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# A new exact algorithm to solve the Multi-trip vehicle routing problem with time windows and limited duration

Florent Hernandez

*Cemagref UMR ITAP, 361 rue JF Breton, Montpellier, France , LIRMM UMR 5506, 161 rue Ada 34392, Montpellier France,  
florent.hernandez@lirmm.fr,*

Dominique Feillet

*Ecole des Mines de Saint-Etienne, CMP Georges Charpak, F-13541 Gardanne, France,  
feillet@emse.fr,*

Rodolphe Giroudeau

*LIRMM UMR 5506, 161 rue Ada 34392, Montpellier France,  
rodolphe.giroudeau@lirmm.fr,*

Olivier Naud

*Cemagref UMR ITAP, 361 rue JF Breton, Montpellier, France,  
olivier.naud@cemagref.fr,*

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## Abstract

This article tackles the multi-trip vehicle routing problem with time windows and limited duration. A trip is a timed route such that a succession of trips can be assigned to one vehicle. We provide a two-phase exact algorithm to solve it. The first phase enumerates possible ordered lists of client matching trip maximum duration criterion. The second phase uses a Branch and Price scheme to generate and choose best set of trips to visit all customers. We propose a set covering formulation as the column generation master problem, where columns (variables) represent trips. The sub-problem selects appropriate timing for trips and has a pseudo-polynomial complexity. Computational results on Solomon's benchmarks are presented. The computational times obtained with our new algorithm are much lower than the ones obtained in the sole exact algorithm previously published on this problem.

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**Keywords :** Vehicle routing, Time windows, Multi-trip, Column Generation, Dynamic programming, Branch and Price

Instance	Root Solution	Solution	Total Time	Root time	Phase1 time	iter	column
c201-25	646.51	659.02	1.561	0.046	0	64	124
c202-25	634.772	653.37	45.819	0.499	0.031	564	555
c203-25	626.017	646.4	247.189	1.795	0.031	1133	854
c204-25	592.06	602.46	252.825	5.09	0.047	673	1268
c205-25	607.913	636.39	38.325	0.141	0	1209	304
c206-25	603.333	636.39	637.612	0.296	0.015	17034	576
c207-25	588.783	603.22	98.273	0.718	0.015	1159	725
c208-25	597.348	613.2	38.154	0.39	0.015	580	484
r201-25	757.79	762.43	0.234	0.046	0	8	174
r202-25	645.78	645.78	0.796	0.453	0.031	7	635
r203-25	620.177	621.97	2.216	0.89	0.078	12	859
r204-25	575.655	579.68	5.026	1.561	0.062	20	1005
r205-25	626.48	634.09	0.827	0.202	0.015	21	375
r206-25	596.74	596.74	0.686	0.515	0.031	4	713
r207-25	583.658	585.74	3.496	0.577	0.046	18	783
r208-25	575.616	579.68	6.681	1.53	0.047	23	1181
r209-25	598.107	602.39	1.67	0.281	0.016	22	562
r210-25	620.293	636.15	7.914	0.359	0.031	73	870
r211-25	568.54	575.91	25.805	1.295	0.046	198	1538
rc201-25	984.438	988.05	1.373	0.11	0	72	342
rc202-25	837.557	881.49	25.664	0.483	0.031	894	880
rc203-25	705.217	749.15	64.271	0.327	0.031	814	1088
rc204-25	-	-	-	-	-	-	-
rc205-25	808.579	840.35	3.746	0.249	0.015	137	598
rc206-25	726.097	761.03	35.703	0.281	0	2006	767
rc207-25	646.457	738.87	71807.37	0.859	0.031	452858	8212
rc208-25	-	-	-	-	-	-	-

Table 1: Results on the Solomon’s benchmark (25 customers) with  $t_{max}$  value set (75; 220)

instances have not been solved within a limit on computing time set to 30 hours.

Tables 1 and 2 present these results. For each instance, we have the root solution cost and the solution cost of the Branch and Price scheme used in Phase 2, the computation time for both phases (Total time), the computation time of Branch and Price root in Phase 2 (Root time), the computation time of Phase 1 (Phase1 time), the number of iteration (iter) and the number of generated columns (column).

We can note, as for the VRPTW, there is great variation for time resolution between instances of the same class. We can also note a significant increase of total computation time between the instances with 25 and 50 customers.

#### 5.4 Impact of the dominance rule in phase 1

In this section, we evaluate the impact of the dominance rule when feasible trips are generated with the elementary shortest path algorithm with resource constraints in Phase 1. The number of available vehicles ( $U$ ) was set to 2 for these tests.

In Table 3, for each instance, we compare the following criteria : the number of generated structures in Phase 1 (# trips), the computation time for both phases (Total time) and the computation time of Phase 1 (Phase1 time), with (Dom) and without (No Dom) dominance

Instance	Root Solution	Solution	Total Time	Root time	Phase1 time	iter	column
c201-50	1309.63	1324.32	1.912	0.17	0.016	14	343
c202-50	1280.44	1310.79	6067.16	1.882	0.25	12085	1635
c203-50	1236.3	1247.77	386.395	9.503	0.5	176	2681
c204-50	1181.61	1195.51	3351.04	63.501	0.796	620	6824
c205-50	1245.19	1265.61	771.309	0.4	0.031	3310	683
c206-50	1241.5	1262.47	6121.5	0.781	0.063	14776	1025
c207-50	1203.8	1216.24	1675.43	3.434	0.187	2038	1624
c208-50	1231.31	1249	4781.94	1.201	0.109	8830	1284
r201-50	1397.07	1405.52	6.529	0.19	0.109	69	699
r202-50	1221.82	1229.91	86.394	9.583	0.766	101	3791
r203-50	1101.63	1104.51	67.246	18.646	2.156	30	5314
r204-50	1010.65	1031.72	22044.9	45.245	5.266	9733	7452
r205-50	1219.64	1230.26	63.471	2.032	0.297	228	1930
r206-50	1150.62	1154.53	34.229	16.453	1.625	15	4887
r207-50	1086.15	1094.83	830.624	26.448	3.016	481	6056
r208-50	1010.65	1031.72	28145.3	57.142	5.375	11521	9702
r209-50	1126.47	1143.91	1619.44	8.342	0.765	2552	3629
r210-50	1152.64	1162.14	273.593	17.915	1.64	268	4871
r211-50	-	-	-	-	-	-	-
rc201-50	1814.12	1876.06	16.153	0.28	0.015	432	463
rc202-50	1678.02	1763.48	3538.58	0.991	0.078	30361	1098
rc203-50	-	-	-	-	-	-	-
rc204-50	1406.73	1457.3	33563.8	7.791	0.281	68145	3699
rc205-50	1698.02	1780.1	4160.9	0.59	0.047	39122	1880
rc206-50	-	-	-	-	-	-	-
rc207-50	-	-	-	-	-	-	-
rc208-50	-	-	-	-	-	-	-

Table 2: Results on the Solomon's benchmark (50 customers) with  $t_{max}$  value set (75;220)