

## Introduction

Path planning is used in humanoid robotics, mobile robotics, etc. Usually the planned motions have to satisfy some constraints. The violation of one constraint may destroy the robots. Therefore, the discretization of these constraints is very important to ensure the results validity and safety. So we introduce a safe way for constraint discretization.

## Path planning under constraints

To generate a path, the usual solution is to solve a constrained optimization problem which takes into account an objective function  $F(x,t)$  and some constraints  $g(x,t)$

$$\min \int F_{(x,t)} dt ; \forall j, \forall t, \in [0, T] g_{j(x,t)} \leq 0 ; x_i^{\min} \leq x_i \leq x_i^{\max}$$

## Constraints discretization

Starting from the parameter vector  $X$ , we compute the joint position, velocity, acceleration and torques and the value of the Zero Moment Point (ZMP), which are continuous functions of time.

$$(X) \rightarrow \{q(t), \dot{q}(t), \ddot{q}(t), \Gamma(t), ZMP(t)\}$$

However, the optimization algorithm needs the evaluation of these values over a time grid. Consequently these constraints must be discretized.

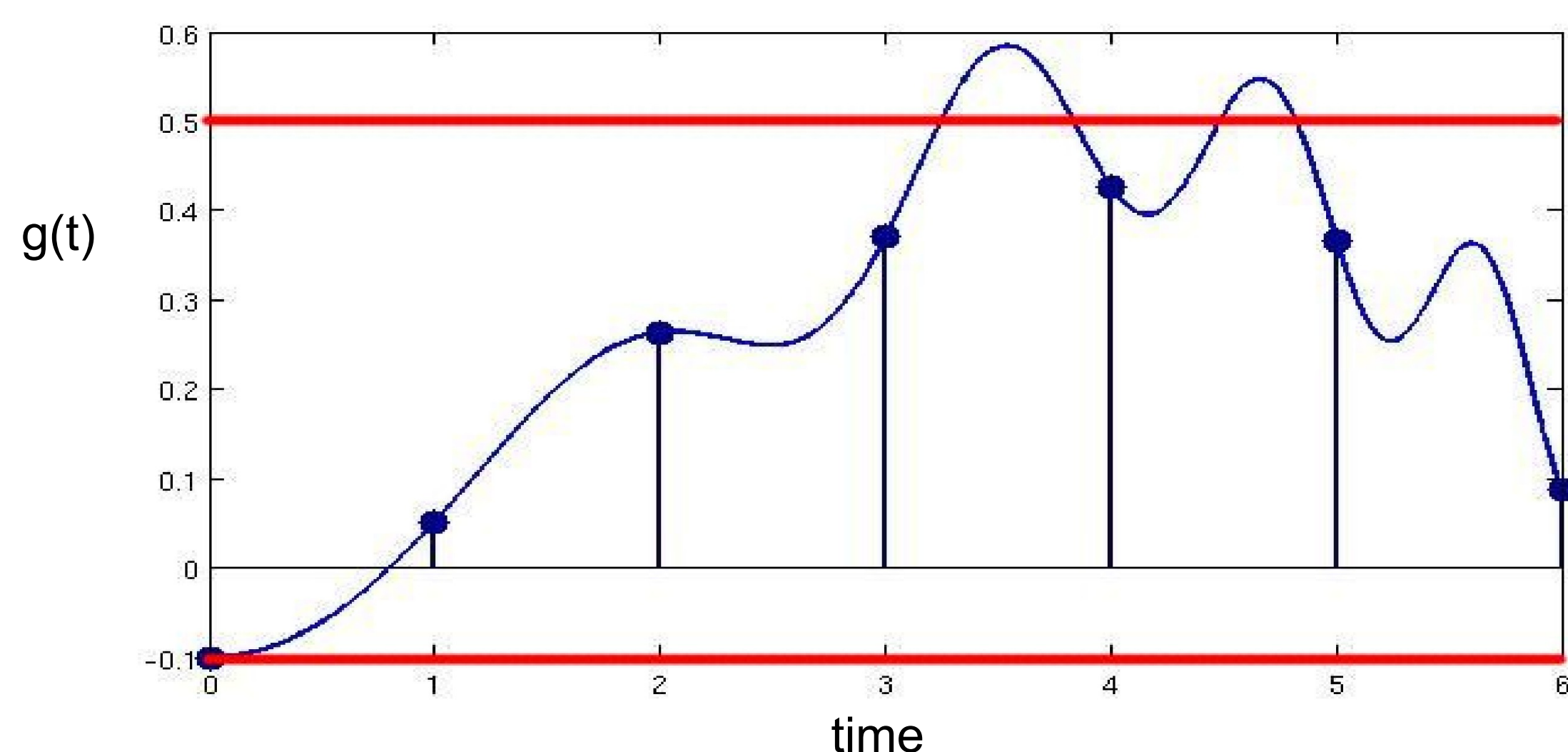


Fig 1 : Time-point discretization of a function

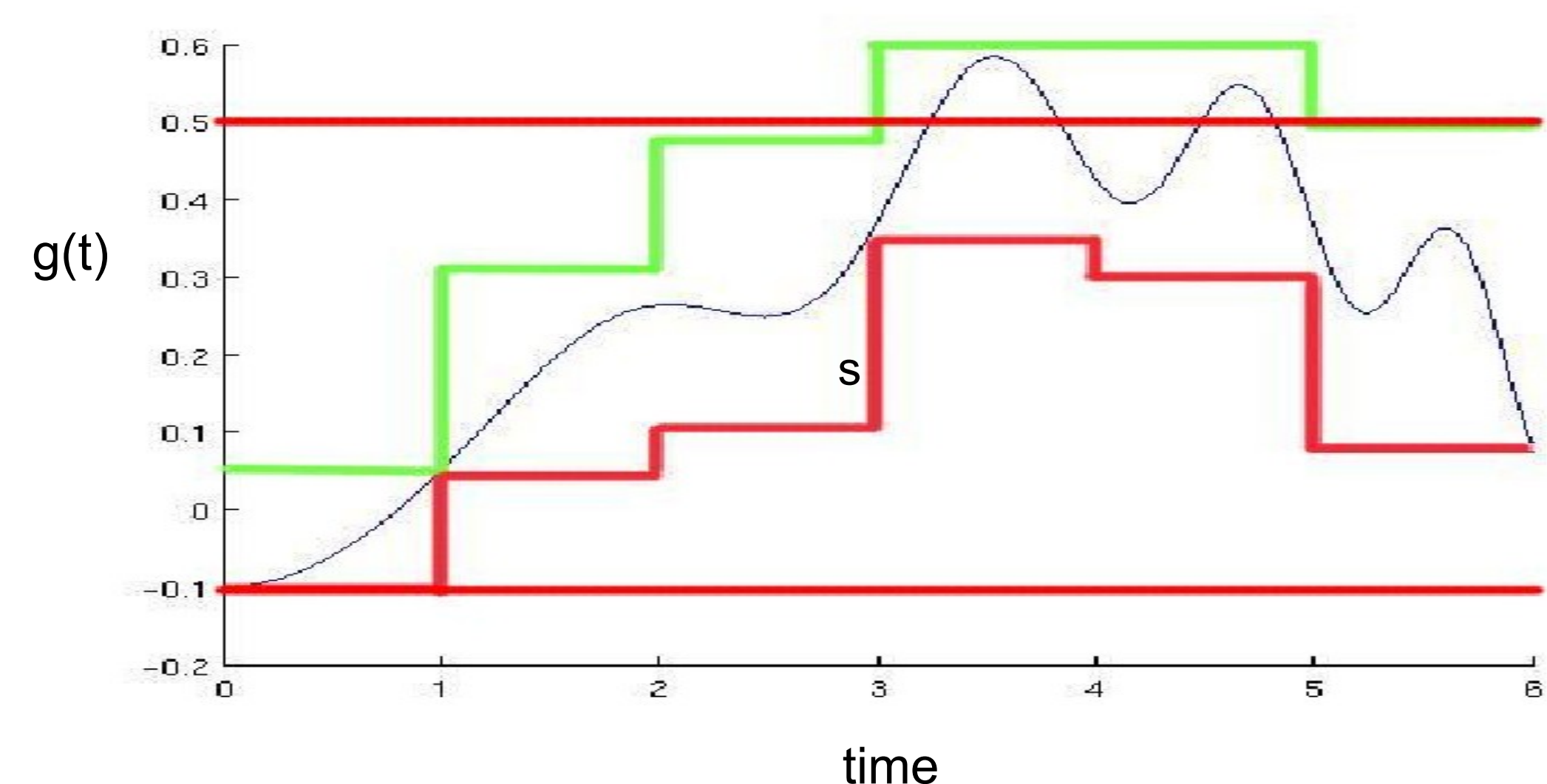


Fig 2 : Time-interval discretization of a function

Discretization is the process of transferring continuous models and equations into discrete counterparts.

Usually, discretization consists on picking up several time-points for the function to be discretized (Cf. Fig 1).

The selected values, given to the optimization algorithm, satisfy the constraints but the continuous function violates them. This solution may be considered as an optimal one, whereas it can cause the fall or the loss of the robot.

We introduce a time-interval discretization using interval analysis (Cf Fig2). The main idea is to bound the function with a minimum and a maximum value during a time-interval.

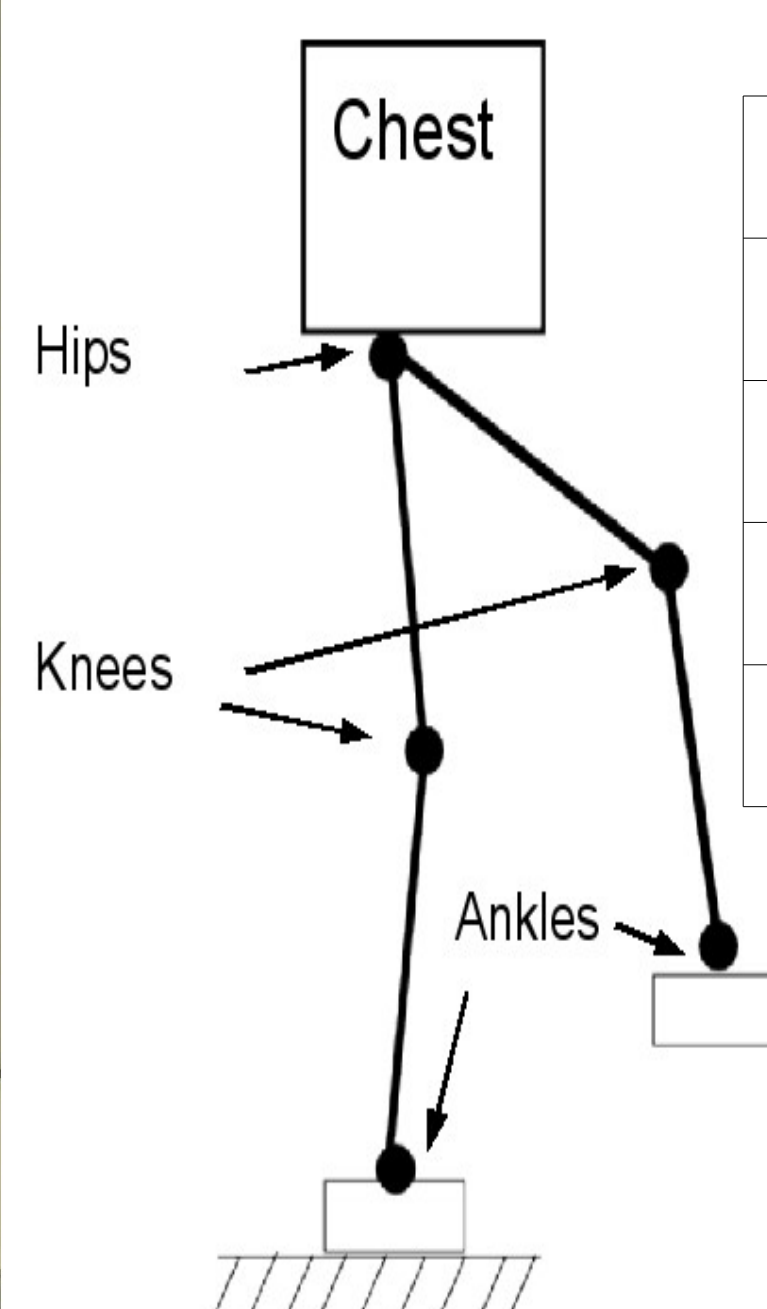
The selected values, given to the optimization algorithm, violate the constraints. So the optimization algorithm must compute a new vector of parameters ( $X$ ), which will satisfy all the constraints.

## A one-step optimization

Time-point discretization :

Nq	Nzmp	Toptim	Tmotion	Nb Viol	% Max Viol
5	5	16 s	2,33 s	1036	13,7
10	5	148 s	2,31 s	1028	7,8
20	5	424 s	2,45 s	1035	1,2
50	5	9912 s	2,42 s	621	0,2

Constraints are violated



Time-interval discretization :

Nq	Nzmp	Toptim	Tmotion	Nb Viol	% Max Viol
5	5	371 s	2,52 s	0	0
10	5	1264 s	2,27 s	0	0
20	5	574 s	2,44 s	0	0
10	10	1841 s	2,19 s	0	0

All constraints are satisfied

We want a step length of 5cm with the minimal motion duration.

We use a 2D model with 6 degrees of freedom where the upper limbs are modeled as a mass.

## Conclusion and future work

The time-interval discretization is applied for the path planning of 5cm-length step for the humanoid robot HOAP3. It allows to obtain safe paths. Future work will address gait motions with double support phases.