Nonlinear identification of skeletal muscle dynamics with Sigma-Point Kalman Filter for model-based FES

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Abstract—A model-based FES would be very helpful for the adaptive movement synthesis of spinal-cord-injured patients. For the fulfillment, we need a precise skeletal muscle model to predict the force of each muscle. Thus, we have to estimate many unknown parameters in the nonlinear muscle system. The identification process is essential for the realistic force prediction. We previously proposed a mathematical muscle model of skeletal muscle which describes the complex physiological system of skeletal muscle based on the macroscopic Hill-Maxwell and microscopic Huxley concepts. It has an original skeletal muscle model to enable consideration for the muscular masses and the viscous frictions caused by the muscle-tendon complex. In this paper, we present an experimental identification method of biomechanical parameters using Sigma-Point Kalman Filter applied to the nonlinear skeletal muscle model. Result of the identification shows its effective performance. The evaluation is provided by comparing the estimated isometric force with experimental data with the stimulation of the rabbit medial gastrocnemius muscle. This approach has the advantage of fast and robust computation, that can be implemented for online application of FES control.

I. INTRODUCTION

Functional Electrical Stimulation (FES) is well known as an effective technique to evoke artificial contractions of paralyzed skeletal muscles. It has been employed as a general method in modern rehabilitation medicine to partially restore motor function for the patients with upper neural lesions [1], [2]. Recently, the rapid progress in microprocessor technology provided the means for computer-controlled FES systems [3], [4], [5], which enable flexible programming of stimulation sequences. A fundamental problem concerning FES is to handle the high complexity and nonlinearity of the neuro-musculo-skeletal system [6], [7]. Moreover, effect such as muscle fatigue, spasticity, and limited force in the stimulated muscle complicate the control task further. The use of mathematical model would improve the development of neuropa-thetics by using optimized operation for individual patients. A mathematical model may enable to describe the relevant characteristics of the patient’s skeletal muscle and predict the precise force against certain stimulation. Therefore it can enhance the design and functions of control strategies applied to FES. Until now, a great variety of muscle models has been proposed over the years, differing in the intended application, mathematical complexity, level of structure considered, and fidelity to the biological facts. Some of them have been attempted to exhibit the microscopic or macroscopic functional behavior like Huxley [8] and Hill [9]. The distribution-moment model [10] constitutes a bridge between the microscopic and macroscopic levels. It is a model for sarcomeres or whole muscle which is extracted via a formal mathematical approximation from Huxley cross-bridge models. Models integrating geometry of the tendon and other macroscopic consideration can be found in [11]. A study, based on Huxley and Hill-Maxwell type model by Bestel-Sorine [12], proposed an explanation of how the beating of cardiac muscle may be performed through a chemical control input. It was connected to the calcium dynamics in muscle cell that stimulates the contractile element of the model. Starting with this concept, we adapted it to the striated muscle [13]. We proposed a musculotendinous model considering the muscular masses and viscous frictions in muscle-tendon complex. This model is represented by differential equations where the outputs are the muscle active stiffness and force. The model input represents the actual electrical signal as provided by the stimulator in FES.

Under general FES, you have to make detailed empirical tuning by actually stimulating the patient’s muscle for each task. If this adjustment can be calculated in the simulation, and if we can find best signal pattern using virtual skeletal muscle, it would be very helpful for the movement synthesis for paraplegic patient. However, to perform this simulation, a precise skeletal muscle model is required to produce the well-predicted force of each muscle. The skeletal muscle dynamics are highly nonlinear, and we have to identify many unknown physiological and biomechanical parameters. The principal objective of this study is then to develop an experimental identification method to identify unknown internal parameters from the limited information. This process is essential for realistic force prediction in the skeletal muscle modeling for FES. For the parameter estimation in our muscle model, the force information corresponding to isometric contractions was used along with the electrical input. Sigma-Point Kalman Filter (SPKF) was applied to the in-vivo rabbit experimental data to identify internal states in the nonlinear dynamics of skeletal muscle. SPKF has higher accuracy and consistency for nonlinear estimation than Extended Kalman Filter (EKF). The identification protocol and the detailed results are described to show the feasibility of our approach and the quality of the identification.

II. SKELETAL MUSCLE MODEL

Our approach is to provide a knowledge model based on the physiological reality to obtain meaningful internal parameters. Basically, our muscle model is composed of two elements in different nature: i) activation model which...
describes how an electrical stimulus generates an Action Potential (AP) and initiates the contraction, ii) mechanical model which describes the dynamics in force scale (Fig. 1). For the detail on the muscle modeling, previously published articles should be referenced [13], [14]. Here, a brief summary of the model and necessary information for the identification are given.

A. Activation Model

The activation model describes the electrical activity of muscle. This part of muscle model represents the excitation-contraction phenomena of muscle and is composed of two sub-models, fiber recruitment model and dynamic activation model. The static recruitment model determines the percentage of recruited motor units and depends on the pulse width \( PW \) and pulse amplitude of the signal \( I \) generated by the stimulator. The recruitment curve is usually approximated by a sigmoid function. The recruitment level determines which units are recruited or not. High threshold values correspond to motor units that are remote from the electrodes. The recruitment rate \( \alpha(PW,I) \) can be assumed as static value when \( PW \) and \( I \) remain constant.

The dynamic activation model was considered as the underlying physiological processes which describes the chemical input signal \( u \) that brings one muscle cell into contraction. Muscle contraction is initiated by an AP along the muscle fiber membrane, which goes deeply into the cell through T-tubules. It causes Calcium releases that induce the contraction process when the concentration goes above a threshold and is sustained till the concentration goes down this threshold again. Hatze [15] gives an example of Calcium dynamics \([Ca^{2+}]\) modeling. As we focus on the recruitment and the mechanical phenomena, we choose to use a simpler model that renders the main characteristics of this dynamics. The contraction-relaxation cycle is then triggered by the \([Ca^{2+}]\) to be defined, and associated with two phases: i) contraction, ii) active relaxation. We use a delayed (\( \tau \)) model to take into account the propagation time of the AP and an average time delay due to the calcium dynamics.

B. Mechanical Model

The model is based on the macroscopic Hill-Maxwell type model and the microscopic description of Huxley [8]. The link between the scales is obtained using the distribution moment technique used also by Zahalak [10]. The model is then composed of passive elements, and a Contractile Element \( E_c \) controlled by input commands: the chemical input \( u \) as suggested by Bestel-Sorine [12] for the cardiac muscle at the sarcomere scale, and the recruitment rate \( \alpha \) at the fiber scale as shown in Fig. 2.

Especially, in order to express isometric contractions whereas the skeleton is not actuated; our muscle model is introduced with masses \( m \) (kg) and linear viscous dampers \( \lambda \) (Ns/m) to ensure energy dissipation. On both sides of \( E_c \), there are elastic springs \( k_s \) (N/m) and viscous dampers to express visco-elasticity of muscle-tendon complex. The parallel element \( k_p \) mainly represents surrounding tissues, but it can be omitted in isometric contraction mode. And we assume the symmetric form that the two masses and passive elements are identical. \( L_{s0} \) and \( L_s \) (m) are the lengths of \( E_c \) and \( k_s \), especially \( L_{s0} \) and \( L_{s0} \) at the rest state. At first, we can define the relative length variation as positive when the length is increasing, as in (1). Especially, in case of isometric contraction, there is the relationship of (2):

\[
\varepsilon_s = \frac{L_s - L_{s0}}{L_{s0}} \quad \varepsilon_c = \frac{L_c - L_{c0}}{L_{c0}}
\]

\[
2L_{s0}\varepsilon_s + L_{c0}\varepsilon_c = 0
\]

The dynamical equation of one of the masses is given by (3). \( F_c \) and \( k_s \) are internal variables that express the force and the stiffness of \( E_c \) respectively. The force \( F_s \) of the whole muscle model is the sum of the spring force \( F_s \) and the damping force \( F_d \). When we measure the tension of skeletal muscle during in-vivo condition, the experimental force corresponds to \( F_c \). When we take the ratio of \( F_c \) and \( F_s \), \( L_{s0} \) is offset and it can be written as (5). (6) shows the relationinal equation in Laplace transform. From the relationship, the differential equation (7) can be obtained.

\[
mL_{s0}\ddot{\varepsilon}_s = F_c - k_sL_{s0}\varepsilon_s - \lambda L_{s0}\ddot{\varepsilon}_s
\]

\[
F_c = F_s + F_d = k_sL_{s0}\varepsilon_s + \lambda L_{s0}\ddot{\varepsilon}_s
\]

\[
\frac{F_c}{L_c} = \frac{m\ddot{\varepsilon}_s + \lambda \dot{\varepsilon}_s + k_s\varepsilon_s}{L_{s0}}
\]

\[
\mathcal{L}[F_c] = m\ddot{\varepsilon}_s + \lambda \dot{\varepsilon}_s + k_s\varepsilon_s
\]

\[
m\ddot{\varepsilon}_c + \lambda \dot{\varepsilon}_c + k_s\varepsilon_c = \lambda \ddot{\varepsilon}_c + k_sF_c
\]

Finally, in the isometric contraction, differential equations of this model can be described as follows:

\[
\dot{k}_c = -k_c|u| + \alpha k_m|u| + k_c|\varepsilon_c|
\]

\[
\dot{\varepsilon}_c = -F_c|u| + \alpha F_m|u| - F_c|\varepsilon_c| + L_{c0}k_c\varepsilon_c
\]

\[
\ddot{F}_c = -\frac{\lambda}{m} \dot{F}_c - \frac{k_s}{m} F_c + \frac{k_s}{m} F_c
\]

\[
\ddot{\varepsilon}_c = \frac{2F_c}{mL_{c0}} - \frac{k_s}{m} \dot{\varepsilon}_c - \frac{\lambda}{m} \ddot{\varepsilon}_c
\]
The dynamics of the contractile element itself correspond to (8) and (9). For the detail, you should refer to [12][13]. \( k_m \) and \( F_m \) are the maximum values for \( k_c \) and \( F_c \) respectively. From (2), (3) and (7), the differential equations of \( F_c \) and \( \varepsilon_c \) are obtained as in (10), (11). The internal state vector of this system should be set as \( x = [ k_c \ F_c \ \varepsilon_c \ \dot{\varepsilon}_c ] \).

### III. EXPERIMENTAL IDENTIFICATION

In this ongoing study, we will develop a method to identify only the parameters in the mechanical part of skeletal muscle model. The input controls of the model are the constant static recruitment rate \( \alpha \) and the chemical control input \( u \) from the activation model. These two controls are computed from FES input signal. It should be mentioned that the experiment was performed with constant FES parameters for pulse width and intensity of electrical stimulation so that the recruitment rate is constant. In addition, calcium dynamics in our model induces a time delay and an "on/off" control so that a correct data processing can get rid of this modeling. The trigger of \( u \) signal can be calculated by the timing of electrical stimulation.

In isometric contraction, the differential equations of skeletal muscle dynamics are straightforwardly given in (8)-(11). In this case, \( k_c \ F_c \ \varepsilon_c \) are unknown time-varying values and \( m \ \lambda \ L_{a0} \) are unknown static parameters to be estimated. For the identification of this model, it is a nonlinear state-space model, and many state-variables are not measurable. And then in-vivo experimental data includes some noises. That is why we need an efficient recursive filter that estimates the state of a dynamic system from a series of noisy measurements.

#### A. Sigma-Point Kalman Filter

For this kind of nonlinear identification, Extended Kalman Filter (EKF) was well-known as standard method. In EKF, the nonlinear equation should be linearized to the first order with partial derivatives (Jacobian matrix) around a mean of the state. The optimal Kalman filtering is then applied to the linearized system. When the model is highly nonlinear, EKF may give particularly poor performance and an easy divergence. In skeletal muscle dynamics, its state-space is dramatically changed between contraction and relaxation phase. At this time, partial derivatives will be incorrect due to the discontinuity. Therefore, we introduced Sigma-Point Kalman Filter (SPKF). The initial idea was proposed by Julier [16], and well described by Merwe [17]. SPKF uses a deterministic sampling technique known as the unscented transform to pick a minimal set of sample points (called sigma points) around the mean. These sigma points are propagated through the true nonlinearity. This approach results in approximations that are accurate to at least the second order in Taylor series expansion. In contrast, EKF results only in first order accuracy.

An outline of the SPKF algorithm is described. For the detail, you should refer to [17][18]. The general Kalman framework involves estimation of the state of a discrete-time nonlinear dynamic system,

\[
\begin{align*}
\mathbf{x}_{k+1} &= \mathbf{f}(\mathbf{x}_k, \mathbf{v}_k) \quad (12) \\
\mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k, \mathbf{n}_k) \quad (13)
\end{align*}
\]

where \( \mathbf{x}_k \) represents the internal state of the system to be estimated and \( \mathbf{y}_k \) is the only observed signal. The process noise \( \mathbf{v}_k \) drives the dynamic system, and the observation noise is given by \( \mathbf{n}_k \). The filter starts by augmenting the state vector to \( L \) dimensions, where \( L \) is the sum of dimensions in the original state, model noise and measurement noise. The corresponding covariance matrix is similarly augmented to a \( L \) by \( L \) matrix. In this form, the augmented state vector \( \bar{\mathbf{x}}_k \) and covariance matrix \( \mathbf{P}_k \) can be defined as in (14)-(15).

\[
\begin{align*}
\bar{\mathbf{x}}_k &= E[\mathbf{x}_k] = [\tilde{\mathbf{x}}_k^T \ \mathbf{v}_k^T \ \mathbf{n}_k^T]^T \quad (14) \\
\mathbf{P}_k &= E[(\mathbf{x}_k - \bar{\mathbf{x}}_k)(\mathbf{x}_k - \bar{\mathbf{x}}_k)^T] = \left[ \begin{array}{ccc} \mathbf{P}_{x_k} & 0 & 0 \\ 0 & \mathbf{R}_{v_k} & 0 \\ 0 & 0 & \mathbf{R}_{n_k} \end{array} \right] \quad (15)
\end{align*}
\]

where \( \mathbf{P}_x \) is the state covariance, \( \mathbf{R}_v \) is the process noise covariance, \( \mathbf{R}_n \) is the observation noise covariance.

In the process update, the \( 2L+1 \) sigma points are computed based on a square root decomposition of the prior covariance as in (16), where \( \gamma = \sqrt{L + \lambda} \) and \( \lambda \) is found using \( \lambda = \alpha^2(L + \kappa) - \kappa \) is chosen in \( 0 < \alpha < 1 \) which determines the spread of the sigma-points around prior mean and \( \kappa \) is usually chosen equal to 0. The augmented sigma point matrix is formed by the concatenation of the state sigma point matrix, the process noise sigma point matrix, and the measurement noise sigma point matrix, such that \( \mathbf{X}^a = [(\mathbf{X}^x)^T \ (\mathbf{X}^v)^T \ (\mathbf{X}^m)^T]^T \). The sigma point weights to be used for mean and covariance estimates are defined as in (17). The optimal value of 2 is usually assigned to \( \beta \).

\[
\begin{align*}
\mathbf{X}_{0,k-1}^a &= \tilde{\mathbf{x}}_k^+ - 1 \\
\mathbf{X}_{i,k-1}^a &= \tilde{\mathbf{x}}_k^+ + \gamma \left( \sqrt{\mathbf{P}}_{k-1} \right)_i \quad i = 1, \ldots, L \quad (16) \\
\mathbf{X}_{i,k-1}^a &= \tilde{\mathbf{x}}_k^+ - \gamma \left( \sqrt{\mathbf{P}}_{k-1} \right)_{i-L \ldots 0} \quad i = L+1, \ldots, 2L \\
\omega_0^a &= \lambda/(L + \lambda) \\
\omega_i^a &= \omega_0^a + (1 - \alpha^2 + \beta) \\
\omega_i^a &= \omega_0^a + 1/(2(L + \lambda)) \quad i = 1, \ldots, 2L
\end{align*}
\]

where \( \left( \sqrt{\mathbf{P}}_{k-1} \right)_i \) is the \( i \)-th column of the square root of the covariance \( \mathbf{P}_{k-1} \). The square root of a symmetric matrix is typically calculated by Cholesky factorization. Then these sigma-points are propagated through the nonlinear function. Predicted mean and covariance are computed as in (19)(20) and predicted observation is calculated like (22).

\[
\begin{align*}
\mathbf{X}_{i,k-1}^x &= \mathbf{f}(\mathbf{X}_{i,k-1}^x, \mathbf{X}_{i,k-1}^m) \quad (18) \\
\bar{\mathbf{x}}_k &= \sum_{i=0}^{2L} \omega_i^a \mathbf{X}_{i,k-1}^x \\
\mathbf{P}_{x_k} &= \sum_{i=0}^{2L} \omega_i^a (\mathbf{X}_{i,k-1}^x - \bar{\mathbf{x}}_k) (\mathbf{X}_{i,k-1}^x - \bar{\mathbf{x}}_k)^T \quad (20)
\end{align*}
\]
\[ Y_{k|k-1} = h(Y_{k|k-1}^+, Y_{k|k-1}^-) \]  
\[ \hat{y}_k^- = \sum_{i=0}^{2L} \omega_i \gamma_{i,k|k-1} \]  

The predictions are then updated with new measurements by first calculating the measurement covariance and state-measurement cross correlation matrices, which are then used to determine the Kalman gain. Finally, updated estimate and covariance are decided through this kalman gain as below.

\[ P_{\hat{y}_k} = \sum_{i=0}^{2L} \omega_i (\Delta_{i,k|k-1}^- - \hat{y}_k^-)(\Delta_{i,k|k-1}^- - \hat{y}_k^-)^T \]  
\[ P_{x_k | y_k} = \sum_{i=0}^{2L} \omega_i (\Delta_{i,k|k-1}^- - \hat{x}_k^-)(\Delta_{i,k|k-1}^- - \hat{x}_k^-)^T \]  
\[ K_k = P_{x_k | y_k} P_{\hat{y}_k}^{-1} \]  
\[ \hat{x}_k = \hat{x}_k^- + K_k (y_k - \hat{y}_k^-) \]  
\[ P_{x_k} = P_{x_k}^- - K_k P_{\hat{y}_k} K_k^T \]

These process update and measurement update should be recursively calculated in \( k = 1, ..., \infty \) until the end point of the measurement.

B. Experimental data for identification

To obtain the experimental data, the Medial Gastrocnemius (MG) muscle of a rabbit was investigated and the muscle force against the electrical stimulation was measured in isometric conditions. It was performed on New-Zealand white rabbits at SMI (research center for Sensory-Motor Interaction) in the Aalborg University as depicted in Fig. 3. Anesthesia was induced and maintained with periodic intramuscular doses of a cocktail of 0.15mg/kg Midazolam (Dormicum, Alpharma A/S), 0.03mg/kg Fentanyl and 1mg/kg Fluraxan (combined in Hypnorm, Janssen Pharmaceutica) [19]. The left leg of the rabbit was anchored at knee and ankle joints to a fixed mechanical frame using bone pins placed through the distal epiphyses of the femur and tibia. Tendon of medial gastrocnemius muscle was attached to the arm of a motorized lever system (Dual-mode system 310B Aurora Scientific Inc.) as shown in Fig. 4. The position and force of the lever arm were recorded. An initial muscle-tendon length was established by flexing the ankle to 90°. A bipolar cuff electrode was implanted around the sciatic nerve, allowing to stimulate the MG muscle. Data acquisition was performed with 48 kHz sampling rate. For the identification of mechanical parameters, the signal input was synchronized to the beginning of the force response so that the delays induced by action potential propagation and calcium dynamics were cancelled.

IV. RESULT OF IDENTIFICATION

In order to facilitate the convergence of the identification, the estimation process has been split into two steps. In the first step, we only estimate geometrical parameter \( Lc0 \). In the second step, we estimate the dynamic parameters: \( m \) and \( \lambda \). The biomechanical muscle model to be identified is presented as (8)-(11). We define the state vectors for geometric and dynamic estimations in SPKF as follows:

\[ \begin{bmatrix} x_G \\ x_A \end{bmatrix} = \begin{bmatrix} k_c & F_c & F_e & \varepsilon_c & \ddot{\varepsilon}_c & Lc0 \\ k_c & F_c & F_e & \varepsilon_c & \ddot{\varepsilon}_c & m & \lambda \end{bmatrix} \]

A. Parameter Identification

The stimulation signal input used for the estimation is composed of two successive pulses at 20Hz which amplitude is 105µA and pulse width is 300µs. The stimulation signal input is used to prepare the two control inputs \( (\alpha, \nu) \) of the mechanical model. The current amplitude and pulse width were selected to recruit the maximum of muscular fibers, then \( \alpha \) is approximated to 90% for the fiber recruitment. The stiffness \( k_c \) has been estimated separately from the experiments achieved on the isolated muscle after the force acquisition. The stiffness is taken as equal to the slope of linear line of the passive length-force relationship. \( k_{c0} \) was 4200N/m. \( F_m \) and \( k_m \) can be obtained knowing the force response of muscle to a stimulation pattern with maximum value of signal parameters (in frequency, amplitude, pulse width). In this case, \( k_{m0}=10000N/m \) and \( F_m=15N \) were used.

The experimental muscle force against the doublet stimulation is plotted in Fig.5 with red line. It was used for parameter estimation as measurement updates for \( F_e \) in SPKF. The blue curve is the estimated muscle force of \( F_e \). The part surrounded by the rectangle was magnified at the upper right side to indicate that the resultant estimated \( F_e \)
was well filtered against the noisy experimental data. Fig. 6 is the error covariance for $F_c$. Figs. 7 and 8 show the estimated parameter $L_{c0}$ and the error covariance. Both of figures show the convergence of the estimation. The evolutions of internal state values for $\varepsilon_c$ and $F_c$ are obtained as in Figs 9 and 10. From these behaviors, we can confirm that the contractile element of the model is successfully shrunken following the dynamics of differential equations under the estimation process.

After the complete estimation process for geometric and dynamic parameters, the estimated values are: $L_{c0} = 6.86$ cm, $m = 19.2$ g, $\lambda = 19.4$ Ns/m. As seen from resulted computational behavior in graphs, the internal state vectors of skeletal model converged well to stationary values. We tested the estimation from 4 different values for initial states, same results could be obtained in stable conditions. The estimated length of contractile element showed close value to its measured length 6.5cm of the isolated skeletal muscle.

B. Model Cross-validation

A cross-validation of the identified model was carried out to confirm the validity of this method on data that have not been used for the estimation. The resultant muscle force was simulated using the identified values and the information of stimulation such as electrical intensity, pulse width and
frequency. Fig. 11 shows the measured force response of the MG muscle of the rabbit and the simulated force with the identified model. The model could predict the nonlinear force properties of stimulated muscle quite well. There is a good agreement between the measured and the predicted force, but when the muscle fatigue appears, the experimental force is lower than that of simulation. The small difference at the third wave can be considered as an error coming from modeling without fatigue factor. However, we could confirm the effectiveness of the identification and it would contribute for the realistic force prediction. For expression of fatigue phenomena [20], we need to make a further investigation for the reproduction.

V. CONCLUSIONS

An identification method for biomechanical parameters of nonlinear skeletal muscle model has been proposed and developed. This method is based on the in-vivo experimental data acquisition. It could contribute for the prediction of the nonlinear force of stimulated muscle under FES. The comparison of experimental muscle force and the simulated force shows the feasibility of the identification. In this study, the estimation was computed by Sigma-Point Kalman Filter. We tested it also by Extended Kalman Filter [14]. In EKF, it highly depends on initial values of state vector and it was not so easy to get the converged results. SPKF gives much more stable performance than EKF. SPKF has a great advantage for high nonlinear system. In addition, this approach performs so fast computation that it could be implemented for online application of FES control. In the actual application for FES, some human parameters are gradually changed following the physiological condition of the patient. Therefore, the function of on-line identification is considered as a key factor for the future of model-based FES.

The future work will concern the identification protocols based on experiments on paraplegic patients in different modes: isometric, isotonic, isokinetic contractions. From the representative data set like the torque, the angle of the joint, and the EMG during the stimulation, we aim at identifying the model and obtaining the inner unknown parameters to enable an accurate muscle force prediction under FES for paraplegia.

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