Generic Method for Deriving the General Shaking Force Balance Conditions of Parallel Manipulators with Application to a Redundant Planar 4-RRR Parallel Manipulator

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Generic Method for Deriving the General Shaking Force Balance Conditions of Parallel Manipulators with Application to a Redundant Planar 4-RRR Parallel Manipulator

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Abstract— *This paper proposes a generic method for deriving the general shaking force balance conditions of parallel manipulators. Instead of considering the balancing of a parallel manipulator link-by-link or leg-by-leg, the architecture is considered altogether.*

The first step is to write the linear momentum of each element. The second step is to substitute the derivatives of the loop equations, by which the general force balance conditions are obtained. Subsequently specific kinematic conditions are investigated in order to find advantageous, simple balance solutions.

As an example, the method is applied to a planar 4-RRR parallel manipulator, for which the force balance conditions and solutions are discussed and illustrated for each step respectively. By including the loop equations, linear relations of the motion among mechanism elements lead to an increase of balance possibilities. For specific kinematic conditions, additional linear relations among the motion of mechanism elements may be obtained, resulting in another increase of balance possibilities. For the latter, symmetric motion is an important feature for which a 4-RRR manipulator is advantageous.

Keywords: Shaking Force Balancing, Linear Momentum, Parallel Manipulator, Machine Vibrations

I. Introduction

When mechanisms (i.e. manipulators or robots) are running at high speeds, shaking forces and shaking moments are a prominent cause of machine vibrations. Instead of applying damping to reduce the influence of these vibrations, with dynamic balancing the mechanism is designed to exert no shaking forces and shaking moments at all. Therefore dynamically balanced mechanisms exhibit, among others, reduced wear, noise, and fatigue [1], increased accuracy [2], [3], and increased payload capacity [4].

Disadvantages of dynamic balancing are the often considerable increase of mass, inertia, and complexity of the mechanism [5]. These are the main reasons that there are few studies on dynamic balancing of multi-degree-of-freedom (multi-DoF) parallel mechanisms, which are already complex mechanisms by themselves because of their parallel structures.

To omit the complexity of using the loop equations, most studies on parallel mechanisms are involved with balancing of each mechanism link or each manipulator leg individually, including [2], [6], [7], [8], [9], [10]. A class of low-mass force balanced mechanisms of which the mechanism is considered altogether without the need of loop equations is presented in [11].

When the loop equations are not considered, the number of balance solutions that can be found is limited. With loop equations the linear relations of the motion among mechanism elements are included, resulting in an increased number of balance solutions. For example deriving the force balance conditions with the *Linear Independent Vector Method* (LIV-method) [12], which considers the loop equations, results in a general description of the force balance from which the complete set of general force balance conditions can be derived. The LIV-method is well applicable to parallel mechanisms, for which [12] also shows a few examples. However the derivation of the balance conditions is cumbersome and specific for each mechanism.

Shaking force balancing by using linear momentum equations and shaking moment balancing by using angular momentum equations have shown to be systematic and intuitive approaches for finding the complete set of balance conditions [11], [13]. A mechanism is force balanced when the linear momentum is conserved, and a mechanism is moment balanced when the angular momentum is conserved for any motion of the mechanism.

This paper proposes a generic method for deriving the general (i.e. complete set of) shaking force balance conditions of parallel manipulators by considering the loop equations. In addition, it will be shown that advantageous balance solutions can be found for specific kinematic conditions, an opportunity that is usually overlooked.

The method is applied to a planar 4-RRR parallel manipulator, which, if in addition to being kinematically redundant has actuation redundancy, can become a high performance robot [14], [15]. This is the case since actuation redundancy leads to superior kinetostatic performances. The kinematic redundancy of the 4-RRR manipulator will show to be advantageous for dynamic balancing as compared with a 3-RRR manipulator.

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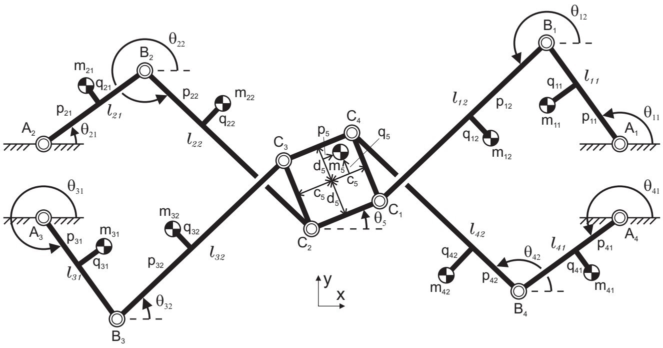

Fig. 1. Redundant Planar 4-RRR Parallel Manipulator with its Parameters. Joints C_i are located at the Platform for having a Maximal Force Transmission to the Platform.

First the linear momentum equations without loop equations are investigated (leg-by-leg approach). Subsequently the derivatives of the loop equations are substituted, gaining the general force balance conditions. Then specific kinematic conditions are investigated and a resulting prototype manipulator is presented.

II. Force balance solutions without loop equations

Figure 1 shows the general topology of the planar 4-RRR parallel manipulator with its parameters. It has four legs i with each two links with lengths l_{i1} and l_{i2} of which the angular rotations with the x-axis are θ_{i1} and θ_{i2} respectively. The platform has width $2c_5$, height $2d_5$, and orientation θ_5. The fixed pivots at the base are A_i, the joints between two links of a single leg are B_i and the joints of the legs with the platform are C_i. For having a maximal force transmission to the platform, joints C_i are located at the platform as indicated for which links B_iC_i are crossing one another. The center-of-mass (CoM) of each link is defined by the parameters p_{ij} and q_{ij} along and perpendicular to the link respectively, with j being the number of the link of a leg. The CoM of the platform is defined by p_5 and q_5 with respect to the center of the platform.

In order to have a shaking force balanced manipulator (i.e. to have the base be without shaking forces), the linear momentum of all moving elements with respect to the base needs to be conserved (to be constant). The positions of the link CoMs can be written in $[x, y]^T$ notation as

$$\mathbf{r}_{i1} = \begin{bmatrix} a_i + p_{i1}c\theta_{i1} - q_{i1}s\theta_{i1} \\ b_i + p_{i1}s\theta_{i1} + q_{i1}c\theta_{i1} \end{bmatrix}$$

$$\mathbf{r}_{i2} = \begin{bmatrix} a_i + l_{i1}c\theta_{i1} + p_{i2}c\theta_{i2} - q_{i2}s\theta_{i2} \\ b_i + l_{i1}s\theta_{i1} + p_{i2}s\theta_{i2} + q_{i2}c\theta_{i2} \end{bmatrix}$$

with $A_i = [a_i, b_i]^T$. To simplify the equations by omitting θ_5, the platform CoM is modeled by four masses μ_{5i} which distribute m_5 among the joints C_i. The positions of C_i write

$$\mathbf{r}_{5i} = \begin{bmatrix} a_i + l_{i1}c\theta_{i1} + l_{i2}c\theta_{i2} \\ b_i + l_{i1}s\theta_{i1} + l_{i2}s\theta_{i2} \end{bmatrix}$$

Deriving the linear momentum equations [5], [13] of the manipulator then results in

$$\begin{aligned}
\mathbf{P}_O &= \sum_{i=1}^{4} (m_{i1}\dot{\mathbf{r}}_{i1} + m_{i2}\dot{\mathbf{r}}_{i2} + \mu_{5i}\dot{\mathbf{r}}_{5i}) \\
&= \begin{bmatrix} -\lambda_{111}\sin\theta_{11} - \lambda_{112}\cos\theta_{11} \\ \lambda_{111}\cos\theta_{11} - \lambda_{112}\sin\theta_{11} \end{bmatrix} \dot{\theta}_{11} + \\
&\quad \begin{bmatrix} -\lambda_{211}\sin\theta_{21} - \lambda_{212}\cos\theta_{21} \\ \lambda_{211}\cos\theta_{21} - \lambda_{212}\sin\theta_{21} \end{bmatrix} \dot{\theta}_{21} + \\
&\quad \begin{bmatrix} -\lambda_{311}\sin\theta_{31} - \lambda_{312}\cos\theta_{31} \\ \lambda_{311}\cos\theta_{31} - \lambda_{312}\sin\theta_{31} \end{bmatrix} \dot{\theta}_{31} + \\
&\quad \begin{bmatrix} -\lambda_{411}\sin\theta_{41} - \lambda_{412}\cos\theta_{41} \\ \lambda_{411}\cos\theta_{41} - \lambda_{412}\sin\theta_{41} \end{bmatrix} \dot{\theta}_{41} + \\
&\quad \begin{bmatrix} -\lambda_{121}\sin\theta_{12} - \lambda_{122}\cos\theta_{12} \\ \lambda_{121}\cos\theta_{12} - \lambda_{122}\sin\theta_{12} \end{bmatrix} \dot{\theta}_{12} + \\
&\quad \begin{bmatrix} -\lambda_{221}\sin\theta_{22} - \lambda_{222}\cos\theta_{22} \\ \lambda_{221}\cos\theta_{22} - \lambda_{222}\sin\theta_{22} \end{bmatrix} \dot{\theta}_{22} + \\
&\quad \begin{bmatrix} -\lambda_{321}\sin\theta_{32} - \lambda_{322}\cos\theta_{32} \\ \lambda_{321}\cos\theta_{32} - \lambda_{322}\sin\theta_{32} \end{bmatrix} \dot{\theta}_{32} + \\
&\quad \begin{bmatrix} -\lambda_{421}\sin\theta_{42} - \lambda_{422}\cos\theta_{42} \\ \lambda_{421}\cos\theta_{42} - \lambda_{422}\sin\theta_{42} \end{bmatrix} \dot{\theta}_{42} \qquad (1)
\end{aligned}$$

with

$$\begin{aligned} \lambda_{i11} &= m_{i1}p_{i1} + m_{i2}l_{i1} + \mu_{5i}l_{i1} & \lambda_{i12} &= m_{i1}q_{i1} \\ \lambda_{i21} &= m_{i2}p_{i2} + \mu_{5i}l_{i2} & \lambda_{i22} &= m_{i2}q_{i2} \end{aligned}$$

and

$$\mu_{51} + \mu_{52} + \mu_{53} + \mu_{54} = m_5 \qquad (2)$$

$$\mu_{51} - \mu_{52} - \mu_{53} + \mu_{54} = \frac{m_5 p_5}{c_5} \qquad (3)$$

$$-\mu_{51} - \mu_{52} + \mu_{53} + \mu_{54} = \frac{m_5 q_5}{d_5} \qquad (4)$$

Equations (2-4) define the distributed masses at C_i by which the CoM of the distributed masses and their summed value remain equal to the position of the CoM and the value of the mass of the platform. Due to the redundant leg, there are four distributed masses and with three equations then one of them is independent. When μ_{54} is chosen as independent parameter, the other μ_{5i} are calculated with

$$\mu_{51} = m_5\left(\frac{1}{2} + \frac{p_5}{2c_5}\right) - \mu_{54} \qquad (5)$$

$$\mu_{52} = m_5\left(-\frac{p_5}{2c_5} - \frac{q_5}{2d_5}\right) + \mu_{54} \qquad (6)$$

$$\mu_{53} = m_5\left(\frac{1}{2} + \frac{q_5}{2d_5}\right) - \mu_{54} \qquad (7)$$

Already various balance solutions can be obtained. For Eq. (1) to be constant (zero) for all motion of the manipulator, all λ_{ijk}

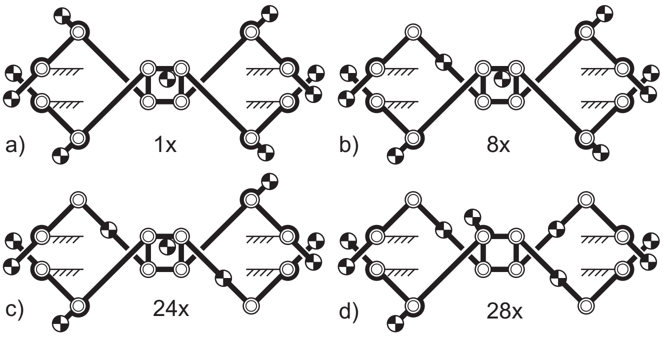

Fig. 2. Topologies of Force Balance Solutions obtained without using the Loop Equations (Leg-by-Leg Approach).

need to be zero. This results in the following force balance conditions.

$$\begin{aligned} q_{i1} &= 0 && (8)\\ q_{i2} &= 0 && (9)\\ m_{i1}p_{i1} + m_{i2}l_{i1} + \mu_{5i}l_{i1} &= 0 && (10)\\ m_{i2}p_{i2} + \mu_{5i}l_{i2} &= 0 && (11) \end{aligned}$$

Having q_{ij} be zero means that the link CoMs are in line with their links. Four types of balance solutions are discussed individually.

A. *Solutions for dependent link COM parameters*

To satisfy Eqs. (10) and (11), p_{ij} can be chosen to be the dependent balance parameters which then are calculated with

$$p_{i1} = \frac{-m_{i2}l_{i1} - \mu_{5i}l_{i1}}{m_{i1}}, \quad p_{i2} = \frac{-\mu_{5i}l_{i2}}{m_{i2}}$$

p_{ij} will be negative when the other parameters are positive. This represents the balance topology shown in Fig. 2a which can be interpreted as adding a countermass to each link. According to Eqs. (2-4), the platform CoM then needs to be within the area bounded by its joints. When the platform CoM is outside this area, one or more p_{i2} will have to be positive.

B. *Solutions for one independent link COM parameter*

To satisfy Eqs. (10) and (11), one p_{ij} can be chosen to be independent instead of one μ_{5i} which then is dependent. One of eight possibilities is when p_{22} is independent, for which μ_{52} is determined by

$$\mu_{52} = \frac{-p_{22}m_{22}}{l_{22}}$$

The other parameters except p_{22} then are calculated with

$$p_{i1} = \frac{-m_{i2}l_{i1} - \mu_{5i}l_{i1}}{m_{i1}}, \quad p_{i2} = \frac{-\mu_{5i}l_{i2}}{m_{i2}}$$

The resulting balance topology is shown in Fig. 2b. According to Eqs. (2-4), the combined CoM of the platform and mass μ_{52} at C_2 then needs to be within the area bounded by the three joints of the independent μ_{5i}.

C. *Solutions for two independent link COM parameters*

To satisfy Eqs. (10) and (11), two p_{ij} can be chosen to be independent instead of two μ_{5i} which then are dependent. One of twenty-four possibilities is when p_{22} and p_{42} are independent, for which μ_{52} and μ_{54} are determined by

$$\mu_{52} = \frac{-p_{22}m_{22}}{l_{22}}, \quad \mu_{54} = \frac{-p_{42}m_{42}}{l_{42}}$$

The other parameters except p_{22} and p_{42} then are calculated with

$$p_{i1} = \frac{-m_{i2}l_{i1} - \mu_{5i}l_{i1}}{m_{i1}}, \quad p_{i2} = \frac{-\mu_{5i}l_{i2}}{m_{i2}}$$

The resulting balance topology is shown in Fig. 2c. According to Eqs. (2-4), the combined CoM of the platform and the masses μ_{52} and μ_{54} at their joints C_2 and C_4 respectively, then needs to be on the line between the two joints of the independent μ_{5i}.

D. *Solutions for three independent link COM parameters*

To satisfy Eqs. (10) and (11), three p_{ij} can be chosen to be independent instead of three μ_{5i} which then are dependent. One of twenty-eight possibilities is when p_{12}, p_{22}, and p_{42} are independent, for which μ_{51}, μ_{52}, and μ_{54} are determined by

$$\mu_{51} = \frac{-p_{12}m_{12}}{l_{12}}, \quad \mu_{52} = \frac{-p_{22}m_{22}}{l_{22}}, \quad \mu_{54} = \frac{-p_{42}m_{42}}{l_{42}}$$

The other parameters then are calculated with

$$p_{i1} = \frac{-m_{i2}l_{i1} - \mu_{5i}l_{i1}}{m_{i1}}, \quad p_{32} = \frac{-\mu_{53}l_{32}}{m_{32}}$$

The resulting balance topology is shown in Fig. 2d. According to Eqs. (2-4), with only one independent μ_{5i} the combined CoM of the platform and the three dependent masses μ_{51}, μ_{52}, and μ_{54} at their joints C_1, C_2, and C_4 respectively, needs to be exactly at the joint of the independent μ_{5i}, C_3.

III. General force balance solutions by including loop equations

For the 4-RRR manipulator there are three independent loop equations which can be written with

$$\begin{aligned} \mathbf{r}_{51} &= \mathbf{r}_{52} + 2c_5 \begin{bmatrix} c\theta_5 \\ s\theta_5 \end{bmatrix} \\ \mathbf{r}_{51} &= \mathbf{r}_{53} + 2c_5 \begin{bmatrix} c\theta_5 \\ s\theta_5 \end{bmatrix} + 2d_5 \begin{bmatrix} s\theta_5 \\ -c\theta_5 \end{bmatrix} \\ \mathbf{r}_{51} &= \mathbf{r}_{54} + 2d_5 \begin{bmatrix} s\theta_5 \\ -c\theta_5 \end{bmatrix} \end{aligned}$$

The time derivatives of these equations are

$$l_{11}\dot{\theta}_{11} \begin{bmatrix} -s\theta_{11} \\ c\theta_{11} \end{bmatrix} + l_{12}\dot{\theta}_{12} \begin{bmatrix} -s\theta_{12} \\ c\theta_{12} \end{bmatrix} = l_{21}\dot{\theta}_{21} \begin{bmatrix} -s\theta_{21} \\ c\theta_{21} \end{bmatrix} + l_{22}\dot{\theta}_{22} \begin{bmatrix} -s\theta_{22} \\ c\theta_{22} \end{bmatrix} + 2c_5\dot{\theta}_5 \begin{bmatrix} -s\theta_5 \\ c\theta_5 \end{bmatrix}$$

$$(12)$$

$$l_{11}\dot{\theta}_{11}\begin{bmatrix} -s\theta_{11} \\ c\theta_{11} \end{bmatrix} + l_{12}\dot{\theta}_{12}\begin{bmatrix} -s\theta_{12} \\ c\theta_{12} \end{bmatrix} = l_{31}\dot{\theta}_{31}\begin{bmatrix} -s\theta_{31} \\ c\theta_{31} \end{bmatrix} + l_{32}\dot{\theta}_{32}\begin{bmatrix} -s\theta_{32} \\ c\theta_{32} \end{bmatrix} + 2c_5\dot{\theta}_5\begin{bmatrix} -s\theta_5 \\ c\theta_5 \end{bmatrix} + 2d_5\dot{\theta}_5\begin{bmatrix} c\theta_5 \\ s\theta_5 \end{bmatrix} \tag{13}$$

$$l_{11}\dot{\theta}_{11}\begin{bmatrix} -s\theta_{11} \\ c\theta_{11} \end{bmatrix} + l_{12}\dot{\theta}_{12}\begin{bmatrix} -s\theta_{12} \\ c\theta_{12} \end{bmatrix} = l_{41}\dot{\theta}_{41}\begin{bmatrix} -s\theta_{41} \\ c\theta_{41} \end{bmatrix} + l_{42}\dot{\theta}_{42}\begin{bmatrix} -s\theta_{42} \\ c\theta_{42} \end{bmatrix} + 2d_5\dot{\theta}_5\begin{bmatrix} c\theta_5 \\ s\theta_5 \end{bmatrix} \tag{14}$$

These loop equations can be substituted in the linear momentum equation Eq. (1) in various ways. A practical choice is to first eliminate θ_5 and $\dot{\theta}_5$ by which two equations remain. Choosing the CoMs of links l_{12} and l_{32} to be independent parameters, the loop equations are rewritten as

$$\begin{aligned}\begin{bmatrix} s\theta_{12} \\ c\theta_{12} \end{bmatrix}\dot{\theta}_{12} = &-\frac{l_{11}}{l_{12}}\begin{bmatrix} s\theta_{11} \\ c\theta_{11} \end{bmatrix}\dot{\theta}_{11} + \frac{l_{21}}{Ul_{12}}\begin{bmatrix} c\theta_{21} \\ -s\theta_{21} \end{bmatrix}\dot{\theta}_{21} + \\ &\frac{l_{22}}{Ul_{12}}\begin{bmatrix} c\theta_{22} \\ -s\theta_{22} \end{bmatrix}\dot{\theta}_{22} + \frac{l_{41}}{Ul_{12}}\begin{bmatrix} -c\theta_{41} \\ s\theta_{41} \end{bmatrix}\dot{\theta}_{41} + \\ &\frac{l_{42}}{Ul_{12}}\begin{bmatrix} -c\theta_{42} \\ s\theta_{42} \end{bmatrix}\dot{\theta}_{42} + \frac{d_5 l_{21}}{c_5 Ul_{12}}\begin{bmatrix} s\theta_{21} \\ c\theta_{21} \end{bmatrix}\dot{\theta}_{21} + \\ &\frac{d_5 l_{22}}{c_5 Ul_{12}}\begin{bmatrix} s\theta_{22} \\ c\theta_{22} \end{bmatrix}\dot{\theta}_{22} + \frac{c_5 l_{41}}{d_5 Ul_{12}}\begin{bmatrix} s\theta_{41} \\ c\theta_{41} \end{bmatrix}\dot{\theta}_{41} + \\ &\frac{c_5 l_{42}}{d_5 Ul_{12}}\begin{bmatrix} s\theta_{42} \\ c\theta_{42} \end{bmatrix}\dot{\theta}_{42}\end{aligned} \tag{15}$$

$$\begin{aligned}\begin{bmatrix} s\theta_{32} \\ c\theta_{32} \end{bmatrix}\dot{\theta}_{32} = &-\frac{l_{31}}{l_{32}}\begin{bmatrix} s\theta_{31} \\ c\theta_{31} \end{bmatrix}\dot{\theta}_{31} + \frac{l_{21}}{l_{32}}\begin{bmatrix} s\theta_{21} \\ c\theta_{21} \end{bmatrix}\dot{\theta}_{21} + \\ &\frac{l_{22}}{l_{32}}\begin{bmatrix} s\theta_{22} \\ c\theta_{22} \end{bmatrix}\dot{\theta}_{22} + \frac{l_{41}}{l_{32}}\begin{bmatrix} s\theta_{41} \\ c\theta_{41} \end{bmatrix}\dot{\theta}_{41} + \\ &\frac{l_{42}}{l_{32}}\begin{bmatrix} s\theta_{42} \\ c\theta_{42} \end{bmatrix}\dot{\theta}_{42} + \frac{l_{21}}{Ul_{32}}\begin{bmatrix} -c\theta_{21} \\ c\theta_{21} \end{bmatrix}\dot{\theta}_{21} + \\ &\frac{l_{22}}{Ul_{32}}\begin{bmatrix} -c\theta_{22} \\ c\theta_{22} \end{bmatrix}\dot{\theta}_{22} + \frac{l_{41}}{Ul_{32}}\begin{bmatrix} c\theta_{41} \\ -s\theta_{41} \end{bmatrix}\dot{\theta}_{41} + \\ &\frac{l_{42}}{Ul_{32}}\begin{bmatrix} c\theta_{42} \\ -s\theta_{42} \end{bmatrix}\dot{\theta}_{42} - \frac{d_5 l_{21}}{c_5 Ul_{32}}\begin{bmatrix} s\theta_{21} \\ c\theta_{21} \end{bmatrix}\dot{\theta}_{21} - \\ &\frac{d_5 l_{22}}{c_5 Ul_{32}}\begin{bmatrix} s\theta_{22} \\ c\theta_{22} \end{bmatrix}\dot{\theta}_{22} - \frac{c_5 l_{41}}{d_5 Ul_{32}}\begin{bmatrix} s\theta_{41} \\ c\theta_{41} \end{bmatrix}\dot{\theta}_{41} - \\ &\frac{c_5 l_{42}}{d_5 Ul_{32}}\begin{bmatrix} s\theta_{42} \\ c\theta_{42} \end{bmatrix}\dot{\theta}_{42}\end{aligned} \tag{16}$$

with $U = \frac{c_5}{d_5} + \frac{d_5}{c_5}$. The resulting linear momentum equations Eq. (22) after substituting these equations in Eq. (1) are presented in the Appendix. When the λ_{ijk} in Eq. (22) are replaced with their original values, the twelve conditions for which the linear momentum equations are zero for all motion become

$$\begin{aligned}
&m_{11}p_{11} + m_{12}l_{11}(1 - \tfrac{p_{12}}{l_{12}}) &&= 0 \\
&m_{31}p_{31} + m_{32}l_{31}(1 - \tfrac{p_{32}}{l_{32}}) &&= 0 \\
&m_{11}q_{11} - m_{12}\tfrac{l_{11}}{l_{12}}q_{12} &&= 0 \\
&m_{31}q_{31} - m_{32}\tfrac{l_{31}}{l_{32}}q_{32} &&= 0 \\
&m_{21}p_{21} + m_{22}l_{21} + m_{12}\left(\tfrac{-l_{21}}{Ul_{12}}q_{12} + \tfrac{d_5 l_{21}}{c_5 Ul_{12}}p_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{21}}{l_{32}}p_{32} + \tfrac{l_{21}}{Ul_{32}}q_{32} - \tfrac{d_5 l_{21}}{c_5 Ul_{32}}p_{32}\right) + \\
&\quad \tfrac{d_5 l_{21}}{c_5 U}(\mu_{51} - \mu_{53}) + (\mu_{52} + \mu_{53})l_{21} &&= 0 \\
&m_{41}p_{41} + m_{42}l_{41} + m_{12}\left(\tfrac{l_{41}}{Ul_{12}}q_{12} + \tfrac{c_5 l_{41}}{d_5 Ul_{12}}p_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{41}}{l_{32}}p_{32} - \tfrac{l_{41}}{Ul_{32}}q_{32} - \tfrac{c_5 l_{41}}{d_5 Ul_{32}}p_{32}\right) + \\
&\quad \tfrac{c_5 l_{41}}{d_5 U}(\mu_{51} - \mu_{53}) + (\mu_{54} + \mu_{53})l_{41} &&= 0 \\
&m_{22}p_{22} + m_{12}\left(\tfrac{-l_{22}}{Ul_{12}}q_{12} + \tfrac{d_5 l_{22}}{c_5 Ul_{12}}p_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{22}}{l_{32}}p_{32} + \tfrac{l_{22}}{Ul_{32}}q_{32} - \tfrac{d_5 l_{22}}{c_5 Ul_{32}}p_{32}\right) + \\
&\quad \tfrac{d_5 l_{22}}{c_5 U}(\mu_{51} - \mu_{53}) + (\mu_{52} + \mu_{53})l_{22} &&= 0 \\
&m_{42}p_{42} + m_{12}\left(\tfrac{l_{42}}{Ul_{12}}q_{12} + \tfrac{c_5 l_{42}}{d_5 Ul_{12}}p_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{42}}{l_{32}}p_{32} - \tfrac{l_{42}}{Ul_{32}}q_{32} - \tfrac{c_5 l_{42}}{d_5 Ul_{32}}p_{32}\right) + \\
&\quad \tfrac{c_5 l_{42}}{d_5 U}(\mu_{51} - \mu_{53}) + (\mu_{54} + \mu_{53})l_{42} &&= 0 \\
&m_{21}q_{21} + m_{12}\left(\tfrac{l_{21}}{Ul_{12}}p_{12} + \tfrac{d_5 l_{21}}{c_5 Ul_{12}}q_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{21}}{l_{32}}q_{32} - \tfrac{l_{21}}{Ul_{32}}p_{32} - \tfrac{d_5 l_{21}}{c_5 Ul_{32}}q_{32}\right) + \\
&\quad \tfrac{l_{21}}{U}(\mu_{51} - \mu_{53}) &&= 0 \\
&m_{41}q_{41} + m_{12}\left(\tfrac{-l_{41}}{Ul_{12}}p_{12} + \tfrac{c_5 l_{41}}{d_5 Ul_{12}}q_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{41}}{l_{32}}q_{32} + \tfrac{l_{41}}{Ul_{32}}p_{32} - \tfrac{c_5 l_{41}}{d_5 Ul_{32}}q_{32}\right) + \\
&\quad \tfrac{l_{41}}{U}(\mu_{53} - \mu_{51}) &&= 0 \\
&m_{22}q_{22} + m_{12}\left(\tfrac{l_{22}}{Ul_{12}}p_{12} + \tfrac{d_5 l_{22}}{c_5 Ul_{12}}q_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{22}}{l_{32}}q_{32} - \tfrac{l_{22}}{Ul_{32}}p_{32} - \tfrac{d_5 l_{22}}{c_5 Ul_{32}}q_{32}\right) + \\
&\quad \tfrac{l_{22}}{U}(\mu_{51} - \mu_{53}) &&= 0 \\
&m_{42}q_{42} + m_{12}\left(\tfrac{-l_{42}}{Ul_{12}}p_{12} + \tfrac{c_5 l_{42}}{d_5 Ul_{12}}q_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{42}}{l_{32}}q_{32} + \tfrac{l_{42}}{Ul_{32}}p_{32} - \tfrac{c_5 l_{42}}{d_5 Ul_{32}}q_{32}\right) + \\
&\quad \tfrac{l_{42}}{U}(\mu_{53} - \mu_{51}) &&= 0
\end{aligned} \tag{17}$$

These are the *general force balance conditions* of the planar 4-RRR parallel manipulator.

In Fig. 3 a variety of force balance topologies is shown that can be obtained from these balance conditions. One important difference of these results with the results of Fig. 2 is that the link CoMs are not restricted to be along the lines through the joints (q_{ij} can be nonzero). In addition, more mass parameters are independent. For instance when the mass of each element is known, for the solutions of Fig. 2 there are three independent mass position parameters while for the solutions of Fig. 3 there are six independent mass position parameters. For the latter generally holds that there are two independent mass position parameters per loop equation, while for the former two of them

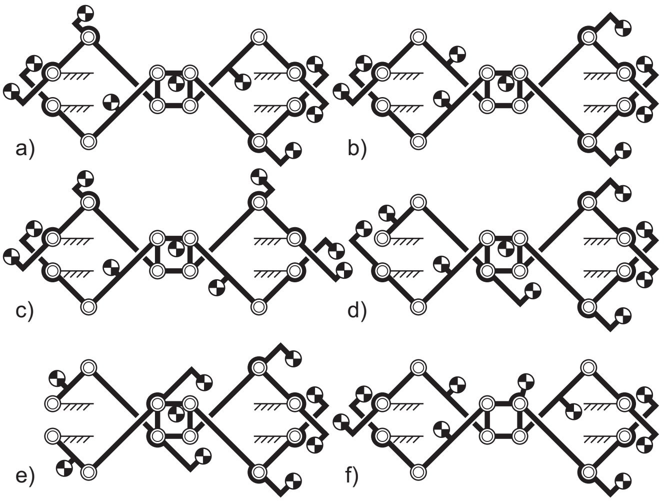

Fig. 3. A Selection of Force Balance Topologies from the General Force Balance Conditions.

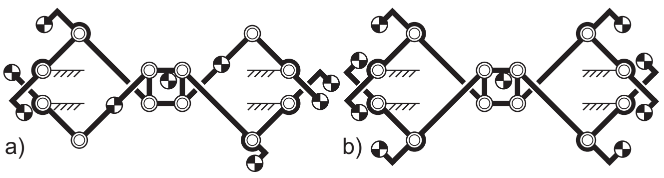

Fig. 4. a) Balance Topology when the CoMs of l_{12} and l_{32} are on the Line through their Joints; b) Symmetric Balance Topology.

are because of the mass distribution of the platform (by which in fact a linear relation between the motion of the platform and of the links was already introduced) and one is because of the redundant leg. If for the six mass position parameters the position of the CoM of the platform is defined together with $q_{i2} = 0$ or $q_{i1} = 0$, then the solutions of Fig. 2 are obtained.

Figure 4a shows a balance topology when the CoMs of l_{12} and l_{32} are on the line through their joints. Figure 4b shows a fully symmetric topology. The advantage of this topology with respect to the topology of Fig. 2a is that there is more design freedom.

The general force balance conditions are valid for any condition of the kinematics. Each link and the platform can have any dimension and each pivot can have any position. In addition, the force balance conditions also hold for planar 1-RRR, 2-RRR, and 3-RRR (parallel) manipulators. Legs of the 4-RRR can be 'taken out' by simply filling out zero mass values. As long as two legs remain, any mass arrangement of the parallel manipulator is possible. For a 1-RRR mechanism, which is serial, the position of the CoM of each element is determined.

IV. Force balance solutions for specific kinematic conditions

The previous section showed that linear relations among the motion of mechanism elements enhance the balance possibilities. Since the 4-RRR manipulator has three-DoF only, the linear momentum equations Eq. (22) could be rewritten to solely depend on three angular velocities. However, the six velocity terms are generally nonlinearly related and therefore this will not influence the general force balance conditions of Eq. (17). But for some specific kinematic conditions, relations among the velocity terms do become linear.

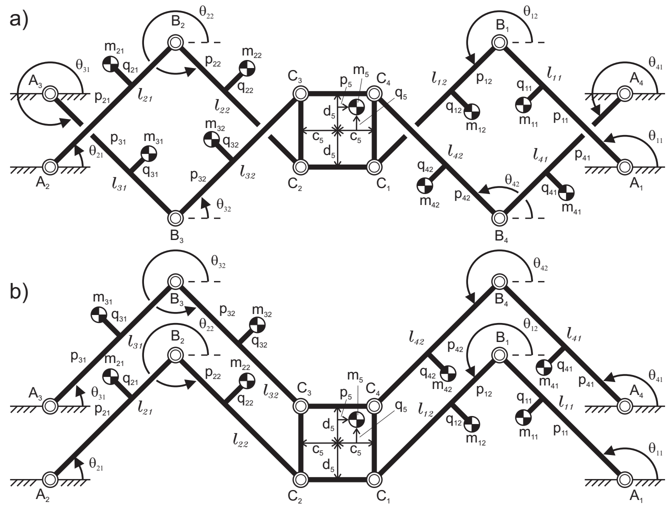

Fig. 5. Specific Kinematic Conditions for which for a Nonrotating Platform Velocity terms a) $\dot{\theta}_{11}$ and $\dot{\theta}_{42}$, $\dot{\theta}_{12}$ and $\dot{\theta}_{41}$, $\dot{\theta}_{21}$ and $\dot{\theta}_{32}$, and $\dot{\theta}_{22}$ and $\dot{\theta}_{31}$; and b) $\dot{\theta}_{11}$ and $\dot{\theta}_{41}$, $\dot{\theta}_{12}$ and $\dot{\theta}_{42}$, $\dot{\theta}_{21}$ and $\dot{\theta}_{31}$, and $\dot{\theta}_{22}$ and $\dot{\theta}_{32}$, become linearly dependent.

Two specific kinematic conditions are shown in Fig. 5 for which for each case four velocity terms become linearly dependent. For the configuration of Fig. 5a the velocity terms of $\dot{\theta}_{11}$ and $\dot{\theta}_{42}$, $\dot{\theta}_{12}$ and $\dot{\theta}_{41}$, $\dot{\theta}_{21}$ and $\dot{\theta}_{32}$, and $\dot{\theta}_{22}$ and $\dot{\theta}_{31}$ are linearly dependent for a nonrotating platform ($\theta_5 = 0$ and $\dot{\theta}_5 = 0$ for all motion). This is the case when for the link lengths hold: $l_{11} = l_{42}$, $l_{12} = l_{41}$, $l_{21} = l_{32}$, and $l_{22} = l_{31}$ and the pivots A_i are located such that the following links remain parallel for all motion of the mechanism: $l_{11} \parallel l_{42}$, $l_{12} \parallel l_{41}$, $l_{21} \parallel l_{32}$, and $l_{22} \parallel l_{31}$. This implies that the line through pivots A_1 and A_4 and the line through A_2 and A_3 are parallel to the line through C_1 and C_4 and the line through C_2 and C_3, respectively.

The force balance conditions for these specific kinematic conditions can be derived by substituting the relations

$$\begin{array}{rlrlrlrl} \theta_{11} = & \theta_{42} & \dot{\theta}_{11} = & \dot{\theta}_{42} & \theta_{12} = & \theta_{41} & \dot{\theta}_{12} = & \dot{\theta}_{41} \\ \theta_{21} = & \theta_{32} & \dot{\theta}_{21} = & \dot{\theta}_{32} & \theta_{22} = & \theta_{31} & \dot{\theta}_{22} = & \dot{\theta}_{31} \\ \theta_5 = & 0 & \dot{\theta}_5 = & 0 & & & & \end{array} \tag{18}$$

in the linear momentum equations Eq. (22). The loop equations already included some of these relations. The resulting linear momentum equations Eq. (23) now have become dependent on four velocity terms. The conditions for which these equations are zero are the force balance conditions

$$
\begin{aligned}
&m_{11}p_{11} + m_{12}l_{11}(1 - \tfrac{p_{12}}{l_{12}}) + m_{42}p_{42} + \\
&\quad m_{12}\left(\tfrac{l_{42}}{Ul_{12}}q_{12} + \tfrac{c_5 l_{42}}{d_5 U l_{12}}p_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{42}}{l_{32}}p_{32} - \tfrac{l_{42}}{Ul_{32}}q_{32} - \tfrac{c_5 l_{42}}{d_5 U l_{32}}p_{32}\right) + \\
&\quad \tfrac{c_5 l_{42}}{d_5 U}(\mu_{51} - \mu_{53}) + (\mu_{54} + \mu_{53})l_{42} &&= 0 \\
&m_{21}p_{21} + m_{22}l_{21} + m_{12}\left(\tfrac{-l_{21}}{Ul_{12}}q_{12} + \tfrac{d_5 l_{21}}{c_5 U l_{12}}p_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{21}}{l_{32}}p_{32} + \tfrac{l_{21}}{Ul_{32}}q_{32} - \tfrac{d_5 l_{21}}{c_5 U l_{32}}p_{32}\right) + \\
&\quad \tfrac{d_5 l_{21}}{c_5 U}(\mu_{51} - \mu_{53}) + (\mu_{52} + \mu_{53})l_{21} &&= 0 \\
&m_{31}p_{31} + m_{32}l_{31}(1 - \tfrac{p_{32}}{l_{32}}) + m_{22}p_{22} + \\
&\quad m_{12}\left(\tfrac{-l_{22}}{Ul_{12}}q_{12} + \tfrac{d_5 l_{22}}{c_5 U l_{12}}p_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{22}}{l_{32}}p_{32} + \tfrac{l_{22}}{Ul_{32}}q_{32} - \tfrac{d_5 l_{22}}{c_5 U l_{32}}p_{32}\right) + \\
&\quad \tfrac{d_5 l_{22}}{c_5 U}(\mu_{51} - \mu_{53}) + (\mu_{52} + \mu_{53})l_{22} &&= 0 \\
&m_{41}p_{41} + m_{42}l_{41} + m_{12}\left(\tfrac{l_{41}}{Ul_{12}}q_{12} + \tfrac{c_5 l_{41}}{d_5 U l_{12}}p_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{41}}{l_{32}}p_{32} - \tfrac{l_{41}}{Ul_{32}}q_{32} - \tfrac{c_5 l_{41}}{d_5 U l_{32}}p_{32}\right) + \\
&\quad \tfrac{c_5 l_{41}}{d_5 U}(\mu_{51} - \mu_{53}) + (\mu_{54} + \mu_{53})l_{41} &&= 0 \\
&m_{11}q_{11} - m_{12}\tfrac{l_{11}}{l_{12}}q_{12} + m_{42}q_{42} + \\
&\quad m_{12}\left(\tfrac{-l_{42}}{Ul_{12}}p_{12} + \tfrac{c_5 l_{42}}{d_5 U l_{12}}q_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{42}}{l_{32}}q_{32} + \tfrac{l_{42}}{Ul_{32}}p_{32} - \tfrac{c_5 l_{42}}{d_5 U l_{32}}q_{32}\right) + \\
&\quad \tfrac{l_{42}}{U}(\mu_{53} - \mu_{51}) &&= 0 \\
&m_{21}q_{21} + m_{12}\left(\tfrac{l_{21}}{Ul_{12}}p_{12} + \tfrac{d_5 l_{21}}{c_5 U l_{12}}q_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{21}}{l_{32}}q_{32} - \tfrac{l_{21}}{Ul_{32}}p_{32} - \tfrac{d_5 l_{21}}{c_5 U l_{32}}q_{32}\right) + \\
&\quad \tfrac{l_{21}}{U}(\mu_{51} - \mu_{53}) &&= 0 \\
&m_{31}q_{31} - m_{32}\tfrac{l_{31}}{l_{32}}q_{32} + m_{22}q_{22} + \\
&\quad m_{12}\left(\tfrac{l_{22}}{Ul_{12}}p_{12} + \tfrac{d_5 l_{22}}{c_5 U l_{12}}q_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{22}}{l_{32}}q_{32} - \tfrac{l_{22}}{Ul_{32}}p_{32} - \tfrac{d_5 l_{22}}{c_5 U l_{32}}q_{32}\right) + \\
&\quad \tfrac{l_{22}}{U}(\mu_{51} - \mu_{53}) &&= 0 \\
&m_{41}q_{41} + m_{12}\left(\tfrac{-l_{41}}{Ul_{12}}p_{12} + \tfrac{c_5 l_{41}}{d_5 U l_{12}}q_{12}\right) + \\
&\quad m_{32}\left(\tfrac{l_{41}}{l_{32}}q_{32} + \tfrac{l_{41}}{Ul_{32}}p_{32} - \tfrac{c_5 l_{41}}{d_5 U l_{32}}q_{32}\right) + \\
&\quad \tfrac{l_{41}}{U}(\mu_{53} - \mu_{51}) &&= 0
\end{aligned}
\tag{19}
$$

The twelve general force balance conditions of Eq. (17) have reduced to eight force balance conditions, which in fact are a combination of the twelve force balance conditions. Therefore the balance solutions of Fig. 3 and Fig. 4 are still valid. With respect to Eq. (17), there are four additional independent mass position parameters (p_{ij} and q_{ij}), yielding new balance possibilities.

Figures 6a and 6b show possible resulting balance topologies. For these topologies the positions of the CoM of the platform and the CoMs of all links l_{i2} can be chosen arbitrary, while the CoMs of links l_{i1} are determined for balance. This means that if countermasses are used, they only need to be applied on the links that are directly pivoted to the base, which is an advantageous feature for low mass addition [16]. It is also possible to have some links l_{i2} contain countermasses instead of links l_{i1}. However, floating masses increase the complexity of the structural design of the manipulator considerably.

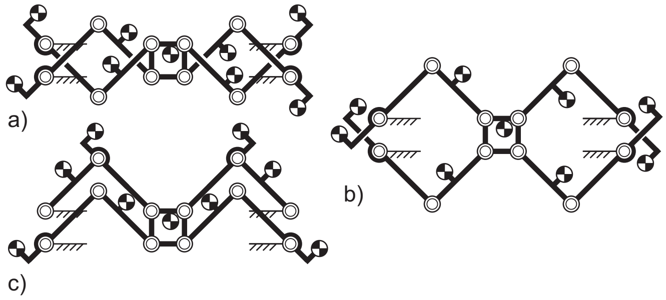

Fig. 6. Force Balance Topologies for a-b) the Kinematic Conditions of Fig. 5a; c) the Kinematic Conditions of Fig. 5b.

For the configuration of Fig. 5b the velocity terms of $\dot\theta_{11}$ and $\dot\theta_{41}$, $\dot\theta_{12}$ and $\dot\theta_{42}$, $\dot\theta_{21}$ and $\dot\theta_{31}$, and $\dot\theta_{22}$ and $\dot\theta_{32}$ are linearly dependent for a nonrotating platform ($\theta_5 = 0$ and $\dot\theta_5 = 0$ for all motion). This is when for the link lengths hold: $l_{11} = l_{41}$, $l_{12} = l_{42}$, $l_{21} = l_{31}$, and $l_{22} = l_{32}$ and the pivots A_i are located such that the following links remain parallel for all motion of the mechanism: $l_{11} \parallel l_{41}$, $l_{12} \parallel l_{42}$, $l_{21} \parallel l_{31}$, and $l_{22} \parallel l_{32}$. This implies that the line through pivots A_1 and A_4 and the line through A_2 and A_3 are parallel to the line through C_1 and C_4 and the line through C_2 and C_3, respectively.

The force balance conditions for this configuration can be obtained from the linear momentum equations Eq. (22) in an equivalent way as for the configuration of Fig. 5a by substituting the relations

$$
\begin{array}{llllllll}
\theta_{11} = & \theta_{41} & \dot\theta_{11} = & \dot\theta_{41} & \theta_{12} = & \theta_{42} & \dot\theta_{12} = & \dot\theta_{42} \\
\theta_{21} = & \theta_{31} & \dot\theta_{21} = & \dot\theta_{31} & \theta_{22} = & \theta_{32} & \dot\theta_{22} = & \dot\theta_{32} \\
\theta_{5} = & 0 & \dot\theta_{5} = & 0 & & & &
\end{array}
\tag{20}
$$

In this case the force balance conditions result in

$$
\begin{aligned}
&m_{11}p_{11} + m_{12}l_{11}(1 - \tfrac{p_{12}}{l_{12}}) + m_{41}p_{41} + m_{42}l_{41} + \\
&\quad m_{12}\left(\tfrac{l_{41}}{Ul_{12}}q_{12} + \tfrac{c_5 l_{41}}{d_5 U l_{12}}p_{12}\right) + \\
&\qquad m_{32}\left(\tfrac{l_{41}}{l_{32}}p_{32} - \tfrac{l_{41}}{Ul_{32}}q_{32} - \tfrac{c_5 l_{41}}{d_5 U l_{32}}p_{32}\right) + \\
&\quad \tfrac{c_5 l_{41}}{d_5 U}(\mu_{51} - \mu_{53}) + (\mu_{54} + \mu_{53})l_{41} &&= 0 \\
&m_{31}p_{31} + m_{32}l_{31}(1 - \tfrac{p_{32}}{l_{32}}) + m_{21}p_{21} + m_{22}l_{21} + \\
&\quad m_{12}\left(\tfrac{-l_{21}}{Ul_{12}}q_{12} + \tfrac{d_5 l_{21}}{c_5 U l_{12}}p_{12}\right) + \\
&\qquad m_{32}\left(\tfrac{l_{21}}{l_{32}}p_{32} + \tfrac{l_{21}}{Ul_{32}}q_{32} - \tfrac{d_5 l_{21}}{c_5 U l_{32}}p_{32}\right) + \\
&\quad \tfrac{d_5 l_{21}}{c_5 U}(\mu_{51} - \mu_{53}) + (\mu_{52} + \mu_{53})l_{21} &&= 0 \\
&m_{22}p_{22} + m_{12}\left(\tfrac{-l_{22}}{Ul_{12}}q_{12} + \tfrac{d_5 l_{22}}{c_5 U l_{12}}p_{12}\right) + \\
&\qquad m_{32}\left(\tfrac{l_{22}}{l_{32}}p_{32} + \tfrac{l_{22}}{Ul_{32}}q_{32} - \tfrac{d_5 l_{22}}{c_5 U l_{32}}p_{32}\right) + \\
&\quad \tfrac{d_5 l_{22}}{c_5 U}(\mu_{51} - \mu_{53}) + (\mu_{52} + \mu_{53})l_{22} &&= 0 \\
&m_{42}p_{42} + m_{12}\left(\tfrac{l_{42}}{Ul_{12}}q_{12} + \tfrac{c_5 l_{42}}{d_5 U l_{12}}p_{12}\right) + \\
&\qquad m_{32}\left(\tfrac{l_{42}}{l_{32}}p_{32} - \tfrac{l_{42}}{Ul_{32}}q_{32} - \tfrac{c_5 l_{42}}{d_5 U l_{32}}p_{32}\right) + \\
&\quad \tfrac{c_5 l_{42}}{d_5 U}(\mu_{51} - \mu_{53}) + (\mu_{54} + \mu_{53})l_{42} &&= 0
\end{aligned}
$$

$$
\begin{aligned}
& m_{11}q_{11} - m_{12}\tfrac{l_{11}}{l_{12}}q_{12} + m_{41}q_{41} + \\
& \quad m_{12}\left(\tfrac{-l_{41}}{Ul_{12}}p_{12} + \tfrac{c_5 l_{41}}{d_5 U l_{12}}q_{12}\right) + \\
& \quad\quad m_{32}\left(\tfrac{l_{41}}{l_{32}}q_{32} + \tfrac{l_{41}}{Ul_{32}}p_{32} - \tfrac{c_5 l_{41}}{d_5 U l_{32}}q_{32}\right) + \\
& \tfrac{l_{41}}{U}(\mu_{53} - \mu_{51}) && = 0 \\
& m_{31}q_{31} - m_{32}\tfrac{l_{31}}{l_{32}}q_{32} + m_{21}q_{21} + \\
& \quad m_{12}\left(\tfrac{l_{21}}{Ul_{12}}p_{12} + \tfrac{d_5 l_{21}}{c_5 U l_{12}}q_{12}\right) + \\
& \quad\quad m_{32}\left(\tfrac{l_{21}}{l_{32}}q_{32} - \tfrac{l_{21}}{Ul_{32}}p_{32} - \tfrac{d_5 l_{21}}{c_5 U l_{32}}q_{32}\right) + \\
& \tfrac{l_{21}}{U}(\mu_{51} - \mu_{53}) && = 0 \\
& m_{22}q_{22} + m_{12}\left(\tfrac{l_{22}}{Ul_{12}}p_{12} + \tfrac{d_5 l_{22}}{c_5 U l_{12}}q_{12}\right) + \\
& \quad\quad m_{32}\left(\tfrac{l_{22}}{l_{32}}q_{32} - \tfrac{l_{22}}{Ul_{32}}p_{32} - \tfrac{d_5 l_{22}}{c_5 U l_{32}}q_{32}\right) + \\
& \tfrac{l_{22}}{U}(\mu_{51} - \mu_{53}) && = 0 \\
& m_{42}q_{42} + m_{12}\left(\tfrac{-l_{42}}{Ul_{12}}p_{12} + \tfrac{c_5 l_{42}}{d_5 U l_{12}}q_{12}\right) + \\
& \quad\quad m_{32}\left(\tfrac{l_{42}}{l_{32}}q_{32} + \tfrac{l_{42}}{Ul_{32}}p_{32} - \tfrac{c_5 l_{42}}{d_5 U l_{32}}q_{32}\right) + \\
& \tfrac{l_{42}}{U}(\mu_{53} - \mu_{51}) && = 0
\end{aligned}
\tag{21}
$$

Also for this configuration eight force balance conditions are obtained which are a combination of the twelve conditions of Eq. (17). Figure 6c shows a possible balance topology in which the positions of the CoMs of the platform, of two links l_{i1}, and of two links l_{i2} can be chosen arbitrary. When applying countermasses, a countermass is needed for each set of links l_{11} and l_{41}, links l_{12} and l_{42}, links l_{21} and l_{31}, and links l_{22} and l_{32}. A disadvantage of this topology is that countermasses at links l_{i2} cannot be omitted.

For many high performance pick and place manipulators the platform does not need to rotate. The rotational degree of freedom, however, may be a necessity to be able to move the platform with accurate translations, compensating for tolerances and production inaccuracies.

Figure 7 shows a prototype manipulator, currently being developed and tested, which is derived from the balance topology of Fig. 6b. The configuration of Fig. 6b is in a singular position, but this is solved by having platform joint C_1 be coincident with C_2 and C_3 be coincident with C_4. An advantage of this configuration with respect to the other configurations is that no arms need to cross one another. Four countermasses are applied near the base pivots, which are the rotational axes of direct drive motors.

The specific kinematic conditions found for the 4-RRR manipulator could be partly valid for the 3-RRR manipulator when two of the three legs are moving symmetrically. However, the third leg cannot be included and the kinematic conditions to have two legs move symmetrically will cause serious problems for the force transmission to the platform. Applying the specific kinematic conditions to a planar 2-RRR parallel manipulator results in a well balanced configuration when the two pivots at the base are coincident and the two joints at the platform are coincident, as illustrated in Fig. 8.

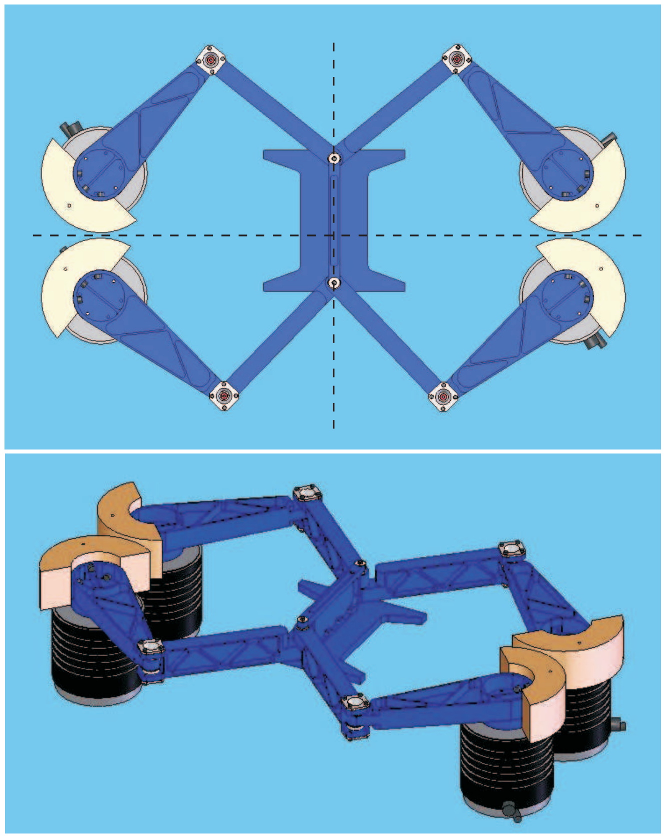

Fig. 7. Prototype Manipulator derived from Fig. 6b with Joint C_1 Coincident with C_2 and C_3 Coincident with C_4 (Patent Pending).

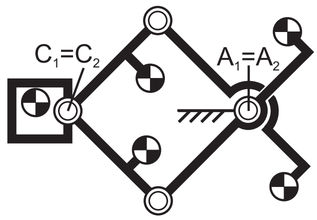

Fig. 8. Planar 2-RRR Parallel Manipulator derived from Fig. 6b with only Two Legs and having the Base Pivots be Coincident and the Platform Joints be Coincident. When the CoM of the Platform is at the Joint, the Platform can also rotate without influencing the Force Balance.

V. Discussion

The positions of the CoMs of the mechanism elements were described along the shortest way towards the base by which the least time dependent parameters are involved for each of them, which is common practice. It is also possible to write the position of for example the CoM of link l_{12} along leg three. This, however, increases the complexity of the calculations considerably. For the linear momentum equations with the loop equations included, the choice of description will not affect the obtained results. For the linear momentum equations without the loop equations this choice will lead to balance solutions which are less compact, less symmetric, and with increased counter-mass addition.

With the substitution of the loop equations care must be taken that the parameters of the platform c_5 and d_5 remain fully coupled. For instance when the loop equations Eqs. (12-14) are substituted in Eq. (1) for θ_{12}, $\dot{\theta}_{12}$, θ_{42}, and $\dot{\theta}_{42}$, c_5 and d_5 become partly decoupled. This reduces the complexity of the linear momentum equations, but it also restricts the balance solutions that can be derived.

Although this paper does not investigate the shaking moment balancing, it is noted that the 4-RRR manipulator has also advantageous moment balance features. If the manipulator is symmetrically arranged, for example equivalent to the prototype manipulator of Fig. 7, and links l_{i1} have equal inertia, links l_{i2} have equal inertia, and the platform is symmetric, then for the specific kinematic conditions the manipulator is completely moment balanced for motion along the lines of symmetry. Motion aside these lines will result in relatively low shaking moments.

VI. Conclusion

A generic method for deriving the general shaking force balance conditions of parallel manipulators was proposed which considers the robot architecture as a whole, rather than link by link or leg by leg. The focus of the method is to find and include linear relations among velocity terms of the linear momentum equations. Therefore the loop equations and specific kinematic conditions were considered.

The method was applied to a planar 4-RRR parallel manipulator, for which the force balance conditions and solutions were discussed and illustrated. A prototype was presented, featuring a simple balance solution for perfect force balance and advantageous moment balancing, taking advantage of kinematic redundancy, symmetric arrangements, and the intended manipulator task.

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Appendix

The linear momentum equation Eq. (1) after substitution of the loop equations Eq. (15) and (16) writes

$$
\begin{aligned}
\mathbf{P}_O = \quad & \begin{bmatrix} (-\lambda_{111}+\frac{l_{11}}{l_{12}}\lambda_{121})s\theta_{11}+(-\lambda_{112}+\frac{l_{11}}{l_{12}}\lambda_{122})c\theta_{11} \\ (\lambda_{111}-\frac{l_{11}}{l_{12}}\lambda_{121})c\theta_{11}+(-\lambda_{112}+\frac{l_{11}}{l_{12}}\lambda_{122})s\theta_{11} \end{bmatrix}\dot{\theta}_{11}+ \\
& \begin{bmatrix} (-\lambda_{211}+\frac{l_{21}}{Ul_{12}}\lambda_{122}-\frac{d_5l_{21}}{c_5Ul_{12}}\lambda_{121}-\frac{l_{21}}{l_{32}}\lambda_{321}-\frac{l_{21}}{Ul_{32}}\lambda_{322}+\frac{d_5l_{21}}{c_5Ul_{32}}\lambda_{321})s\theta_{21}+ \\ (-\lambda_{212}-\frac{l_{21}}{Ul_{12}}\lambda_{121}-\frac{d_5l_{21}}{c_5Ul_{12}}\lambda_{122}-\frac{l_{21}}{l_{32}}\lambda_{322}+\frac{l_{21}}{Ul_{32}}\lambda_{321}+\frac{d_5l_{21}}{c_5Ul_{32}}\lambda_{322})c\theta_{21} \\ (\lambda_{211}-\frac{l_{21}}{Ul_{12}}\lambda_{122}+\frac{d_5l_{21}}{c_5Ul_{12}}\lambda_{121}+\frac{l_{21}}{l_{32}}\lambda_{321}+\frac{l_{21}}{Ul_{32}}\lambda_{322}-\frac{d_5l_{21}}{c_5Ul_{32}}\lambda_{321})c\theta_{21}+ \\ (-\lambda_{212}-\frac{l_{21}}{Ul_{12}}\lambda_{121}-\frac{d_5l_{21}}{c_5Ul_{12}}\lambda_{122}-\frac{l_{21}}{l_{32}}\lambda_{322}+\frac{l_{21}}{Ul_{32}}\lambda_{321}+\frac{d_5l_{21}}{c_5Ul_{32}}\lambda_{322})s\theta_{21} \end{bmatrix}\dot{\theta}_{21}+ \\
& \begin{bmatrix} (-\lambda_{311}+\frac{l_{31}}{l_{32}}\lambda_{321})s\theta_{31}+(-\lambda_{312}+\frac{l_{31}}{l_{32}}\lambda_{322})c\theta_{31} \\ (\lambda_{311}-\frac{l_{31}}{l_{32}}\lambda_{321})c\theta_{31}+(-\lambda_{312}+\frac{l_{31}}{l_{32}}\lambda_{322})s\theta_{31} \end{bmatrix}\dot{\theta}_{31}+ \\
& \begin{bmatrix} (-\lambda_{411}-\frac{l_{41}}{Ul_{12}}\lambda_{122}-\frac{c_5l_{41}}{d_5Ul_{12}}\lambda_{121}-\frac{l_{41}}{l_{32}}\lambda_{321}+\frac{l_{41}}{Ul_{32}}\lambda_{322}+\frac{c_5l_{41}}{d_5Ul_{32}}\lambda_{321})s\theta_{41}+ \\ (-\lambda_{412}+\frac{l_{41}}{Ul_{12}}\lambda_{121}-\frac{c_5l_{41}}{d_5Ul_{12}}\lambda_{122}-\frac{l_{41}}{l_{32}}\lambda_{322}-\frac{l_{41}}{Ul_{32}}\lambda_{321}+\frac{c_5l_{41}}{d_5Ul_{32}}\lambda_{322})c\theta_{41} \\ (\lambda_{411}+\frac{l_{41}}{Ul_{12}}\lambda_{122}+\frac{c_5l_{41}}{d_5Ul_{12}}\lambda_{121}+\frac{l_{41}}{l_{32}}\lambda_{321}-\frac{l_{41}}{Ul_{32}}\lambda_{322}-\frac{c_5l_{41}}{d_5Ul_{32}}\lambda_{321})c\theta_{41}+ \\ (-\lambda_{412}+\frac{l_{41}}{Ul_{12}}\lambda_{121}-\frac{c_5l_{41}}{d_5Ul_{12}}\lambda_{122}-\frac{l_{41}}{l_{32}}\lambda_{322}-\frac{l_{41}}{Ul_{32}}\lambda_{321}+\frac{c_5l_{41}}{d_5Ul_{32}}\lambda_{322})s\theta_{41} \end{bmatrix}\dot{\theta}_{41}+ \\
& \begin{bmatrix} (-\lambda_{221}+\frac{l_{22}}{Ul_{12}}\lambda_{122}-\frac{d_5l_{22}}{c_5Ul_{12}}\lambda_{121}-\frac{l_{22}}{l_{32}}\lambda_{321}-\frac{l_{22}}{Ul_{32}}\lambda_{322}+\frac{d_5l_{22}}{c_5Ul_{32}}\lambda_{321})s\theta_{22}+ \\ (-\lambda_{222}-\frac{l_{22}}{Ul_{12}}\lambda_{121}-\frac{d_5l_{22}}{c_5Ul_{12}}\lambda_{122}-\frac{l_{22}}{l_{32}}\lambda_{322}+\frac{l_{22}}{Ul_{32}}\lambda_{321}+\frac{d_5l_{22}}{c_5Ul_{32}}\lambda_{322})c\theta_{22} \\ (\lambda_{221}-\frac{l_{22}}{Ul_{12}}\lambda_{122}+\frac{d_5l_{22}}{c_5Ul_{12}}\lambda_{121}+\frac{l_{22}}{l_{32}}\lambda_{321}+\frac{l_{22}}{Ul_{32}}\lambda_{322}-\frac{d_5l_{22}}{c_5Ul_{32}}\lambda_{321})c\theta_{22}+ \\ (-\lambda_{222}-\frac{l_{22}}{Ul_{12}}\lambda_{121}-\frac{d_5l_{22}}{c_5Ul_{12}}\lambda_{122}-\frac{l_{22}}{l_{32}}\lambda_{322}+\frac{l_{22}}{Ul_{32}}\lambda_{321}+\frac{d_5l_{22}}{c_5Ul_{32}}\lambda_{322})s\theta_{22} \end{bmatrix}\dot{\theta}_{22}+ \\
& \begin{bmatrix} (-\lambda_{421}-\frac{l_{42}}{Ul_{12}}\lambda_{122}-\frac{c_5l_{42}}{d_5Ul_{12}}\lambda_{121}-\frac{l_{42}}{l_{32}}\lambda_{321}+\frac{l_{42}}{Ul_{32}}\lambda_{322}+\frac{c_5l_{42}}{d_5Ul_{32}}\lambda_{321})s\theta_{42}+ \\ (-\lambda_{422}+\frac{l_{42}}{Ul_{12}}\lambda_{121}-\frac{c_5l_{42}}{d_5Ul_{12}}\lambda_{122}-\frac{l_{42}}{l_{32}}\lambda_{322}-\frac{l_{42}}{Ul_{32}}\lambda_{321}+\frac{c_5l_{42}}{d_5Ul_{32}}\lambda_{322})c\theta_{42} \\ (\lambda_{421}+\frac{l_{42}}{Ul_{12}}\lambda_{122}+\frac{c_5l_{42}}{d_5Ul_{12}}\lambda_{121}+\frac{l_{42}}{l_{32}}\lambda_{321}-\frac{l_{42}}{Ul_{32}}\lambda_{322}-\frac{c_5l_{42}}{d_5Ul_{32}}\lambda_{321})c\theta_{42}+ \\ (-\lambda_{422}+\frac{l_{42}}{Ul_{12}}\lambda_{121}-\frac{c_5l_{42}}{d_5Ul_{12}}\lambda_{122}-\frac{l_{42}}{l_{32}}\lambda_{322}-\frac{l_{42}}{Ul_{32}}\lambda_{321}+\frac{c_5l_{42}}{d_5Ul_{32}}\lambda_{322})s\theta_{42} \end{bmatrix}\dot{\theta}_{42}
\end{aligned} \tag{22}
$$

After substitution of Eqs. (18), the linear momentum for the specific configuration of Fig. 4a becomes

$$
\begin{aligned}
\mathbf{P}_O = \quad & \begin{bmatrix} (-\lambda_{111}+\frac{l_{11}}{l_{12}}\lambda_{121}-\lambda_{421}-\frac{l_{42}}{Ul_{12}}\lambda_{122}-\frac{c_5l_{42}}{d_5Ul_{12}}\lambda_{121}-\frac{l_{42}}{l_{32}}\lambda_{321}+\frac{l_{42}}{Ul_{32}}\lambda_{322}+\frac{c_5l_{42}}{d_5Ul_{32}}\lambda_{321})s\theta_{11}+ \\ (-\lambda_{112}+\frac{l_{11}}{l_{12}}\lambda_{122}-\lambda_{422}+\frac{l_{42}}{Ul_{12}}\lambda_{121}-\frac{c_5l_{42}}{d_5Ul_{12}}\lambda_{122}-\frac{l_{42}}{l_{32}}\lambda_{322}-\frac{l_{42}}{Ul_{32}}\lambda_{321}+\frac{c_5l_{42}}{d_5Ul_{32}}\lambda_{322})c\theta_{11} \\ (\lambda_{111}-\frac{l_{11}}{l_{12}}\lambda_{121}+\lambda_{421}+\frac{l_{42}}{Ul_{12}}\lambda_{122}+\frac{c_5l_{42}}{d_5Ul_{12}}\lambda_{121}+\frac{l_{42}}{l_{32}}\lambda_{321}-\frac{l_{42}}{Ul_{32}}\lambda_{322}-\frac{c_5l_{42}}{d_5Ul_{32}}\lambda_{321})c\theta_{11}+ \\ (-\lambda_{112}+\frac{l_{11}}{l_{12}}\lambda_{122}-\lambda_{422}+\frac{l_{42}}{Ul_{12}}\lambda_{121}-\frac{c_5l_{42}}{d_5Ul_{12}}\lambda_{122}-\frac{l_{42}}{l_{32}}\lambda_{322}-\frac{l_{42}}{Ul_{32}}\lambda_{321}+\frac{c_5l_{42}}{d_5Ul_{32}}\lambda_{322})s\theta_{11} \end{bmatrix}\dot{\theta}_{11}+ \\
& \begin{bmatrix} (-\lambda_{211}+\frac{l_{21}}{Ul_{12}}\lambda_{122}-\frac{d_5l_{21}}{c_5Ul_{12}}\lambda_{121}-\frac{l_{21}}{l_{32}}\lambda_{321}-\frac{l_{21}}{Ul_{32}}\lambda_{322}+\frac{d_5l_{21}}{c_5Ul_{32}}\lambda_{321})s\theta_{21}+ \\ (-\lambda_{212}-\frac{l_{21}}{Ul_{12}}\lambda_{121}-\frac{d_5l_{21}}{c_5Ul_{12}}\lambda_{122}-\frac{l_{21}}{l_{32}}\lambda_{322}+\frac{l_{21}}{Ul_{32}}\lambda_{321}+\frac{d_5l_{21}}{c_5Ul_{32}}\lambda_{322})c\theta_{21} \\ (\lambda_{211}-\frac{l_{21}}{Ul_{12}}\lambda_{122}+\frac{d_5l_{21}}{c_5Ul_{12}}\lambda_{121}+\frac{l_{21}}{l_{32}}\lambda_{321}+\frac{l_{21}}{Ul_{32}}\lambda_{322}-\frac{d_5l_{21}}{c_5Ul_{32}}\lambda_{321})c\theta_{21}+ \\ (-\lambda_{212}-\frac{l_{21}}{Ul_{12}}\lambda_{121}-\frac{d_5l_{21}}{c_5Ul_{12}}\lambda_{122}-\frac{l_{21}}{l_{32}}\lambda_{322}+\frac{l_{21}}{Ul_{32}}\lambda_{321}+\frac{d_5l_{21}}{c_5Ul_{32}}\lambda_{322})s\theta_{21} \end{bmatrix}\dot{\theta}_{21}+ \\
& \begin{bmatrix} (-\lambda_{311}+\frac{l_{31}}{l_{32}}\lambda_{321}-\lambda_{221}+\frac{l_{22}}{Ul_{12}}\lambda_{122}-\frac{d_5l_{22}}{c_5Ul_{12}}\lambda_{121}-\frac{l_{22}}{l_{32}}\lambda_{321}-\frac{l_{22}}{Ul_{32}}\lambda_{322}+\frac{d_5l_{22}}{c_5Ul_{32}}\lambda_{321})s\theta_{31}+ \\ (-\lambda_{312}+\frac{l_{31}}{l_{32}}\lambda_{322}-\lambda_{222}-\frac{l_{22}}{Ul_{12}}\lambda_{121}-\frac{d_5l_{22}}{c_5Ul_{12}}\lambda_{122}-\frac{l_{22}}{l_{32}}\lambda_{322}+\frac{l_{22}}{Ul_{32}}\lambda_{321}+\frac{d_5l_{22}}{c_5Ul_{32}}\lambda_{322})c\theta_{31} \\ (\lambda_{311}-\frac{l_{31}}{l_{32}}\lambda_{321}+\lambda_{221}-\frac{l_{22}}{Ul_{12}}\lambda_{122}+\frac{d_5l_{22}}{c_5Ul_{12}}\lambda_{121}+\frac{l_{22}}{l_{32}}\lambda_{321}+\frac{l_{22}}{Ul_{32}}\lambda_{322}-\frac{d_5l_{22}}{c_5Ul_{32}}\lambda_{321})c\theta_{31}+ \\ (-\lambda_{312}+\frac{l_{31}}{l_{32}}\lambda_{322}-\lambda_{222}-\frac{l_{22}}{Ul_{12}}\lambda_{121}-\frac{d_5l_{22}}{c_5Ul_{12}}\lambda_{122}-\frac{l_{22}}{l_{32}}\lambda_{322}+\frac{l_{22}}{Ul_{32}}\lambda_{321}+\frac{d_5l_{22}}{c_5Ul_{32}}\lambda_{322})s\theta_{31} \end{bmatrix}\dot{\theta}_{31}+ \\
& \begin{bmatrix} (-\lambda_{411}-\frac{l_{41}}{Ul_{12}}\lambda_{122}-\frac{c_5l_{41}}{d_5Ul_{12}}\lambda_{121}-\frac{l_{41}}{l_{32}}\lambda_{321}+\frac{l_{41}}{Ul_{32}}\lambda_{322}+\frac{c_5l_{41}}{d_5Ul_{32}}\lambda_{321})s\theta_{41}+ \\ (-\lambda_{412}+\frac{l_{41}}{Ul_{12}}\lambda_{121}-\frac{c_5l_{41}}{d_5Ul_{12}}\lambda_{122}-\frac{l_{41}}{l_{32}}\lambda_{322}-\frac{l_{41}}{Ul_{32}}\lambda_{321}+\frac{c_5l_{41}}{d_5Ul_{32}}\lambda_{322})c\theta_{41} \\ (\lambda_{411}+\frac{l_{41}}{Ul_{12}}\lambda_{122}+\frac{c_5l_{41}}{d_5Ul_{12}}\lambda_{121}+\frac{l_{41}}{l_{32}}\lambda_{321}-\frac{l_{41}}{Ul_{32}}\lambda_{322}-\frac{c_5l_{41}}{d_5Ul_{32}}\lambda_{321})c\theta_{41}+ \\ (-\lambda_{412}+\frac{l_{41}}{Ul_{12}}\lambda_{121}-\frac{c_5l_{41}}{d_5Ul_{12}}\lambda_{122}-\frac{l_{41}}{l_{32}}\lambda_{322}-\frac{l_{41}}{Ul_{32}}\lambda_{321}+\frac{c_5l_{41}}{d_5Ul_{32}}\lambda_{322})s\theta_{41} \end{bmatrix}\dot{\theta}_{41}
\end{aligned} \tag{23}
$$