Path Following Control for an Eel-Like Robot

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HAL Id: lirmm-00106474
https://hal-lirmm.ccsd.cnrs.fr/lirmm-00106474v1

Submitted on 16 Oct 2006

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Abstract— **We investigate the problem of controlling the motion of an eel like-robot. Recent work has shown promising results for this type of systems, opening new issues in the field of efficient propulsion and high manoeuvrability systems. The motion planning is decoupled in the two subproblems of thrust generation and heading control. In this paper we investigate a new type of autonomous gait generation, explicitly controlling the local system curvatures. The solution is then coupled with a path following controller, using the virtual target principle. The controller design is based on Lyapunov techniques. Simulations illustrate the performances of the proposed solution.**

I. INTRODUCTION

Since a long time, biologically inspired solutions have motivated a large number of robotic applications. In the domain of underwater robotics, the question of efficient and low consumption underwater propulsion, in conjunction with a high manoeuvrability is an active field of research. A whale tail is an 'unreachable' reference in terms of thrust efficiency; the lamprey is able of incredible maneuvers. The marine biologists have studied this type of locomotion, from the lamprey to the tuna fish, and have identified the body shape evolution to produce a thrust along the body axis. From a robotics point of view, this body shape evolution is a propagating wave (also called 'gait'), adjusting the signal phase according to the joint situation located on each vertebrae. From these results, a large number of interesting robotic applications have been performed in the last decades. The first question to be solved, concerning the model derivation of such a flexible system, is generally solved considering a highly coupled, non linear, hyper redundant model. The motion control problem is decoupled into three sub-problems:

- the gait generation,
- the control of the joint actuation in order to follow the previous reference,
- the adaptation of the gait parameters according to the system situation with respect to the main path the system has to reach and follow.

The gait generation is a crucial question in this problem. Indeed, a bad chosen gait will agitate the robot, without guarantying that the maximum number of the system elements (vertebrae) are involved in the movement, loosing by the way the desired efficiency, in terms of thrust and manoeuvrability.

The solutions of the literature propose some actuation gaits, directly controlling the joints as a trajectory tracker, or a kinematic joint control to fo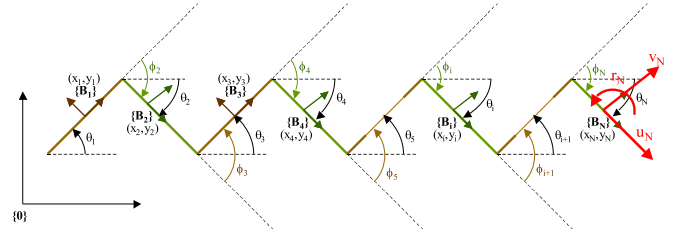llow the desired body shape ([1], [3], [2]). The loop is closed on the gait parameters, adapting them in function of the system situation with respect to the main goal (path). An interresting optimal approach is described in [4]. Neural approach has been also tested, and promising results can be found in [5] and [6].

In this paper, we propose a method to drive the system according to a reference body curvature, which is not time dependant and that explicitly takes into account the system dynamics. The control is derived using Lyapunov theory and Backstepping techniques, in order to guarantee the system to asymptotically converge to the desired shape defined in function of the curvilinear abscissa of each joints on the reference to be followed. The reference parameters are then adjusted in order for the system to reach a desired path, at a desired forward velocity. The path following algorithm is using the 'virtual target' guidance principle.

The paper is organized as follow : chapter II describes the system modeling, chapter III introduces the control design method we are proposing and chapter IV indicates some simulation results to illustrate the performances of our solution.

II. SYSTEM MODELING

We model the eel-like robot as a planar, serial chain of N links of length d_i, mass m_i and inertia I_i, where $i = 1...N$. Referring to the figure 1, we define two groups of kinematic variables called *configuration variables* $\dot{q}$ and *shape variable* $\dot{s}$, denoted as :

$$\dot{q} = \left[u_1 v_1 r_1 \dot{\phi}_2 \dot{\phi}_3 ... \dot{\phi}_{N-1} \dot{\phi}_N \right]^T$$
$$\dot{s} = \left[u_2 v_2 u_3 v_3 ... u_N v_N \right]^T \qquad (1)$$

where u_i is the forward velocity of the link i, v_i is the side-slip velocity, and r_i the rotational velocity, expressed in the body axis $\{B_i\}$. ϕ_i is the relative angle between the link $i - 1$ and i. Note that the variables ϕ_i can be called

Fig. 1. Problem Pose

articular variables, and that the group q defines a necessary and sufficient set to describe the system situation.

A. Hydrodynamics Approximation

We model each link of the eel-like robot as a cylinder of length d_i, radius L_i and mass m_i. The significant hydrodynamical forces applied on the systems are 3 types :

- perpendicular drag effect : we assume that differential pressure only acts perpendicularly to the body. Note that this is the force that produces the system thrust. We consider the approximation used in [2].

$$F_i^v = -\mu_i v_i$$

where F_i is the perpendicular drag force acting in the link i, moving with a side-slip velocity v_i ; μ_i is the perpendicular drag coefficient of the considered system element.

- Parallel drag effect : due to the streamlined nature of the body of the eel, the friction coefficient in the body direction is negligible with respect to the perpendicular direction ([2], [6]). Nevertheless, this effect is significant on the first link (also called *head*) for forward movement, and on the last segment for backward movement (*tail*).

$$F_1^u = \begin{cases} -\nu_1 u_1 & \text{if} \quad u_1 > 0 \\ 0 & \text{if} \quad u_1 \le 0 \end{cases}$$
$$F_N^u = \begin{cases} -\nu_N u_N & \text{if} \quad u_1 < 0 \\ 0 & \text{if} \quad u_N \ge 0 \end{cases}$$
$$F_i^u = \quad 0 \text{ for } i = 2 \text{ to } N - 1$$

where F_1^u and F_N^u are the parallel drag effect acting on the head and the tail, respectively, ν_i ,$i = 1, N$ are the parallel drag coefficient, and u_i denotes the forward speed of the ith element.

- Added mass : the added mass term is estimated considering the cylindrical shape of the system components. We refer to [7] for an approximation of this effect parameters. The inertial matrix for the ith element, expressed in the body frame $\{B_i\}$ is written as :

$$M_i = \begin{bmatrix} m_i^x & 0 & 0 \\ 0 & m_i^y & 0 \\ 0 & 0 & I^i \end{bmatrix}$$

where m_i^x and m_i^y is the wet mass (intertial and added mass)in the x and y direction of the body axis $\{B_i\}$, and I_i is the wet moment of inertial the ith element.

B. Model Derivation

Thanks to the variable definition in (1) the system Lagrangian is written.

$$L = \frac{1}{2} \sum_{i=1}^{N} \dot{\eta}_i^T M_i \dot{\eta}$$

where $\dot{\eta}_i = [u_i v_i r_i]$ denotes the ith element velocity expressed in the body frame $\{B_i\}$. The previous Lagrangian expression is rewritten as :

$$L = \frac{1}{2} \dot{q}^T M^1 \dot{q} + \frac{1}{2} \dot{s}^T M^2 \dot{s}$$

where

$$M^1 = \begin{bmatrix} m_1^x & 0 & 0 & 0 & \dots & 0 \\ 0 & m_1^y & 0 & 0 & \dots & 0 \\ 0 & 0 & a_1 & a_2 & \dots & I_N \\ 0 & 0 & a_2 & a_2 & \dots & I_N \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & I_N & I_N & \dots & I_N \end{bmatrix}$$

with $a_i = \sum_{k=i}^{N} I_k$ and

$$M^2 = \begin{bmatrix} m_2^x & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & m_2^y & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & m_3^x & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & m_3^y & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & m_N^x & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & m_N^y \end{bmatrix}$$

The consideration of the following kinematic relations in the body frames :

$$\begin{aligned} u_i &= u_1 \cos(\theta_i - \theta_1) + v_1 \sin(\theta_i - \theta_1) \\ &\quad + \dot{\theta}_1 \frac{d_1}{2} \sin(\theta_i - \theta_1) \\ &\quad + \sum_{k=2}^{i-1} \dot{\theta}_k d_k \sin(\theta_i - \theta_k) \\ v_i &= -u_1 \sin(\theta_i - \theta_1) + v_1 \cos(\theta_i - \theta_1) \\ &\quad + \dot{\theta}_1 \frac{d_1}{2} \cos(\theta_i - \theta_1) \\ &\quad + \sum_{k=2}^{i-1} \dot{\theta}_k d_k \cos(\theta_i - \theta_k) + \dot{\theta}_i \frac{d_i}{2} \end{aligned}$$

with $i = 2...N$, leads the extraction of the matrix β, expressing the relation between the configuration and shape variables.

$$\dot{s} = \beta \dot{q}$$

The matrix β can be easily computed with the following intermediate relations :

$$\dot{s} = \beta_\theta [u_1 v_1 \dot{\theta}_1 \dot{\theta}_2 ... \dot{\theta}_3]^T$$
$$[u_1 v_1 \dot{\theta}_1 \dot{\theta}_2 ... \dot{\theta}_3]^T = T_\theta^\phi \dot{q}$$
$$\beta = \beta_\theta T_\theta^\phi$$

where

$$\beta_\theta = \begin{bmatrix} c_1^2 & s_1^2 & \frac{d_1}{2} s_1^2 & 0 & 0 & \dots & 0 \\ -s_1^2 & c_1^2 & \frac{d_1}{2} c_1^2 & \frac{d_2}{2} & 0 & \dots & 0 \\ c_1^3 & s_1^3 & \frac{d_1}{2} s_1^3 & d_2 s_2^3 & 0 & \dots & 0 \\ -s_1^3 & c_1^3 & \frac{d_1}{2} c_1^3 & d_2 c_2^3 & \frac{d_3}{2} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ c_1^N & s_1^N & \frac{d_1}{2} s_1^N & d_2 s_2^N & d_3 s_3^N & \dots & 0 \\ -s_1^N & c_1^N & \frac{d_1}{2} c_1^N & d_2 s_2^N & d_3 s_3^N & \dots & \frac{d_N}{2} \end{bmatrix}$$

with $c_k^i = \cos(\theta_i - \theta_k)$, $s_k^i = \sin(\theta_i - \theta_k)$ and

$$T_\theta^\phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 1 & 1 & 1 & \dots & 0 \\ 0 & 0 & 1 & 1 & 1 & \dots & 1 \end{bmatrix}$$

The Lagrangian is then rewritten.

$$L = \frac{1}{2}\dot{q}^T \left(M^1 + \beta^T M^2 \beta\right) \dot{q}$$

The dynamic model is derived using

$$F^{ext} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q}$$

to obtain

$$F^{ext} = (M^1 + \beta^T M^2 \beta)\ddot{q} + (\beta^T M^2 \dot{\beta})\dot{q}$$

And the expression for $F^{ext} = F^1 + \beta^T F^2$ yields

$$(M^1 + \beta^T M^2 \beta)\ddot{q} = F^1 + \beta^T F^2 - (\beta^T M^2 \dot{\beta})\dot{q} \quad (2)$$

where $F^1 = (F_1^u, F_1^v, 0, \tau_2, ..., \tau_N)$ are the forces that act in the dimensions defined by the configuration variable q, and $F^2 = (F_2^u, F_2^v, ..., F_N^u, F_N^v)$ contains forces that act in the dimensions defined by the dependent variable s. $\Gamma = (\tau_2, ..., \tau_N)$ is the vector of control torques.

III. CONTROL DESIGN

The control problem is decoupled in 3 sub problems : the thrust generation, the heading control and the combination with a path following requirement.

A. Thrust Control

The thrust is obtained with a periodic signal propagating along the system body. This reference signal is called 'gait'. A classic approach consists in imposing to the join actuators a sinusoidal based reference of the form : $\phi_i = A_i \sin(w_i t + \psi_i)$ as in [8]. The problem is to adjust the gait parameters A_i, w_i and ψ_i to obtain an efficient thrust generation. Note the work of W. Saintval [4] on an optimization method to evaluate these gait parameters.

We are proposing to use a different approach in the gait generation. The gait we are proposing is not explicitly time-dependent and controls the local curvature of the system. This has two advantages.

- The fact that our gait does not explicitly consider the time variable leads to an autonomous system (if the desired velocity is not time dependent), and allows for the use of mathematical tool reserved for autonomous systems.
- Controlling the local curvature is a further step to mimic the behavior of fishes which use a baro-receptors line, deployed along their body, that allows them to locally control the quality of the flow.

Let s_i the curvilinear coordinate of the articulation i, along the system body; s_1 denotes the curvilinear position of the head and s_{N+1} is the position of the extremity of the tail. The initial values for s_i are : $s_1|_{t=0} = 0$, $s_2|_{t=0} = d_1$, $s_3|_{t=0} = d_1 + d_2,...,s_{N+1}|_{t=0} = \sum_{k=1}^{N} d_k$. The parameterized desired curvature along the system body is expressed as $C^R(s) = A\sin(ws)$. The discretization on each system articulation yields :

$$C_i^R = A\sin(ws_i)$$

Let C_i^E the system curvature of the articulation i.

$$C_i^E = \frac{\theta_i - \theta_{i-1}}{d_i + d_{i+1}} = \frac{\phi_i}{d_i + d_{i+1}}$$

The control objective is to drive the system curvature C_i^E to the reference C_i^R, for $i = 2, ...N$.

1) Kinematic Reference Definition:

Let $V_1 = \frac{1}{2}\sum_{i=2}^{N}(C_i^R - c_i^E)^2$ be a Lyapunov candidate. Straighforward computation shows that the choice

$$\dot{C}_i^E = \dot{C}_i^R + K_1(C_i^R - C_i^E), i = 2...N$$

yields $\dot{V}_1 = -K_1\sum_{i=2}^{N}(C_i^R - C_i^E)^2 \leq 0$. Let C_i^{Kin} be the kinematic reference for the dynamic control

$$\dot{C}_i^{Kin} = \dot{C}_i^R + K_1(C_i^R - C_i^E), i = 2...N$$

2) Dynamic Control Design:

Let $V_2 = \frac{1}{2}\sum_{i=2}^{N}(\dot{C}_i^{Kin} - \dot{C}_i^E)^2]$ be a Lyapunov candidate. The derivation of V_2 shows that the choice

$$\ddot{C}_i^E = \ddot{C}_i^{Kin} + K_2(\dot{C}_i^{Kin} - \dot{C}_i^E)$$

yields $\dot{V}_2 \leq 0$. Let C_i^{Dyn} be the dynamic reference.

$$\ddot{C}_i^{Dyn} = \ddot{C}_i^{Kin} + K_2(\dot{C}_i^{Kin} - \dot{C}_i^E)$$

This yields to a desired joint acceleration

$$\ddot{\phi}_i^R = (d_i + d_{i-1})\ddot{C}_i^{Dyn}$$

The dynamical model of the equation (2) is rewritten

$$\ddot{q} = \mathcal{M}\left(F^1 + \mathcal{B}\right)$$

where $\mathcal{M} = (M^1 + \beta^T M^2 \beta)^{-1}$ and $\mathcal{B} = \beta^T F^2 - (\beta^T M^2 \dot{\beta})\dot{q}$. The matrices $\mathcal{M}$ and $\mathcal{B}$ are decomposed in:

$$\mathcal{M} = \left[\begin{array}{c|c} \mathcal{M}_1 & \mathcal{M}_2 \\ (3\times 3) & (3\times(N-1)) \\ \hline \mathcal{M}_3 & \mathcal{M}_4 \\ ((N-1)\times 3) & ((N-1)\times(N-1)) \end{array}\right]$$

$$\mathcal{B} = \left[\begin{array}{c} \mathcal{B}_1 \\ (3\times 1) \\ \hline \mathcal{B}_2 \\ ((N-1)\times 1) \end{array}\right]$$

The control is then written as :

$$\Gamma = \mathcal{M}_4^{-1}\left(\ddot{\phi}^R - \mathcal{M}_3(f_1 + \mathcal{B}_1)\right) - \mathcal{B}_2$$

where $f_1 = [F_1^u F_1^v 0]^T$.

This control imposes the system to 'slide' on the sinusoidal curvature reference with a curvilinear velocity $\dot{s}$.

3) Velocity control:

The thrust control objective is to make the system undulate until a forward desired velocity u_d is reached. This is done in regulating the variable s_i as :

$$\dddot{s}_i = K_s(u_d - u_1) \tag{3}$$

where K_s is a positive gain, and the reference amplitude A as :

$$\dot{A}_i = K_A(u_d - u_1) \tag{4}$$

where K_A is a positive gain and $i = 2..N$. The previous choices are intuitively justified. Deeper theoretical work has to be done to prove the convergence of u_1 to u_d. Simulation results (cf chapter IV) indicate the desired behavior.

B. Heading Control

The heading control is generally done with considering an offset term ψ_i^{offs} in the thrust reference : $\phi_i = A_i \sin(w_i t + \psi_i) + \psi_i^{offs}$ as in [8]. In our case, the heading control is done in a similar way. An offset term C_i^{offs} is added on the reference curvature.

$$C_i^R = A \sin(ws_i) + C_i^{offs}$$

where C_i^{offs} is a function of the heading error

$$C_i^{offs} = K_c(\psi_d - \theta_1) \tag{5}$$

where ψ_d is the desired heading. Note that the equations (3), (4) and (5) are considering error functions (to be regulated to zero) just considering the head position (link 1). Simulation results indicate a good system behavior, nevertheless, a deeper theoretical study has to be made to prove that the error functions are effectively vanishing.

C. Path Following Control

The path following algorithm we are considering here is inspired by the one developed for terrestrial [9] or marine vehicles [10]. This solution admits an intuitive explanation : a path following controller should look at i) the distance from the vehicle to the path and ii) the angle between the velocity vector and the tangent to the path, and reduce both to zero. This motivates the development of a kinematic model of the system in terms of a Serret-Frenet frame $\{F\}$ that moves along the path; $\{F\}$ plays the role of a body axis of a 'virtual target' that should be tracked by the system. The curvilinear evolution of the virtual target is fundamental. Indeed, as intuition suggests, considering the virtual target as the closest point on the path leads to a singularity that drastically reduces the control performances [11]. In our solution, the virtual target has its own motion control algorithm. Consider figure 2, where the considered system reference is the gravity center of the first link. P is an arbitrary point on the path associated with P, consider the corresponding Serret-Frenet frame $\{F\}$. The signed curvilinear abcissa of the moving target P is denoted σ. The first link position can be expressed in $\{F\}$ as (σ_1, ρ_1). θ_C
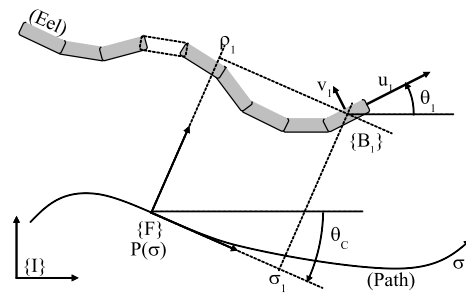

Fig. 2. Path Following Problem Pose

denotes the absolute heading of the moving target and θ_1 is the first link heading. The eel-like system is a marine vehicle and can side-slip ($v_1 \neq 0$). Then according to the results in [10] the evolution of the moving target should consider the total velocity of the system. Nevertheless, for reason of simplicity, we are considering that v_1 is negligible with respect to u_1. This assumption is not valid if u_1 is small, that is related to initial conditions. Then, according to the results in [9] the evolution of the moving target is chosen as :

$$\dot{\sigma} = u_1 \cos \theta + K_\sigma \sigma_1$$

where $\theta = \theta_1 - \theta_C$. Note the cooperative behavior of the moving target : when $\theta = \pi/2$, the moving target velocity is reduced to $\dot{\sigma} = K_\sigma \sigma_1$ that warranties its convergence to the closest point on the path.

The guidance strategy is chosen according to [11], considering the approach angle $\delta(\rho_1, u_1) = -sign(u_1)\delta_a \tanh(k_\delta \rho_1)$, where δ_a denotes the asymptotic incidence and k_δ is a positive gain. The control objective is to drive the system in order to regulate the quantity $\delta - \theta$ to zero. The heading control (5) is modified.

$$C_i^{offs} = K_c(\delta - \theta)$$

IV. SIMULATION RESULTS

We have simulated a 10 identical links eel-like robot using Matlab Software. The system and control parameters are chosen according to : $d_i = 0.2m$, $L_i = 0.05m$, $K_1 = K_2 = 30$, $K_A = 0.5$, $K_c = 2$, $u_d = 2m/s$, $\delta_a = \pi/2$ and $w = \pi$. The path is designed as a spline curve passing through chosen waypoints. The figure 3 shows the head position trajectory. It clearly shows that the system converges to the desired path. The figure 4 shows the evolution of the head position expressed in the Serret-Frenet frame, attached to the virtual target. The figure 5 displays the head velocities, u_1 is the forward velocity, v_1 the side slip velocity and $v_{t1} = \sqrt{u_1^2 + v_1^2}$, and shows the convergence to the desired forward velocity. The table I displays snapshots of the simulation at different instant. The first picture of the table I shows the initial shape at $t = 0$. An initialization procedure has been implemented such that, during the 2 first seconds, the equation (3) is forced to $\dddot{s} = 0$. This implies that the system shape is converging to the sinusoidal curvature profile. The second picture of the table I shows the system state at the end of the initialization
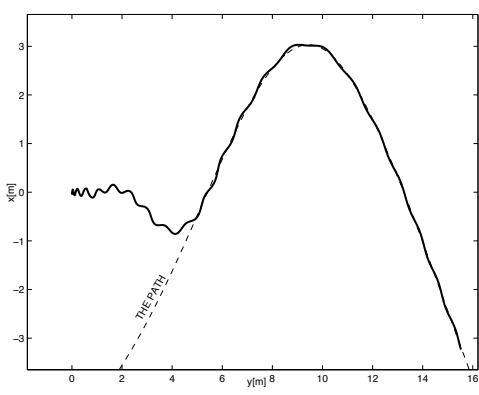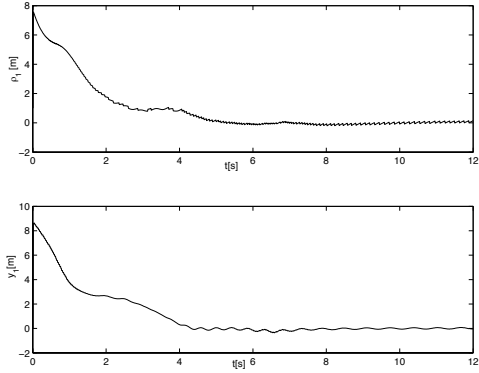
Fig. 3.   System trajectory

Fig. 5.   Head velocity

Fig. 4.   Relative distance system/target evolution

Fig. 6.   System trajectory

procedure, at $t = 2s$. Then, the control of the virtual target is established according to (3). The third picture shows the approach of the vehicle to the path, at $t = 4.3s$. As expected, the system goes to the desired path with an incidence angle guided by the approach angle δ. When the system is far form the path, the incidence is guided to $\delta_a = \pi/2$. As the system is approaching the path, the incidence is reduced until the system reaches the path. Then, the desired incidence is null, as shown in the fourth picture ($time = 5.5s$). The fifth ($time = 7s$) and sixth ($time = 9.1s$) pictures are showing the system following the desired path.

One of the major advantages of this type of eel-like system is its maneuvers ability. We have proceed to a second simulation where the added heading control term of the equation (5) is forced to $C_i^{offs} = 1$. The figure 6 displays the system trajectory, while figure 7 shows the convergence to the desired velocity. This simulation indicates a very interesting radius of curvature of the resulting trajectory. The trajectory is included in a circle of radius $0.7m$, with a forward velocity equals to $2m/s$. Generally, AUV type vehicles exhibits a minimal radius of curvature about $10m$. This warranties further study on this issue.

V. CONCLUSION

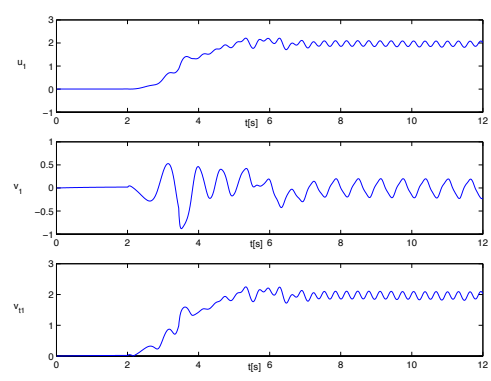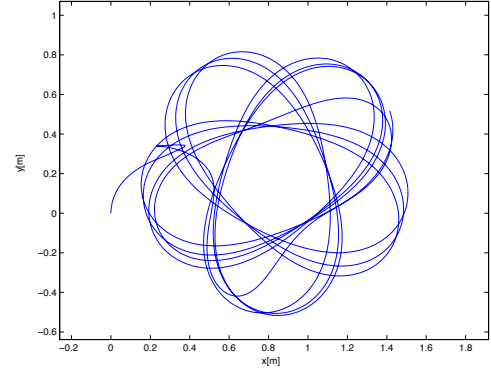We have developed a new type of autonomous gait generation, explicitly controlling the local system curvature. The control design of the system is based on Lyapunov methods, and warranties the convergence of the system shape to the desired curvature profile. The system heading control is done in modifying the reference curvature profile, in order to guarantee the system to be driven to a desired heading. This control is then coupled with a path-following requirement, using the *virtual target* principle. Simulations illustrate the system behavior and show the convergence of the eel like robot to the desired path. This kind of system has two major advantages :

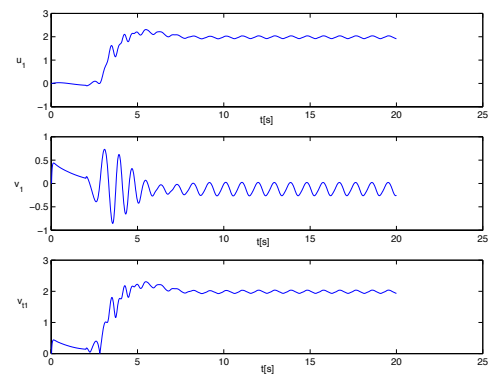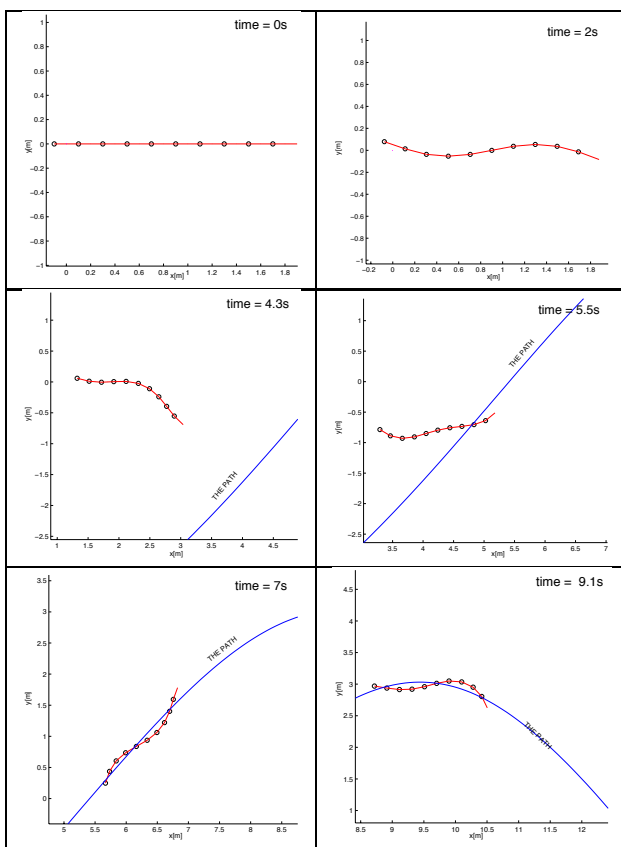- maneuvrability : we have proceed to a simulation that il-

Fig. 7.   Head velocity

TABLE I

SIMULATION SNAPSHOTS

lustrates the expected performance of this kind of system. We are imposing a constant heading error and the system is constantly trying to correct this error, while traveling at a forward velocity equals to $2m/s$. Simulations indicates that the system trajectory is included in a circle of radius $0.7m$. This minimal radius of curvature of the resulting trajectory could also be improved with deeper study on the gait generation.

- Propulsion efficiency : undulatory type locomotion are expected to be very efficient. From the bio-mimetic inspiration, we could say that a good controller should insure the local fluid flow quality along the body, as fishes do using a baro-receptors line deployed all along their body. From now, the existing solutions impose a predefined gait that guides the system shape, and induces propulsion. Then the system efficiency is directly dependent of this

gait choice. Nevertheless, we are working now on a gait adaptation strategy in order to combine a nominal shape respect with local forces control consideration. Then it should be envisioned to explicitly study the system propulsion efficiency, and compare it to existing solution based on propellers.

This work is proposing a new way to study gait generation. For reason of simplicity, some assumptions have been made that forbids the elaboration of a formal proof of convergence. Nevertheless, we think that the problem is now well posed to design a control algorithm that respects global convergence requirements, based on a formal proof derivation. This warrants further research on the subject.

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