, we suppose that the user is able to understand that if $cat(Tom) \wedge miaw(Tom)$ is present in the set, then queries $cat(Tom)$ and $\exists x cat(x)$ are both true. To sum up, we suppose that the other methods are used to explain reasoning under *consistent* knowledge and we use argumentation to explain reasoning under *inconsistent* knowledge.

Related work. Note that this is the first work studying the link between semantics used in argumentation (stable, preferred, grounded) and semantics used in inconsistent ontological knowledge base query answering (AR, IAR, ICR). There is not much related work. However, we review some papers that study similar issues.

For instance, the link between maximal consistent subsets of a knowledge base and stable extensions of the corresponding argumentation system was shown by Cayrol [11]. That was the first work showing this type of connection between argument-based and non argument-based reasoning. This result was generalised [20] by studying the whole class of argumentation systems corresponding to maximal consistent subsets of the propositional knowledge base. The link between the ASPIC system [18] and the Argument Interchange Format (AIF) ontology [13] has recently been studied [7]. Another related paper comprises constructing an argumentation framework with ontological knowledge allowing two agents to discuss the answer to queries concerning their knowledge (even if it is inconsistent) without one agent having to copy all of their ontology to the other [9]. While those papers are in the area of our paper, none of them is related to the study of the links between different semantics for inconsistent ontological KB query answering and different argumentation semantics.s

Future work. We plan to answer different questions, like: Can other semantics from argumentation theory yield different results? Are those results useful for inconsistent ontological KB query answering? What happens in the case when preferences are present? What is the link between having preferences on databases and having preferences on arguments? More generally speaking, we want to examine how the knowledge representation community could benefit from other results from argumentation theory and whether the argumentation community could use some open problems in the knowledge representation as inspiration for future work.

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