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# ACCELEROMETER BASED IDENTIFICATION OF MECHANICAL SYSTEMS

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## Abstract

*This paper deals with a comparison of sensor location and nature in the identification of physical parameters for mechanical systems with lumped elasticities. The identification model is a linear model in relation to a minimal set of parameters. The dynamic parameters are estimated by using the solution of weighted least squares of an over determined linear system obtained from the sampling of the dynamic model along a closed loop tracking trajectory. An experimental study exhibits the identification results depending on two types of sensors (position, acceleration) and different locations (motor, load).*

## I. Introduction

The needs in term of precision and velocity of complex machines (machine tools, robots, ...), require an accurate dynamic modelling to increase the quality of their simulation to improve their design and their control [1]. The presented works concern the identification of the inertia, stiffness and frictions parameters of the dynamic model for these systems. In the last years, the use of subspaces identification methods for the estimation of flexible modes through the direct dynamic model are the object of numerous researches [2][3]. However, most of these investigations do not underline the difficulty to estimate physical parameters. The identification technique presented here uses the inverse dynamic model of the system which is a linear model with respect to the dynamic parameters. These parameters are estimated by a method of weighted least squares [4][5][6]. In the case of flexible systems, flexible degrees of freedom are not all measured. It is a major difficulty compared with the rigid multi body systems where all the joint positions are measured. The proposed method takes into account this problem. More particularly, we focus on the influence of

the location and the type of sensors on the identification results of physical parameters. This article is divided into four sections. Section 2 deals with the modelling of flexible mechanical systems. Section 3 presents the method of identification by weighted least squares and the practical aspects of the method in term of data acquisition and filtering. Section 4 is dedicated to the experimental closed loop identification of a linear machine tools axis. The study shows the identification results depending on two types of sensors, i.e. position and/or acceleration, and different locations i.e. on motor and/or load.

## II. Modelling

### A. Dynamic model

To illustrate our approach, without loss of generality, this paper will deal, with the identification of mechanical systems modelled by two inertia and one stiffness. The equations of Newton-Euler allow to calculate the inverse model expressing the motor torque according to the state and to its derivatives:

$$\Gamma_m = J_m \ddot{q}_m + F_{vm} \dot{q}_m + F_{sm} \text{sign}(\dot{q}_m) + K(q_m - q_c) + F_{vmc} (\dot{q}_m - \dot{q}_c) \quad (1)$$

$$0 = J_c \ddot{q}_c - K(q_m - q_c) - F_{vmc} (\dot{q}_m - \dot{q}_c) \quad (2)$$

Where:  $q_m$ ,  $\dot{q}_m$ ,  $\ddot{q}_m$  are respectively the position, the velocity and the acceleration of the motor rotor.  $F_{vm}$ ,  $F_{sm}$  are respectively the global coefficients of viscous frictions and Coulomb frictions of the motor rotor.  $J_m$  is the of inertia of the motor rotor.  $K$  is the global stiffness of the mechanical transmission between the motor rotor and the load, which is characterised by a model reduced to the first flexible mode.  $q_c$ ,  $\dot{q}_c$ ,  $\ddot{q}_c$  are respectively the position, the velocity and the acceleration of the load.

$F_{vmc}$  is the coefficient of internal damping of materials which compose the kinematic chain.  $J_c$  is the inertia of the load.

## B. Standard identification model

The dynamic model (1) and (2) can be rewritten in a relation linear to the dynamic parameters as following:

$$y_s = D_s X_s \quad (3)$$

When the measurements of the elastic variable and the rigid variable are available, the equation (7) is obtained from (1) and (2) with:

$$y_s = \begin{bmatrix} \Gamma_m \\ 0 \end{bmatrix} \quad (4)$$

$$D_s = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = \begin{pmatrix} \ddot{q}_m & \dot{q}_m & \text{sign}(\dot{q}_m) & 0 & (\dot{q}_m - \dot{q}_c) & (q_m - q_c) \\ 0 & 0 & 0 & \ddot{q}_c & -(\dot{q}_m - \dot{q}_c) & -(q_m - q_c) \end{pmatrix} \quad (5)$$

$$X_s = (J_m \quad F_{vm} \quad F_{sm} \quad J_c \quad F_{vmc} \quad K)^T \quad (6)$$

## C. Minimal model of identification

The dynamic model (1) and (2) leads to a minimal model of identification which depends on available measurements and which is linear in relation to the parameters to be identified:

$$y = D X \quad (7)$$

$y$  is the measurement vectors,  $D$  is called the regressor,  $X$  is the vector of the unknown parameters.

# III. Identification method

## A. Weighted Least Squares

The method of identification developed for the manipulator robots is applied for the flexible systems. The vector  $X$  is estimated as the solution of the Weighted Least Squares (WLS) of an over determined system obtained from the sampling at the various moments  $t_i$ ,  $i=1, \dots, r=ne$  of the system (7) [4][5][6][7]:

$$Y = W X + \rho \quad (8)$$

Where:  $W$  ( $rxNp$ ) is the observation matrix obtained by sampling of  $D$ ,  $Np$  is the number of parameters to identify,  $Y$  is a vector ( $rx1$ ) which contains the motor

torque,  $\rho$  is the vector of the errors ( $rx1$ ).

The unicity of the solution depends on the rank of the observation matrix. The loss of rank can come from two origins:

- A structural rank deficiency which stands for any samples in  $W$ . This problem of identifiability is resolved by using the basic parameters which supply a minimal representation of the model [9][10].

- data rank deficiency due to a bad choice of noisy samples in  $W$ . This is the problem of optimal measurement strategies which is solved using closed loop identification to track exciting trajectories [8][11][12].

Calculating the WLS solution of (8) from noisy discrete measurements or estimations of derivatives, may lead to bias because  $W$  and  $Y$  are non independent random matrices. Then it is essential to filter data in  $Y$  and  $W$ , before computing the WLS solution.

## B. Acquisition and filtering of the data

Joints velocities and accelerations are estimated by a band pass filtering  $F$  of the position, obtained by the product of a low pass filter  $H$  in both the forward and reverse direction (Butterworth) and from a derivative filter  $D_c$ , obtained by a central difference algorithm, without phase shift. The cut-off frequency  $\omega_H$  of the low pass filter  $H$  should be chosen to avoid any distortion of magnitude on the filtered signals in the range  $[0, \omega_{dyn}]$ . A second filtering is implemented to eliminate the high frequencies noises in the motor torque. The vector  $Y$  and each column of  $W$  are filtered (parallel filtering) by a low pass filter and are resampled at a lower rate, keeping one sample over  $n_d$  because there is no more signal in the range  $[\omega_H, \omega_s/2]$ . This step is not sensitive to filter distortion because error introduced by this filtering process is the same in each member of the linear system (8).

## C. Tuning of filters

The key point is to choose the cut-off frequency  $\omega_H$  and  $\omega_s$  to keep useful signal of the dynamic behavior of the system in the filter bandwidth. In [5], the author proposes to choose the sampling frequency  $\omega_s$  of measurements in practice, if possible, such as:

$$\omega_s \geq 100\omega_{dyn} \quad (9)$$

Where  $\omega_{dyn}$  is the bandwidth of the position closed loop.

A strategy of tuning for the frequency  $\omega_H$  and the sampling frequency  $\omega_s$  is presented in [4]. This method suggests to bound the distortion of amplitude introduced by the derivative filter and the low pass filter at a frequency fixed with regard to the dynamics of the system.

#### IV. Experimental identification

An experimental identification is performed on a didactic testing bed EMPS300 (ElectroMechanical Positioning System) similar to a linear axis of robot or machine tool. This testing bed is composed of the following elements (figure 1):

- An electronic of power constituted by a four quadrants converter feeding a DC motor with permanent magnets by a pulse width modulated (PWM) tension with a frequency of 17KHz. A loop of current allows the control in current (and in torque) of the motor with a bandwidth at  $-45^\circ$  of 630Hz,
- A DC motor with permanent magnets, an encoder and a flexible coupling,
- A load in translation,
- An accelerometer, placed on the load, supplies an information about the acceleration of the load,
- An incremental encoder, placed in the extremity of the screw, supplies an information about the angular position of the screw.

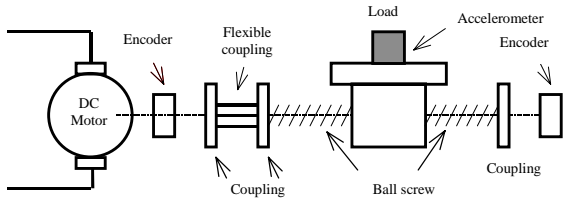


Fig. 1: Experimental device

Figure 2 gives a mass-spring representation of the system. This class of system can be also described by using the modelling of the flexible manipulator robots generalized to multi-bodies systems with lumped elasticity [13]. The use of a symbolic calculation software dedicated to the robotics such as SYMORO + [14] allows to generate in a systematic, fast and optimized way the geometrical, kinematic and dynamic models.

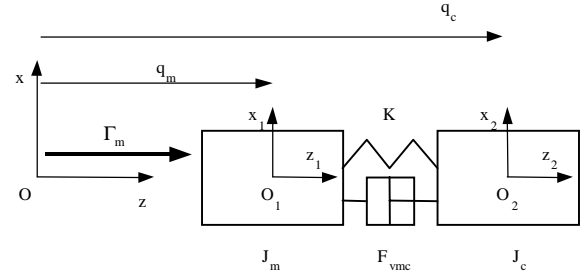


Fig. 2: Mass-spring representation of the system

The sampling frequency for the acquisition of the measurements is equal to 5kHz, which corresponds to about 50 times the frequency of the flexible mode of the system. A closed loop identification, using classical position and velocity feedback control, has been performed. A chirp sweeping a bandwidth around the estimated frequency of the flexible mode which is estimated to  $f_1 = 100\text{Hz}$  and speed trapezoidal trajectories are used to identify the process.

##### A. Identification with load and motor positions

The identification model is given by (4)(5)(6). The results of the experimental identification are reported in the table 1. The estimated parameters are given with their confidence interval and their relative standard deviation. Standard deviations  $\sigma_{\hat{x}_i}$  are estimated using classical and simple results from statistics, considering the matrix  $W$  to be a deterministic one, and  $p$  to be a zero mean additive independent noise, with standard deviation  $\sigma_p$  such that :

$$C_{pp} = \sigma_p^2 I_{rxr} \quad (10)$$

Where  $I_{rxr}$  is the matrix identity ( $rxr$ ).

The covariance matrix of the estimation error and standard deviations can be calculated by:

$$C_{\hat{x}\hat{x}} = \sigma_p^2 [W^T W]^{-1} \quad (11)$$

$\sigma_{\hat{x}_i}^2 = C_{\hat{x}\hat{x}_{ii}}$ , is the  $i^{\text{th}}$  diagonal coefficient of  $C_{\hat{x}\hat{x}}$ .

The relative standard deviation  $\% \sigma_{\hat{x}_r}$  is given by:

$$\% \sigma_{\hat{x}_r} = 100 \frac{\sigma_{\hat{x}_i}}{\hat{X}_i} \quad (12)$$

A parameter with  $\% \sigma_{\hat{x}_r} \geq 10 \%$  can be removed from the model because it is not identifiable on the given trajectory and it poorly increases the relative norm error. The maximum order of derivatives is  $nd=2$ . Experimental results in table 1 show that a good estimation is obtained with a Butterworth frequency  $\omega_H = 1443 \text{ rad.s}^{-1}$ . From the table 1, we notice that the dynamic parameter  $J_m$ ,  $F_{vm}$ ,  $F_{sm}$ ,  $J_c$ ,  $K$  presents a very small relative standard deviation, which translates the good identification of these parameters. The coefficient  $F_{vmc}$  is identifiable but the confidence granted to the identified value is lesser high than for the other parameters.

Parameters	$\hat{X}$	$2^* \sigma_{\hat{x}}$	$\% \sigma_{\hat{x}_r}$
$J_m \text{ (kg.m}^2\text{)}$	9.87e-6	2.98e-8	0.1510
$F_{vm} \text{ (N.m/(rad/s))}$	5.64e-5	1.29e-6	1.1419
$F_{sm} \text{ (N.m)}$	3.61e-3	9.47e-5	1.3125
$J_c \text{ (kg.m}^2\text{)}$	7.08e-7	3.82e-8	2.6959
$F_{vmc} \text{ (N.m/(rad/s))}$	2.31e-3	2.54e-5	0.5495
$K \text{ (N.m/rad)}$	2.96e-1	1.39e-2	2.3424

TAB. 1 : Identification results

## B. Identification with load acceleration and motor position

A more realistic alternative concerning the measurement of flexible degrees of freedom in industrial applications is to use an accelerometer to measure the load acceleration. The position measurement of the load is now assumed not available. The measured load acceleration could be integrated twice to estimate the velocity and the position but it raises the problem of the estimation of initial conditions. An other solution is to use the derivative of the torque to proceed to the identification. The non linear function sign in (1) is a problem for the derivative. We suggest doing an identification by using an uncoupled excitation of the mechanical parameters of the system. The minimal model of identification is given by:

$$y = DX = \begin{bmatrix} y^1 \\ y^2 \end{bmatrix} = \begin{bmatrix} D^1 \\ D^2 \end{bmatrix} X \quad (13)$$

Where  $D^1$  and  $D^2$  are the regressors associated to two excitations,  $y^1$  is a measurement vector coming from the chirp signal.  $y^2$  is a measurement vector built from the tracking of speed trapezoidal trajectories. This motion

allows to identify the Coulomb frictions without exciting in a significant way the flexibility. The expression (13) is defined by:

$$y = \begin{pmatrix} y^1 \\ y^2 \end{pmatrix} = \begin{pmatrix} \ddot{\Gamma}_m \\ 0 \\ -\dot{\Gamma}_m \end{pmatrix} \quad (14)$$

$$D = \begin{pmatrix} D^1 \\ D^2 \end{pmatrix} = \begin{pmatrix} \ddot{q}_m & \ddot{q}_m & 0 & 0 & (\ddot{q}_m - \ddot{q}_c) & (\ddot{q}_m - \ddot{q}_c) \\ 0 & 0 & 0 & \ddot{q}_c & -(\ddot{q}_m - \ddot{q}_c) & -(\ddot{q}_m - \ddot{q}_c) \\ \ddot{q}_m & \ddot{q}_m & \text{sign}(\dot{q}_m) & \ddot{q}_c & 0 & 0 \end{pmatrix} \quad (15)$$

$$X = (J_m \quad F_{vm} \quad F_{sm} \quad J_c \quad F_{vmc} \quad K)^T \quad (16)$$

For this model the maximum order of derivatives is  $nd=4$ . Experimental results in table 2 are obtained with a Butterworth frequency  $\omega_H = 1394 \text{ rad.s}^{-1}$ .

Parameters	$\hat{X}$	$2^* \sigma_{\hat{x}}$	$\% \sigma_{\hat{x}_r}$
$J_m \text{ (kg.m}^2\text{)}$	9.24e-6	7.73e-8	0.4184
$F_{vm} \text{ (N.m/(rad/s))}$	3.87e-5	1.77e-6	2.2844
$F_{sm} \text{ (N.m)}$	6.23e-3	1.35e-4	1.0839
$J_c \text{ (kg.m}^2\text{)}$	6.76e-7	5.46e-8	4.0356
$F_{vmc} \text{ (N.m/(rad/s))}$	4.86e-4	3.27e-5	3.3644
$K \text{ (N.m/rad)}$	3.80e-1	3.40e-2	4.4696

TAB. 2 : Identification results

## C. Experimental validation

The validation of the identification consists in comparing the experimental signals of the position, and the current with those obtained by simulation of the direct model (17)(18)(19) in closed loop:

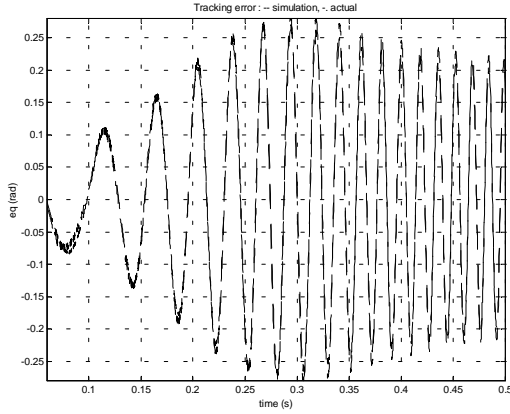
$$\begin{pmatrix} \dot{q}_m \\ \dot{q}_c \\ \ddot{q}_m \\ \ddot{q}_c \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K}{J_m} & -\frac{K}{J_m} & -\frac{F_{vmc} + F_{vm}}{J_m} & -\frac{F_{vmc}}{J_m} \\ \frac{K}{J_c} & -\frac{K}{J_c} & \frac{F_{vmc}}{J_c} & -\frac{F_{vmc}}{J_c} \end{pmatrix} \begin{pmatrix} q_m \\ q_c \\ \dot{q}_m \\ \dot{q}_c \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{J_m} \\ 0 \end{pmatrix} \Gamma_m + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{J_m} \\ 0 \end{pmatrix} \Gamma_c \quad (17)$$

$$y = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} q_m \\ q_c \\ \dot{q}_m \\ \dot{q}_c \end{pmatrix} \quad (18)$$

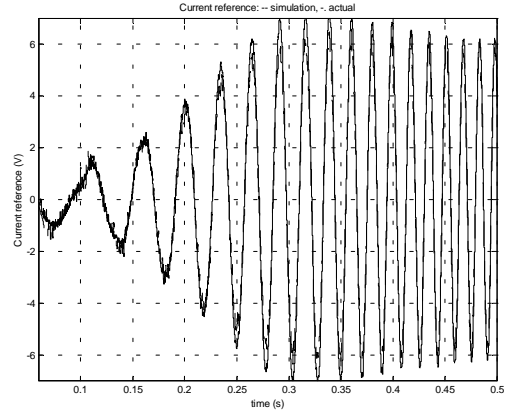
$$\Gamma_C = F_{sm} \text{sign}(\dot{q}_m) \quad (19)$$

The experimental results for the identification model (4) (5)(6) (and respectively (14)(15)(16)) are used to simulate the model of the figure 3 (4). On figures 3 and 4, we present a comparison between the simulated and

the actual tracking error of the load (Figures 3.a and 4.a). Figures 3.b and 4.b are dedicated to a comparison between the simulated and the actual motor current. The figures 3 and 4 show the simulation and the measurements are very close, this means a good identification of the parameters for the bench EMPS300.

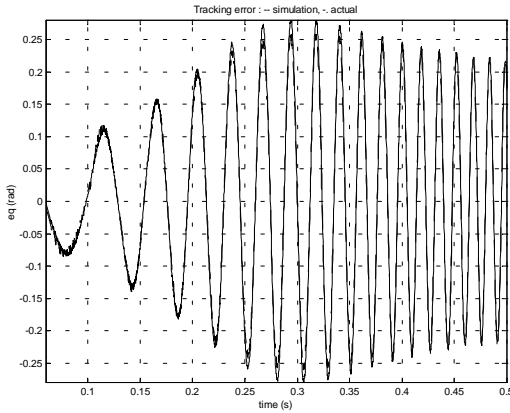


3.a)

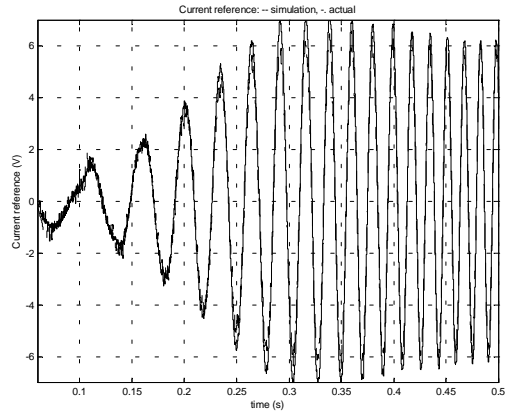


3.b)

Fig. 3: Closed loop validation with the model (4)(5)(6)



4.a)



4.b)

Fig. 4: Closed loop validation with the model (14)(15)(16)

## V. Conclusion

This paper presented a identification method for the physical parameters of the mechanical systems with lumped elasticity. These parameters are estimated by using the solution of weighted least squares of an over

determined system model linear with regard to a minimal set of parameters and obtained from the sampling of the dynamic model along an closed loop tracking trajectory and using two different sets of data. At first, the motor and the load position are available, secondly the motor position and the load acceleration are used. An experimental study on a mechanical system shows the

efficiency of the method.

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