A Futuristic Monorail Tramway Stabilized by an Inertia Wheel

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Abstract— **This paper presents a futuristic transportation mean: a monorail tramway, running on a narrow rail, stabilized by an inertia wheel. As the required infrastructure is very light, this transportation mean is an interesting answer to traffic saturation problems at which the agglomerations are exposed. To keep its balance, this tramway uses an inertial wheel controlled in real time, and driven by a regenerative actuator and driver, for consumption purpose. This paper is a preliminary study for this project. The analysis of the dynamics of the tramway in the transversal plane is made. The demonstration of the stabilization concept feasibility is done using the inverted pendulum, the most popular tool of the control theory, and more particularly, the inverted inertia wheel pendulum. The mechanical principles used here are classical but unique for this kind of application. Unlike in the other studies of inertia wheel pendulums, the localizations of the centers of masses were decided taking into account the real mass distribution (passengers and the tram structure) and optimized to minimize power consumption. Additionally, instead of using an encoder to measure the orientation of the pendulum, an inclinometer was preferred. Concerning the control part, the Linear Quadratic Gaussian control was chosen to stabilize the inverted pendulum because this control law allows controlling a system even with a partial knowledge of its state. A discussion regarding state observability is carried out showing that, even if in theory the angular velocity of the inertia wheel is enough to observe the full state of the system, in practice a sensor able to measure the orientation of the pendulum needs to be added. Experimental results are given.**

Keywords— **monorail tramway, inverted pendulum, inertia wheel, underactuated system, non-linear system, LQ (Linear Quadratic), LQG (Linear Quadratic Gaussian)**

I. INTRODUCTION

NOWADAYS, the world of urban transportation is in mutation phase. Indeed, lots of agglomerations have decided to promote public transports, to the detriment of individual vehicles. This strategy is an answer to traffic saturation problems. And to diversify mass transportation means, adding tramways is a solution. Additionally tramways can participate to the reputation of an agglomeration and then, can attract new investments, e.g. tourists, society, manufactures, etc. To do so, it is necessary to rely on an exclusive tram concept, so that an agglomeration distinguishes itself from its competitors. There are a lot of monorail tram forms in the world; their principal differences are on the technologies used for the propulsion and the sustentation. The proposed tramway is very innovative: it only requires a narrow rail as it is stabilized by an inertia wheel. This reduces the cost of infrastructures.

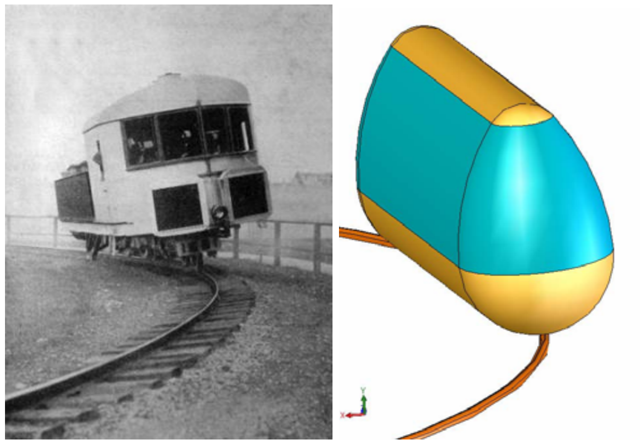

Fig. 1: left part, the monorail of L.Brennan and Scherl (1907), a pendular monorail train stabilized by a gyroscope; right part: CAD view of the futuristic monorail tramway stabilized by an inertia wheel

Fig. 1 shows a realization of a pendular monorail train running on a narrow rail. Of course, the picture is very old, but it shows that this project is not an unreasonable idea. Unlike the train on this figure which uses a mechanically controlled gyroscope to keep its balance, we decided to use an inertia wheel for the tramway. Hence, this solution seemed very promising from the energetic point of view, as electrical actuators without gearbox, coupled with regenerative drivers could be chosen to act directly on the inertia wheel. (As the torque to tilt the gyroscope is very high, this solution seemed not to be possible with the gyroscope). This will be validated in the future.

The demonstration of the stabilization concept feasibility is done using the inverted pendulum, the most popular tool of the control theory. The mechanical principles used are classical but unique for this kind of application. The inverted pendulum is an ideal tool to test very quickly and cheaply, different control methods, going from the most classic to the most innovative. Once tested on this simple mechanism, the control laws can be extended to much more complex systems, e.g. launch vehicles, fighter aircrafts, boats, biped robots, etc, as their dynamics are of the same type.

There are a lot of inverted pendulum forms; the most popular are: simple inverted pendulum [1]-[5], double inverted pendulum [6], Furuta inverted pendulum [7], Inertia Wheel Pendulum (IWP) [8]-[10], etc. The equivalent dynamics of the proposed tramway is the one of the Inertia Wheel Pendulum. This equivalence is valid strictly on transversal plane.

It is necessary to specify that the other particularities of this study are the positions of the centers of mass. Hence, the masses are not homogenously distributed, in order to simulate the real distribution of masses concerning tram passengers and to the tram architecture. Furthermore, the position of the inertia wheel relatively to the pole is optimized in order to reduce mechanical torques when balancing the pendulum.

This paper is organized as follows: Section II presents the system architecture and its specificities. Section III introduces the IPSIW mathematical model. Section IV exposes the state measurements problem. Section V gives an overview of the control problems and the main steps in the design of the control laws. Section VI proposes some simulation and practical results. Finally, we conclude with some remarks in section VII.

II. PRESENTATION OF THE SYSTEM

As a preliminary study we focused on the dynamics of the train in the transversal plane. The system turned out to be an inverted pendulum stabilized by an inertia wheel

A. *Why a special geometry for the study of the monorail tramway?*

We demonstrate in this section, the interests to use a non conventional geometry to study the monorail tramway. Unlike the others studies, this geometry is thought, to take into account the real system characteristics and more particularly, the masses distribution of the passengers and structure.

B. *Classical Geometry*

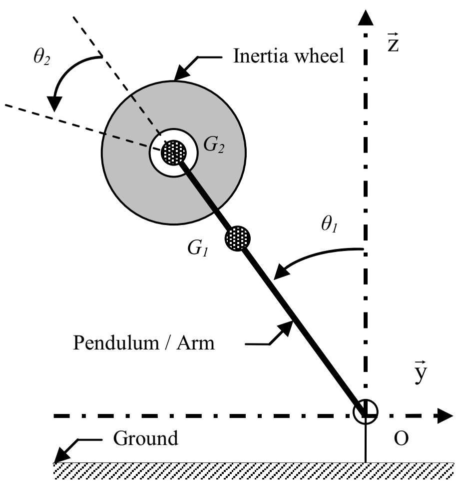

Fig. 2: Classical geometry of the Inertia Wheel Pendulum

The Inertia Wheel Pendulum is an underactuated mechanical system because it has more Degrees of Freedom (DoF) than actuators, i.e. two DoF (pivot joints) for one actuator (acting on the inertia wheel). The IWP is composed of three elements: a frame, an arm or pendulum and an inertia wheel. The arm is in free rotation relatively to the frame while the inertia wheel is actuated (its rotation axis is on the arm). Fig. 2 introduces the classical geometry of the IWP commonly used.

C. *Modified Geometry*

The geometry presented in Fig. 2 is not optimized for industrial applications. In fact, it is imperative to minimize the mechanical power in play in order to minimize consumption. The total resistant moment of the system is given by:

$$M_{IWP} = M_{Pendulum} + M_{InertiaWheel} \quad (1)$$

where the resistant moment of the pendulum is

$$M_{Pendulum} = m_1 g l_1 \sin\theta_1 , \quad (2)$$

and the resistant moment of the inertia wheel is

$$M_{InertiaWheel} = m_2 g l_2 \sin\theta_1 . \quad (3)$$

TABLE I
PARAMETERS AND VARIABLES OF THE MODEL

Symbol	Unit	Description
m_i	kg	Mass
G_i	no unit	Geometrical point representing the center of mass
l_i	m	Distance from the origin to the center of mass
i_i	kg·m^2	Inertia
θ_i	rad	Angle position
g	m/s^2	Acceleration due to gravity
τ	N·m	Torque applied on the inertia wheel

$i \in \{1, 2\}$, 1: inverted pendulum and 2: inertia wheel

Equation (2) and (3) show that it is possible to reduce the total resistant moment when playing with the position of the centers of mass G_1 and G_2 (In the tramway, G_1 characterizes the masses of the passengers and the tram structure, and G_2 characterizes the inertia wheel mass), and more particularly, in decreasing l_1 and l_2. Moreover, the masses distribution must be similar with the real system, in order to take into account the real distribution of masses concerning tram passengers and the tram architecture. This distribution of masses is essentially concentrated at the bottom of the tram structure. Fig. 3 introduces the modified geometry of the IWP used in this study.

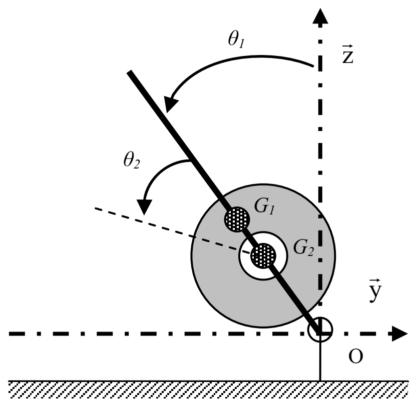

Fig. 3: Optimized geometry of the Inertia Wheel Pendulum, in order to reduce the total resistant torque.

III. MATHEMATICAL MODELING

A. *Nonlinear Model*

The mathematical model of the IWP can be derived using the Euler-Lagrange formalism. This approach involves determining the kinetic and the potential energies of the system in terms of generalized coordinates. The Euler-Lagrange equations can be written as follows:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}}\right] - \frac{\partial L}{\partial x} = Q_x \quad (2)$$

with:

$$x=\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix},\ Q_x=\begin{bmatrix}0\\ \tau\end{bmatrix} \text{ and } L=T-V$$

L: Lagrangian, T: Kinetic energy, V: Potential energy, Q_x: generalized forces not taken into account in T and V, x: generalized coordinates.

The total kinetic energy of system T is as follows:

$$T=T_{Pendulum}+T_{InertiaWheel} \tag{4}$$

where the kinetic energy of the pendulum is

$$T_{Pendulum}=\frac{1}{2}\left(m_1 l_1{}^2+i_1\right)\dot{\theta}_1{}^2,$$

and the kinetic energy of the inertia wheel is

$$T_{InertiaWheel}=\frac{1}{2}m_2 l_2{}^2\dot{\theta}_1{}^2+\frac{1}{2}i_2\left(\dot{\theta}_1+\dot{\theta}_2\right)^2.$$

The potential energy of the system V is given by:

$$V=V_{Pendulum}+V_{InertiaWheel} \tag{5}$$

where the potential energy of the pendulum is

$$V_{Pendulum}=m_1 l_1 g\cos\theta_1,$$

and the potential energy of the inertia wheel is

$$V_{InertiaWheel}=m_2 l_2 g\cos\theta_1.$$

The Lagrangian is obtained using (4) and (5):

$$L=\frac{1}{2}I\dot{\theta}_1{}^2+\frac{1}{2}i_2\left(\dot{\theta}_1+\dot{\theta}_2\right)^2-\overline{ml}g\cos\theta_1 \tag{6}$$

where

$$I=m_1 l_1{}^2+m_2 l_2{}^2+i_1 \text{ and } \overline{ml}=m_1 l_1+m_2 l_2.$$

The following dynamics equations of the system are obtained from the Euler-Lagrange equations (1):

$$\left(I+i_2\right)\ddot{\theta}_1+i_2\ddot{\theta}_2-\overline{ml}g\sin\theta_1=0 \tag{7}$$

$$i_2\left(\ddot{\theta}_1+\ddot{\theta}_2\right)=\tau \tag{8}$$

Equations (7) and (8) can be written in a compact manner:

$$\begin{bmatrix}I+i_2 & i_2\\ i_2 & i_2\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}+\begin{bmatrix}-\overline{ml}g\sin\theta_1\\ 0\end{bmatrix}=\begin{bmatrix}0\\ \tau\end{bmatrix} \tag{9}$$

B. Linear Model

The IWP state space representation is obtained when linearizing the dynamics model (or nonlinear model) around the unstable equilibrium point (pendulum in stopped upright position). State equations are given by:

$$\begin{cases}\dot{X}=AX+Bu\\ Y=CX\end{cases} \tag{10}$$

where:

$$X=\begin{bmatrix}\theta_1 & \dot{\theta}_1 & \dot{\theta}_2\end{bmatrix}^T \text{ and } u=\tau$$

In the neighbourhood of the unstable equilibrium point $X=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$, the linearization involves the following approximation: $\sin\theta_1\approx\theta_1$. From (5) and (6), we obtain the state equation matrixes:

$$A=\begin{bmatrix}0 & 1 & 0\\ a & 0 & 0\\ -a & 0 & 0\end{bmatrix} \tag{11}$$

$$B=\begin{bmatrix}0\\ b_1\\ b_2\end{bmatrix} \tag{12}$$

$$C=\begin{bmatrix}c_{11} & 0 & 0\\ 0 & c_{22} & 0\\ 0 & 0 & c_{33}\end{bmatrix} \tag{13}$$

where:

$a=\frac{\overline{mlg}}{I}$, $b_1=-\frac{1}{I}$, $b_2=\frac{I+i_2}{Ii_2}$, and:

$c_{11}\in\{0,1\}$; $c_{22}\in\{0,1\}$ and $c_{33}\in\{0,1\}$ depending if the corresponding state parameter is observed or not.

IV. State Measurements

A. State Measurements Problem

We demonstrate in this section that the angular speed measurement accessibility of the inertia wheel is a necessary and sufficient condition in order to have a fully observable system (a permanent access to the system state), and this, whatever are the accessibilities of the other measurements.

Beside this, we also worked on a way to obtain directly the state measures. The angular position measurement of the pendulum is very difficult to obtain because it is difficult (not to say impossible) to use a rotational sensor located directly on the rail. Indeed, the IWP's rotation axis which models the contact wheel/rail is a unilateral pivot joint and not a bilateral pivot joint (see Fig. 4).

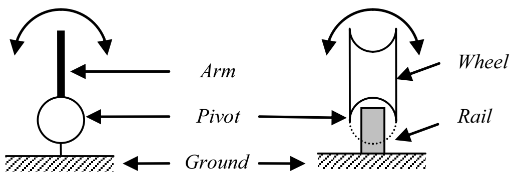

Fig. 4: (a) classical bilateral pivot joint and (b) unilateral pivot joint

There are a few alternative solutions to make this measurement, e.g. tilt sensor, accelerometer or gyroscope. But, these three sensors have got one of the following disadvantages: insensitivity to the strong dynamics, uncertain reliability, or temporal drift. Having a reliable measurement, with a very large bandwidth, is possible, by mixing one or several sensors with a calculator. We selected the FAS-G inclinometer proposed by MicroStrain, which merge data given by two accelerometers and one gyrometer, to give a good estimation of the tilting angle in real time.

To optimize the prototype, it is essential to determine, with the observability analysis, the minimum number of sensor required to know the state of the system during its operation. Defining the sensors number consists in defining the output matrix C.

B. Observability analysis

The degree of observability can be determined using the Kalman's criterion: A system is fully observable, if the rank of the observability matrix O_b is equal to the system dimension n:

$$O_b = \begin{bmatrix} C \\ CA \\ \dots \\ CA^{n-1} \end{bmatrix} \quad (14)$$

The dimension of the system is three (n=3), and the rank of matrix O_b is equivalent to the rank of matrix $\hat{O}_{b,}$ as the redundant columns of matrix O_b were supresed:

$$\hat{O}_b = \begin{bmatrix} C_{11} & 0 & 0 & 0 & aC_{22} & -aC_{33} & 0 \\ 0 & C_{22} & 0 & C_{11} & 0 & 0 & -aC_{33} \\ 0 & 0 & C_{33} & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad (15)$$

Looking at the structure of matrix $\hat{O}_b$ shows that the angular speed measurement accessibility of the inertia wheel is a necessary and sufficient condition in order to have a fully observable system, and this, whatever are the accessibilities of the other measurements. In fact, when C_{33} is equal to one, the column vectors 3, 6 and 7 of the $\hat{O}_b$ matrix are linearly independent, and then the rank of O_b is equal to the system dimension, i.e. three.

Hence, the system is fully observable as long as the velocity of the inertia wheel is measured.

C. Output Matrix

As we can see in the precedent section of this chapter, the system is fully observable when the angular speed measurement of the inertia wheel is accessible. Therefore, the output matrix C can be written in a more compact form than (13):

$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad (16)$$

We will in the section dedicated to experimental results, that even if, from a theoretical point of view, measuring the velocity of the inertia wheel is enough to estimate the full state of the system, in practice, an inclinometer has to be used to get a direct measure of the orientation angle and the orientation velocity of the pendulum.

V. OPTIMAL CONTROL DESIGN

A. Control Principle

Traditionally, the control of inverted pendulums consists in bringing the pendulum, from its stable equilibrium position to its unstable equilibrium position, and to maintain it in this state. But, for this study, we have considered only the stabilization phase of the inverted pendulum, because it has been decided *a priori* that the angular position of monorail tramway could only vary in the range of ±10° with regard to the vertical line.

To solve the problem regulation, we used the Linear Quadratic Gaussian synthesis (LQG). The control law is determined by minimizing the following quadratic criterion:

$$J = \mathrm{E}\left[\int_0^{\infty} \left(X^T Q X + u^2 R \right) dt \right] \quad (17)$$

where Q is the state weighting matrix, R is the control weighting matrix and E is the mathematical expectation.

The solution of this problem is obtained with the separation principle, i.e. using two steps:

- Firstly, the optimal state of X is searched using the Kalman filter method;
- Secondly, this estimate is used (like if it is the exact measurement of the state vector) to solve the linear optimal control problem with the Linear Quadratic synthesis (LQ).

B. Linear States Estimator

Observability analysis showed that it is possible to reconstruct all system states with a linear states estimator, when the angular speed measurement of the inertia wheel is accessible. In this case, the states estimator is a Kalman filter and its mathematical representation is given by:

$$\dot{\hat{X}} = A\hat{X} + Bu + K_f \left(Y - C\hat{X} \right) \quad (18)$$

where $\hat{X}$ is the optimal estimation of X and K_f is the estimation gain matrix.

The gain matrix K_f is given by the following equation

$$K_f = P_f C^T V^{-1} \quad (19)$$

with matrix P_f representing the positive definite solution of the following algebraic Riccati equation

$$P_f A^T + A P_f - P_f C^T V^{-1} C P_f + W = 0 \quad (20)$$

and with, $P_f = P_f^{\,T}$

where V is the output noise covariance matrix and W is the state noise covariance matrix.

To simplify the adjustment of noise covariance matrixes V and W, the value of V was set to BB^T and an adapted value of W was searched to find the adequate gain matrix K_f. Hence, the output noise covariance matrix is equal to:

$$V = \begin{bmatrix} 0 & 0 & 0 \\ 0 & b_1^{\,2} & b_1 b_2 \\ 0 & b_1 b_2 & b_2^{\,2} \end{bmatrix} \quad (21)$$

C. Linear Quadratic Synthesis

The LQ synthesis consists in searching the gain matrix K_c, such as the state feedback controller stabilizes the IPSIW around its unstable equilibrium position, and this, for whatever the disturbances interacting with the system. The state feedback controller equation is:

$$u = -K_c \hat{X} \quad (22)$$

The gain matrix K_c is given by the following equation

$$K_c = R^{-1} B^T P_c\,, \quad (23)$$

with matrix P_c representing the positive definite solution of the following algebraic Riccati equation

$$A^T P_c + P_c A - P_c B R^{-1} B^T P_c + Q = 0\,. \quad (24)$$

To simplify the adjustment of weighting matrixes Q and R, the value of Q was set to $C^T C$ and an adapted value of R was searched to find the adequate gain matrix K_c. Hence, the state weighting matrix is equal to:

$$Q=\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix} \quad (25)$$

VI. Experimental Results

A prototype of the Inertia Wheel Inverted Pendulum was built to test the different control strategies, and to validate the project feasibility with realistic mass repartitions, realistic characteristics for actuators, sensors, etc.

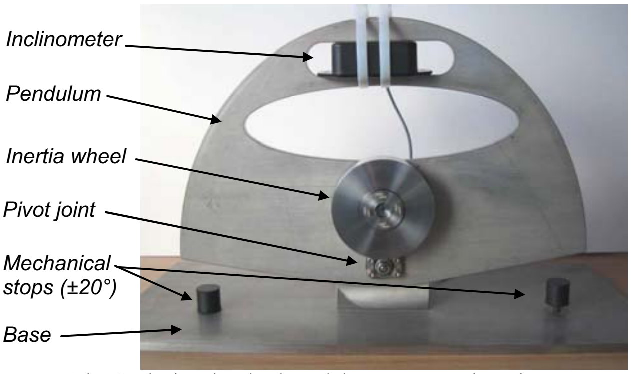

Fig. 5: The inertia wheel pendulum, prototype in action

A. Real values for the IWP prototype

Table 2 presents the real values of the IWP parameters. These values are taken from the prototype developed for the experimentations. They where decided applying a scale factor on the values of the full scale virtual prototype. It is to note that the dynamic behavior of this scale model won't be exactly the same, as the one of the full scale realization, as the intensity of gravity should also be decreased with the same scale factor (which is not possible of course).

TABLE 2
REAL VALUES FOR THE PROTOTYPE OF THE IWP

Symbol	Unit	Value
m_1	kg	3.228
m_2	kg	0.86422
l_1	m	$63.54\cdot10^{-3}$
l_2	m	$52\cdot10^{-3}$
i_1	$kg\cdot m^2$	$3.0427222\cdot10^{-2}$
i_2	$kg\cdot m^2$	$7.9866\cdot10^{-4}$

B. Experimental results

We tested the LQG control strategy explained above, and noticed that even if, from a theoretical point of view, observing the velocity of the inertia wheel is enough to estimate the state of the model, in practice it doesn't work as there might be some friction on the pivot joints disturbing the measures. So instead of using a Kalman filter to estimate the states we added an inclinometer (the FAS-G by MicroStrain, introduced in section IV.A) to measure the angular position of the pendulum and its angular velocity.

Then, we tuned the system: at first using Matlab Simulink, to get the matrix gain K_c with the methodology explained above, and in a second step, adjusting the gains by hand. We chose values to get a good compromise between respond speed (about 3 s to recover from a perturbation of amplitude 10°), system stability, and power consumption (less than 40W of mechanical power to recover from the perturbation, and a motor torque compatible with the chosen motor capabilities).

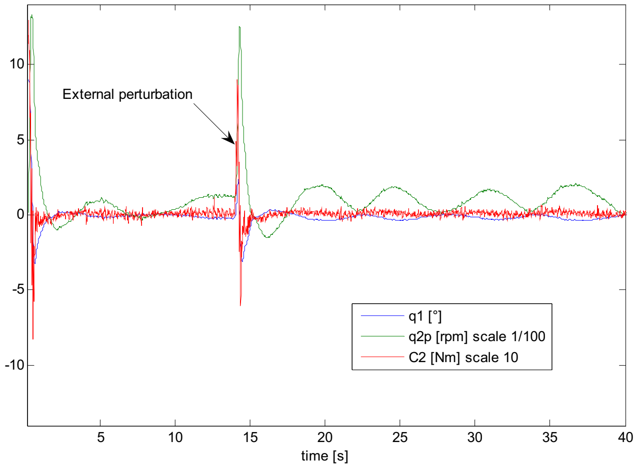

Fig. 6: Experimental results on the Inertia Wheel Pendulum

Fig. 6 shows the obtained results. The control loop was set to 10 ms. Beside some small oscillations that will be suppressed in the future, the overall behavior of the system was good. The control will be improved in the future using different control strategies, such as auto-adaptive control.

VII. Conclusion

The Monorail Tram Stabilized by an Inertia Wheel is a new futuristic transportation mean. It is an answer to traffic saturation problems at witch the agglomerations are exposed. The innovation is in the stabilization system i.e. the inertia wheel. The demonstration of the stabilization concept feasibility is done using the inverted pendulum, the most popular tool of the control theory, and more particularly, the Inertia Wheel Pendulum. The experimentation confirmed the possibility to stabilize the inverted pendulum within a range of ±10° relatively to the vertical line; and this for a realistic mass repartition modeling the monorail tram, and for a reasonable actuator. We also noticed that, even if in theory the angular velocity of the inertia wheel is enough to observe the full state of the system, in practice a sensor able to measure the orientation of the pendulum needed to be added. The Linear Quadratic control was chosen and allowed a good stabilization of the inverted pendulum, even with intense perturbations.

The experimentation has given some satisfying results but the project is not finished because it's necessary to experiment a new prototype in 3 dimensions. Indeed, we want to take into account the real masses distribution in a tram. Furthermore, we want to test another control law more robust to external disturbances e.g. the variation of mass passengers, the variation of gravity center, the wind, etc. The thought control laws are auto-adaptive control laws, where identification and control can be done at the same time.

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