Camera Calibration and 3D Reconstruction Using Interval Analysis

Benoit Telle, Marie-José Aldon, Nacim Ramdani

Camera calibration and 3D reconstruction using interval analysis

Benoît Telle
Marie-Josée Aldon
LIRMM
UMR CNRS/UMII, n.C55060
161, rue ADA
34392 Montpellier - cedex5 - France
telle@lirmm.fr

Nacim Ramdani
CERTES
Universit Paris XII- Val de Marne,
ave G. de Gaulle, 94000 Crteil.
ramdani@univ-paris12.fr

Abstract

This paper deals with the problem of error estimation in 3D reconstruction. It shows how interval analysis can be used in this way for 3D vision applications. The description of an image point by an interval assumes an unknown but bounded localization. We present a new method based on interval analysis tools to propagate this bounded uncertainty. This way of computation can produce guaranteed results since a data is not the most probabilistic value but an interval which contains the true value. We validate our method by computing a guaranteed model for a projective camera, and we achieve a guaranteed 3D reconstruction.

1 Introduction

This paper deals with the problem of error estimation in 3D reconstruction. Classical approaches suggest to use a Gaussian model for error estimation and propagation [8]. Error is seen as an additive noise which creates small enough perturbations to permit the use of maximum likelihood estimators. This approximation allows [2] to evaluate the covariance of the camera model parameters then the scene reconstruction error. Gaussian noise model has been introduced with signal analysis tools. Image processing justifies its use. Nevertheless, camera calibration and 3D reconstruction can be seen as geometric problems. Projective geometry permits to describe perspective effects in camera calibration [1] and improves numerically the 3D reconstruction stability [7]. However a geometrical uncertainty remains unavoidable[11].

The geometrical interpretation of Gaussian uncertainty is made by an ellipsoid. It is centered on pixel position and its dimensions are defined by the covariance matrix. It represents the probability of pixel position for a given confidence. In this paper, pixel coordinates are seen as two unknown but bounded variables. The representation of the pixel uncertainty is a rectangular shape which supports its distribution. It can cover the pixel area or more. Thus, interval analysis tools permit to compute camera calibration and guaranteed 3D reconstruction.

In the first part of this paper we briefly present the projective model that we use for camera calibration, and then, we formulate the problem of 3D reconstruction using a stereovision system. We recall that these two processes may be modeled by using either an homogeneous or a non-homogeneous linear system. Afterward, we describe an original interval-based method that we have developed to propagate bounded data uncertainty when solving homogeneous linear systems. The case of non-homogeneous systems is solved by using an interval analysis tool: Krawczyk contractor. In the last section, experimental results allow us to compare the performance of these two estimation methods applied to camera calibration and 3D reconstruction.

2 Problem formulation

We present here the basic equations of camera calibration and 3D reconstruction. Two kinds of formulation are used: the homogeneous and no homogeneous ones. Resolution of these equations will be the goal of this paper.

2.1 Projective Camera Calibration

Camera calibration consists in determining the (3×4) transformation matrix P that maps a 3D point Q expressed with respect to a scene frame, onto its 2D image q whose coordinates are expressed in pixel units. The camera model that we consider is the standard pinhole model. Points are described with homogeneous coordinates.

$$\begin{aligned} q_i &= (u_i,\ v_i,\ s_i)^t \\ P &= \begin{pmatrix} p_1^t \\ p_2^t \\ p_3^t \end{pmatrix}^t \end{aligned}$$

The relation between a scene point and its image coordinates is the result of the transformation

$$q_i = PQ_i \tag{1}$$

For each couple (q_i, Q_i), equation (1) produces the following system:

$$\begin{pmatrix} u_i \\ v_i \\ s_i \end{pmatrix} \wedge \begin{pmatrix} p_1^t Q_i \\ p_2^t Q_i \\ p_3^t Q_i \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & -s_i Q_i^t & v_i Q_i^t \\ s_i Q_i^t & 0 & -u Q_i^t \\ -v_i Q_i^t & u Q_i^t & 0 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = 0 \tag{2}$$

The unknown vector is $X = \begin{pmatrix} p_1 & p_2 & p_3 \end{pmatrix}^t$. We have to solve equation (2) which has the form $A_i X = 0$. Given n point ($n \geq 6$) [7], we have an overdetermined linear system which is generally solved by least square minimization. It has the homogeneous form:

$$AX = \begin{pmatrix} A_1 \\ ... \\ A_n \end{pmatrix} X = 0$$

The system can also be expressed in a non homogeneous way. We call $\{\overrightarrow{e_u}, \overrightarrow{e_v}, \overrightarrow{e_s}\}$ the basis of the projective image coordinate system, q and Q are the matrix constructed from points q_i and Q_i

$$\begin{aligned} q &= \begin{pmatrix} q_1 & ... & q_n \end{pmatrix} \\ Q &= \begin{pmatrix} Q_1 & ... & Q_n \end{pmatrix} \end{aligned}$$

¿From the relation (1) we obtain:

$$\begin{aligned} & q = \begin{pmatrix} p_1^t \\ p_2^t \\ p_3^t \end{pmatrix} Q \\ \Leftrightarrow \quad & q^t = Q^t \begin{pmatrix} p_1 & p_2 & p_3 \end{pmatrix} \\ \Leftrightarrow \quad & \begin{Bmatrix} Q^t p_1 - q^t e_u = 0 \\ Q^t p_2 - q^t e_v = 0 \\ Q^t p_3 - q^t e_s = 0 \end{Bmatrix} \end{aligned} \tag{3}$$

These three equations form three linear overdetermined systems in the form $AX + B = 0$.

2.2 3D Reconstruction

Problem of 3D reconstruction is expressed in the same way as camera calibration. We suppose we have two models of camera (i.e. a stereoscopic system). The couple of image points $\left(q_i^l, q_i^r\right)$ provided by the couple of cameras $\left(P^l, P^r\right)$ is:

$$\begin{Bmatrix} q_i^l = P^l Q_i \\ q_i^r = P^r Q_i \end{Bmatrix}$$

Points $\{Q_i\}$ have to be computed by the inversion of this linear system. Let us write $[x]_\times$ the matrix equivalent to the cross product on x :

$$x \wedge y = [x]_\times y$$

The homogeneous resolution is realized in the following way:

$$\begin{aligned} & \begin{Bmatrix} q_i^l = P^l Q_i \\ q_i^r = P^r Q_i \end{Bmatrix} \\ \Leftrightarrow \quad & \begin{Bmatrix} 0 = \left[q_i^l\right]_\times P^l Q_i \\ 0 = \left[q_i^r\right]_\times P^r Q_i \end{Bmatrix} \end{aligned} \tag{4}$$

Let us write

$$A = \begin{pmatrix} \left[q_i^l\right]_\times P^l \\ \left[q_i^r\right]_\times P^r \end{pmatrix} \tag{5}$$

As for camera calibration, the system has an homogeneous form: $AQ = 0$. It is usually solved by minimization.

The non homogeneous system is given by the development of the camera model. Let us write:

$$P = \begin{pmatrix} M & V \end{pmatrix}$$

M is a (3×3) matrix, V is a (3×1) vector. We set the scale factor of the 3D point Q_i to the value 1:

$$Q_i = \begin{pmatrix} \widetilde{Q}_i^t & 1 \end{pmatrix}^t$$

System (4) becomes

$$\begin{aligned} & \begin{Bmatrix} 0 = \left[q_i^l\right]_\times^l \begin{pmatrix} M^l & V^l \end{pmatrix} Q_i \\ 0 = \left[q_i^r\right]_\times \begin{pmatrix} M^r & V^r \end{pmatrix} Q_i \end{Bmatrix} \\ \Leftrightarrow \quad & \begin{Bmatrix} 0 = \begin{pmatrix} \left[q_i^l\right]_\times M^l & \left[q_i^l\right]_\times V^l \end{pmatrix} Q_i \\ 0 = \begin{pmatrix} \left[q_i^r\right]_\times M^r & \left[q_i^r\right]_\times V^r \end{pmatrix} Q_i \end{Bmatrix} \end{aligned} \tag{6}$$

Let us write

$$A = \begin{pmatrix} \left[q_i^l\right]_\times M^l \\ \left[q_i^r\right]_\times M^r \end{pmatrix} ;\ B = \begin{pmatrix} \left[q_i^l\right]_\times V^l \\ \left[q_i^r\right]_\times V^r \end{pmatrix}$$

Since the scale factor of Q takes the value 1, equation (6) can be written

$$A\widetilde{Q} + B = 0 \tag{7}$$

Camera calibration and 3D reconstruction involve similar linear systems. In both cases, we have to resolve an equation in the form $AX + B = 0$ or $AX = 0$. These kinds of systems has been quite studied with scalar models. In the next part we propose to resolve them in the case of uncertain but bounded data. Pixel coordinates are described by interval and so matrix A and B are bounded models.

3 Bounded uncertainty propagation

In this part, problem of calibration and reconstruction are formulated with interval analysis description tools.

3.1 Introduction to interval arithmetic

An interval $[q]$ is defined by lower and upper bounds:

$$[q] = [\underline{q}; \overline{q}]$$

An image point $[x]$ is a vector of intervals. It describes the set of possible values for the bounded variable x. Likewise, a matrix $[A]$ is a matrix of intervals. The solution set $\{X\}$ for the linear system $[A]\,X + [B] = 0$ is defined by

$$x \in \{X\} \Leftrightarrow \exists a \in [A]\,, \exists b \in [B] \mid ax + b = 0$$

Intervals do not describe correctly this solution set. Thus, the resolution of these systems is necessarily a particular estimation of the solution.

Several properties characterize an interval and define its arithmetic. The midpoint (or center) and the radius are two descriptive properties. Nevertheless, computation is realized with bounded description of intervals. We propose to use the writing find in [10] as a new way for computing with intervals:

$$\begin{aligned} [q] &= < q; \overleftrightarrow{q} > & (8) \\ q &= \frac{\overline{q} + \underline{q}}{2} \\ \overleftrightarrow{q} &= \frac{\overline{q} - \underline{q}}{2} \end{aligned}$$

¿From this definition (8), it follows the property

$$\begin{aligned} \forall\lambda &\in R^*\,,\, [q] \in IR^n \\ \lambda\,[q] &= < \lambda q; |\lambda|\,\overleftrightarrow{q} > & (9) \\ \lambda\,[q] &= \lambda q + < 0; |\lambda|\,\overleftrightarrow{q} > & (10) \end{aligned}$$

Interval computation makes use of a large set of operators described in [5]. Sum and product are the most current operations for matrix computation:

$$\begin{aligned} [q_1] + [q_2] &= \left[\underline{q_1} + \underline{q_2}; \overline{q_1} + \overline{q_2}\right] \\ [q_1] \times [q_2] &= \begin{bmatrix} \min\left(\underline{q_1}\underline{q_2}, \overline{q_1}\overline{q_2}, \underline{q_1}\overline{q_2}, \overline{q_1}\underline{q_2}\right); \\ \max\left(\underline{q_1}\underline{q_2}, \overline{q_1}\overline{q_2}, \underline{q_1}\overline{q_2}, \overline{q_1}\underline{q_2}\right) \end{bmatrix} \end{aligned}$$

3.2 Solving homogeneous systems

Tools of set theory provide the only one solution $[X] =< 0; 0 >$ for the equation $[A]\,X = 0$. Nevertheless, property (10) permits us to formulate this problem with scalar values and results can be found. Suppose $[A] = \left([a]_{i,j}\right)$ is a $(n \times m)$ interval matrix. Let us construct the diagonal $(m \times m)$ matrix $[B^\tau]$:

$$[B^\tau] = \tau I + \left[B^0\right]$$

with:

$$\begin{aligned} \left[B^0\right] &= \left([b]_{j,j}\right) = \left\{ \bigcup_{k=1\ldots n} < 0; \frac{\tau}{|a_{k,j}|}\overleftrightarrow{a_{k,j}} > \right\} \\ \tau &> 0 \end{aligned}$$

We call $[B^\tau]$ the interval basis associated to the matrix $[A]$. Center of its diagonal elements is τ and each element has a radius which is the largest observed in the associated column of matrix $[A]$. Let us demonstrate the following property:

$$[A] \subset \frac{1}{\tau} A\,[B^\tau] \tag{11}$$

We can verify this inclusion for each element of the matrix:

$$\left(\frac{1}{\tau} A\,[B^\tau]\right)_{i,j} = \frac{1}{\tau} \sum_k A_{i,k}\,([\tau])_{k,j}$$

Since the basis is a diagonal matrix:

$$\begin{aligned} \left(\frac{1}{\tau} A\,[B^\tau]\right)_{i,j} &= \frac{1}{\tau} A_{i,j} \left(\bigcup_k < \tau; \left|\frac{\tau}{a_{k,j}}\right| \overleftrightarrow{a_{k,j}} > \right) \\ &= A_{i,j} < 1; \max_k \left(\left|\frac{1}{a_{k,j}}\right| \overleftrightarrow{a_{k,j}} \right) > \end{aligned}$$

The property (9) applied to element $\left([a]_{i,j}\right)$ ensures that:

$$\begin{aligned} &\left(A_{i,j} < 1; \left|\frac{1}{a_{i,j}}\right| \overleftrightarrow{a_{i,j}} > \right) \\ \subset &\left(A_{i,j} < 1; \max_k \left(\left|\frac{1}{a_{k,j}}\right| \overleftrightarrow{a_{k,j}} \right) > \right) \end{aligned}$$

which demonstrates (11).

We cannot ensure that the solution set of the system is an interval $[X]$. Since interval analysis provides only intervals, the best approximation of the interval solution we can find for vector $[X]$ has a null radius. We can define the interval basis associated to the vector $[X]$ by the diagonal matrix $\left[B_X^1\right]$:

$$\left(\left[B_X^1\right]\right)_{i,i} =< 1; 0 >$$

And so

$$[X] = \left(\left[B_X^1\right]\right) X = X \tag{12}$$

According to the properties (12) and (11) of the interval basis, we can write:

$$[A]\,[X] \;=\; 0 \Rightarrow \frac{1}{\tau} A\,[B^{\tau}]\,\left[B_X^1\right] X = 0 \qquad (13)$$

$$\Rightarrow \; \frac{1}{\tau} A\,[B^{\tau}]\,X = 0 \qquad (14)$$

Since relation (13) is an implication and not an equivalence we can use its contrary proof as a constraint:

$$\frac{1}{\tau} A\,[B^{\tau}]\,X \neq 0 \Rightarrow [A]\,X \neq 0 \qquad (15)$$

And so equation $[A]\,X = 0$ can be written in a more pessimistic way as the necessary following constraint:

$$\frac{1}{\tau} A\,[B^{\tau}]\,X = 0 \qquad (16)$$

Now, the solution of (16) will be an outer enclosure of the exact one. In expression (16), the uncertainty appears explicitly. We can propose different problem formulations.

$$\begin{aligned} & \tfrac{1}{\tau} A\,[B^{\tau}]\,X = 0 \\ \Leftrightarrow \quad & \tfrac{1}{\tau} A\left(\tau I + \left[B^0\right]\right) X = 0 \\ \Leftrightarrow \quad & AX + \tfrac{1}{\tau} A\left[B^0\right] X = 0 \qquad (17) \\ \Leftrightarrow \quad & A\left(X + \tfrac{1}{\tau}\left[B^0\right] X\right) = 0 \qquad (18) \end{aligned}$$

Let be $X_0 \in \left\{\widehat{X}\right\}$ one of the solution set for the scalar system: $AX = 0$. We propose to propagate the model uncertainty to the solution by using the intervals basis in (18)

$$[X_0] = X_0 + \frac{1}{\tau}\left[B^0\right] X_0 \qquad (19)$$

Difficulty remains to find X_0, meanwhile system construction permits to ensure that

$$\exists A_0 \in [A] \mid \dim\left(\ker\left(A_0\right)\right) \neq 0$$

We use the steepest descent method on the determinant of A to define A_0. The aim is to force an eigen value or a singular value of A_0 to be null. This operation adds pessimism to the system. Matrix A_0 becomes the center of our new system. We have the inclusion:

$$[A] \subset [A_0] \subset \frac{1}{\tau} A_0\,[B_0^{\tau}]$$

Finding A_0 is a critical step in the resolution of this problem. It has the drawback to minimize all eigen values of the system at the expense of robustness. Determinant is null if at least one eigen value is null. The steepest descent minimizes all the eigen values. The algorithm can stop near zero without being null. If A_0 cannot be found, the solution X is chosen as the singular vector associated to the smallest singular value of A. Result is a least square approximation [7]. Uncertainty is still valid for accurate data due to pessimism. In that case, we deal with estimation of bounded uncertainty propagation.

3.3 Solving non homogeneous systems

Non homogeneous systems allows the use of set theory properties to solve constraint satisfaction problems (CSP). A CSP is a system which aims to find the set of solutions $\{x\} \subset [x]$ subject to a set of constraints $f(x)$. In our case, these constraints are expressed by the equality $f(x) = 0$.

$$H : (f(x) = 0; x \in [X])$$

In the case of a non homogeneous system, f is the set of linear functions defined by matrix $[A]$ and $[B]$:

$$f([X]) = [A]\,[X] + [B]$$

Such a system may be easily solved by using classical algorithms based on Arc Consistency (AC3, AC4), which have been developped for artificial intelligence applications [4][9]. In the following section, we use interval analysis tools like the Krawczyk contractor [3] which have been adapted by [6] for solving overdetermined linear systems.

4 3D vision using interval analysis

In this section, we apply the previous methods for solving linear systems involved by camera calibration and 3D reconstruction.We assume that the uncertainty source in vision systems is the geometrical indetermination of point localization in the image. The best precision a camera could give is not a point but the rectangular shape associated to the pixel. Description of an image point by an interval is in agreement with this assumption. So, equations (2) are still valid since elementary operations are fully described by interval arithmetic rules.

In order to avoid segmentation and matching problems in the experimental test, points $\{Q_i\}$ are elements of a three dimensional known test pattern (figure 1-a). The length between two points of the square pattern is 33mm, the distance between the test-pattern and the camera is $3m$.

4.1 Application to camera calibration

We can see in figure 2-a the re-projection of the test pattern for an uncertain model of camera computed from the homogeneous system of equations (2). The expression (19) allows us to evaluate data accuracy by adjusting their uncertainty. In that case, point detection has a subpixel accuracy, the radius of uncertainty that we have to used for englobing detected points is 0.5 pixel. The figure 2-b is obtained by using the non-homogeneous resolution of camera model (3). Observed rectangles represent detected points position uncertainty with the computed uncertain camera model. The mean surface of these rectangle is $8,55$ pixels in the case of

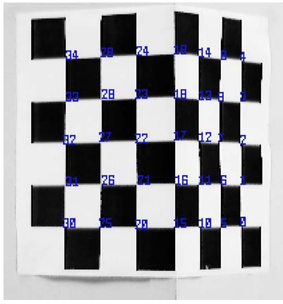


Figure 1. Image of the test pattern

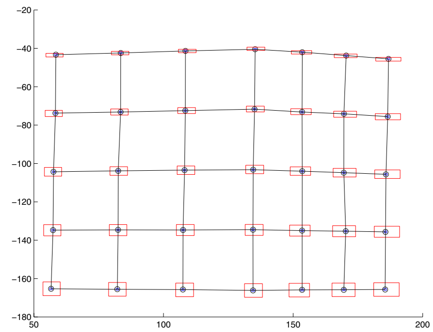
homogeneous calibration and 17.7447 pixels for the non-homogeneous one. Pessimism induced by the two method transforms pixel radius accuracy from 0.5pixel to 1.4 pixels in the first case and to 2.1 pixels in the second case. Camera calibration using homogeneous model provides better results since camera is a projective entity.

4.2 Application to 3D reconstruction

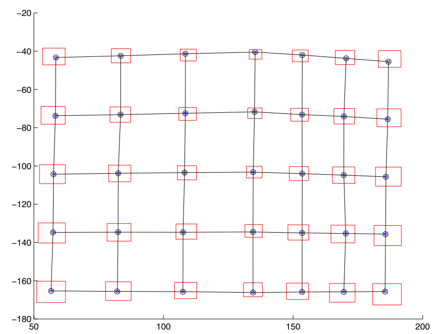
Figures (3-a,b) show the reconstruction of the test pattern in the homogeneous case (4). This reconstruction is obtained by using the uncertain camera model that the homogeneous calibration gave. This homogeneous 3D reconstruction could give good results since the mean volume of uncertainty for a reconstructed point is near zero. However, uncertainty in homogeneous space is useless for our application. The Euclidian representation of the scene require a fixed scale factor. Uncertainty on this term induces high instability. The division operation in interval arithmetic is quite pessimistic. Uncertainty can grow to be infinite if the denominator is an interval which contains 0. Moreover differences in the uncertainty repartition shows the numerical sensitivity of the method.

The reconstruction observed in figure (3-a,b) produces a mean uncertain volume of $703cm^3$ for each 3D reconstructed point. More precisely, the mean dimensions of the box equivalent to a $3m$ far point are about $(21mm \times 26mm \times 635mm)$.

Figure (3-c,d) shows the 3D reconstruction computed with Krawczyk contractor applied to the non homogeneous system (6). We present here the results obtained by using an accurate camera model. Indeed, those obtained with the uncertain camera model gave very large uncertainty volumes. As Newton algorithm, the series need several iterations to converge to solution for non-linear problem. Since our case is linear, the first iteration produces the fixed-point. Thus, reconstruction is quite rapid. The first value radius of the series which initialize algorithm is chosen as large as possible. The mean uncertain volume produced by this method is $1.77cm^3$. So, the mean dimension of the box equivalent to a $3m$ far point are about $(11mm \times 2mm \times 53mm)$. Uncertainty distribution is more uniform than that provided by the previous reconstruction. Moreover, this reconstruction does not take into account uncertainty of the camera model.



(a)



(b)

Figure 2. Re-projection obtain by (a) homogeneous calibration and (b)non-homogeneous calibration

5 Conclusion

Applying interval analysis tools in vision is a way to avoid hypothesis about the error distribution model. The only assumption is that uncertainty is bounded. This kind of assumption is suitable in the context of geometrical computation. Pixels description by using interval permits us to consider them as finite geometrical elements (rectangular shape) rather than probabilistic elements (ellipsoidal representation). Then we have proposed to model intervals by their center and radius rather than by their bounds. From this formulation, interval basis have been introduced. Whereas no guaranteed solution exist for homogeneous

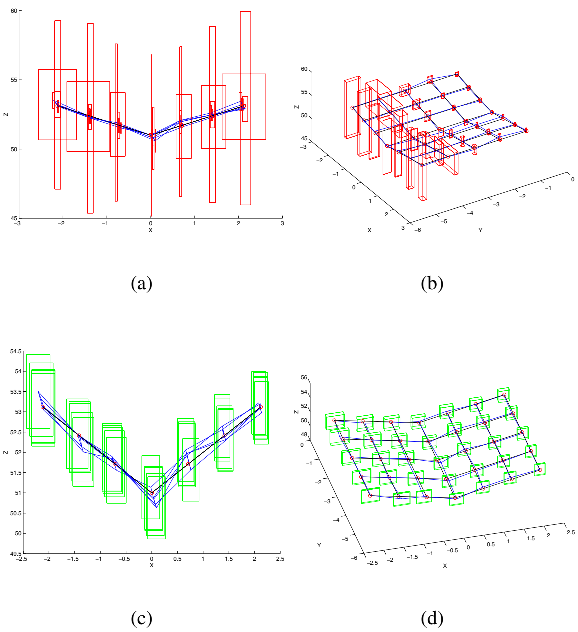


Figure 3. 3D reconstruction: real test pattern and reconstructed one are superposed. (a)X-Z plane view and (b)perspective view of the homogeneous reconstruction, (c-d)3D reconstruction by Krawczyk contractor

bounded systems, this new problem formulation allows us to estimate it. We compute a camera model to validate our method. Hence the uncertainty propagation presents the interest of producing an accurate bounded camera model.

In future works, numerical sensitivity of our algorithm will have to be reduced by data normalization. Moreover, given the performances of the Krawczyk contractor with uncertain camera model, we aims to propose tools for guaranteed 3D reconstruction.

References

[1] P. Gros, R. Hartley, R. Mohr, and L. Quan. How useful is projective geometry? *Computer Vision and Image Understanding: CVIU*, 65(3):442–446, 1997.

[2] E. Grossmann and J. SantosVictor. The precision of 3d reconstruction from uncalibrated views. In *British Machine Vision Conference*, Southampton, 1998.

[3] R. Krawczyk. Newton-algorithmen zur bestimmung von nullstellen mit fehlerschranken. *Computing*, 4:187–201, 1969.

[4] O. Lhomme. *Contribution À la Résolution de Contraintes sur Les Reels Par Propagation D'intervalles*. PhD thesis, Sophia Antipolis, Nice, France, 1994.

[5] L.Jaulin, M.Kieffer, O.Didrit, and E.Walter. *Applied Interval Analysis*. Springer Verlag, London, 2001.

[6] A. Neumaier. A simple derivation of the hansen-bliek-rohn-ning-kearfott enclosure for linear interval equations. *Reliable Computing*, 5:131–136, 1999. Erratum, Reliable Computing 6 (2000), 227.

[7] R.Hartley and A.Zisserman. *Multiple View Geometry in Computer Vision*. Cambridge, 2001.

[8] R.M.Haralick. Propagating covariance in computer vision. In *Procedings of Workshop on Performance Characterisics of Vision Algorithms*, pages 1–12, 1996.

[9] R.Mohr and T.C.Henderson. Arc and path consistency revisited. *Artificial Intelligence*, 28:225–233, 1986.

[10] S.M. Rump. *Intlab - Interval Laboratory*. inst. f. Informatik III, Technical University Hamburg-Harburg Schwarzenbergstr, 1998. http://www.ti3.tu-harburg.de/ rump/intlab/index.html.

[11] W.Eric and L.Grimson. Why stereovision is not always about 3d reconstruction. Technical report, M.I.T. A.I.Laboratory, july 1993.