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An algebraic approach to the German sentence.

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Abstract: We attempt to show that the assignment of appropriate “types” to German verbs will determine the correct word order in German sentences. The types are elements of the “free pregroup” generated by a partially ordered set of basic types, pregroups being a generalization of partially ordered groups. We look at a selection of direct and indirect sentences, including questions and subordinate clauses, but we hope that our method will extend beyond the fragment of German grammar investigated here.

1. Introduction

Pregroups had been conceived in [L 1999] as an algebraic tool to recognize grammatically well-formed sentences in natural languages. In [L 2000] an attempt was made to explain the word order of German sentences with the help of appropriate types assigned to the German verbs. But only the orders SVO (subject, verb, object), SOV and VSO were considered. Here we wish to extend the discussion to include also the orders VOS, OSV and OVS. For the sceptical reader, here are some examples to show that all six permutations can occur in certain contexts:

<i>sie</i> <i>liebt</i> <i>ihn</i>	(SVO)
<i>(dass)</i> <i>sie</i> <i>ihn</i> <i>liebt</i>	(SOV)
<i>liebt</i> <i>sie</i> <i>ihn</i> ?	(VSO)
<i>liebt</i> <i>ihn</i> <i>die Frau</i>	(VOS)
<i>(dass)</i> <i>ihn</i> <i>die Frau</i> <i>liebt</i>	(OSV)
<i><u>ihn</u></i> <i>liebt</i> <i>die Frau</i>	(OVS)

with stress on *ihn*, in reply to

wen *liebt* *die Frau*?

However, the OVS example is a case of “topicalization”, something we will refrain from discussing in this article.

2. Algebraic background

We briefly recall the mathematical machinery. A *pregroup* is a partially ordered monoid in which each element a has a *left adjoint*, i.e. an element a^ℓ satisfying

$$a^\ell a \rightarrow 1 \rightarrow aa^\ell$$

and a *right adjoint* a^r satisfying

$$aa^r \rightarrow 1 \rightarrow a^r a.$$

Here, multiplication is denoted by juxtaposition and the partial order by an arrow. One easily proves various theoretical properties such as the uniqueness of adjoints, the contravariance of adjunction

$$\text{if } a \rightarrow b \text{ then } b^\ell \rightarrow a^\ell \text{ and } b^r \rightarrow a^r,$$

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and certain identities, such as

$$(ab)^\ell = b^\ell a^\ell, (ab)^r = b^r a^r, a^{\ell r} = a = a^{r\ell}, 1^\ell = 1 = 1^r.$$

In linguistic applications we are primarily interested in the *free pregroup* generated by a partially ordered set of so-called *basic types* a, b, c, \dots . From these we form the *simple types*

$$\dots a^{\ell\ell}, a^\ell, a, a^r, a^{rr}, \dots$$

by forming iterated adjoints (although in practice, never more than double adjoints are required). The elements of the free pregroup are strings of simple types, they will just be called *types*. Multiplication is now concatenation of strings and the empty string is the unit 1. Adjoints of strings are formed with the help of the above identities, and the partial order of strings is determined by the partial order of the set of basic strings with the help of contravariance of adjunction, as well as the *contractions*

$$a^\ell a \rightarrow 1, aa^r \rightarrow 1$$

and the *expansions*

$$1 \rightarrow aa^\ell, 1 \rightarrow a^r a.$$

It was proved in [L 1999] that, as long as one merely wishes to show that a string of simple types reduces to a single simple type, say

$$\alpha_1 \dots \alpha_n \rightarrow \beta$$

in the free pregroup, only contractions are needed. However, for other purposes, for example to prove the above identities, which we will feel free to use, the expansions are necessary.

3. Basic types

Our investigation will be limited to a small fragment of German. To start with, we will assign basic types to the subject of a sentence. These will depend on a subscript $p = 1, \dots, 6$ denoting the person, where $p = 1, 2, 3$ stand for the three persons singular and $p = 4, 5, 6$ for the three persons plural. To account for variations in word order, we need to give different types to noun phrases and personal pronouns.

$$\pi_p = \text{subject } (p = 1, \dots, 6)$$

$$\hat{\pi}_p = \text{subject pronoun } (p = 3, 6)$$

$$\nu_p = \text{subject noun phrase } (p = 3, 6)$$

We postulate

$$\hat{\pi}_p \rightarrow \pi_p, \nu_p \rightarrow \pi_p$$

in the partially ordered set of basic types.

Next, we will assign basic types to the objects in German sentences, equivalently, object complements of German verbs. These depend on a subscript $c = 1, \dots, 4$ denoting the *case*, namely nominative, genitive, dative and accusative respectively. While the nominative normally belongs to the subject, it may also occur as the complement of verbs like *sein*, but we will not discuss this further. For some purposes (see Section 10), we also need “pseudo-objects”.

$$o_c = \text{object } c = 1, \dots, 4$$

$$\hat{o}_c = \text{object pronoun}$$

$$n_c = \text{object noun phrase}$$

$$o_c^* = \text{pseudo-object.}$$

We postulate

$$\hat{o}_c \rightarrow o_c, o_c^* \rightarrow o_c, n_c \rightarrow o_c.$$

German verbs may have none, one or two object-complements. But, among the complements we should also count adverbs, such as *gerne, heute, nicht*, and separable prefixes like *mit* in

$$\text{mitgeben, ich gebe... mit.}$$

The type of adverbs should really be divided into a hierarchy of subtypes (see the Duden, Section 1403, [K 1998]), but we shall avoid this complication here and write

$$\alpha = \text{adverb.}$$

We will assign the same type to adverbial phrases, including prepositional phrases. To each separable prefix P we assign the type

δ_p = separable prefix P ,

but we shall omit the subscript in what follows.

The type of sentences and propositional clauses will depend on a subscript $t=1,\dots,4$ denoting the tense-mood: present indicative, past indicative, present subjunctive, past subjunctive. We adopt the following basic types:

s_t = statement in tense-mood t

q_t = question in tense-mood t

t_t = subordinate clause in tense-mood t

\tilde{s}_t = indirect statement in tense-mood t

\tilde{q}_t = indirect question in tense-mood t .

Sometimes the tense-mood is irrelevant, so we introduce neutral types s , q , t , \tilde{s} and \tilde{q} and postulate

$s_t \rightarrow s$, $q_t \rightarrow q$, $t_t \rightarrow t$, $\tilde{s}_t \rightarrow \tilde{s}$, $\tilde{q}_t \rightarrow \tilde{q}$.

Other basic types are

i = infinitival phrase

p_2 = past participle with *haben*

p_2' = past participle with *sein*.

4. Types of verb infinitives

We shall assign types to verbs, as they might be listed in the dictionary². The types of other verb forms can then be calculated, as we shall see in Section 5. The left column in the list below gives infinitival phrases of type i . Optional complements are placed in parentheses. In the right column we list the type of the infinitive of the bare verb without complements, as it might be listed in the dictionary.

<i>(heute) kommen</i>	<i>kommen: (α^r)i</i>
<i>(α)</i>	
<i>(heute) ankommen</i>	<i>kommen: δ^r (α^r)i</i>
<i>(α) δ</i>	
<i>(den Mann/ihn) (heute) sehen</i>	<i>sehen: (α^r) (α_4^r)i</i>
<i>(α_4) (α)</i>	
<i>den Mann/ihn (heute) ansehen</i>	<i>sehen: δ^r (α^r) α_4^r i</i>
<i>α_4 (α) δ</i>	

From now on we shall largely ignore the optional adverb, which will be taken care of by Metarule 1 below.

<i>(dem Mann/ihm) helfen</i>	<i>helfen: (α_3^r)i</i>
<i>(α_3)</i>	
<i>des Mannes/seiner gedenken</i>	<i>gedenken: α_2^r i</i>
<i>α_2</i>	
<i>das Buch/es geben</i>	<i>geben: α_4^r i</i>
<i>α_4</i>	
<i>dem Mann/ihm das Buch geben</i>	<i>geben: $n_4^r \alpha_3^r$ i</i>
<i>α_3 n_4</i>	

² We leave it open whether by "dictionary" is meant a suitable revised version of the Duden or the mental dictionary of the native speaker

<i>das Buch /es mitgeben</i>		<i>geben: $\delta^r o_4^r i$</i>
o_4	δ	
<i>dem Mann /ihm das Buch mitgeben,</i>		<i>geben: $\delta^r n_4^r o_3^r i$</i>
o_3	n_4	δ

Metarule 1: In the type of a bare infinitive, we may insert α^r after δ^r or, in the absence of a separable prefix, at the beginning.³

The bare infinitive in the right column is assigned a type containing certain “valencies”⁴ to bond with corresponding complements in the normal word order, as in the left column. From this type the types of the inflected forms (finite forms and past participle) are to be calculated. So are alternative types to explain deviations from the normal word order, if one of the complements is not a noun phrase, but a personal pronoun. (We have assumed that *jemand* and *etwas* represent noun phrases.) For example, the bare infinitive *geben* may have the valency

$$n_4^r o_3^r = (o_3 n_4)^r \text{ or } \delta^r n_4^r o_3^r = (o_3 n_4 \delta)^r,$$

when the direct object is a noun phrase, but this does not explain the word order of *es jemandem mitgeben*.

The easiest way to handle this is to assign a new type to *geben*, namely $(\hat{o}_4 o_3 \delta)^r i$. So we have:

<i>es</i>	<i>dem Mann</i>	<i>mitgeben</i>		
\hat{o}_4	n_3	$\delta \delta^r o_3^r \hat{o}_4^r i$	$\rightarrow i$	since $n_3 o_3^r \rightarrow o_3 o_3^r \rightarrow 1$

(Recall that $n_3 \rightarrow o_3$, so that the type n_3 of *jemandem* may be replaced by o_3 in this reduction.)

But,

<i>*das Buch</i>	<i>dem Mann</i>	<i>mitgeben</i>		
n_4	n_3	$\delta \delta^r o_3^r \hat{o}_4^r i$	$\nrightarrow i$	since $n_4 \nrightarrow \hat{o}_4$
<i>*dem Mann</i>	<i>es</i>	<i>mitgeben</i>		
n_3	\hat{o}_4	$\delta \delta^r o_3^r \hat{o}_4^r i$	$\nrightarrow i$	
n_3	\hat{o}_4	$\delta \delta^r n_4^r o_3^r i$	$\nrightarrow i$	since $\hat{o}_4 \nrightarrow n_4$

This alternative type assignment will be shared by other bi-transitive verbs such as *zeigen*. We take care of all such new type assignments by a metarule⁵

Metarule 2: If a verb infinitive has type $n_4^r o_3^r i$, then it also has type $o_3^r \hat{o}_4^r i$.

The most frequent combination of two objects as verb complements consists of a dative and an accusative as discussed above. But other combinations are also possible, as in

<i>den Mann</i>	<i>des Diebstahls</i>	<i>beschuldigen</i>
o_4	n_2	$n_2^r o_4^r i$

³ Metarule 1 places the adverb after the object(s), according to our sample sentences. However, the correct position of the adverbs is a more complex story, for example, if the object is an indefinite noun-phrase, then the adverb precedes the object. Compare *den Witz oft machen* with *oft einen Witz machen*. We do not want here to elaborate Metarule 1 to take into account this distinction. All our other metarules remain valid, even if a more refined Metarule 1 would create different infinitive types.

⁴ The word “valency”, based on a chemical analogy, is used in the present sense by the Duden [K1998], but had been used in a similar fashion by C.S. Pierce [1897], when discussing the loose ends or the “unsaturated bonds” of a relation.

⁵ It is tempting to assume a simpler rule instead, Metarule 2': accusative pronouns of type \hat{o}_4 also have type $o_3 n_4 o_3^r$, but it seems that this might cause difficulties in later sections.

The following verbs require an infinitive or a past participle as complement:

<i>wollen:</i>	$i'i$
<i>haben:</i>	$p_2^r i$
<i>sein:</i>	$p_2'^r i$
<i>*werden:</i>	$i'i$

As a future auxiliary, the infinitive of *werden* does not really exist, but it will serve as a template for deriving the types of finite forms, such as *wirst* and *würdest*.

Among possible complements of verbs there are also indirect statements and indirect questions, for example:

<i>hoffen :</i>	$i\tilde{s}^\ell$
<i>fragen :</i>	$(o_4^r)i\tilde{q}^\ell$
<i>wissen :</i>	$i^\ell\tilde{s}^\ell, i\tilde{q}^\ell$

For example, since $\tilde{q}_1 \rightarrow \tilde{q}$, $\hat{o}_4 \rightarrow o_4$

<i>(ich will)ihn</i>	<i>fragen,</i>	<i>wer morgen kommt</i>	
\hat{o}_4	$(o_4^r)i(\tilde{q}^\ell)$	\tilde{q}_1	$\rightarrow i.$

5. Types for inflected verb forms

Presumably, the dictionary lists only infinitives of verbs, together with some clues how to calculate other forms for irregular verbs. (For regular verbs such clues are not necessary.) For a verb V we are interested in its past participle $Part_2(V)$ and the finite forms $C_p(V)$, where $t=1,\dots,4$ stands for the tense-moods and $p=1,\dots,6$ for the persons singular and plural.

Here is the recipe for calculating the types of these forms :

Metarule 3 If the infinitive of V has type $X^r i Y^\ell$, then the past participle

$$Part_2(V) \text{ has type } \begin{cases} X^r p_2 Y^\ell & \text{if the perfect is formed with } haben \\ X^r p_2' Y^\ell & \text{if the perfect is formed with } sein \end{cases}$$

and the finite form

$$C_p(V) \text{ has type } \begin{cases} \pi_p^r s_t Y^\ell X^\ell & \text{in statements} \\ X^r \pi_p^r t_i Y^\ell & \text{in subordinate clauses} \\ q_t Y^\ell X^\ell \pi_p^\ell & \text{in yes/no- questions} \end{cases}$$

Here X and Y are strings of simple types, e.g.

$$X = \xi_1 \dots \xi_n, \quad (n \geq 0).$$

In particular, $Y=1$ is often the empty string. For verbs with separable prefix P , $\xi_n = \delta_p$ (but we usually omit the subscript). An optional adverb would occur as ξ_n or, if $\xi_n = \delta_p$, as ξ_{n-1} .

It is our intention to exploit this and later metarules to analyze various direct and indirect sentences, without overloading the dictionary.

6. Word order in declarative sentences

For the moment, let us concentrate on the special case $Y=1$ in Metarule 3. Here are some illustrations :

<i>Er</i>	<i>kommt.</i>		
$\hat{\pi}_3$	$\pi_3^r s_1$	$\rightarrow \pi_3 \pi_3^r s_1$	$\rightarrow s_1$
$(X=1)$			

$$\begin{array}{l}
\text{Er} \quad \text{kommt} \quad \text{heute} \quad \text{an.} \\
\widehat{\pi}_3 \quad \pi_3^r s_1 \delta^\ell \alpha^\ell \quad \alpha \quad \delta \quad \rightarrow \pi_3 \pi_3^r s_1 \delta^\ell \alpha^\ell \alpha \delta \quad \rightarrow s_1 \\
(X = \alpha\delta, \text{ using Metarule 1})
\end{array}$$

$$\begin{array}{l}
\text{Er} \quad \text{sieht} \quad \text{es/etwas.} \\
\widehat{\pi}_3 \quad \pi_3^r s_1 o_4^\ell \quad o_4 \quad \rightarrow s_1 \\
(X = o_4)
\end{array}$$

$$\begin{array}{l}
\text{Er} \quad \text{sieht} \quad \text{es} \quad \text{an} \\
\widehat{\pi}_3 \quad \pi_3^r s_1 \delta^\ell o_4^\ell \quad \widehat{o}_4 \quad \delta \rightarrow \pi_3 \pi_3^r s_1 \delta^\ell o_4^\ell o_4 \delta \quad \rightarrow s_1 \\
(X = o_4 \delta)
\end{array}$$

$$\begin{array}{l}
\text{Er} \quad \text{gibt} \quad \text{ihr/der Frau} \quad \text{ein Buch} \\
\widehat{\pi}_3 \quad \pi_3^r s_1 n_4^\ell o_3^\ell \quad o_3 \quad n_4 \quad \rightarrow \pi_3 \pi_3^r s_1 n_4^\ell o_3^\ell o_3 n_4 \rightarrow s_1 \\
(X = o_3 n_4)
\end{array}$$

$$\begin{array}{l}
\text{Er} \quad \text{gibt} \quad \text{es} \quad \text{der Frau.} \\
\widehat{\pi}_3 \quad \pi_3^r s_1 o_3^\ell \widehat{o}_4^\ell \quad \widehat{o}_4 \quad n_3 \quad \rightarrow \pi_3 \pi_3^r s_1 o_3^\ell o_3 \quad \rightarrow s_1 \\
(X = \widehat{o}_4 o_3, \text{ using Metarule 2})
\end{array}$$

The following examples involve auxiliary verbs:

$$\begin{array}{l}
\text{Er} \quad \text{wollte} \quad \text{nicht} \quad \text{kommen} \\
\widehat{\pi}_3 \quad \pi_3^r s_2 i^\ell \alpha \quad a^r i \quad \rightarrow s_2 \\
(X = i \text{ using Metarule 1})
\end{array}$$

$$\begin{array}{l}
\text{Sie} \quad \text{hätte} \quad \text{ihn} \quad \text{angesehen} \\
\widehat{\pi}_3 \quad \pi_3^r s_4 p_2^\ell \quad \widehat{o}_4 \quad \delta \delta^r o_4^r p_2 \quad \rightarrow s_4 \\
(X = p_2 ; X = o_4 \delta)
\end{array}$$

$$\begin{array}{l}
\text{Sie} \quad \text{ist} \quad \text{nicht} \quad \text{gekommen} \\
\widehat{\pi}_3 \quad \pi_3^r s_1 p_2'^\ell \quad \alpha \quad \alpha^r p_2' \quad \rightarrow s_1 \\
(X = p_2' ; X = \alpha \text{ using Metarule 1})
\end{array}$$

$$\begin{array}{l}
\text{Sie} \quad \text{würde} \quad \text{es} \quad \text{ihm} \quad \text{nicht} \quad \text{gegeben} \quad \text{haben} \\
\widehat{\pi}_3 \quad \pi_3^r s_4 i^\ell \widehat{o}_4 \quad \widehat{o}_3 \quad \alpha \quad \alpha^r o_3^r \widehat{o}_4^r p_2 \quad p_2^r i \quad \rightarrow s_4 \\
(X = i ; X = \widehat{o}_4 o_3, \text{ Metarule 2})
\end{array}$$

To obtain examples with $Y \neq 1$, we take $Y = \tilde{s}$ or \tilde{q} . Recall that the types of indirect sentences are \tilde{s} or \tilde{q} (to be analyzed in Section 7).

$$\begin{array}{l}
\text{Er} \quad \text{hofft,} \quad \text{dass sie kommt} \\
\widehat{\pi}_3 \quad \pi_3^r s_1 \tilde{s}^\ell \quad \tilde{s} \quad \rightarrow s_1 \\
(X = 1, Y = \tilde{s})
\end{array}$$

$$\begin{array}{l}
\text{Er} \quad \text{hatte} \quad \text{sie} \quad \text{gefragt,} \quad \text{ob sie käme} \\
\widehat{\pi}_3 \quad \pi_3^r s_2 p_2^\ell \quad \widehat{o}_4 \quad o_4^r p_2 \tilde{q}^\ell \quad \tilde{q} \quad \rightarrow s_2 \\
(X = p_2, Y = 1; X = o_4, Y = \tilde{q})
\end{array}$$

7. Word order in subordinate clauses

Indirect statements and questions may be introduced by the complementizers

dass : $\tilde{s}t^\ell$

ob, wieso, wann etc. : $\tilde{q}t^\ell$

respectively⁶. These are then followed by an incomplete subordinate clause of type $t_i \rightarrow t$ with the *SOV* word order, making use of Metarule 3. We start with some examples with $Y = 1$:

dass sie ihr ein Buch gibt
 $\tilde{s}t^\ell \quad \hat{\pi}_3 \quad \hat{o}_3 \quad n_4 \quad n_4^r o_3^r \pi_3^r t_1 \rightarrow \tilde{s}$
 ($X = o_3 n_4$)

ob sie ihm nicht helfen wird
 $\tilde{q}t^\ell \quad \hat{\pi}_3 \quad \hat{o}_3 \quad \alpha \quad \alpha^r o_3^r i \quad i^r \pi_3^r t_1 \rightarrow \tilde{q}$
 (Metarule 1; $X = i$)

For our next few examples we require a new metarule :

Metarule 4: Dative and accusative pronouns of type \hat{o}_c ($c = 3$ or 4) also have type $v_p \hat{o}_c v_p^\ell$ ($p = 3$ or 6).

dass ihr die Frauen das Buch geben
 $\tilde{s}t^\ell \quad v_6 \hat{o}_3 v_6^\ell v_3 \quad n_4 \quad n_4^r o_3^r \pi_6^r t_1 \rightarrow \tilde{s}$
 ($X = o_3 n_4$)

ob es Gisela ihr gegeben hätte
 $\tilde{q}t^\ell \quad v_3 \hat{o}_3 v_3^\ell v_3 \quad \hat{o}_3 \quad o_3^r \hat{o}_4^r p_2 \quad p_2^r \pi_3^r t_4 \rightarrow \tilde{q}$
 ($X = \hat{o}_4 o_3$ using Metarule 2; $X = p_2$).

wieso ihr Gisela das Buch geben will.
 $\tilde{q}t^\ell \quad v_3 \hat{o}_3 v_3^\ell v_3 \quad n_4 \quad (o_3 n_4)^r i \quad i^r \pi_3^r t_1 \rightarrow \tilde{q}t^\ell v_3 \hat{o}_3 o_3^r \pi_3^r t_1 \rightarrow \tilde{q}$
 ($X = o_3 n_4$; $X = i$)

8. Word order in yes-or-no questions.

We again apply Metarule 3, with $Y = 1$:

Kommt der Mann heute an?
 $q_1 \delta^\ell \alpha^\ell \pi_3^\ell v_3 \quad \alpha \quad \delta \rightarrow q_1 \delta^\ell \alpha^\ell \pi_3^\ell \pi_3 \alpha \delta \rightarrow q_1$
 ($X = \alpha \delta$, using Metarule 1)

Hilft die Frau dem Jungen gerne?
 $q_1 \alpha^\ell o_3^\ell \pi_3^\ell v_3 \quad v_3 \quad n_3 \quad \alpha \rightarrow q_1$
 ($X = o_3 \alpha$)

Wird es Gisela der Freundin geben?
 $q_1 i^\ell \pi_3^\ell v_3 \hat{o}_4 v_3^\ell v_3 \quad n_3 \quad o_3^r \hat{o}_4^r i \rightarrow q_1 i^\ell \pi_3^\ell \pi_3 \hat{o}_4 \hat{o}_4^r i \rightarrow q_1$
 ($X = i$; $X = \hat{o}_4 o_3$ using Metarule 2)

⁶ Other indirect questions introduced by *wer, wen, wem*, etc are treated in Section 11

Würde ihm die Frau nicht geholfen haben?
 $q_4 i^\ell \pi_3^\ell \nu_3 \widehat{o}_3 \nu_3^\ell \nu_3 \alpha \alpha^r o_3^r p_2 p_2^r i \rightarrow q_4$
 ($X = i$; $X = o_3 \alpha$)

In addition to the yes-or-no questions, there are what the Duden calls "Ergänzungsfragen". These are questions which ask for a missing subject, object or adverbial phrase.

9. Asking for the subject

To ask for the subject, one uses the question words *wer* or *was*, depending on whether the expected subject is a person or not. To keep this article in reasonable bounds, we will refrain from elaborating this distinction. The word order is the same as in a statement. Thus *wer* or *was* has type $qs^\ell \pi_3$, as in

Er kam heute.
 $\pi_3 \pi_3^r s_2 \alpha^\ell \alpha \rightarrow s_2$
Wer kam heute?
 $qs^\ell \pi_3 \pi_3^r s_2 \alpha^\ell \alpha \rightarrow q$

Es gefällt dir.
 $\pi_3 \pi_3^r s_1 o_3^\ell \widehat{o}_3 \rightarrow s_1$
Was gefällt dir?
 $qs^\ell \pi_3 \pi_3^r s_1 o_3^\ell \widehat{o}_3 \rightarrow q$

Notice that we wrote q rather than q_1 or q_2 , since question words are not marked for tense.⁷ Here are some more examples:

Wer fragt ihn, ob er kommt?
 $qs^\ell \pi_3 \pi_3^r s_1 \tilde{q}^\ell o_4^\ell \widehat{o}_4 \tilde{q}^\ell \pi_3 \pi_3^r t_1$

Wer würde ihm das Buch geben?
 $qs^\ell \pi_3 \pi_3^r s_4 i^\ell \widehat{o}_3 n_4 n_4^r o_3^r i$

In the above examples, $p = 3$, but $p = 6$ may also occur:

⁷ In analogy with questions asking for the object (Section 10), one can also use

wer/was : $q \pi_3^{*\ell\ell} q^\ell$

with a new type π_3^* , postulating $\pi_3^* \rightarrow \pi_3$ and add a new metarule:

Metarule : If the infinitive of V has type $X^r i Y^\ell$, then

$C_{ip}(V)$ has type $q_i \pi_p^{*\ell} Y^\ell X^\ell$ ($p = 3, 6$)

This will account for

Wer kam gestern?
 $q \pi_3^{*\ell\ell} q^\ell q_2 \pi_3^{*\ell} \alpha^\ell \alpha \rightarrow q$

Note that the star on π_p prevents

**Kam heute er/der Mann?*
 $q_2 \pi_3^{*\ell} \alpha^\ell \alpha \pi_3 \rightarrow q_2$, since $\pi_3 \rightarrow \pi_3^*$.

Welche Bücher gefallen dir?

Here the compound expression, *welche Bücher*, should have type $qs^\ell \pi_6$. We shall not discuss this any further.

10. Asking for a verb complement

To ask for an object complement of type o_c , where $c=1,2,3,4$, we use the question words *wer/was, wessen, wem, wen/was*.

The choice in cases $c=1$ and $c=4$ depend on whether we expect the object to be a person or not. We will ignore this distinction in what follows. We shall assign to these question words the type $qo_c^{*\ell} q^\ell$. But first, let us look at some examples illustrating the cases $c=3$ and $c=4$. (The other cases occur more rarely.)

<i>Wem</i>	<i>gabst</i>	<i>du</i>	<i>das Buch?</i>	
$qo_3^{*\ell} q^\ell$	$q_2 o_3^{*\ell} n_4^\ell \pi_2^\ell$	π_2	n_4	
<i>Was</i>	<i>hattest</i>	<i>du</i>	<i>ihm</i>	<i>gegeben?</i>
$qo_4^{*\ell} q^\ell$	$q_2 p_2^\ell \pi_2^\ell$	π_2	\widehat{o}_3	$o_3^r p_2 o_4^{*\ell}$
<i>Wem</i>	<i>wolltest</i>	<i>du</i>	<i>das Buch</i>	<i>geben?</i>
$qo_3^{*\ell} q^\ell$	$q_2 i^\ell \pi_2^\ell$	π_2	n_4	$n_4^r i o_3^{*\ell}$
<i>Wem</i>	<i>wolltest</i>	<i>du</i>	<i>es</i>	<i>geben?</i>
$qo_3^{*\ell} q^\ell$	$q_2 i^\ell \pi_2^\ell$	π_2	\widehat{o}_4	$\widehat{o}_4^r i o_3^{*\ell}$

The reason for the star is to prevent the following:

**Gabst du das Buch ihm?*

**Hattest du ihm gegeben das Buch?*

**wolltest du das Buch geben dem Mann?*

which are not acceptable in German.

The new types of *gabst* and *gegeben* are derivable from the new types of *geben*, but to explain the latter, we require a new metarule, which will allow us to derive them from the old types

geben : $n_4^r o_3^r i$, $o_3^r \widehat{o}_4^r i$.

Metarule 5: If the infinitive of a verb V has type

$V^r z^r U^r i Y^\ell = (U z V)^r i Y^\ell$

where $z \rightarrow o_c$, then it also has types

$V^r U^r i o_c^{*\ell} Y^\ell = (UV)^r i (Y o_c^*)^\ell$ (in direct questions)⁸

$V^r U^r o_c^{*r} i Y^\ell = (o_c^* UV)^r i Y^\ell$ (in indirect questions)⁹

The application to indirect questions will only come up in Section 11. But for now, we may infer, with the help of Metarule 3, that

$Part_2(V) : (UV)^r p_2 (Y o_c^*)^\ell$ or $Part_2(V) : (UV)^r p_2' (Y o_c^*)^\ell$

$C_{ip}(V) : q_i (Y o_c^*)^\ell (UV)^\ell \pi_p^\ell$,

which we used in the examples of direct questions above.

⁸ This type will also be used for the passive voice, Section 13

⁹ This type will also be used in relative clauses, Section 12

The adverbial question words, *wann*, *wie*, etc; transform a yes-or-no question into a more specific one which asks for an adverbial complement. Thus¹⁰

wann, *wie*, *warum*, *wo*...: $q\alpha^\ell$.

The word order is the same as in a yes-or-no question. For example, the yes-or-no question

Siehst du ihn an?
 $q_1\delta^\ell o_4^\ell \pi_2^\ell \quad \pi_2 \quad \hat{o}_4 \quad \delta$

is transformed into the w-question

Wann siehst du ihn an?
 $q\alpha^\ell \quad q_1\delta^\ell o_4^\ell \pi_2^\ell \quad \pi_2 \quad \hat{o}_4 \quad \delta$

Of course,

**siehst du ihn an heute?*

remains ungrammatical, as *siehst* has type $q_1\delta^\ell \alpha^\ell o_4^\ell \pi_2^\ell$, according to Metarules 1 and 3.

11. Indirect questions

An indirect question is either introduced by *ob* or by the same question word as a direct question, but with a different type as follows¹¹:

ob, *wann*, *wie*, *warum*, *wo*...: $\tilde{q}t^\ell$
wer/was: $\tilde{q}t^\ell \pi_3$
wessen: $\tilde{q}t^\ell \pi_p o_2^* \pi_p^\ell$
wem: $\tilde{q}t^\ell \pi_p o_3^* \pi_p^\ell$
wen/was: $\tilde{q}t^\ell \pi_p o_4^* \pi_p^\ell$ ($p=1, \dots, 6$).

Note that the question words, *ob*, *wann*, *wie* etc., precede an incomplete subordinate clause (without complementizer) and transform it into a complete indirect question.

Here are a few examples

ob/wann du ihr das Buch heute geben wirst
 $\tilde{q}t^\ell \quad \pi_2 \quad \hat{o}_3 \quad n_4 \quad \alpha \quad \alpha^r n_4^r o_3^r i \quad i^r \pi_2^r t_1$

wer ihr das Buch heute geben wird
 $\tilde{q}t^\ell \pi_3 \quad \hat{o}_3 \quad n_4 \quad \alpha \quad \alpha^r n_4^r o_3^r i \quad i^r \pi_3^r t_1$

¹⁰ Another solution, similar to object complements, assigns the following type to the question words

wann, *wie* *warum*, *wo*...: $q\alpha^{*\ell} q^\ell$.

Here we assume $\alpha^* \rightarrow \alpha$ and Metarule 5 with α instead of o_c and α^* instead of o_c^* . Then the yes/no question

siehst du ihn heute an?
 $q_1\delta^\ell \alpha^\ell o_4^\ell \pi_2^\ell \quad \pi_2 \quad \hat{o}_4 \quad \alpha \quad \delta$

is transformed into the w-question

wann siehst du ihn an?
 $q\alpha^{*\ell} q^\ell \quad q_1\alpha^{*\ell} \delta^\ell o_4^\ell \pi_2^\ell \quad \pi_2 \quad \hat{o}_4 \quad \delta$

where the star again prevents the ungrammatical

**siehst du ihn an heute?*

¹¹ For completeness sake, we repeat here the types of *ob*, *wann*, *wieso* etc. given in Section 7

<i>wem</i>	<i>du</i>	<i>das Buch</i>	<i>heute</i>	<i>geben</i>	<i>wirst</i>
$\tilde{q}t^\ell \pi_2 o_3^* \pi_2^\ell$	π_2	n_4	α	$\alpha' n_4^r o_3^* i$	$i^r \pi_2^r t_1$
<i>wem</i>	<i>du</i>	<i>es</i>	<i>heute</i>	<i>gegeben</i>	<i>hast</i>
$\tilde{q}t^\ell \pi_2 o_3^* \pi_2^\ell$	π_2	\hat{o}_4	α	$\alpha' \hat{o}_4^r o_3^* p_2$	$p_2^r \pi_2^r t_1$
<i>was</i>	<i>du</i>	<i>ihr</i>	<i>heute</i>	<i>geben</i>	<i>wirst</i>
$\tilde{q}t^\ell \pi_2 o_4^* \pi_2^\ell$	π_2	\hat{o}_3	α	$\alpha' o_3^r o_4^* i$	$i^r \pi_2^r t_1$

12. Relative clauses

Among the many kinds of relative clauses discussed in the Duden [K 1998], we shall here concentrate on those that modify noun phrases and are introduced by a relative pronoun. We recall from our paper [LP 2003] that noun phrases were given the type

$$n_{gnc} \quad (g = 1, 2, 3; n = 1, 2; c = 1, 2, 3, 4),$$

where the subscript g stands for the gender (masculine, feminine, neuter), the subscript n for the number (singular, plural) and the subscript c for the case (nominative, genitive, dative, accusative).

The relative pronouns can be arranged in a matrix

$$R = \begin{pmatrix} \textit{der} & \textit{dessen} & \textit{dem} & \textit{den} \\ \textit{die} & \textit{deren} & \textit{der} & \textit{die} \\ \textit{das} & \textit{dessen} & \textit{dem} & \textit{das} \\ \textit{die} & \textit{deren} & \textit{denen} & \textit{die} \end{pmatrix}$$

where the first three rows represent the genders singular and the last row the plural irrespective of gender. The columns represent the cases. The matrix R is almost the same as the matrix D for the definite article discussed in [loc.cit], except that all genitives and the plural dative contain an extra morpheme $+(s)en$. Let $g' = 1, 2, 3, 4$; the entries of R are denoted $R_{g'c}$.

A noun phrase of type n_{gnc} may be modified by a relative clause starting with $R_{g'c'}$, where the case c' is independent of the case c and

$$g' = \begin{cases} g, & \text{if } n = 1 \\ 4, & \text{if } n = 2 \end{cases}.$$

Its type is given by

$$R_{g'1} : n_{gnc}^r n_{gnc}^t \pi_3$$

$$R_{g'c'} : n_{gnc}^r n_{gnc}^t \pi_p o_c^* \pi_p^\ell, \quad (c' = 2, 3, 4)$$

where p stands for the person of the subject of the relative clause. Note that $R_{g'c'}$ agrees with the noun phrase it modifies in number and gender, but not in case.

To find the type of the finite form, use Metarule 5 to derive types $V^r U^r o_c^* i$ for *geben*, *sehen* etc. from the infinitive $V^r z^r U^r i$. Then take the type for subordinate clauses in Metarule 3. In the same way, the past participle gets the new type $V^r U^r o_c^* p_2$. Notice that in the case where the relative pronoun is the subject, the type $V^r z^r U^r p_2$ derived directly from the $V^r z^r U^r i$ still works.

Here are a couple of illustrations :

<i>den Mann,</i>	<i>der</i>	<i>mir</i>	<i>das Buch</i>	<i>gegeben</i>	<i>hat</i>
n_{114}	$n_{114}^r n_{114}^t \pi_3$	\hat{o}_3	n_4	$n_4^r o_3^r p_2$	$p_2^r \pi_3^r t_1 \rightarrow n_{114}$

<i>die Vorfahren,</i>	<i>deren</i>	<i>wir</i>	<i>heute</i>	<i>gedenken</i>		
n_{124}	$n_{124}^r n_{124}^t \pi_4^* \pi_4^\ell$	π_4	α	$\alpha^r o_2^{*r} \pi_4^r t_1$	$\rightarrow n_{124}$	
<i>die Kinder,</i>	<i>denen</i>	<i>du</i>	<i>es</i>	<i>gabst,</i>		
n_{321}	$n_{321}^r n_{321}^t \pi_2^* \pi_2^\ell$	π_2	\widehat{o}_4	$\widehat{o}_4^r o_3^{*r} \pi_2^r t_2$	$\rightarrow n_{321}$	
<i>die Frauen,</i>	<i>denen</i>	<i>ihr</i>	<i>es</i>	<i>heute geben</i>	<i>werdet</i>	
n_{224}	$n_{224}^r n_{224}^t \pi_5^* \pi_5^\ell$	π_5	\widehat{o}_4	α	$\alpha^r \widehat{o}_4^r o_3^{*r} i^r \pi_5^r t_1$	$\rightarrow n_{224}$
<i>eines Mannes,</i>	<i>den</i>	<i>du</i>	<i>gesehen</i>	<i>hättest</i>		
n_{112}	$n_{112}^r n_{112}^t \pi_2^* \pi_2^\ell$	π_2	$o_4^{*r} p_2$	$p_2^r \pi_2^r t_4$	$\rightarrow n_{112}$	

The relative pronouns *der/die/das, dessen* etc. behave similarly to the question words *wer/was, wessen* etc in indirect questions. Just replace \tilde{q} in the type of *wer/was, wessen* by $n_{gnc}^r n_{gnc}$ to get the type of the corresponding relative pronoun. The order of words in relative clauses is the same as in indirect questions.

13. The passive voice

German admits two kinds of passive: with *werden* or with *sein*. We shall confine attention to the former, in particular to the most frequently occurring passive of the verb with a direct object. All such verbs that form the perfect with *haben* admit a passive:

er hat sie gesehen / sie wird gesehen

er hatte ihr ein Buch gegeben / ein Buch wurde ihr gegeben.

We derive these and many other examples with the help of an appropriate type assignment to the passive auxiliary *werden*. In contrast to the future auxiliary **werden*, which is defective, it has all finite forms, an infinitive and a past participle, although the last is *worden* instead of **geworden*.

$$\text{werden : } \quad o_4^* p_2^r i = o_4^{*r} p_2^r i = (p_2 o_4^*)^r i \quad ^{12}$$

Hence, by Metarule 3,

$$\begin{array}{ll} \text{worden :} & o_4^* p_2^r p_2' \\ C_{ip}(\text{werden}) : & \pi_p^r s_i o_4^{*ll} p_2^\ell \quad \text{in statements} \\ & q_i o_4^{*ll} p_2^\ell \pi_p^\ell \quad \text{in yes-or-no questions} \\ & o_4^* p_2^r \pi_p^r t_i \quad \text{in subordinate clauses.} \end{array}$$

Here are some examples

$$\begin{array}{lll} \text{Sie} & \text{wurde} & \text{gesehen} \\ \widehat{\pi}_3 & \pi_3^r s_2 o_4^{*ll} p_2^\ell & p_2 o_4^* \end{array}$$

$$\begin{array}{lll} \text{Wurde} & \text{sie} & \text{gesehen?} \\ q_2 o_4^{*ll} p_2^\ell \pi_3^\ell & \widehat{\pi}_3 & p_2 o_4^* \end{array}$$

$$\begin{array}{llll} \text{(ich glaubte), dass} & \text{sie} & \text{gesehen} & \text{würde} \\ \tilde{s} t^\ell & \widehat{\pi}_3 & p_2 o_4^* & o_4^* p_2^r \pi_3^r t_4 \end{array}$$

¹² This is an instance where the string $X = p_2 o_4^*$ of the infinitive $X^r i$ includes a non-basic type.

Sie *wird* *gesehen* *werden.*
 $\widehat{\pi}_3$ $\pi_3 s_1 i^\ell$ $p_2 o_4^{*\ell}$ $o_4^* p_2^r i$

Sie *ist* *gesehen* *worden.*
 $\widehat{\pi}_3$ $\pi_3 s_1 p_2'$ $p_2 o_4^{*\ell}$ $o_4^* p_2^r p_2'$

We may extend the discussion from purely transitive verbs to verbs which admit an indirect object as well as a direct one:

Es *wurde* *ihr* *gegeben.*
 $\widehat{\pi}_3$ $\pi_3^r s_2 o_4^{*\ell\ell} p_2^\ell$ \widehat{o}_3 $o_3^r p_2 o_4^{*\ell}$

Wurde *es* *ihr* *gegeben?*
 $q_2 o_4^{*\ell\ell} p_2^\ell \pi_3^\ell$ $\widehat{\pi}_3$ \widehat{o}_3 $o_3^r p_2 o_4^{*\ell}$

Wurde *ihr* *das Buch* *gegeben?*
 $q_2 o_4^{*\ell\ell} p_2^\ell \pi_3^\ell$ $v_3 o_3 v_3^\ell$ v_3 $o_3^r p_2 o_4^{*\ell}$

(Ich glaubte), dass *es* *ihr* *gegeben* *würde.*
 $\tilde{s}t^\ell$ $\widehat{\pi}_3$ \widehat{o}_3 $o_3^r p_2 o_4^{*\ell}$ $o_4^* p_2^r \pi_3^r t_4$

To obtain the type $o_3^r p_2 o_4^{*\ell}$ of the past participle *gegeben*, use Metarule 5 to derive *geben*: $o_3^r i o_4^{*\ell}$ from the infinitive *geben*: $(o_3 n_4)^r i$ and then apply Metarule 3. In the following two examples, the Metarule 5 has to be used twice:

Wem *wurde* *es* *gegeben?*
 $q o_3^{*\ell} q^\ell$ $q_2 o_4^{*\ell\ell} p_2^\ell \pi_3^\ell$ $\widehat{\pi}_3$ $p_2 o_4^* o_3^{*\ell}$

(Metarule 3, applied to *geben*: $i o_4^{*\ell} o_3^{*\ell}$, from *geben*: $o_3^r i o_4^{*\ell}$, by Metarule 5)

(Ich weiß), wem *es* *gegeben* *wurde.*
 $\tilde{q}t^\ell \pi_p o_3^* \pi_p^\ell$ $\widehat{\pi}_3$ $o_3^* p_2 o_4^{*\ell}$ $o_4^* p_2^r \pi_3^r t_2$

(Metarule 3, applied to *geben*: $o_3^* i o_4^{*\ell}$, also from *geben*: $o_3^r i o_4^{*\ell}$ by Metarule 5)

Finally, let us look at a more complex example:

Sie *wurde* *gefragt,* *ob* *sie* *gesehen worden* *sei.*
 $\widehat{\pi}_3$ $\pi_3^r s_2 o_4^{*\ell\ell} p_2^\ell$ $p_2 o_4^* \tilde{q}^\ell$ $\tilde{q}t^\ell$ $\widehat{\pi}_3$ $p_2 o_4^{*\ell}$ $o_4^* p_2^r p_2'$ $p_2^r \pi_3^r t_3$

14. Concluding remarks

We have attempted to show how word order in German sentences is triggered by the types assigned to words in the dictionary. A similar project had been carried out by Ballmer [B 1975] in the context of Bar-Hillel's [BH 1964] formulation of categorical grammar with two basic types only. Here we follow [L 2000] in using the pregroup approach, with a much larger set of basic types, wedding morphology to syntax, to do justice to such features as person, tense and case. The present account differs from that in [L 2000] where *werden* had been assigned the type $p_2^r o_4^{*r} i$, hence we need no longer assert that German requires double right adjoints.

We want the dictionary to only list infinitives of verbs, together with their types, and leave the types of other verb forms to be calculated by a number of metarules. We have also presented one metarule for calculating alternative types of oblique personal pronouns.

Both authors spoke German in early childhood and have relied on introspection to "recall" aspects of German grammar. In case of doubt, they have consulted the standard German reference work on grammar, the Duden [K 1998]. The edition consulted has 912 pages, hence only a tiny fraction of German grammar had been

covered here. For the same reason, it was not possible to address many, if not most, of the concerns of modern linguists working within the Chomskyan paradigm.¹³

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¹³ Bierwisch [1996] had looked at German grammar from a ‘minimalist’ point of view, but did not address the problem of different word orders, which is our main concern.