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An Exact Method to Solve the Multi-Trip Vehicle Routing Problem with Time Windows

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École Polytechnique de Montréal (MAGI) and CIRRELT, Canada

TRANSP-OR Seminar

August 2012

Summary

- 1 Problem statement
- 2 Branch and Price algorithm
- 3 Results
- 4 Conclusion

Instance

- an oriented graph $G = (V, A)$.
 - $V = \{0, \dots, n\}$ with 0 the depot and $1, \dots, n$ the customers
 - a cost c_{ij} and a travel time t_{ij} for each arc $(i, j) \in A$
 - for each customer $i \in \{1, \dots, n\}$
 - demand d_i
 - service time st_i
 - a time windows $[a_i, b_i]$
 - U vehicles allowed
 - a capacity Q
 - planning time horizon $[0, T]$
-
- Each customer must be visited within its time window
 - vehicles may arrive earlier and wait before start the service

Objective:

Find a set of trips with minimal cost visiting all customers and respecting capacity and time windows constraints such that :

- two trips are not traveled at the same time by the same vehicle
 - at most U vehicles are used
- A trip is portion of a vehicle route issued from the depot and coming back to the depot

Meta-heuristics

- Fleishmann (1990) : first idea of multi-trip
- Tabu search : Taillard, Laporte and Gendreau (1996), Brandao and Mercer (1998)
- Genetic algorithm : Salhi and Petch (2004)
- Decomposition approach : Battarra, Monaci and Vigo (2009)

Exact methods for a variant where a limit duration is imposed on the trip

- Azi et al (2007 and 2010) and Macedo et al (2011)
 - Limit duration decrease the complexity that allows the use of a specific strategy
 - ⇒ In our problem there is no limit duration

MTVRPTW vs VRPTW

MTVRPTW \Rightarrow variant of the vehicle routing problem with time windows (VRPTW)

VRPTW

- Visit all customers (graph covering)
- 1 demand and 1 service time per customer
- 1 time windows per customer
- 1 cost and 1 travel time between each customer

MTVRPTW

VRPTW

- unlimited fleet
- 1 vehicle = 1 route

MTVRPTW

- limited number of vehicles
- 1 vehicle = many trips

A set covering problem

Like VRPTW

- Linear relaxation of explicit formulation is very weak
- \Rightarrow Formulation where variables represent trips

MTVRPTW \neq VRPTW

- Temporal constraints appear between two trips
- \Rightarrow Trips must be located in time

MTVRPTW

- Trips definition is extended
- \Rightarrow Structure definition

Definitions of *structure* and *trip*

Structure definition

A structure is defined by:

- sequence of visited customers
- length / cost
- duration
- time window $[\mathcal{A}, \mathcal{B}]$ where \mathcal{A} is the depot earliest departure time and \mathcal{B} is the depot latest arrival time for which this structure is valid and its duration is minimal (i.e., minimum waiting time)

Trip definition

A trip is defined by a structure and:

- start and end times

Many trips with different schedules can be derived from every structure

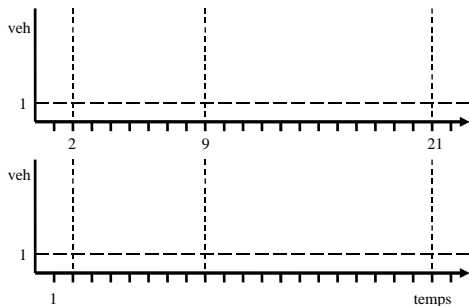
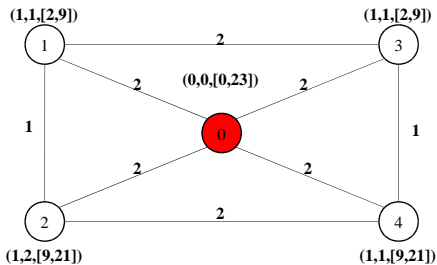
A set covering formulation for VRPTW

- Ω a set of feasible trips, fixed in time
- θ_k indicates the number of times where trip r_k is selected for covering, c_k cost of trip r_k
- $a_{ik} = 1$ if the customer i is visited by r_k , 0 else

$$\begin{aligned} & \text{minimize } \sum_{r_k \in \Omega} c_k \theta_k \\ & \sum_{r_k \in \Omega} a_{ik} \theta_k \geq 1 \quad (i \in V \setminus \{0\}) \\ & \theta_k \in \mathbb{N} \quad (r_k \in \Omega) \end{aligned}$$

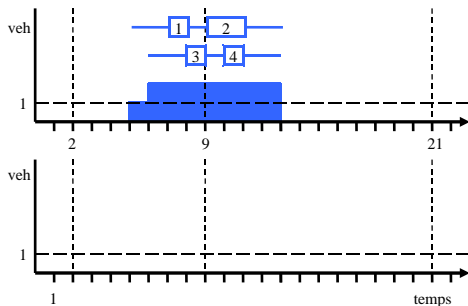
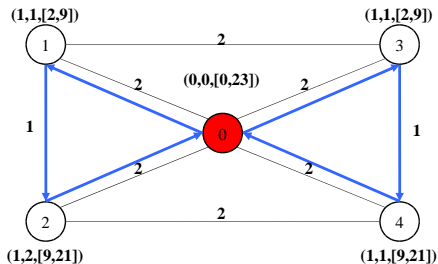
How to model the temporal constraints ?

Trip succession



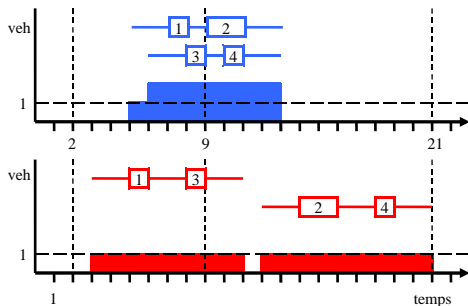
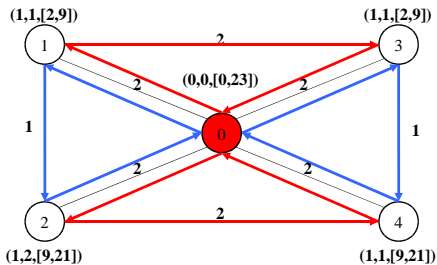
- 1 vehicle with a capacity of 2

Trip succession



- 1 vehicle with a capacity of 2
- VRPTW : Solution cost 10, **not feasible**

Trip succession



- 1 vehicle with a capacity of 2
- VRPTW : Solution cost 10, **not feasible**
- MTRPTW : Solution cost 12, **feasible**

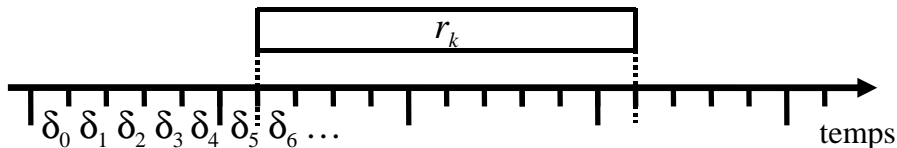
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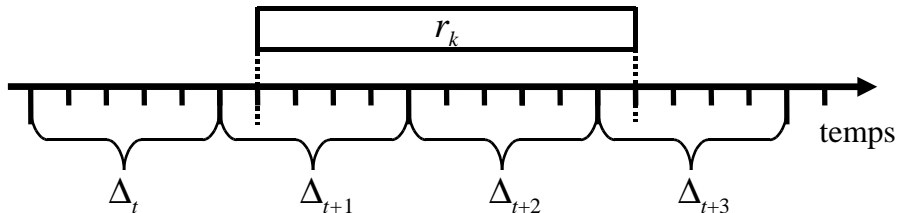
One time constraint by instant ?

- one time interval δ_t by instant $\Rightarrow b_{tk}$ are binary
- combinatorial explosion of constraint number related to temporal precision



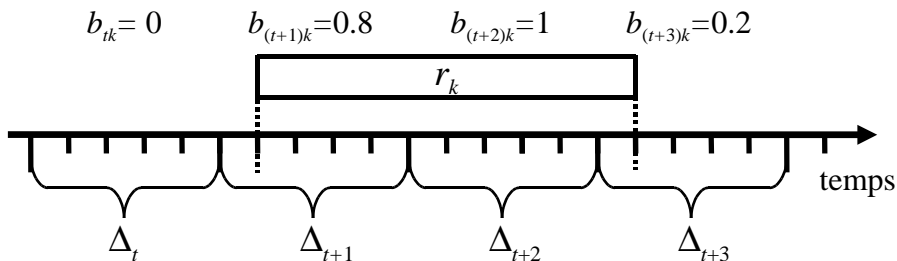
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- one time interval δ_t by instant $\Rightarrow b_{tk}$ are binary
- **combinatorial explosion of constraint number related to temporal precision**
- time interval length = minimal duration of possible trips



One time constraint by instant ?

- one time interval δ_t by instant $\Rightarrow b_{tk}$ are binary
- **combinatorial explosion of constraint number related to temporal precision**
- time interval length = minimal duration of possible trips
- $\Rightarrow b_{tk} \in [0, 1]$ become the fraction of time interval Δ_t occupied by trip r_k



A set covering formulation for MTRPTW

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Time interval length = minimal duration of possible trips

- relaxation of the problem

Ω is a set of feasible trips, fixed in time

- too many variables

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- relaxation of the problem

- \Rightarrow if the solution found is not feasible then the problem have to be solved with one time interval δ_t by instant

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Ω is a set of feasible trips, fixed in time

- too many variables

- \Rightarrow adressed by column generation

What is the column generation ?

Inspiration

Simplex algorithm

- Only basic variables are interesting

How does a nonbasic variable becomes a basic variable ?

- In minimisation case : only if its reduced cost is negative

Method

The column generation consists to solve iteratively two problems.

- the restricted master problem = problem restricted to a sub set of variables (basic and nonbasic)
- the pricing problem = Find new nonbasic variables that can become basic

Stop when the pricing problem don't find new nonbasic variables that can improve the solution.

Problem decomposition

- Restricted master problem : Set covering problem where the integrity constraints are relaxed, reduced to a subset of variables Ω_w
- Subproblem : Find a negative reduced cost variable \Rightarrow Elementary shortest path problem with resource constraints (ESPPRC)

Problem decomposition

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Reduced cost

$$c_k^r = c_k - \sum_{i \in V \setminus \{0\}} a_{ik} \lambda_i + \sum \Delta_t b_{tk} \mu_t$$

- λ_i dual value associated to customer i
- μ_t dual value associated to time interval Δ_t

Dynamic programming :

- labels = $L_{num} = (T_{num}^1, \dots, T_{num}^n)$
- each node has a label list
- during label extension
 - create a new label and insert it in corresponding node label list
 - set resource consumption
 - check that the resource constraints are meet
- stop when no more label can be extended

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Problem

- too many generated labels

Solution

- apply a dominance relation after each extension on corresponding label list

Objective 1

- take into account the loading times

The loading times are at the depot before departure

- add a customer to a trip \Rightarrow delay time of service for the previous customers

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Subproblem

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Objective 2

- generate a trip with minimal reduced cost

Subproblem

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Objective 2

- generate a trip with minimal reduced cost

avoid the time interval associated with a dual value μ_t not null

- take into account all possible departure time

Subproblem

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Objective 2

- generate a trip with minimal reduced cost

avoid the time interval associated with a dual value μ_t not null

- take into account all possible departure time
 - **too many labels**
 - \Rightarrow group labels that represent the same structure and select a representative

Subproblem: Label groups and representative label

Label group definition

A group of labels is a set of labels that represent the same partial path and whose arrival-to-destination dates belong to the same time interval

How to select a representative label

two main rules

- it can be dominated by an other label if and only if all labels of its group are dominated by this label
- it must accept all extensions accepted by at least one label of its group

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In our previous studies

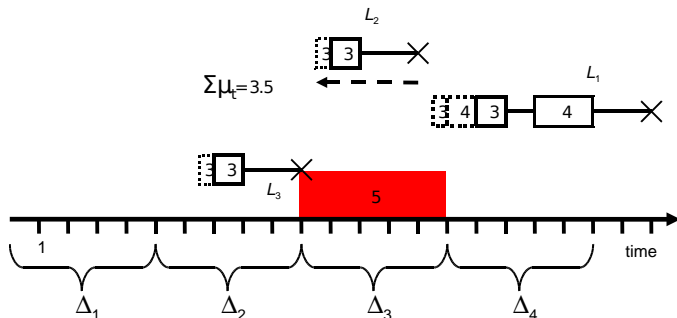
- To compare the time dependent reduced cost of two different labels L_k and $L_{k'}$ during the dominance relation process we use this formula
- $$c_k^r + \sum_{t > h_{k'}}^{t = h_k} \mu_t \leq c_{k'}^r$$

Subproblem: Select a representative label

Select relative to the reduced cost

Let L_2 and L_3 in the same group (they represent the same partial path)

- reduced cost formula : $c_k^r + \sum_{t>h_{k'}}^{t=h_k} \mu_t \leq c_{k'}^r$
- $c_1^r = 3$; $c_3^r = 5$; $c_2^r = c_3^r + 3.5 = 8.5$
- comparisons :

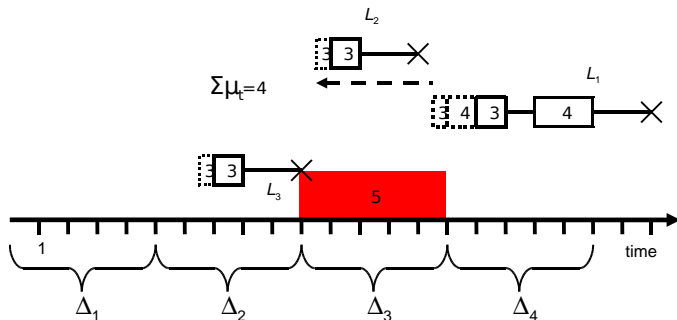


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- comparisons : $c_1^r + 4 < c_2^r$

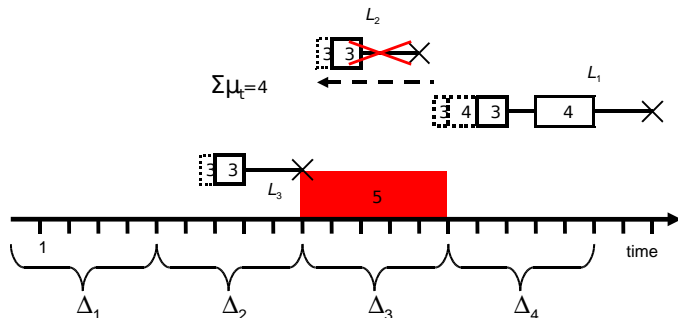


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- comparisons : $3 + 4 < 8.5$

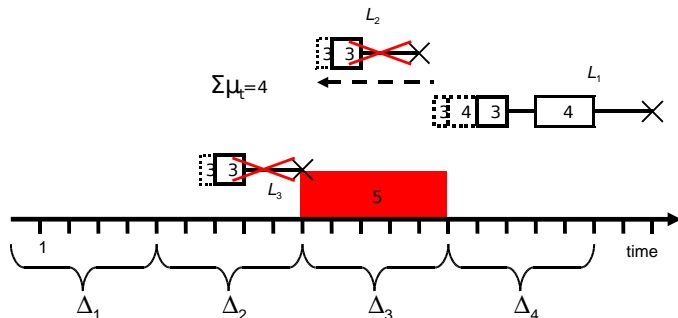


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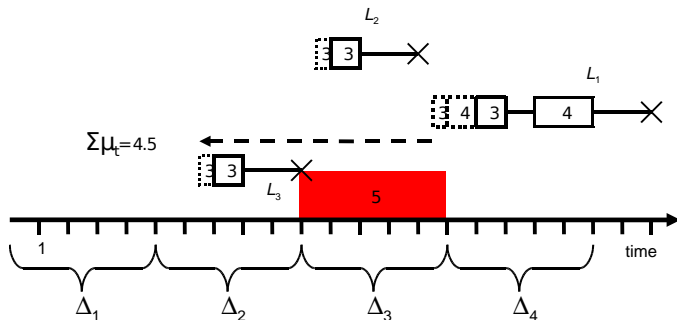


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- $c_1^r = 3$; $c_3^r = 5$; $c_2^r = c_3^r + 3.5 = 8.5$
- comparisons : $c_1^r + 4.5 > c_3^r$

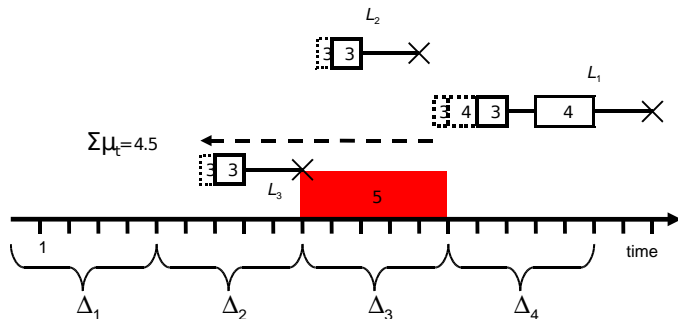


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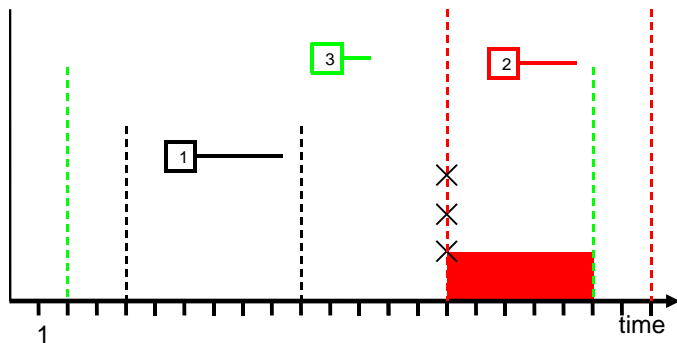
- reduced cost formula : $c_k^r + \sum_{t>h_{k'}}^{t=h_k} \mu_t \leq c_{k'}^r$
- $c_1^r = 3$; $c_3^r = 5$; $c_2^r = c_3^r + 3.5 = 8.5$
- comparisons : $3 + 4.5 > 5$



Subproblem: Select a representative label

Select relative to the possible extensions

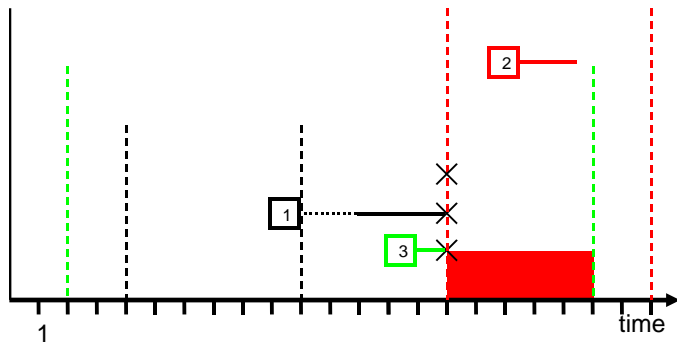
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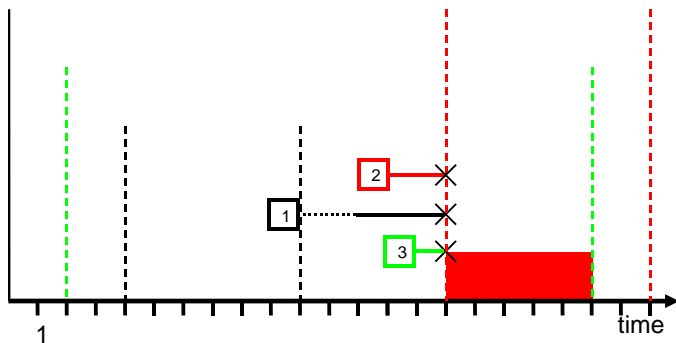
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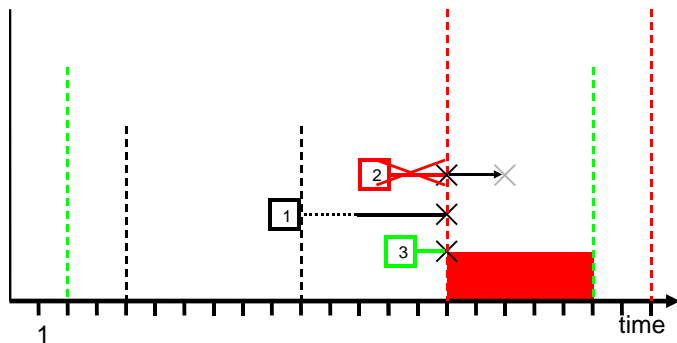
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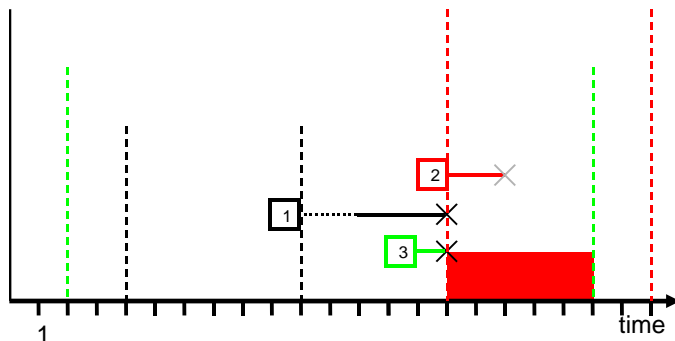
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- all labels of the group have to take into account \Rightarrow retardation



Subproblem: Select a representative label

Select relative to the possible extensions

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- all labels of the group have to take into account \Rightarrow retardation



New parameter : retardation of the arrival time to the depot

New definition of representative labels

Définition

A path p from j to 0 is represented by the label:

$L_p = (c_p^r, h_p, q_p, d_p, rd_p, V_p^1, \dots, V_p^n)$, where:

- c_p^r is the reduced cost of p
- h_p is the starting time of the service of j
- q_p is the carried quantity
- d_p is the arrival time to the depot
- rd_p is the possible retardation of the arrival time of the depot
- $V_p^i = 1$ if the customer i is unreachable by p , 0 else

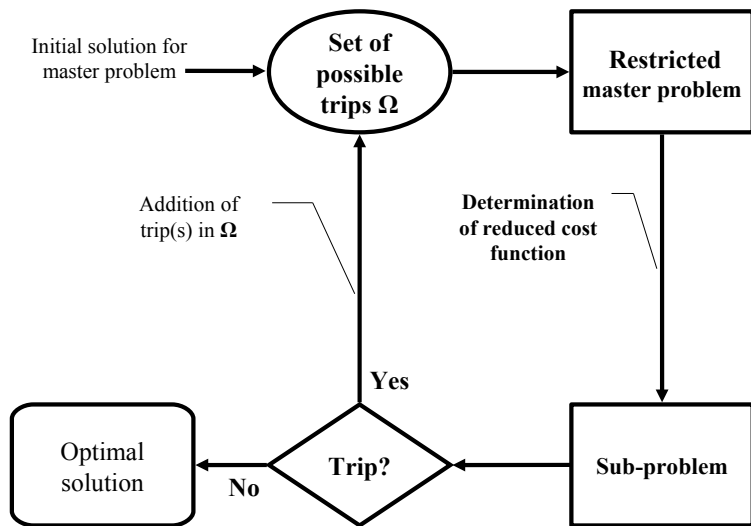
Initialisation

- One label is created at the end of the planning time horizon and one label at the beginning of each time interval with a not null dual value

Label L_1 dominates label L_2 if and only if:

- $q_1 \leq q_2$ (carried quantity)
- All customers unreachable by L_1 are not by L_2 too
- $h_2 + rd_2 \leq h_1$ (starting time of the service)
- $c_1^r + \sum_{t>h_2}^{t=h_1} \mu_t \leq c_2^r$

Column generation scheme



Recall

Branch and bound	Branch and Price
Simplexe	Column generation

Recall

Branch and bound	Branch and Price
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Branching on arcs

- select an arc (i,j) with a fractional flow
- $x_{ij} = 1 \Rightarrow \theta_k = 0$ if trip k visits customer i or j without using arc (i,j)
- $x_{ij} = 0 \Rightarrow \theta_k = 0$ if trip k uses arc (i,j)

Recall

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Problem

- having all arcs with an integer flow ($x_{ij} \in \mathbb{N}$) does not imply that the solution is integer ($\theta_k \in \{0, 1\}$)

Recall

Branch and bound	Branch and Price
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Problem

- having all arcs with an integer flow ($x_{ij} \in \mathbb{N}$) does not imply that the solution is integer ($\theta_k \in \{0, 1\}$)
- \Rightarrow call a repair strategy

Case when the flow matrix is integer and some variables fractional

- The flow matrix is such that every customer has an unique successor
- The flow matrix represents a set of structures
- In the actual fractional solution some structures are represented by several trips with different time positions

We consider the following issue:

- Is it possible to assign a single time position to every structure?
- Equivalent to determining the existence of an integer solution supported by the integer flow matrix

Solved using a VRPTW modeling

- Nodes: structures
- Arcs: feasible successions

Instance of VRPTW

- customers
 - service time
 - demande
 - time window
- arcs (i, j)
 - travel time
 - cost

Our branching problem

- structures
 - duration
 - time window
- arcs (i, j)
 - cost of the structure j

Instance of VRPTW

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Branch and Price: case of an integer flow matrix

Instance of VRPTW

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 - service time
 - demande
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- arcs (i, j)
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 - cost

Our branching problem

- structures
 - duration
 - 0
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- arcs (i, j)
 - 0
 - cost of the structure j

We solve it with:

- Standard branch and price scheme

if a solution of the VRPTW is found

- Update the upper bound
- prune the node

If no VRPTW solution exists

- Select an arc not involved in previous branching constraints
- Branch on this arc

Hardware and software

- Language: C++
- Solver: GLPK (open source)
- Computer: Intel Core2Duo E7300 2.66GHz, 3Gb RAM

Benchmarks

- based on Solomon's instances: R2, RC2, C2
- 25 customers: 2 vehicles allowed
- 50 customers: 4 vehicles allowed
- computation time limited to 30h per instance

Instances with 25 customers

Instances	% GAP			Time (sec)		
	Min	Max	Avg	Min	Max	Avg
C	0.55	4.85	2.27	12	371	170
R	0.44	4.70	2.41	22	3769	1006
RC	3.52	8.98	5.41	9	20038	12537

- 25/27 instances closed
- great variation of computation times and GAPs

Instances with 50 customers

Instances	% GAP	Time (sec)
C	-	-
R201-50	1.58	237
R202-50	2.42	78880
R205-50	2.39	24062
RC201-50	2.24	662
RC202-50	2.40	99346

- Only 5/27 instances closed
- great variation of computation times

Conclusion

- Master problem: time constraint between trips
 - time aggregation
 - all solution found with time aggregation are feasible that imply a good relaxation of time constraint
- Sub-problem: time dependent reduced cost
 - appropriate dynamic programming \Rightarrow representative labels

Perspectives

- Problem with a great temporal dependence
 - mainly solved with a structural branching scheme
 - \Rightarrow improve the branching scheme with temporal branching politic

Thank you for your attention

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