On the Form-Closure Capability of Robotic Underactuated Hand

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***Abstract*—This paper presents a new method to study the capability of an underactuated hand to produce form-closed grasps. First, the stability behaviours of different underactuated parallel-jaw grippers are analyzed and compared according to their actuation and transmission mechanisms. Then, both notions of 1st order and 2nd order form-closure are revisited for underactuated hands, since in this particular case the assumption of fixed contacts made in the original definition is false. Therefore, constraints imposed by non-backdrivable mechanisms are introduced into the model of the whole grasp. Finally, a simple geometrical condition, necessary and sufficient for 1st order form-closure is proposed. This permits to conclude on the minimum number of non-backdrivable mechanisms required to produce 1st order form-closed grasps using an underactuated hand.**

***Keywords*—robotic hand, underactuation, form-closure, non-backdrivability.**

I. Introduction

In advanced research for robotic hands, two main fields can be identified that are manipulation and grasping. The first has led to dexterous hands with several actuators (more than six) such as the Utah/MIT hand [1], the Stanford/JPL hand [2], the Belgrade/USC hand [3], DLR hands [4]. The main drawback of those hands is the high cost of their control architecture which requires many actuators and sensors. Meanwhile, efforts have been made to design grasping hands with mechanical and control architectures that are simple enough to be commercialized, as for example prostheses for amputees or industrial grippers for pick-and-place operations. Therefore, many researchers have used underactuation as a strategy to reduce the number of actuators while preserving the capability of the hand to adapt its shape to the grasped object (in order to increase the total contact surface). Good examples of such an approach are those of Barrett Hand [5], RTR II Hand [6], SARAH and MARS Hands [7] (both designed for space applications).

Surprisingly, very few underactuated hands have found success as industrial grippers, probably because they can lead to somewhat non-intuitive behaviours and produce non-stable grasps. Such phenomena have been recently studied in [8] for a single underactuated finger with *n* phalanxes. In [9], the author demonstrates that, in some configurations of the finger, phalanx forces are negative. The finger is then not in static equilibrium because of unilateral contacts. This results in initiating an "ejection phenomenon", which either stops when a so-called "equilibrium position" is reached or carries on until actual ejection occurs. However, as previously stated in [10], the condition of static equilibrium of fingers is not sufficient to conclude on the stability or on the closure properties of a grasp. Despite these progresses and the interest of researchers in both force-closure and form-closure, very few extensions [11] of these properties have been made yet to study the stability of a **whole grasp** exerted by an underactuated hand.

The force-closure property is related to the capability of a grasp to actively control contact forces in order to counteract any external efforts exerted on the object, whereas form-closure is related to the capability of a grasp to completely restrain an object, not relying on the magnitude of contact forces or on friction forces but only on the geometric property of a set of unilateral contacts. We choose to investigate this last property because it represents an optimal grasp to be performed when using an underactuated hand, as it does not require any control of contact forces. Furthermore, because forces are not involved in the definition, it means that it is independent from friction effects and that, unlike power grasps, large grasp forces are not required.

In section II, the concept of underactuation is briefly recalled. Section III illustrates with a simple example that the actuation and transmission mechanisms can strongly influence the stability behaviour of an "underactuated grasp" (a grasp exerted by an underactuated hand will be called so in this paper). In order to extend this study to more complex hands, general definitions of 1st order and 2nd order form-closure for underactuated grasps are revisited in section IV. In section V, a unified approach is proposed to study the form-closure of an underactuated grasp. Therefrom, we deduce a simple geometrical condition, necessary and sufficient for 1st order form-closure and finally conclude on the minimum number of non-backdrivable mechanisms required to produce 1st order form-closed grasps.

II. Underactuation

A mechanism is said to be underactuated when it has fewer actuators than "configuration variables" [12] (*i.e.* independent parameters able to characterize all feasible motions of the mechanism). Underactuation in robotic hands is used as a strategy to reduce the number of actuators while preserving the hand capability to adapt its shape to the grasped object. This can be accomplished thanks to the use of : (i) differential transmission mechanisms such as "four-bar linkages" [7], pulley-cable [13] or cam-cable mechanisms [14], (ii) compliant mechanisms, where non-rigid bodies such as springs are used to share the actuation force among fingers [6] or (iii) triggered mechanisms such as the one used in the BarrettHand [5].

In the scope of our study we will consider underactuated mechanisms using a differential mechanism and one or more non-backdrivable mechanisms in the transmission of motion.

"A differential mechanism is a mechanism in which the amount of dynamical inputs from three ports acts in balance" [13]. From a kinetostatic point of view this results in:

$$T_1/a_1 = T_2/a_2 = T_3/a_3 \,, \quad (1)$$

$$a_1.d\theta_1 + a_2.d\theta_2 + a_3.d\theta_3 = 0 \,, \quad (2)$$

where T_i, $i \in \{1,2,3\}$, are the torques (forces) of a three-port differential mechanism and $d\theta_i$ are the angle displacements. Parameters a_i, $i \in \{1,2,3\}$, describe the transmission characteristics and may depend on the configuration of the mechanism: $a_i = a_i(\theta_1, \theta_2, \theta_3)$.

It is recalled that a transmission mechanism is said to be non-backdrivable when motion can be transmitted only from the input to the output axis. Considering the example given in Fig. 1, the input torque T_{in} is positive during the closing process, the constraint imposed by the non-backdrivable mechanism (when assumed ideal) is expressed as following:

$$\dot{\theta}_{out} \geq 0 \,, \quad (3)$$

where $\dot{\theta}_{out}$ is the output velocity of the toothed wheel.

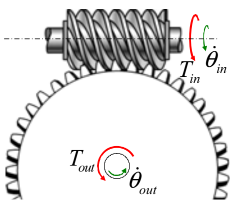

Figure 1. Scheme of a non-backdrivable worm-gear mechanism.

III. Form-closure behaviours of an underactuated parallel-jaw gripper

In this section, it is pointed out how the kind of underactuation and transmission mechanisms used between fingers and between phalanxes can influence the capability of an underactuated hand to produce form-closed grasps. This will be first illustrated with the simpler case of a parallel-jaw gripper. Three different underactuated mechanisms are analyzed and compared according to their form-closure capability. It is recalled that a grasp is said to be form-closed if the object is completely immobilized by a set of contact constraints [15]. In this definition, contacts are supposed to be fixed in space, which is false in case of an underactuated hand. Therefore, new constraints are considered that are imposed by non-backdrivable mechanisms. Combining these constraints with contact constraints permits ton conclude on the form-closure of the object.

A. *Case 1: a differential mechanism with variable distribution ratio*

The mechanism drawn on Fig. 2 uses pulleys and cables to achieve underactuation. The distribution ratio of forces $R = \| f_1 / f_2 \|$ is not constant and depends on the configuration of the differential mechanism. A static analysis leads to:

$$R = \left\| \frac{\sin(\theta_2 - \theta_0)}{\sin(\theta_1 + \theta_0)} \right\| , \quad (4)$$

The actuation force is equally distributed when the object is centred in the hand ($\theta_1 = \theta_2$ and $\theta_0 = 0$).

In order to do the modelling necessary to study form-closure, we consider that the actuator pulling on cable 0 prevents it from going upward (which would result in the opening of the hand). Hence, it is similar to consider a fictitious non-backdrivable mechanism located between the actuator and the rest of the transmission device which imposes the following constraint during the closing sequence:

$$dl \leq 0 \,, \quad (5)$$

where l is the length of cable 0 between points A and O_0 and dl is an infinitesimal variation of l.

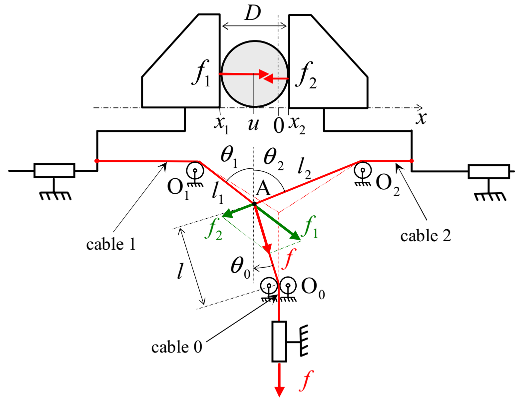

Figure 2. Scheme of a parallel-jaw gripper using a differential mechanism with a variable distribution ratio. Cables 1 and 2 are connected to cable 0 at point A. The actuation force f is applied on cable 0 and distributed among cables 1 and 2.

As seen on Fig. 3, if the object is not centered in the hand but on the left side, then the grasp is not form-closed since it can move to the right. When the object is centered in the hand, the grasp is 2^{nd} order form-closed. In this particular position, any infinitesimal motion du implies $dl = 0$ and is then not prohibited by the non-backdrivable mechanism. However, when considering second-order effects, we find $d^2l/du^2 > 0$, meaning that this equilibrium position is stable, in the sense that if the object moves to the left or to the right, it tends to come back to the central position. The resulting grasp is said to be 2nd order form-closed since only 2nd order terms permit to conclude on the form-closure of the hand.

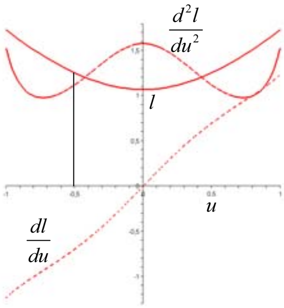

Figure 3. Representation of l, dl/du and d^2l/du^2 as a function of the object's position u, assuming that both fingers are contacting the object.
(i) when the object is on the left side (*e.g.* $u=-0.5$), the grasp is not form-closed, since it can move to the right: $du>0$ implies $dl<0$ which is permitted by the non-backdrivable mechanism. In the contrary case, if it moves to the left, $du<0$ implies $dl>0$, this motion is prohibited by the non-backdrivable mechanis.
(ii) when the object is centered in the hand ($u=0$), 1st order terms are null $dl/du=0$, but 2nd order terms are positive $d^2l/du^2>0$, meaning that any motion du is prohibited by the non-backdrivable mechanism. The grasp is 2nd order form-closed, since only 2nd order terms permit to conclude on the form-closure of the grasp.

B. *Case 2: a differential mechanism with constant distribution ratio*

The mechanism shown on Fig. 4 uses pulleys and cables to achieve underactuation with a constant force distribution. A non-backdrivable mechanism imposes $dl \le 0$. Once both fingers are in contact with the object, any motion of the object du implies $dl=0$, whatever the position of the object u. Contrary to the previous mechanism, we have $d^2l/du^2=0, \forall u$. This mechanism is incapable to produce form-closed grasps.

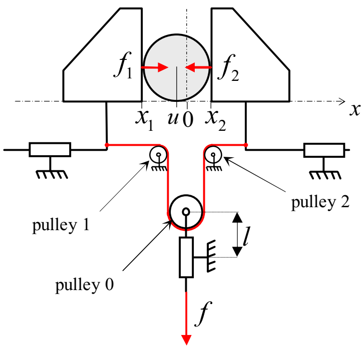

Figure 4. Scheme of a parallel-jaw gripper using a differential mechanism with constant distribution ratio.

C. *Case 3: a differential mechanism with two non-backdrivable mechanisms*

This mechanism (Fig. 5) uses a traditional differential mechanism to achieve underactuation. Motion is transmitted to fingers via two worm-driven rack mechanisms that are non-backdrivable and impose the constraints: $dx_1 \ge 0$ and $dx_2 \le 0$ during the closing sequence. This implies that once both fingers are in contact with the object, any motion of the object du is prohibited by one or the other non-backdrivable mechanism. As a conclusion, this mechanism can produce 1st order form-closed grasps, whatever the position of the object u.

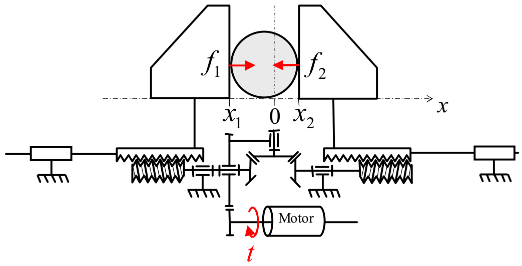

Figure 5. Scheme of a parallel-jaw gripper using a differential mechanism with two non-backdrivable transmission mechanisms.

D. *Conclusion*

The form-closure capability of three underactuated parallel-jaw grippers have been compared and analysed according to their underactuation and transmission mechanisms (see the recapitulation in Tab. 1). In this simple case, the stability behaviors could be intuitively identified. The same kinds of behaviors will be encountered in more complex underactuated hands that use multiple fingers and phalanxes. In the following, a method is proposed in order to analyze the form-closure behavior of an underactuated hand using multiple fingers and phalanxes.

TABLE I. STABILITY BEHAVIORS OF DIFFERENT UNDERACTUATED PARALLEL-JAW GRIPPERS

	case 1	**case 2**	**case 3**
Form-closure capable	Yes, in a single position	No	Yes
Type of equilibrium	2nd order	$\varnothing$	1st order

IV. AN EXTENSION OF THE FORM-CLOSURE PROPERTY FOR UNDERACTUATED HANDS

In order to generalize the previous study of form-closure for underactuated hands with N fingers and M phalanxes per finger, we propose an extension of both 1st order and 2nd order form-closure properties for underactuated hands. First, the original definition of form-closure is given. This definition is not adapted to treat underactuated grasps since it is based on the assumption that contacts are fixed in space. Hence, a new definition is proposed that considers constraints that are imposed by non-backdrivable mechanisms. A particular point is made of using the same mathematical formalism for both definitions so that numerous results already demonstrated for the original definition can be used again later in our case.

For the sake of clarity, only translations of the object in the plane are considered. We believe, this method can be extended to the spatial case and generalized to rotations.

A. Original definition of form-closure

Form-closure is the ability of a set of unilateral contact constraints to completely restrain motions of a grasped object.

The term "contact constraint" simply relates the fact that each part of the gripper that is in contact with the object cannot penetrate it, when assuming rigid bodies.

As stated in [15], in many cases it is sufficient to study the first-order approximation of contact inequalities that can be written as following:

$$\mathbf{dy} = \mathbf{P}\,\mathbf{du} \geq \mathbf{0}\,, \tag{6}$$

where $\mathbf{du}$ is an infinitesimal displacement of the object and $\mathbf{dy}$ is the vector that contains infinitesimal displacements of phalanxes. Each component dy_i of $\mathbf{dy}$ is approximated to the 1[st] order by the orthogonal projection of $\mathbf{du}$ onto the normal of the i^{th} phalanx $\mathbf{n}_i$. $\mathbf{P}$ is the projection matrix whose rows are the normal vectors of each phalanx expressed in the hand's base frame: $\mathbf{P} = [\mathbf{n}_1 \quad \ldots \quad \mathbf{n}_c]^T$, where c is the number of contact constraints. It follows therefrom the next conditions:

Assuming that contacts are fixed, a grasp is said to be 1[st] order form-closed if and only if for any motion $\mathbf{du}$ of the object, at least one contact constraint is violated, which gives:

$$\forall \mathbf{du} \in \mathbb{R}^d, \mathbf{du} \neq [\mathbf{0}], \exists i \in \{1,\ldots,c\}, \text{such that } dy_i < 0\,, \tag{7}$$

where d is the dimension of the object's configuration space, 3 for planar motions and 6 for spatial motions. In our case, $d = 2$ for simplification purpose.

A sufficient condition for the grasp not to be form-closed is:

$$\exists \mathbf{du} \in \mathbb{R}^d, \mathbf{du} \neq [\mathbf{0}], \text{such that } \forall i \in \{1,\ldots,c\}\ dy_i > 0\,, \tag{8}$$

Finally, a necessary condition for higher order form-closure is:

$$\exists \mathbf{du} \in \mathbb{R}^d, \mathbf{du} \neq [\mathbf{0}], \text{such that } \forall i \in \{1,\ldots,c\}\ dy_i \geq 0\,, \tag{9}$$

If (9) is satisfied and (8) is not, it means there exists at least one zero component $dy_i = 0$. The first order approximation is then not sufficient to conclude on the form-closure of the grasp and higher order effects have to be taken into account.

B. Extension of form-closure for underactuated hands

The conditions given previously are based on the assumption that contact points (*i.e.* phalanxes) are fixed relatively to the hand's base frame. This is true if the control position of each phalanx is considered ideal (*i.e.* with infinite rigidity). Anyway, in case of underactuated hands, the position of each phalanx cannot be controlled independently. Thus, a new condition for 1[st] order form-closure is proposed that takes into account non-backdrivable mechanisms:

A grasp is said to be 1[st] order form-closed if and only if for any infinitesimal displacement of phalanxes $\mathbf{dy}$ *that does not cause interpenetration of phalanxes with the object, at least one of the constraints imposed by the non-backdrivable mechanisms is violated.*

Such a displacement that does not cause interpenetration can be defined as the combination of displacements of two distinct domains D_1 and D_2:

$$\mathbf{dy} = \mathbf{dy_u} + \mathbf{d\varepsilon}\,, \tag{10}$$

where $\mathbf{dy_u} \in D_1$ and $\mathbf{d\varepsilon} \in D_2$.

The first domain D_1 contains displacements of phalanxes such that they accompany a motion $\mathbf{du} \in \mathbb{R}^d$ of the object (see Fig. 6-a). The resulting displacement of the i^{th} phalanx can be approximated to the first-order as the projection of $\mathbf{du}$ along the normal vector of the phalanx $\mathbf{n}_i$: $dy_{u_i} = \mathbf{n}_i^T \times \mathbf{du}$, (see Fig. 6-b). If the i^{th} phalanx is not in contact with the object, then $dy_{u_i} = 0$. More generally, the following relation can be written:

$$\mathbf{dy_u} = \mathbf{S}\,\mathbf{P}\,\mathbf{du}\,, \tag{11}$$

where $\mathbf{P}$ is now $p \times d$, with p the total number of phalanxes. $\mathbf{S}$ is a diagonal matrix, $s_{ii} = 1$ if the i^{th} phalanx is in contact with the object, otherwise $s_{ii} = 0$.

The second field D_2 contains displacements of phalanxes such that the contact is lost between the phalanx and the object $d\varepsilon_i \leq 0$ if the i^{th} phalanx was in contact with the object (Fig. 7). If the i^{th} phalanx is not in contact with the object, then it can move back and forth without penetrating the object, $d\varepsilon_i \in \mathbb{R}$. Considering a hand with four phalanxes, all phalanxes contacting the object except phalanx 3, one gets $\{d\varepsilon_1, d\varepsilon_2, d\varepsilon_3, d\varepsilon_4\} \in D_2 = \{\mathbb{R}^- \times \mathbb{R}^- \times \mathbb{R} \times \mathbb{R}^-\}$.

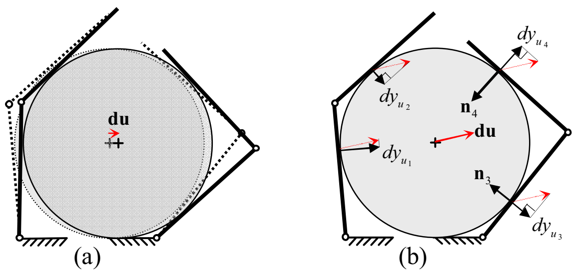

Figure 6. Representation of displacements of phalanxes $\mathbf{dy_u} \in D_1$.
(a) shows the displacements of phalanxes, when accompanying an infinitesimal motion of the object $\mathrm{du} \in \Re^d$, *i.e.* while keeping the distance between the phalanx and the object surface equal to zero in case the phalanx is contacting the object,
(b) such a displacement $\mathbf{dy}_{\mathbf{u}_i}$ can be approximated by the projection of the vector $\mathbf{du}$ along the normal of the considered phalanx $\mathbf{n}_i$.

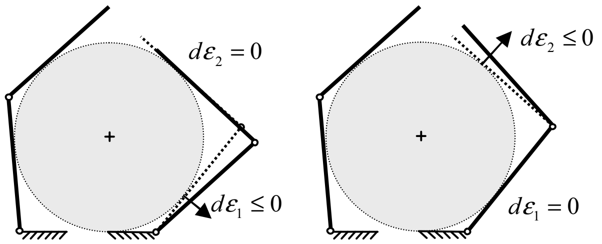

Figure 7. Representation of displacements of phalanxes $\mathbf{d\varepsilon} \in D_2$.

As a result, a necessary and sufficient condition for 1st order form-closure of an underactuated grasp can be reformulated:

$$\forall \mathbf{du} \in \mathbb{R}^d, \forall \mathbf{d\varepsilon} \in D_2, \begin{bmatrix} \mathbf{d\varepsilon} \\ \mathbf{du} \end{bmatrix} \neq [\mathbf{0}], \exists i \in \{1,\ldots,k\}, \text{such that } dq_i < 0\,, \quad (12)$$

where $\mathbf{dq}$ is the vector that contains infinitesimal variations of the parameters of the non-backdrivable mechanisms and k is the number of these mechanisms.

A sufficient condition for no form-closure of an underactuated grasp is:

$$\exists \mathbf{du} \in \mathbb{R}^d, \exists \mathbf{d\varepsilon} \in D_2, \begin{bmatrix} \mathbf{d\varepsilon} \\ \mathbf{du} \end{bmatrix} \neq [\mathbf{0}], \text{ such that } \forall i \in \{1,\ldots,k\}\ dq_i > 0\,, \quad (13)$$

Finally, a necessary condition for 2nd order form-closure of an underactuated grasp is:

$$\exists \mathbf{du} \in \mathbb{R}^d, \exists \mathbf{d\varepsilon} \in D_2, \begin{bmatrix} \mathbf{d\varepsilon} \\ \mathbf{du} \end{bmatrix} \neq [\mathbf{0}], \text{ such that } \forall i \in \{1,\ldots,k\}\ dq_i \ge 0\,, \quad (14)$$

C. Conclusion

In this section, the definition of form-closure has been revisited for underactuated grasps, since the original definition is not adapted to treat this case. Therefore, constraints that are imposed by non-backdrivable mechanisms were introduced. Finally, a necessary and sufficient condition for 1st order form-closure, a sufficient condition for no form-closure and a necessary condition for 2nd order form-closure of an underactuated grasp have been formulated It should be noted that this new condition (12) also includes the original property of 1st form-closure (7), meaning that if a grasp is not 1st order form-closed in the original sense, then condition (12) will not be either satisfied.

V. A UNIFIED APPROACH FOR 1ST ORDER FORM-CLOSURE

In this section, a unified approach is proposed to study the 1st order form-closure of an underactuated grasp. This formalism permits to treat in the same manner cases where not all fingers are contacting the object and cases where the object is contacting the palm. It is first shown how relations are manipulated in order to find the same kind of mathematical problem as the one that has to be solved for the original 1st order form-closure. Thus, theoretical results that have already been demonstrated for original form-closure can be extended to our case. Therefrom, a simple geometrical condition necessary and sufficient for 1st order form-closure is formulated. This permits us to lay down some requirements on the minimum number of non-backdrivable mechanisms needed to achieve 1st order form-closure. Our approach is then illustrated with the case of a two-fingered hand with two phalanxes per finger.

A. A unified approach

A 1st order approximation of infinitesimal variations of non-backdrivable parameters is given as following:

$$\mathbf{dq} = \mathbf{A}\,\mathbf{dy}\,, \quad (15)$$

As explained in the previous section, any displacement of phalanxes that does not induce interpenetration of phalanxes with the object, when approximated to the 1st order, can be written under the following form:

$$\mathbf{dy} = \mathbf{S}\,\mathbf{P}\,\mathbf{du} + \mathbf{d\varepsilon}\,, \quad (16)$$

where $\mathbf{du} \in \mathbb{R}^d$ and $\mathbf{d\varepsilon} \in D_2$.

$$\mathbf{dq} = \begin{bmatrix} \mathbf{A} & \mathbf{A\,S\,P} \end{bmatrix} \begin{bmatrix} \mathbf{d\varepsilon} \\ \mathbf{du} \end{bmatrix}, \quad (17)$$

A "constraint vector" $\mathbf{d\tilde{q}}$ is introduced containing every constraint of the problem, *i.e.* constraints that are imposed by non-backdrivable mechanisms: $dq_i \ge 0$ and contact constraints: $d\varepsilon_i \le 0$. In the same manner, if the object is contacting the palm, a new component is added to the vector $\mathbf{d\tilde{q}}$. The palm is treated in a different manner than phalanxes, since it is fixed relatively to the hand's base frame. This vector $\mathbf{d\tilde{q}}$ is built so that each component $d\tilde{q}_i$ has to be positive or null otherwise it is violated. This leads to:

$$\mathbf{d\tilde{q}} = \mathbf{M} \begin{bmatrix} \mathbf{d\varepsilon} \\ \mathbf{du} \end{bmatrix} \text{ with } \mathbf{M} = \begin{bmatrix} \mathbf{A} & \mathbf{A\,S\,P} \\ -\mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{n}_{\mathrm{p}}^T \end{bmatrix}, \quad (18)$$

where $\mathbf{n}_{\mathrm{p}}$ is the unit vector normal to the palm.

The necessary and sufficient condition for 1st order form-closure (12) becomes:

$$\forall \begin{bmatrix} \mathbf{d\varepsilon} \\ \mathbf{du} \end{bmatrix} \neq [\mathbf{0}] \in \mathbb{R}^{p+d}, \exists i \in \{1,\ldots,k+c\}, \text{such that } d\tilde{q}_i < 0\,, \quad (19)$$

B. A geometrical condition for 1st order form-closure

As stated in [16], a grasp is 1st order form-closed (using the original definition) if and only if the polytope $Poly(\mathbf{n}_i)$, whose vertices are the vectors normal to the phalanxes (rows of $\mathbf{P}$), contains in its interior the origin of $\mathbb{R}^d$ (see Fig. 8).

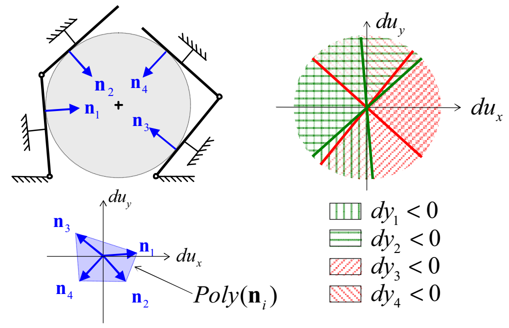

Figure 8. A 1st order form-closed grasp, according to the original definition, *i.e.* assuming that phalanxes are fixed. The hatched halfspaces $dy_i < 0$ represent prohibited motions. Since the whole plane is hatched, it means that any infinitesimal motion $\mathbf{du}$ is prohibited by at least one contact constraint. This is also clearly shown by the polytope $Poly(\mathbf{n}_i)$ that contains in its interior the origin of the plane (du_x, du_y). The grasp depicted is then 1st order form-closed, assuming that contacts are fixed.

By analogy, based on (19), a geometric condition necessary and sufficient for 1st order form-closure of an underactuated grasp is:

A grasp is 1st order form-closed if and only if the polytope $Poly(\mathbf{m}_i)$*, whose vertices are the rows of matrix* $\mathbf{M}$*, contains the origin of* $\mathbb{R}^{p+d}$ *in its interior.*

$$\mathbf{M} = \left[\mathbf{m}_1 \quad \dots \quad \mathbf{m}_{k+c}\right]^T, \tag{20}$$

Reuleaux [17] and Somov [18] proved that at least $d+1$ contact constraints are required for the original 1st order form-closure. This means, that matrix $\mathbf{M}$ needs at least $p+d+1$ non-zero rows for 1st order form-closure. This leads to the following inequality, necessary for 1st order form-closure:

$$k + c \geq p + d + 1, \tag{21}$$

where c is the number of contact constraints, comprising the phalanxes and the palm.

C. *Case of a two fingers – two phalanxes hand*

Let's first introduce notations that will be used in the following:

$d\,{}^i y_j$ is the infinitesimal displacement of the j^{th} phalanx of the i^{th} finger relatively to the hand's base frame;

${}^i k_j$ is the contact location on the j^{th} phalanx of the i^{th} finger;

${}^i l_j$ is the length of the j^{th} phalanx of the i^{th} finger;

${}^i\theta_j$ defines joint coordinate of the j^{th} phalanx of the i^{th} finger;

${}^i\theta_a$ defines joint coordinate of the bar a_i;

The case study considers a two-fingered hand with two phalanxes per finger Fig. 9. This hand is actually a simplified version of the SARAH Hand [7] in the plane, with only two phalanxes per finger. Fingers are underactuated using a "four-bar linkage" mechanism. A classical differential mechanism is used in order to distribute the motor torque on each finger.

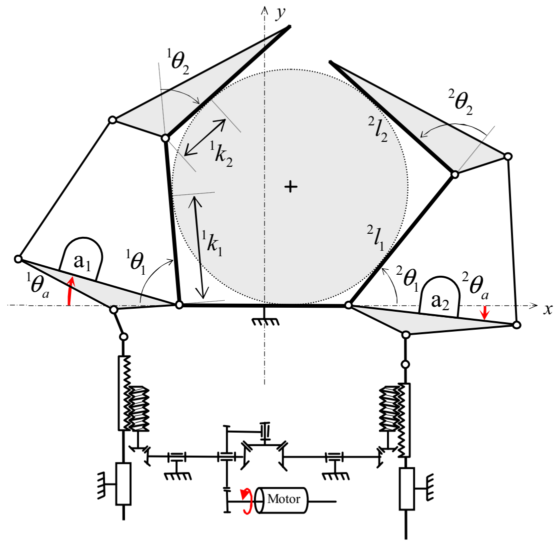

Figure 9. A simplified version of the SARAH Hand in the plane with two phalanxes per finger.

Two non-backdrivable mechanisms are introduced so that $d\,{}^1\theta_a, d\,{}^2\theta_a \geq 0$ during the closing sequence of the hand. For the sake of clarity, only translations of the grasped object are studied. The rotation around the *z*-axis is not taken into account since we consider grasps of circular objects.

The kinematic relation between non-backdrivable parameters $\mathbf{dq} = \left[d\,{}^1\theta_a \quad d\,{}^2\theta_a\right]^T$ and displacements of phalanxes of both fingers $\mathbf{dy} = \left[d\,{}^1y_1 \quad d\,{}^1y_2 \quad d\,{}^2y_1 \quad d\,{}^2y_2\right]^T$ is determined using the general approach proposed by Birglen in [8] extended to infinitesimal displacements and considering the whole hand:

$$\mathbf{A} = \begin{bmatrix} {}^1\tilde{\mathbf{T}}'\,{}^1\mathbf{J}^{-1} & \mathbf{0} \\ \mathbf{0} & {}^2\tilde{\mathbf{T}}'\,{}^2\mathbf{J}^{-1} \end{bmatrix}_{2\times 4}, \tag{22}$$

where ${}^i\tilde{\mathbf{T}}' = \left[1 \quad -{}^iR\right]$ is the simplified transmission matrix of the i^{th} finger (when neglecting the spring torques), ${}^iR$ is the transmission ratio. ${}^i\mathbf{J}$ is the jacobian matrix of the i^{th} finger and is given by the following expression:

$${}^i\mathbf{J} = \begin{bmatrix} {}^ik_1 & 0 \\ {}^ik_2 + {}^il_1 cos\,{}^i\theta_2 & {}^ik_2 \end{bmatrix}, \tag{23}$$

The projection matrix $\mathbf{P}$ is given as following:

$$\mathbf{P} = \begin{bmatrix} \sin{}^1\theta_1 & \cos{}^1\theta_1 \\ \sin({}^1\theta_1 + {}^1\theta_2) & \cos({}^1\theta_1 + {}^1\theta_2) \\ -\sin{}^2\theta_1 & \cos{}^2\theta_1 \\ -\sin({}^2\theta_1 + {}^2\theta_2) & \cos({}^2\theta_1 + {}^2\theta_2) \end{bmatrix}_{4\times 2}, \tag{24}$$

If each of the four phalanxes is in contact with the object, then $\mathbf{S} = \mathbf{I}_{4\times 4}$, and matrix $\mathbf{M}$ is 6×6. As previously said, such a grasp cannot be 1st order form-closed since $p+d+1=7$

rows are needed for $\mathbf{M}$. In case the object is contacting the palm, $\mathbf{M}$ becomes 7×6, satisfying then the preliminary necessary condition (21) on the minimum number of rows.

In the following, the form-closure behavior of the hand is analyzed according to the configuration of the grasped object. In order to simplify the study and to permit the visualization of the phenomena, the size of matrix $\mathbf{M}$ is reduced to 2×2. Therefore, it is first verified that the non-zero components ${}^{j}A_{i}$ of matrix $\mathbf{A}$ are strictly positive. Hence, each component of $\mathbf{A}\,\mathbf{d\varepsilon}$ is negative or null for any displacement of the phalanxes $\mathbf{d\varepsilon}\in D_2$ ($\mathbf{d\varepsilon}\in\mathbb{R}^{p}$ since each phalanx is considered to be in contact with the object). In other words, assuming the object fixed ($\mathbf{du}=\mathbf{0}$), any displacement of phalanxes is prohibited by non-backdrivable mechanisms (as shown for the second finger in Fig. 10).

$$\mathbf{A}=\begin{bmatrix} {}^{1}A_{1} & {}^{1}A_{2} & 0 & 0 \\ 0 & 0 & {}^{2}A_{1} & {}^{2}A_{2} \end{bmatrix}, \tag{25}$$

Where ${}^{i}A_{1}=\frac{1}{{}^{i}k_{1}}+{}^{i}R.\left(\frac{{}^{i}k_{2}+{}^{i}l_{1}.\cos{}^{i}\theta_{2}}{{}^{i}k_{1}.{}^{i}k_{2}}\right)$ and ${}^{i}A_{2}=-\frac{{}^{i}R}{{}^{i}k_{2}}$, (26)

Since $\mathbf{A}\,\mathbf{d\varepsilon}\le\mathbf{0},\ \forall\mathbf{d\varepsilon}\in D_2$, the following inequality can be written:

$$\mathbf{dq}=\mathbf{A}\,\mathbf{d\varepsilon}+\mathbf{A}\,\mathbf{I}_{4\times4}\,\mathbf{P}\,\mathbf{du}\le\mathbf{A}\,\mathbf{P}\,\mathbf{du}, \tag{27}$$

Then, the size of matrix $\mathbf{M}=\mathbf{A}\,\mathbf{P}$ is reduced to 2×2.

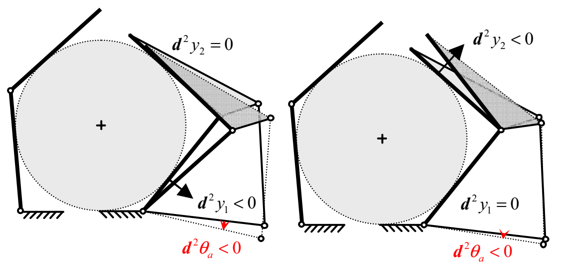

Figure 10. Any backward motion of the phalanxes induces $d^2\theta_a<0$ and is then prohibited by the non-backdrivable mechanism.

The necessary and sufficient condition for first-order form-closure (19) becomes:

$$\forall\,\mathbf{du}\ne[\mathbf{0}]\in\mathbb{R}^{d},\ \exists i\in\{1,\ldots,k\},\text{ such that } dq_i<0, \tag{28}$$

Four different cases are identified (see Fig. 11):

Case (a): $\mathbf{m}_1$ and $\mathbf{m}_2$ are not collinear. This implies that there exists an object's motion $\mathbf{du}$ for which $d^1\theta_a, d^2\theta_a>0$. The grasp is then not form-closed.

While closing the gripper (*i.e.* $d^1\theta_a, d^2\theta_a>0$), if the object does not encounter the palm on his path (see case c), the object moves until it reaches an equilibrium position described in the next case.

Case (b): $\mathbf{m}_1$ and $\mathbf{m}_2$ are collinear and matrix $\mathbf{M}$ is singular. Our 1st order analysis does not permit to conclude on the form-closure of the grasp since if $\mathbf{du}$ is orthogonal to $\mathbf{m}_i$, $d^1\theta_a=d^2\theta_a=0$. In conclusion, 2nd order terms have to be taken into account in order to check if the grasp is 2nd order form-closed.

Case (c): In this case, the object is contacting the palm. Then, a new constraint appears that is $du_y\ge0$. A new row $[0\quad1]$ is added to matrix $\mathbf{M}$ which becomes 3×2. Vectors $\mathbf{m}_1$ and $\mathbf{m}_2$ are not collinear, the field of possible motions given by the non-backdrivable mechanisms is the same as the one depicted for case a and is directed towards the palm. The grasp is then 1st order form-closed. As seen on Fig. 11, the origin of $\mathbb{R}^{d}$ is contained in the interior of the polytope $Poly(\mathbf{m}_i)$.

Case (d): In case the object is contacting the palm ($\mathbf{M}$ is 3×2), $\mathbf{m}_1$ and $\mathbf{m}_2$ are collinear, the object can move infinitesimally along a single line, in one direction. The other direction is pointing towards the palm. As seen on Fig. 11, the origin of $\mathbb{R}^{d}$ is not contained in the interior of the polytope $Poly(\mathbf{m}_i)$ but on its boundary. No conclusion can be given on the form-closure of the grasp, 2nd order terms have to be taken into account.

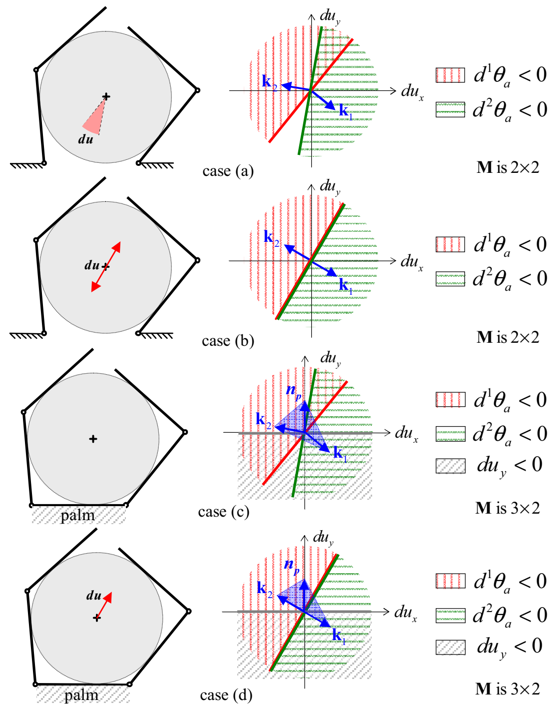

Figure 11. Different form-closure behaviors of the underactuated hand depicted on Fig. 9 according to the configuration of the grasped object. (a) the grasp is not form-closed because there exists $\mathbf{du}$ such that variations of the non-backdrivable parameters are positive, (b) it is not possible to conclude on the form-closure of the grasp, 2nd order effects must be taken into account, (c) the grasp is form-closed because the origin of the space (du_x, du_y) is contained in the interior of the polytope $Poly(\mathbf{m}_i)$, (d) it is not possible to conclude on the form-closure of the grasp, 2nd order effects must be taken into account.

D. Conclusion

It has been shown in this section that a necessary and sufficient condition for 1st order form-closure is that the polytope whose vertex are the rows of matrix $\mathbf{M}$ contains the origin of $\mathbb{R}^{p+d}$. Using the works of Reuleaux and Somov, this permitted to say that at least $p-c+d+1$ non-backdrivable mechanisms are required to achieve 1st order form-closure. This was illustrated with the case of a two fingered hand with two phalanxes per finger using two non-backdrivable mechanisms. Such a hand can produce 1st order form-closed grasps, only if all 4 phalanxes and the palm are contacting the object. In the contrary case, only higher order form-closed grasps can be expected.

VI. Conclusion

In this paper, a method has been proposed to study the form-closure property of an underactuated hand using differential mechanisms for underactuation between fingers and between phalanxes. With the simple case of a parallel-jaw gripper, we first illustrated the influence of the transmission and actuation mechanisms on the form-closure behavior. In order to extend this study to more complex underactuated hands, we revisited both concepts of 1st order and 2nd order form closure. Since the original definition is based on the assumption that contacts are fixed relatively to the hand's base frame, we proposed a new definition of form-closure adapted to underactuated hands. Therefore, constraints that are imposed by non-backdrivable mechanisms were introduced. We then proposed a unified approach that considers the whole grasp and a simple geometrical condition necessary and sufficient for 1st order form-closure. Therefrom, we concluded on the minimum number of non-backdrivable mechanisms required to produce 1st order form-closed grasps.

The presented method is local, in the sense that it permits to study form-closure of a grasp of a given object in a given configuration. This study can be extended to determine the domain of configurations for which the grasp is form-closed This would help in the grasp synthesis to determine possible configurations of the hand for which the grasp of a known object is 1st order form-closed. It is also possible, using this method at the design stage, to determine the kinematic parameters of an underactuated hand so that it achieves form-closed grasps of objects that have a particular shape (cylindrical, spherical, planar, …) and whose dimensions belong to a given interval. To the best of the author's knowledge, form closure of a whole grasp exerted by an underactuated hand is studied here for the first time.

References

[1] S. C. Jacobsen, E. K. Iversen, D. F. Knutti, R. T. Johnson, and K. B. Biggers, "Design of the UTAH/MIT dextrous hand," *in Proc. of ICRA86: IEEE International Conference on Robotics and Automation*, San Francisco, CA, USA, 1986, pp. 1520–1532.

[2] J. K. Salisbury and J. J. Craig, "Articulated hands: Force control and kinematic issues," *The International Journal of Robotics Research*, Vol. 1, No. 1, pp. 4-17, 1982.

[3] G. Bekey, R. Tomovic, and I. Zeljkovic, "Control architecture for the Belgrade/USC Hand," in Dextrous Robot Hands, S.T. Venkataraman and T. Iberall (Ed.), Springer-Verlag, 1989.

[4] J. Butterfass, M. Grebenstein, H. Liu, and G. Hirzinger, "DLR-hand II: next generation of a dextrous robot hand," *in proc. of ICRA 2001: IEEE International Conference on Robotics and Automation*, Seoul, Korea, May 1-26, 2001, pp. 109-114.

[5] W.T. Townsend, "The BarrettHand grasper – programmably flexible part handling and assembly," *Industrial Robot: An International Journal*, MCB Uiversity Press, Vol. 27, No. 3, pp. 181–188, 2000.

[6] B. Massa, S. Roccella, M. C. Carrozza, and P. Dario, "Design and Development of an Underactuated Prosthetic Hand," *in Proc. Of ICRA2002: IEEE International Conference on Robotics & Automation*, Washington, DC, May 2002, pp. 3374-3379.

[7] T. Laliberté, L. Birglen, and C. Gosselin, "Underactuation in Robotic Grasping Hands," *Japanese Journal of Machine Intelligence and Robotic Control*, Special Issue on Underactuated Robots, pp. 77-87, Vol. 4, No. 3, September 2002.

[8] L. Birglen and C. Gosselin, "Kinetostatic analysis of underactuated fingers," *IEEE Transactions on Robotics and Automation,* vol. 20, no. 2, pp. 211-221, April 2004.

[9] Birglen, L. and Gosselin, C. M., "On the Force Capability of Underactuated Fingers," *in Proc. Of ICRA2003: IEEE International Conference on Robotics & Automation*, Taipei, Taiwan, September, 2003.

[10] L. Birglen, "Analysis and Control of Underactuated Robotic Hands," Thesis, Robotics Laboratory, Laval University, Quebec, Canada, 2004.

[11] L. Minzhou, M. Tao, W. Xiaohua, and Y. Yong, "Grasp Characteristics of An Underactuated Robot Hand," *in Proc. Of ICRA2004: IEEE International Conference on Robotics & Automation*, New Orleans, LA, April 2004, pp. 2236-2241.

[12] R. Olfati-Saber, "Nonlinear control of underactuated mechanical systems with application to robotics and aerospace vehicles," Thesis, Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA, February 2001.

[13] Hirose S., "Connected Differential Mechanism and its Applications", *in Proc. of Int. Conf. of Advanced Robotics*, Tokyo, Japan, September 1985.

[14] S. Krut, "A force-isotropic underactuated finger," *in Proc. of ICRA2005: IEEE International Conference on Robotics & Automation*, Barcelona, Spain, April 2005, pp. 2325-2330.

[15] A. Bicchi and V. Kumar. "Robotic Grasping and Manipulation." In S. Nicosia, B. Siciliano, A. Bicchi, and P. Valigi (eds.), editors, *Ramsete: Articulated and mobile robots for services and Technology*, volume 270, chapter 4, pages 55-74. Springer-Verlag, Berlin Heidelberg, Germany, 2001.

[16] E. Rimon and J. Burdick, "On force and form closure for multiple finger grasps," *in Proc. Of ICRA1996: IEEE International Conference on Robotics & Automation*, pages 1795--1800, Minneapolis, May 1996.

[17] F.Reuleaux, *Kinematics of machinery*, New York: Dover, 1963. First published in German, 1875.

[18] P. Somov, "Über Gebiete von Schraubengeschwindigkeiten eines starren Körpers bei verschiedener Zahl von Stützflächen," *Zeitschrift für Mathematik und Physik*, vol. 45, 1900.