

## Combinatorics of Periods in Strings

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# Combinatorics of Periods in Strings

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Here, we consider a central notion of word combinatorics and string algorithmics: the periods of a string. A *period* is an offset (*i.e.*, a shift) at which a word can overlap itself. A word may have several periods, which we call its *set of periods*, and distinct words of the same length may share the same period set. When denoted by a binary string, a period set is called the *autocorrelation* of a word. In the early 80's, Guibas and Odlyzko provided the first investigation of the structure of period sets [3, 2] and characterized them. Considering the set  $\Gamma_n$  of all period sets of strings of length  $n$  over a finite alphabet, they showed that  $\Gamma_n$  is independent of the alphabet (provided the cardinality of  $\Sigma \geq 2$ ).

Pursuing the goal of finding an enumeration algorithm for  $\Gamma_n$ , we study further the properties of  $\Gamma_n$  and exhibit the redundancy in period sets. It enables us to introduce the notion of an *irreducible period set* and to elucidate the structure of both  $\Gamma_n$  and the set of all irreducible period sets, denoted  $\Lambda_n$ . We then propose the first efficient enumeration algorithm for  $\Gamma_n$ . We also exhibit a relation between the number of binary partitions of  $n$  and the number of distinct period sets (*i.e.*, the cardinality of  $\Gamma_n$ ). It allows us to improve upon the previously known asymptotic lower bounds on the cardinality of  $\Gamma_n$  [3]. Additionally, from these results we derive a new recurrence to compute the population of a period set, as well as an algorithm to sample uniformly irreducible and classical period sets.

All above mentioned results were published in [6, 7]. Related entries of the Encyclopedia of Integer Sequences [8] are A018819 and A000123. This study has been extended to partial words [1]. The enumeration algorithm found applications for the computation of several statistics about the vocabulary of strings, like the number of missing words of length  $n$  in a text or the number of common words between two texts [4, 5].

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