Robust Estimation of Parallel Robot with Interval Analysis

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Robust estimation of parallel robot dynamic parameters with interval analysis

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ABSTRACT

This paper deals with the application of interval analysis for outer bounding the physical parameters of parallel robots. The robot is modeled with classical Lagrange equation which leads to an inverse dynamic model linear with respect to the parameters. Assuming the error additive on input (motor torque), the problem is termed as a constraint propagation one and the solution is performed with a preconditionned Gauss-Seidel contractor modified in order to be used with over-determined linear systems. Experimental results are exhibited for a fully parallel robot with 4 degrees of freedom.

Keywords: outer enclosure, Gauss-Seidel contractor, parameters estimation, robots dynamics.

1. INTRODUCTION

Robust estimation of physical parameters (*e.g.* for robot dynamic model) are often necessary for computing robust model based control algorithms. This robust identification can be performed through bounded errors techniques (Walter and Pronzato, 1997) (Maksarov and Norton, 2002) (Poignet *et al.*, 2003). Bounded errors may encompass significant structural errors that cannot be accounted for by random variables or noise errors without specifying any statistical properties. Among the bounded error methods, interval arithmetic provides efficient tools to compute guaranteed estimation or robust control (Jaulin *et al.*, 2001) (Jaulin *et al.*, 2002). In such a context, the solution is a set of parameter vector consistent with measurement data, prior error bounds and modeling hypotheses.

In this paper, the problem of dynamic robot parameter estimation is expressed with a model which is linear with respect to the physical parameters. The estimation problem is then stated as a linear interval constraint satisfaction problem (CSP). Interval fixed-point contractors make it then possible to compute a smallest box outer-bounding the solution set (Jaulin *et al.*, 2001).

Fig. 1. H4 robot.

Experimental results are exhibited for a 4-dof parallel mechanism – the H4 robot - (Company and Pierrot, 1999; Pierrot et al, 2001). Figure 1 shows a photograph of the H4 parallel robot. This machine is based on 4 independent active chains between the base and the nacelle; each chain is actuated by a brushless direct drive motor fixed on the base and equipped with an incremental position encoder. Thanks to its design, the mechanism is able to provide high performance. However in order to achieve high speed and acceleration for pick-and-place applications or precise motion in machining tasks, advanced model based robust controllers are often required to increase the performances of the robot, which justify these works on guaranteed estimation.

The paper is organized as follows : Section 2 is dedicated to the geometric, kinematics and dynamic modelling of the H4 robot. Section 3 details the interval methods. Section 4 exhibits major experimental results on a fully parallel robot. Finally, conclusions are given in section 5.

2. MODELLING

2.1. Geometric and kinematics modelling

The Jacobian matrix and the forward geometric model are required to compute the dynamic model (see section 2.2)

(Khalil, and Dombre, 2002). Therefore we briefly present the way of computing the different relationship necessary to obtain these model and matrix. The design parameters of the robot are described on Figure 2 where the following parameters have been chosen:

$\alpha_1 = 0$; $\alpha_2 = \pi$; $\alpha_3 = 3\pi/2$; $\alpha_4 = 3\pi/2$
$\boldsymbol{u_1} = \boldsymbol{u_y}$; $\boldsymbol{u_2} = -\boldsymbol{u_y}$; $\boldsymbol{u_3} = \boldsymbol{u_x}$; $\boldsymbol{u_4} = \boldsymbol{u_x}$

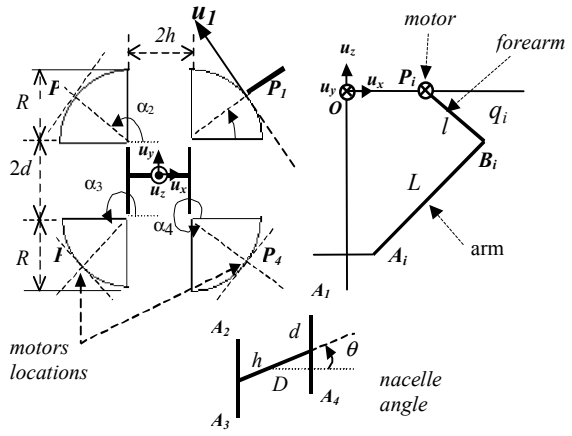

Fig. 2. Design parameters.

The angles α_i describe the position of the four motors, L is the length of arms, l is the length of the forearms, θ the nacelle's angle, and d and h are the half lengths of the "H" forming the nacelle. $\boldsymbol{O}$ is the origin of the base frame and D is the origin of the nacelle frame. R gives the motor's position. The $\boldsymbol{A_iB_i}$ segments represent the arms of the robot and $\boldsymbol{P_iB_i}$ the forearm segments. The joint positions are represented by q_i.

To obtain the geometric model, we need to express the different points of the mechanical system with respect to the origin $\boldsymbol{O}$. The origin is fixed in the middle of the nacelle with the coordinates (x, y, z). In the Cartesian space, the end effector position is given by (x, y, z, θ).

$$\mathbf{OD} = \begin{bmatrix} x & y & z \end{bmatrix}^T \tag{1}$$

The vector that joins the absolute origin O and all of the forearms to the nacelle is:

$$\boldsymbol{OA_i} = \boldsymbol{OD} + \boldsymbol{DA_i} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \boldsymbol{DA_i} \tag{2}$$

The $\boldsymbol{DA_i}$ segments can be expressed as:

$$\boldsymbol{DA_1} = \begin{bmatrix} h\cos\theta \\ h\sin\theta + d \\ 0 \end{bmatrix}; \boldsymbol{DA_2} = \begin{bmatrix} -h\cos\theta \\ -h\sin\theta + d \\ 0 \end{bmatrix} \tag{3}$$

$$\boldsymbol{DA_3} = \begin{bmatrix} -h\cos\theta \\ -h\sin\theta - d \\ 0 \end{bmatrix}; \boldsymbol{DA_4} = \begin{bmatrix} h\cos\theta \\ h\sin\theta - d \\ 0 \end{bmatrix} \tag{4}$$

Moreover, the vector that links the absolute origin and all of the arms to the forearms is:

$$\boldsymbol{OB_i} = \boldsymbol{OP_i} + \boldsymbol{P_iB_i} \tag{5}$$

with:

$$\boldsymbol{P_iB_i} = \begin{bmatrix} l\cos q_i \cos\alpha_i \\ l\cos q_i \sin\alpha_i \\ -l\sin q_i \end{bmatrix} \tag{6}$$

and actuator locations are:

$$\boldsymbol{OP_1} = \begin{bmatrix} h + R\cos\alpha_1 \\ d + R\sin\alpha_1 \\ 0 \end{bmatrix}; \boldsymbol{OP_2} = \begin{bmatrix} -h + R\cos\alpha_2 \\ d + R\sin\alpha_2 \\ 0 \end{bmatrix} \tag{7}$$

$$\boldsymbol{OP_3} = \begin{bmatrix} -h + R\cos\alpha_3 \\ -d + R\sin\alpha_3 \\ 0 \end{bmatrix}; \boldsymbol{OP_4} = \begin{bmatrix} h + R\cos\alpha_4 \\ -d + R\sin\alpha_4 \\ 0 \end{bmatrix} \tag{8}$$

Finally, arms coordinates are given by:

$$\mathbf{A_iB_i} = \mathbf{A_iO} + \mathbf{OB_i} \tag{9}$$

The analytical forward position relationship is difficult to compute. Up to now, the simplest model we have got is a 8th degree polynomial equation. The forward model is then computed iteratively using the classical formula:

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \mathbf{J}(\mathbf{x}_n, \mathbf{q}_n)\,[\mathbf{q} - \mathbf{q}_n] \tag{10}$$

Where $\mathbf{q}$ is the convergence point and $\mathbf{J}$ is the robot Jacobian matrix. If the mechanism is not in a singular configuration, this expression is derived as follows (Company and Pierrot, 1999; Pierrot et al, 2001):

$$\mathbf{J} = \mathbf{J}_x^{-1}\,\mathbf{J}_q \tag{11}$$

Where:

$$\mathbf{J}_x = \begin{bmatrix} A_1B_{1x} & A_1B_{1y} & A_1B_{1z} & (\mathbf{DC_1} \times \mathbf{A_1B_1})_z \\ A_2B_{2x} & A_2B_{2y} & A_2B_{2z} & (\mathbf{DC_1} \times \mathbf{A_2B_2})_z \\ A_3B_{3x} & A_3B_{3y} & A_3B_{3z} & (\mathbf{DC_2} \times \mathbf{A_3B_3})_z \\ A_4B_{4x} & A_4B_{4y} & A_4B_{4z} & (\mathbf{DC_2} \times \mathbf{A_4B_4})_z \end{bmatrix} \quad (12)$$

$$\mathbf{J}_q = diag((\mathbf{P}_i\mathbf{B}_i \times \mathbf{A}_i\mathbf{B}_i).\mathbf{u_{mi}}) \quad , \quad i = 1,...4 \quad (13)$$

$\boldsymbol{DC_i}$ $(i = \{1,2\})$ is the distance between the center of the nacelle and the center of the half lengths of the "H" that forms the nacelle.

2.2. Dynamic modelling

In first approximation, the dynamic model is computed by considering physical dynamics. Indeed, the drive torques are mainly used to move the motor inertia, the fore-arms and the arms and the nacelle equipped with a machining tool. Because of the design, the fore-arm inertia can be considered as a part of the motor inertia and the arm (manufacturing in carbon materials) effects are neglected [8], [9]. A simple friction model is added considering viscous and Coulomb friction.

If $\mathbf{\Gamma_{mot}}$ is the (4x1) actuator torque vector, the basic equation of dynamics can be written as :

$$\Gamma_{mot} = \mathbf{I}_{mot}\ddot{\mathbf{q}} + \mathbf{J}^T\mathbf{M}(\ddot{\mathbf{x}} - G) + \mathbf{F}_v\dot{\mathbf{q}} + \mathbf{F}_s sign(\dot{\mathbf{q}}) \quad (14)$$

where $\mathbf{I_{mot}}$ represents the motor's inertia matrix including the forearm's inertia, $\mathbf{M}$ a matrix containing the mass of the nacelle and its inertia, $\dot{\mathbf{q}}$ is the (4x1) joint velocity vector, $\ddot{\mathbf{q}}$ is the (4x1) joint acceleration vector , $\ddot{\mathbf{x}}$ is the (4x1) vector of cartesian accelerations $\begin{bmatrix} \ddot{x} & \ddot{y} & \ddot{z} & \ddot{\theta} \end{bmatrix}^T$, and G the gravity constant. Thanks to the design, the forearm's inertia is taken into account in the motor's inertia. $\mathbf{F_v}$ are the viscous friction coefficients and $\mathbf{F_c}$ are the Coulomb friction.

With:

$$\mathbf{I}_{mot} = \begin{bmatrix} I_{mot1} & 0 & 0 & 0 \\ 0 & I_{mot2} & 0 & 0 \\ 0 & 0 & I_{mot3} & 0 \\ 0 & 0 & 0 & I_{mot4} \end{bmatrix} \quad (15)$$

$$\mathbf{M}_{mot} = \begin{bmatrix} M_{nac} & 0 & 0 & 0 \\ 0 & M_{nac} & 0 & 0 \\ 0 & 0 & M_{nac} & 0 \\ 0 & 0 & 0 & I_{bc} \end{bmatrix} \quad (16)$$

The dynamic equation can be rewritten as a relation linear in the dynamic parameters. By introducing $\mathbf{J}^T = \begin{bmatrix} \mathbf{J}_{43} & \mathbf{J}_4 \end{bmatrix}$, it follows:

$$\mathbf{\Gamma}_{mot} = \begin{bmatrix} \mathbf{q} & \mathbf{J}_{43} \begin{bmatrix} x \\ y \\ z - G \end{bmatrix} & \mathbf{J}_4\theta & \mathbf{q} & sign(\mathbf{q}) \end{bmatrix} \boldsymbol{\theta} \quad (17)$$

where $\boldsymbol{\theta}$ is the vector of parameters:

$$\boldsymbol{\theta} = [I_{mot1} \;\; I_{mot2} \;\; I_{mot3} \;\; I_{mot4} \;\; M_{nac} \;\; I_{bc} \;\; F_{v1} \;\; F_{v2} \;\; F_{v3} \;\; F_{v4} \;\; F_{s1} \;\; F_{s2} \;\; F_{s3} \;\; F_{s4}]^T \quad (18)$$

Only the torque input $\mathbf{\Gamma_{mot}}$ and motor position $\mathbf{q}$ are directly measured. As acceleration measurement $\ddot{\mathbf{x}}$ is not available, $\ddot{\mathbf{x}}$ is evaluated by:

$$\ddot{\mathbf{x}} = \mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}} \quad (19)$$

where $\mathbf{J}$ depends on $\mathbf{x}$ and $\mathbf{q}$, is computed using a central difference algorithm.

3. PARAMETER BOUNDING

3.1. Bounded error context

When the statistical properties of the random variable used to model the actual disturbances acting on model inputs or outputs remain unattainable, it is still possible to compute values for the bounds between the output of a model $y_k^m(\cdot)$ and some actual measurements y_k. Indeed, the sensors used for data measurements are frequently characterized with a prior maximum measurement error. Under the hypothesis of additive noise, actual model output can be related to actual data as follows:

$$y_k = y_k^m(\boldsymbol{\theta}^*) + \varepsilon_k^* \quad , \quad k = 1 \ldots N \quad (20)$$

where N is the number of observations and $\boldsymbol{\theta}^*$ is the unknown *true* parameter vector to be identified and $\{\varepsilon^*\}$ an output error sequence assumed to be stationary and bounded but otherwise unknown. The error sequence thus satisfies the following inequality:

$$\forall k = 1 \ldots N, \; -1 \le \varepsilon_k^* \le 1 \quad (21)$$

This description is known as the standard form with a normalized error, and it is always possible to transform the case where the upper and lower prior error bounds are different to such a form.

A $(p \times 1)$ parameter vector $\boldsymbol{\theta}$ is said to be feasible, if and only if the output error is enclosed in the prior bounds.

Consequently, the issue of the bounded-error set estimation is to compute the set, known as *the posterior feasible set*, defined as

$$S=\left\{\boldsymbol{\theta}\in Q\,\middle|\;\forall k=1\ldots N,\;-1\le y_k-y_k^m(\boldsymbol{\theta})\le 1\right\} \tag{22}$$

where the prior parameter search space $Q\subseteq\mathbb{R}^p$.

When the model is linear in parameter, which is the case of the robot inverse dynamic model, it is written as:

$$y_k^m=\mathbf{d}_k^T\boldsymbol{\theta} \tag{23}$$

The parameters set compatible with the datum at observation k is a strip Π_k defined by:

$$\Pi_k=\left\{\boldsymbol{\theta}\in\mathbb{R}^p\,\middle|\;\left|\mathbf{y}_k-\mathbf{d}_k^T\boldsymbol{\theta}\right|\le 1\right\} \tag{24}$$

The posterior feasible set is the intersection of a prior search space Q and N strips $\boldsymbol{\Pi}_k$:

$$S=\left\{\boldsymbol{\theta}\in Q\,\middle|\;\forall k=1\ldots N,\;\left|y_k-\mathbf{d}_k^T\boldsymbol{\theta}\right|\le 1\right\} \tag{25}$$

Consequently, characterizing the solution parameter vectors set (25) consists on solving the following *Constraint Satisfaction Problem* (*CSP*)

$$H:\left(\mathbf{W}\boldsymbol{\theta}-\mathbf{Y}=0,\quad\mathbf{Y}\in[\mathbf{Y}],\quad\boldsymbol{\theta}\in Q\right) \tag{26}$$

where $\mathbf{W}=[\mathbf{d}_1^T,\mathbf{d}_2^T,...,\mathbf{d}_N^T]^T$, $[\mathbf{Y}]=[\mathbf{Y}-\mathbf{1},\mathbf{Y}+\mathbf{1}]$ and $\mathbf{Y}=[\mathbf{y_1},\mathbf{y_2},...,\mathbf{y_N}]^T$. One can write equivalently

$$H:\left(\mathbf{W}\boldsymbol{\theta}-[\mathbf{Y}]=0,\quad\boldsymbol{\theta}\in Q\right) \tag{27}$$

The solution vector $[\boldsymbol{\theta}]$ is given as an interval vector. The width of each component of the interval solution vector indicates the uncertainty associated with the identified physical parameter.

In fact, due to the large number of data available, the system described by (27) is over-determined. It can however be re-written with square matrices by using the following method introduced in (Rump, 2002):

$$\mathrm{H}:\left(\mathbf{A}\mathbf{X}-[\mathbf{b}]=0,\quad\mathbf{X}\in X\right) \tag{28}$$

where

$$\mathbf{A}=\begin{bmatrix}\mathbf{W} & -eye(N)\\ zeros(p) & \mathbf{W}^T\end{bmatrix} \tag{29}$$

and

$$[\mathbf{b}]=\begin{bmatrix}[\mathbf{Y}]\\ zeros(p,1)\end{bmatrix} \tag{30}$$

where $eye(N)$ is an $(N\times N)$ identity matrix, $zeros(p)$ is an $(p\times p)$ matrix of zeros and $zeros(p,1)$ an $(p\times 1)$ vector of zeros. The solution vector is then given by the first p components of the [**X**] vector.

3.2 Fixed-point contractors

Interval CSPs can be solved with contractors. An operator C_H is a contractor for the CSP H if it satisfies:

$$\begin{aligned}&\forall[\mathbf{x}]\in Q\quad C_H([\mathbf{x}])\subset[\mathbf{x}] && \text{(contractance)}\\ &\forall[\mathbf{x}]\in Q\quad C_H([\mathbf{x}])\cap S=[\mathbf{x}]\cap S && \text{(correctness)}\end{aligned} \tag{31}$$

where $\cap$ is the intersection of two boxes (Jaulin *et al.*, 2001).

A solver for a *CSP* $\mathrm{H}:(\mathbf{f}(\mathbf{x})=\mathbf{0},\mathbf{x}\in[\mathbf{x}])$ is an algorithm $\boldsymbol{\Psi}$ such that:

$$\mathbf{f}(\mathbf{x})=\mathbf{0}\Leftrightarrow\mathbf{x}=\boldsymbol{\Psi}(\mathbf{x}) \tag{32}$$

According to the fixed point theorem and using (32), if the series $\mathbf{x}_{k+1}=\boldsymbol{\Psi}(\mathbf{x}_k)$ converges towards $\mathbf{x}_\infty$, then $\mathbf{x}_\infty$ shall contain the solution of H.

For linear square systems, a well known method is given by the following interval *Gauss-Seidel* contractor, where the matrices [**A**] and [**b**] can be interval ones.

Algorithm C_{GS} **:**

$$\begin{pmatrix}[\mathbf{A}]\\ [\mathbf{b}]\\ [\mathbf{p}]\end{pmatrix}\rightarrow\begin{pmatrix}[\mathbf{A}]\\ [\mathbf{b}]\\ [\mathbf{p}]\cap(diag([\mathbf{A}]))^{-1}([\mathbf{b}]-extdiag([\mathbf{A}])[\mathbf{p}])\end{pmatrix}$$

Where: $\mathbf{A}=diag(\mathbf{A})+extdiag(\mathbf{A})$.

Remark : All the diagonal element of the A matrix must be non-null.

In practice, a more efficient version of the *Gauss-Seidel* contractor can be obtained by preconditioning the **A** matrix. The new algorithm is presented in the following:

Algorithm C_{GSP} **:**

$\mathbf{A}_0 := mid([\mathbf{A}])$;

$[\mathbf{A}'] := \mathbf{A}_0^{-1}[\mathbf{A}]$;

$[\mathbf{b}'] := \mathbf{A}_0^{-1}[\mathbf{b}]$;

$C_{GS}(\mathbf{A}'\mathbf{p} - \mathbf{b}' = \mathbf{0}, [\mathbf{A}'], [\mathbf{p}], [\mathbf{b}'])$;

$[\mathbf{b}] := \mathbf{A}_0[\mathbf{b}'] \cap [\mathbf{b}]$;

$[\mathbf{A}] := \mathbf{A}_0[\mathbf{A}'] \cap [\mathbf{A}]$.

Where *mid* stands for the point value taken at the centre of the boxes and C_{GS} is the Gauss-Seidel contractor (Jaulin *et al.*, 2001).

4. EXPERIMENTAL RESULTS

4.1. Experimental data

Joint position $\mathbf{q}$ and the current reference $\mathbf{V_T}$ (the control input) are collected at a 1000Hz sample rate while the robot is tracking exciting trajectories containing both slow (for friction) and high dynamics (for inertia). These trajectories ensure a low condition number. The identification is performed by using a closed-loop joint PI control. The torques are computed using a linear relationship between torque $\mathbf{\Gamma_{mot}}$ and voltage $\mathbf{V_T}$ where G_T is the amplifier gain:

$$\Gamma_{mot} = G_T V_T \tag{33}$$

Joint velocities and accelerations for computing the regressor are estimated by a band pass filtering of the position. The band pass filtering is obtained by the product of a low pass filter in both the forward and the reverse direction (Butterworth) and a derivative filter obtained by a central difference algorithm, without phase shift. A parallel filtering is implemented to reject the high frequency ripples of the measured motor torques. Practical aspects of the derivative estimation and data filtering are completely detailed in (Poignet and Gautier, 2001).

4.2. Estimated parameters

The prior bounds on motor torques are tuned by taking into account prior information on motors. They were chosen prior to the computation as 10% of measurement range of the torques (±15Nm).

Table 1 contains the estimated parameter boxes. Prior values for motor inertia and nacelle mass and inertia are known by design. The initial boxes are equal to [0.0, 2.0]. Table 2 exhibits a comparison with the weighted least square estimation results (Vivas *et al.*, 2003). The estimated parameter boxes contain the prior values as well as the weighted least square solution. The estimated bounds are tight enough to be considered as a good estimation.

Table 1. Estimated parameter boxes.
Values are expressed in USI.

Parameter	Estimated boxes	*A priori* values
I_{mot1}	[0.0009, 0.0238]	0.012
I_{mot2}	[0.0044, 0.0257]	0.012
I_{mot3}	[0.0096, 0.0320]	0.012
I_{mot4}	[0.0098, 0.0319]	0.012
M_{nac}	[0.7650, 1.2338]	1.0
I_{bc}	[0.0000, 0.0071]	0.0008
F_{v1}	[0.0000, 0.4690]	/
F_{v2}	[0.1733, 0.2554]	/
F_{v3}	[0.0000, 0.2856]	/
F_{v4}	[0.0000, 0.2664]	/
F_{c1}	[0.0000, 1.6068]	/
F_{c2}	[0.0000, 1.6300]	/
F_{c3}	[0.2245, 1.0952]	/
F_{c4}	[0.2264, 1.1014]	/

Table 2. Weighted least square estimated parameters
Values are expressed in USI.

Parameters	Estimated values	$\%\sigma_{\hat{x}r}$
I_{mot1}	0.0141	2.6286
I_{mot2}	0.0120	3.0444
I_{mot3}	0.0153	1.6939
I_{mot4}	0.0213	1.1933
M_{nac}	1.0492	0.4236
I_{bc}	0.0030	3.5049
F_{v1}	0.1636	5.6781
F_{v2}	0.0560	15.5674
F_{v3}	0.0930	6.5734
F_{v4}	0.0917	6.4301
F_{c1}	1.1453	2.0450
F_{c2}	1.0950	2.0563
F_{c3}	0.7222	2.8366
F_{c4}	0.9932	2.0451

Where $\%\sigma_{\hat{x}r}$ stands for the relative standard deviation. Complete details of the computation are given in (Vivas *et al.*, 2003).

5. CONCLUSION

This paper exhibits relevant results for robust dynamic identification of parallel robots stated as an interval constraint satisfaction problem. The solution set is outer-bounded with the pre-conditioned interval *Gauss-Seidel* contractor.

For the parallel robot under study, the estimated parameter boxes contain the prior values. The uncertainties derived for the parameter remain large.

Further work will concern *i*) the study of the influence of the prior error bounds on the posterior uncertainty, *ii*) the possibility of accounting for data outliers and *iii)* the case where the regressor is no longer exact but subject to bounded uncertainty.

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