# Path Following Control for an Eel-Like Robot 

Lionel Lapierre, Bruno Jouvencel

## To cite this version:

Lionel Lapierre, Bruno Jouvencel. Path Following Control for an Eel-Like Robot. OCEANS, Jul 2005, Brest, France. pp.460-465, 10.1109/OCEANSE.2005.1511759 . lirmm-00106474

HAL Id: lirmm-00106474 https://hal-lirmm.ccsd.cnrs.fr/lirmm-00106474

Submitted on 16 Oct 2006

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Path Following Control for an Eel-like Robot 

Lionel Lapierre<br>LIRMM<br>Montpellier, France<br>Email: lapierre@lirmm.fr

Bruno Jouvencel<br>LIRMM<br>Montpellier, France<br>Email: jouvencel@lirmm.fr


#### Abstract

We investigate the problem of controlling the motion of an eel like-robot. Recent work has shown promising results for this type of systems, opening new issues in the field of efficient propulsion and high manoeuvrability systems. The motion planning is decoupled in the two subproblems of thrust generation and heading control. In this paper we investigate a new type of autonomous gait generation, explicitly controlling the local system curvatures. The solution is then coupled with a path following controller, using the virtual target principle. The controller design is based on Lyapunov techniques. Simulations illustrate the performances of the proposed solution.


## I. Introduction

Since a long time, biologically inspired solutions have motivated a large number of robotic applications. In the domain of underwater robotics, the question of efficient and low consumption underwater propulsion, in conjunction with a high manoeuvrability is an active field of research. A whale tail is an 'unreachable' reference in terms of thrust efficiency; the lamprey is able of incredible maneuvers. The marine biologists have studied this type of locomotion, from the lamprey to the tuna fish, and have identified the body shape evolution to produce a thrust along the body axis. From a robotics point of view, this body shape evolution is a propagating wave (also called 'gait'), adjusting the signal phase according to the joint situation located on each vertebrae. From these results, a large number of interesting robotic applications have been performed in the last decades. The first question to be solved, concerning the model derivation of such a flexible system, is generally solved considering a highly coupled, non linear, hyper redundant model. The motion control problem is decoupled into three sub-problems:

- the gait generation,
- the control of the joint actuation in order to follow the previous reference,
- the adaptation of the gait parameters according to the system situation with respect to the main path the system has to reach and follow.
The gait generation is a crucial question in this problem. Indeed, a bad chosen gait will agitate the robot, without guarantying that the maximum number of the system elements (vertebrae) are involved in the movement, loosing by the way the desired efficiency, in terms of thrust and manoeuvrability.

The solutions of the literature propose some actuation gaits, directly controlling the joints as a trajectory tracker, or a kinematic joint control to follow the desired body shape ([1],


Fig. 1. Problem Pose
[3], [2]). The loop is closed on the gait parameters, adapting them in function of the system situation with respect to the main goal (path). An interresting optimal approach is described in [4]. Neural approach has been also tested, and promising results can be found in [5] and [6].
In this paper, we propose a method to drive the system according to a reference body curvature, which is not time dependant and that explicitly takes into account the system dynamics. The control is derived using Lyapunov theory and Backstepping techniques, in order to guarantee the system to asymptotically converge to the desired shape defined in function of the curvilinear abscissa of each joints on the reference to be followed. The reference parameters are then adjusted in order for the system to reach a desired path, at a desired forward velocity. The path following algorithm is using the 'virtual target' guidance principle.
The paper is organized as follow : chapter II describes the system modeling, chapter III introduces the control design method we are proposing and chapter IV indicates some simulation results to illustrate the performances of our solution.

## II. System Modeling

We model the eel-like robot as a planar, serial chain of $N$ links of length $d_{i}$, mass $m_{i}$ and inertia $I_{i}$, where $i=1 \ldots N$. Referring to the figure 1 , we define two groups of kinematic variables called configuration variables $\dot{q}$ and shape variable $\dot{s}$, denoted as :

$$
\begin{gather*}
\dot{q}=\left[u_{1} v_{1} r_{1} \dot{\phi}_{2} \dot{\phi}_{3} \ldots \dot{\phi}_{N-1} \dot{\phi}_{N}\right]^{T}  \tag{1}\\
\dot{s}=\left[u_{2} v_{2} u_{3} v_{3} \ldots u_{N} v_{N}\right]^{T}
\end{gather*}
$$

where $u_{i}$ is the forward velocity of the link $i, v_{i}$ is the side-slip velocity, and $r_{i}$ the rotational velocity, expressed in the body axis $\left\{B_{i}\right\} . \phi_{i}$ is the relative angle between the link $i-1$ and $i$. Note that the variables $\phi_{i}$ can be called
articular variables, and that the group $q$ defines a necessary and sufficient set to describe the system situation.

## A. Hydrodynamics Approximation

We model each link of the eel-like robot as a cylinder of length $d_{i}$, radius $L_{i}$ and mass $m_{i}$. The significant hydrodynamical forces applied on the systems are 3 types:

- perpendicular drag effect : we assume that differential pressure only acts perpendicularly to the body. Note that this is the force that produces the system thrust. We consider the approximation used in [2].

$$
F_{i}^{v}=-\mu_{i} v_{i}
$$

where $F_{i}$ is the perpendicular drag force acting in the link $i$, moving with a side-slip velocity $v_{i} ; \mu_{i}$ is the perpendicular drag coefficient of the considered system element.

- Parallel drag effect : due to the streamlined nature of the body of the eel, the friction coefficient in the body direction is negligible with respect to the perpendicular direction ([2], [6]). Nevertheless, this effect is significant on the first link (also called head) for forward movement, and on the last segment for backward movement (tail).

$$
\begin{aligned}
& F_{1}^{u}=\left\{\begin{array}{ccc}
-\nu_{1} u_{1} & \text { if } & u_{1}>0 \\
0 & \text { if } & u_{1} \leq 0
\end{array}\right. \\
& F_{N}^{u}=\left\{\begin{array}{ccc}
-\nu_{N} u_{N} & \text { if } & u_{1}<0 \\
0 & \text { if } & u_{N} \geq 0
\end{array}\right. \\
& F_{i}^{u}=\text { for } i=2 \text { to } N-1
\end{aligned}
$$

where $F_{1}^{u}$ and $F_{N}^{u}$ are the parallel drag effect acting on the head and the tail, respectively, $\nu_{i}, i=1, N$ are the parallel drag coefficient, and $u_{i}$ denotes the forward speed of the $i^{\text {th }}$ element.

- Added mass : the added mass term is estimated considering the cylindrical shape of the system components. We refer to [7] for an approximation of this effect parameters. The inertial matrix for the $i^{t h}$ element, expressed in the body frame $\left\{B_{i}\right\}$ is written as:

$$
M_{i}=\left[\begin{array}{ccc}
m_{i}^{x} & 0 & 0 \\
0 & m_{i}^{y} & 0 \\
0 & 0 & I^{i}
\end{array}\right]
$$

where $m_{i}^{x}$ and $m_{i}^{y}$ is the wet mass (intertial and added mass)in the $x$ and $y$ direction of the body axis $\left\{B_{i}\right\}$, and $I_{i}$ is the wet moment of inertial the $i^{t h}$ element.

## B. Model Derivation

Thanks to the variable definition in (1) the system Lagrangian is written.

$$
L=\frac{1}{2} \sum_{i=1}^{N} \dot{\eta}_{i}^{T} M_{i} \dot{\eta}
$$

where $\dot{\eta}_{i}=\left[u_{i} v_{i} r_{i}\right]$ denotes the $i^{t h}$ element velocity expressed in the body frame $\left\{B_{i}\right\}$. The previous Lagrangian expression is rewritten as :

$$
L=\frac{1}{2} \dot{q}^{T} M^{1} \dot{q}+\frac{1}{2} \dot{s}^{T} M^{2} \dot{s}
$$

where

$$
M^{1}=\left[\begin{array}{cccccc}
m_{1}^{x} & 0 & 0 & 0 & \ldots & 0 \\
0 & m_{1}^{y} & 0 & 0 & \ldots & 0 \\
0 & 0 & a_{1} & a_{2} & \ldots & I_{N} \\
0 & 0 & a_{2} & a_{2} & \ldots & I_{N} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & I_{N} & I_{N} & \ldots & I_{N}
\end{array}\right]
$$

with $a_{i}=\sum_{k=i}^{N} I_{k}$ and

$$
M^{2}=\left[\begin{array}{ccccccc}
m_{2}^{x} & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & m_{2}^{y} & 0 & 0 & \ldots & 0 & 0 \\
0 & 0 & m_{3}^{x} & 0 & \ldots & 0 & 0 \\
0 & 0 & 0 & m_{3}^{y} & \ldots & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \\
0 & 0 & 0 & 0 & \ldots & m_{N}^{x} & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 & m_{N}^{y}
\end{array}\right]
$$

The consideration of the following kinematic relations in the body frames :

$$
\begin{aligned}
u_{i}= & u_{1} \cos \left(\theta_{i}-\theta_{1}\right)+v_{1} \sin \left(\theta_{i}-\theta_{1}\right) \\
& +\dot{\theta}_{1} \frac{d_{1}}{2} \sin \left(\theta_{i}-\theta_{1}\right) \\
& +\sum_{k=2}^{i-1} \dot{\theta}_{k} d_{k} \sin \left(\theta_{i}-\theta_{k}\right) \\
v_{i}= & -u_{1} \sin \left(\theta_{i}-\theta_{1}\right)+v_{1} \cos \left(\theta_{i}-\theta_{1}\right) \\
& +\dot{\theta}_{1} \frac{d_{1}}{2} \cos \left(\theta_{i}-\theta_{1}\right) \\
& +\sum_{k=2}^{i-1} \dot{\theta}_{k} d_{k} \cos \left(\theta_{i}-\theta_{k}\right)+\dot{\theta}_{i} \frac{d_{i}}{2}
\end{aligned}
$$

with $i=2 \ldots N$, leads the extraction of the matrix $\beta$, expressing the relation between the configuration and shape variables.

$$
\dot{s}=\beta \dot{q}
$$

The matrix $\beta$ can be easily computed with the following intermediate relations :

$$
\begin{gathered}
\dot{s}=\beta_{\theta}\left[u_{1} v_{1} \dot{\theta}_{1} \dot{\theta}_{2} \ldots \dot{\theta}_{3}\right]^{T} \\
{\left[u_{1} v_{1} \dot{\theta}_{1} \dot{\theta}_{2} \ldots \dot{\theta}_{3}\right]^{T}=T_{\theta}^{\phi} \dot{q}} \\
\beta=\beta_{\theta} T_{\theta}^{\phi}
\end{gathered}
$$

where

$$
\beta_{\theta}=\left[\begin{array}{ccccccc}
c_{1}^{2} & s_{1}^{2} & \frac{d_{1}}{2} s_{1}^{2} & 0 & 0 & \ldots & 0 \\
-s_{1}^{2} & c_{1}^{2} & \frac{d_{1}}{2} c_{1}^{2} & \frac{d_{2}}{2} & 0 & \ldots & 0 \\
c_{1}^{3} & s_{1}^{3} & \frac{d_{1}}{2} s_{1}^{3} & d_{2} s_{2}^{3} & 0 & \ldots & 0 \\
-s_{1}^{3} & c_{1}^{3} & \frac{d_{1}}{2} c_{1}^{3} & d_{2} c_{2}^{3} & \frac{d_{3}}{2} & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
c_{1}^{N} & s_{1}^{N} & \frac{d_{1}}{2} s_{1}^{N} & d_{2} s_{2}^{N} & d_{3} s_{3}^{N} & \ldots & 0 \\
-s_{1}^{N} & c_{1}^{N} & \frac{d_{1}}{2} c_{1}^{N} & d_{2} s_{2}^{N} & d_{3} s_{3}^{N} & \ldots & \frac{d_{N}}{2}
\end{array}\right]
$$

with $c_{k}^{i}=\cos \left(\theta_{i}-\theta_{k}\right), s_{k}^{i}=\sin \left(\theta_{i}-\theta_{k}\right)$ and

$$
T_{\theta}^{\phi}=\left[\begin{array}{ccccccc}
1 & 0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 1 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 1 & 0 & 0 & \ldots & 0 \\
0 & 0 & 1 & 1 & 0 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 1 & 1 & 1 & \ldots & 0 \\
0 & 0 & 1 & 1 & 1 & \ldots & 1
\end{array}\right]
$$

The Lagrangian is then rewritten.

$$
L=\frac{1}{2} \dot{q}^{T}\left(M^{1}+\beta^{T} M^{2} \beta\right) \dot{q}
$$

The dynamic model is derived using

$$
F^{e x t}=\frac{d}{d t} \frac{\partial L}{\partial \dot{q}}-\frac{\partial L}{\partial q}
$$

to obtain

$$
F^{e x t}=\left(M^{1}+\beta^{T} M^{2} \beta\right) \ddot{q}+\left(\beta^{T} M^{2} \dot{\beta}\right) \dot{q}
$$

And the expression for $F^{e x t}=F^{1}+\beta^{T} F^{2}$ yields

$$
\begin{equation*}
\left(M^{1}+\beta^{T} M^{2} \beta\right) \ddot{q}=F^{1}+\beta^{T} F^{2}-\left(\beta^{T} M^{2} \dot{\beta}\right) \dot{q} \tag{2}
\end{equation*}
$$

where $F^{1}=\left(F_{1}^{u}, F_{1}^{v}, 0, \tau_{2}, \ldots, \tau_{N}\right)$ are the forces that act in the dimensions defined by the configuration variable $q$, and $F^{2}=\left(F_{2}^{u}, F_{2}^{v}, \ldots, F_{N}^{u}, F_{N}^{v}\right)$ contains forces that act in the dimensions defined by the dependent variable $s . \Gamma=$ $\left(\tau_{2}, \ldots, \tau_{N}\right)$ is the vector of control torques.

## III. Control Design

The control problem is decoupled in 3 sub problems: the thrust generation, the heading control and the combination with a path following requirement.

## A. Thrust Control

The thrust is obtained with a periodic signal propagating along the system body. This reference signal is called 'gait'. A classic approach consists in imposing to the join actuators a sinusoidal based reference of the form : $\phi_{i}=A_{i} \sin \left(w_{i} t+\psi_{i}\right)$ as in [8]. The problem is to adjust the gait parameters $A_{i}, w_{i}$ and $\psi_{i}$ to obtain an efficient thrust generation. Note the work of W. Saintval [4] on an optimization method to evaluate these gait parameters.

We are proposing to use a different approach in the gait generation. The gait we are proposing is not explicitly timedependent and controls the local curvature of the system. This has two advantages.

- The fact that our gait does not explicitly consider the time variable leads to an autonomous system (if the desired velocity is not time dependent), and allows for the use of mathematical tool reserved for autonomous systems.
- Controlling the local curvature is a further step to mimic the behavior of fishes which use a baro-receptors line, deployed along their body, that allows them to locally control the quality of the flow.
Let $s_{i}$ the curvilinear coordinate of the articulation $i$, along the system body; $s_{1}$ denotes the curvilinear position of the head and $s_{N+1}$ is the position of the extremity of the tail. The initial values for $s_{i}$ are $:\left.s_{1}\right|_{t=0}=0,\left.s_{2}\right|_{t=0}=d_{1}$, $\left.s_{3}\right|_{t=0}=d_{1}+d_{2}, \ldots,\left.s_{N+1}\right|_{t=0}=\sum_{k=1}^{N} d_{k}$. The parameterized desired curvature along the system body is expressed as $C^{R}(s)=A \sin (w s)$. The discretization on each system articulation yields :

$$
C_{i}^{R}=A \sin \left(w s_{i}\right)
$$

Let $C_{i}^{E}$ the system curvature of the articulation $i$.

$$
C_{i}^{E}=\frac{\theta_{i}-\theta_{i-1}}{d_{i}+d_{i+1}}=\frac{\phi_{i}}{d_{i}+d_{i+1}}
$$

The control objective is to drive the system curvature $C_{i}^{E}$ to the reference $C_{i}^{R}$, for $i=2, \ldots N$.

## 1) Kinematic Reference Definition:

Let $V_{1}=\frac{1}{2} \sum_{i=2}^{N}\left(C_{i}^{R}-c_{i}^{E}\right)^{2}$ be a Lyapunov candidate. Straighforward computation shows that the choice

$$
\dot{C}_{i}^{E}=\dot{C}_{i}^{R}+K_{1}\left(C_{i}^{R}-C_{i}^{E}\right), i=2 \ldots N
$$

yields $\dot{V}_{1}=-K_{1} \sum_{i=2}^{N}\left(C_{i}^{R}-C_{i}^{E}\right)^{2} \leq 0$. Let $C_{i}^{\text {Kin }}$ be the kinematic reference for the dynamic control

$$
\dot{C}_{i}^{K i n}=\dot{C}_{i}^{R}+K_{1}\left(C_{i}^{R}-C_{i}^{E}\right), i=2 \ldots N
$$

## 2) Dynamic Control Design:

Let $\left.V_{2}=\frac{1}{2} \sum_{i=2}^{N}\left(\dot{C}_{i}^{K i n}-\dot{C}_{i}^{E}\right)^{2}\right]$ be a Lyapunov candidate. The derivation of $V_{2}$ shows that the choice

$$
\ddot{C}_{i}^{E}=\ddot{C}_{i}^{K i n}+K_{2}\left(\dot{C}_{i}^{K i n}-\dot{C}_{i}^{E}\right)
$$

yields $\dot{V}_{2} \leq 0$. Let $C_{i}^{D y n}$ be the dynamic reference.

$$
\ddot{C}_{i}^{D y n}=\ddot{C}_{i}^{K i n}+K_{2}\left(\dot{C}_{i}^{K i n}-\dot{C}_{i}^{E}\right)
$$

This yields to a desired joint acceleration

$$
\ddot{\phi}_{i}^{R}=\left(d_{i}+d_{i-1}\right) \ddot{C}_{i}^{D y n}
$$

The dynamical model of the equation (2) is rewritten

$$
\ddot{q}=\mathcal{M}\left(F^{1}+\mathcal{B}\right)
$$

where $\mathcal{M}=\left(M^{1}+\beta^{T} M^{2} \beta\right)^{-1}$ and $\mathcal{B}=\beta^{T} F^{2}-\left(\beta^{T} M^{2} \dot{\beta}\right) \dot{q}$. The matrices $\mathcal{M}$ and $\mathcal{B}$ are decomposed in:

$$
\begin{gathered}
\mathcal{M}=\left[\begin{array}{c|c}
\mathcal{M}_{1} & \mathcal{M}_{2} \\
(3 \times 3) & (3 \times(N-1)) \\
\hline \mathcal{M}_{3} & \mathcal{M}_{4} \\
((N-1) \times 3) & ((N-1) \times(N-1))
\end{array}\right] \\
\mathcal{B}=\left[\begin{array}{c}
\mathcal{B}_{1} \\
\hline(3 \times 1) \\
((N-1) \times 1)
\end{array}\right]
\end{gathered}
$$

The control is then written as :

$$
\Gamma=\mathcal{M}_{4}^{-1}\left(\ddot{\phi}^{R}-\mathcal{M}_{3}\left(f_{1}+\mathcal{B}_{1}\right)\right)-\mathcal{B}_{2}
$$

where $f_{1}=\left[F_{1}^{u} F_{1}^{v} 0\right]^{T}$.
This control imposes the system to 'slide' on the sinusoidal curvature reference with a curvilinear velocity $\dot{s}$.
3) Velocity control:

The thrust control objective is to make the system undulate until a forward desired velocity $u_{d}$ is reached. This is done in regulating the variable $s_{i}$ as :

$$
\begin{equation*}
\ddot{s}_{i}=K_{s}\left(u_{d}-u_{1}\right) \tag{3}
\end{equation*}
$$

where $K_{s}$ is a positive gain, and the reference amplitude $A$ as :

$$
\begin{equation*}
\dot{A}_{i}=K_{A}\left(u_{d}-u_{1}\right) \tag{4}
\end{equation*}
$$

where $K_{A}$ is a positive gain and $i=2 . . N$. The previous choices are intuitively justified. Deeper theoretical work has to be done to prove the convergence of $u_{1}$ to $u_{d}$. Simulation results (cf chapter IV) indicate the desired behavior.

## B. Heading Control

The heading control is generally done with considering an offset term $\psi_{i}^{o f f s}$ in the thrust reference : $\phi_{i}=A_{i} \sin \left(w_{i} t+\right.$ $\left.\psi_{i}\right)+\psi_{i}^{o f f s}$ as in [8]. In our case, the heading control is done in a similar way. An offset term $C_{i}^{o f f s}$ is added on the reference curvature.

$$
C_{i}^{R}=A \sin \left(w s_{i}\right)+C_{i}^{o f f s}
$$

where $C_{i}^{o f f s}$ is a function of the heading error

$$
\begin{equation*}
C_{i}^{o f f s}=K_{c}\left(\psi_{d}-\theta_{1}\right) \tag{5}
\end{equation*}
$$

where $\psi_{d}$ is the desired heading. Note that the equations (3), (4) and (5) are considering error functions (to be regulated to zero) just considering the head position (link 1). Simulation results indicate a good system behavior, nevertheless, a deeper theoretical study has to be made to prove that the error functions are effectively vanishing.

## C. Path Following Control

The path following algorithm we are considering here is inspired by the one developed for terrestrial [9] or marine vehicles [10]. This solution admits an intuitive explanation : a path following controller should look at i) the distance from the vehicle to the path and ii) the angle between the velocity vector and the tangent to the path, and reduce both to zero. This motivates the development of a kinematic model of the system in terms of a Serret-Frenet frame $\{F\}$ that moves along the path; $\{F\}$ plays the role of a body axis of a 'virtual target' that should be tracked by the system. The curvilinear evolution of the virtual target is fundamental. Indeed, as intuition suggests, considering the virtual target as the closest point on the path leads to a singularity that drastically reduces the control performances [11]. In our solution, the virtual target has its own motion control algorithm. Consider figure 2, where the considered system reference is the gravity center of the first link. $P$ is an arbitrary point on the path associated with $P$, consider the corresponding Serret-Frenet frame $\{F\}$. The signed curvilinear abcissa of the moving target $P$ is denoted $\sigma$. The first link position can be expressed in $\{F\}$ as $\left(\sigma_{1}, \rho_{1}\right) . \theta_{C}$


Fig. 2. Path Following Problem Pose
denotes the absolute heading of the moving target and $\theta_{1}$ is the first link heading. The eel-like system is a marine vehicle and can side-slip $\left(v_{1} \neq 0\right)$. Then according to the results in [10] the evolution of the moving target should consider the total velocity of the system. Nevertheless, for reason of simplicity, we are considering that $v_{1}$ is negligible with respect to $u_{1}$. This assumption is not valid if $u_{1}$ is small, that is related to initial conditions. Then, according to the results in [9] the evolution of the moving target is chosen as :

$$
\dot{\sigma}=u_{1} \cos \theta+K_{\sigma} \sigma_{1}
$$

where $\theta=\theta_{1}-\theta_{C}$. Note the cooperative behavior of the moving target : when $\theta=\pi / 2$, the moving target velocity is reduced to $\dot{\sigma}=K_{\sigma} \sigma_{1}$ that warranties its convergence to the closest point on the path.

The guidance strategy is chosen according to [11], considering the approach angle $\delta\left(\rho_{1}, u_{1}\right)=-\operatorname{sign}\left(u_{1}\right) \delta_{a} \tanh \left(k_{\delta} \rho_{1}\right)$, where $\delta_{a}$ denotes the asymptotic incidence and $k_{\delta}$ is a positive gain. The control objective is to drive the system in order to regulate the quantity $\delta-\theta$ to zero. The heading control (5) is modified.

$$
C_{i}^{o f f s}=K_{c}(\delta-\theta)
$$

## IV. Simulation Results

We have simulated a 10 identical links eel-like robot using Matlab Software. The system and control parameters are chosen according to : $d_{i}=0.2 m, L_{i}=0.05 m, K_{1}=K_{2}=30$, $K_{A}=0.5, K_{c}=2, u_{d}=2 m / s, \delta_{a}=\pi / 2$ and $w=\pi$. The path is designed as a spline curve passing through chosen waypoints. The figure 3 shows the head position trajectory. It clearly shows that the system converges to the desired path. The figure 4 shows the evolution of the head position expressed in the Serret-Frenet frame, attached to the virtual target. The figure 5 displays the head velocities, $u_{1}$ is the forward velocity, $v_{1}$ the side slip velocity and $v_{t 1}=\sqrt{u_{1}^{2}+v_{1}^{2}}$, and shows the convergence to the desired forward velocity. The table I displays snapshots of the simulation at different instant. The first picture of the table I shows the initial shape at $t=0$. An initialization procedure has been implemented such that, during the 2 first seconds, the equation (3) is forced to $\ddot{s}=0$. This implies that the system shape is converging to the sinusoidal curvature profile. The second picture of the table I shows the system state at the end of the initialization


Fig. 3. System trajectory


Fig. 4. Relative distance system/target evolution




Fig. 5. Head velocity


Fig. 6. System trajectory
control design of the system is based on Lyapunov methods, and warranties the convergence of the system shape to the desired curvature profile. The system heading control is done in modifying the reference curvature profile, in order to guarantee the system to be driven to a desired heading. This control is then coupled with a path-following requirement, using the virtual target principle. Simulations illustrate the system behavior and show the convergence of the eel like robot to the desired path. This kind of system has two major advantages :

- maneuvrability: we have proceed to a simulation that il-


Fig. 7. Head velocity


TABLE I
Simulation snapshots
lustrates the expected performance of this kind of system. We are imposing a constant heading error and the system is constantly trying to correct this error, while traveling at a forward velocity equals to $2 \mathrm{~m} / \mathrm{s}$. Simulations indicates that the system trajectory is included in a circle of radius 0.7 m . This minimal radius of curvature of the resulting trajectory could also be improved with deeper study on the gait generation.

- Propulsion efficiency : undulatory type locomotion are expected to be very efficient. From the bio-mimetic inspiration, we could say that a good controller should insure the local fluid flow quality along the body, as fishes do using a baro-receptors line deployed all along their body. From now, the existing solutions impose a predefined gait that guides the system shape, and induces propulsion. Then the system efficiency is directly dependent of this
gait choice. Nevertheless, we are working now on a gait adaptation strategy in order to combine a nominal shape respect with local forces control consideration. Then it should be envisioned to explicitly study the system propulsion efficiency, and compare it to existing solution based on propellers.
This work is proposing a new way to study gait generation. For reason of simplicity, some assumptions have been made that forbids the elaboration of a formal proof of convergence. Nevertheless, we think that the problem is now well posed to design a control algorithm that respects global convergence requirements, based on a formal proof derivation. This warrants further research on the subject.


## Acknowledgment

The authors would like to thank Dr. K.A. McIsaac for his useful advises. This work is supported by the project Robot Anguille of the interdisciplinary program ROBEA of the French National Research Center (CNRS).

## References

[1] H. Vela, K. Morgansen and J. Burdick, Underwater locomotion from oscillatory shape deformations, in Proc. of the 41st IEEE Conference on Decision and Control (CDC), Las Vegas, Nevada USA, December 2002, pp 2074-2080.
[2] K. McIsaac and J. Ostrowski, Motion Planning for Anguiliform Locomotion, in IEEE Transaction on Robotics and Automation, August 2003, 19(4), pp 637-652.
[3] J. Ostrowski and K. McIsaac, Open-loop Verification of Motion Planning for an Underwater Eel-like Robot, in Proc. of the 7th International Symposium on Experimental Robotics (ISER), Honolulu, Hawaii, December 10-13, 2000.
[4] W. Saintval, Optimizing robotic aquatic locomotion, in Proc of the Thirty-fourth Annual Meeting of the Florida Section of the Mathematical Association of America, David Kerr and Bill Rush eds, April 2002, Florida, USA.
[5] D. Sfakiotakis and M. Tsakiris, A simulation environement for undulatory locomotion, in Proc. of the International Conference on Applied Simulation and Modelling, June 28-30, 2004, Rhodes Island, Greece.
[6] O. Ekeberg A combined neuronal and mechanical model of fish swimming , Biological cybernetics, Vol. 69, pp 363-374, 1993.
[7] T. Fossen Guidance and Control of Ocean Vehicles, John Willey \& Sons Inc, New York, 1994.
[8] K. McIsaac and J. Ostrowski, A Geometric Approach to Anguilliform Locomotion: Modeling of an Underwater Eel Robot, in Proc. of the IEEE Conference on Robotics and Automation (ICRA 99), pp. 2843-3848, Detroit Michigan, USA, May 10-14, 1999.
[9] D. Soetanto, L. Lapierre, A. Pascoal,Adaptive, non-singular path following control of a wheeled robot, in Proc. of the IEEE Conference on Decision and Control (CDC 03), Maui, Hawaii, USA, Dec. 2003.
[10] L. Lapierre, D. Soetanto, A. Pascoal,Nonlinear Path Following with application to the Control of Autonomous Underwater Vehicle, in Proc. of the IEEE Conference on Decision and Control (CDC 03), Maui, Hawaii, USA, Dec. 2003.
[11] C. Samson and K. Ait-Abderrahim, Mobile Robot Control Part 1 : Feedback Control of a Nonholonomic Mobile Robot, Technical Report No 1281, INRIA, Sopha-Antipolis, France, June 1991.

