

# An Exact Method to Solve the Multi-Trip Vehicle Routing Problem with Multi Time Windows

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# An Exact Method to Solve the Multi-Trip Vehicle Routing Problem with Time Windows

### F. Hernandez

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#### École Polytechnique de Montréal (MAGI) and CIRRELT, Canada \*\*\*

## TRANSP-OR Seminar

August 2012



2 Branch and Price algorithm

### 3 Results



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## Problem statement

### Instance

- an oriented graph G = (V, A).
- $V = \{0, \cdots, n\}$  with 0 the depot and  $1, \cdots, n$  the customers
- ullet a cost  $c_{ij}$  and a travel time  $t_{ij}$  for each arc  $(i,j)\in A$
- for each customer  $i \in \{1, \cdots, n\}$ 
  - demand  $d_i$
  - service time st<sub>i</sub>
  - a time windows [*a<sub>i</sub>*, *b<sub>i</sub>*]
- U vehicles allowed
- a capacity Q
- planning time horizon [0, T]
- Each customer must be visited within its time window
- vehicles may arrive earlier and wait before start the service

### Objective:

Find a set of trips with minimal cost visiting all customers and respecting capacity and time windows constraints such that :

- two trips are not traveled at the same time by the same vehicle
- at most U vehicles are used

• A trip is portion of a vehicle route issued from the depot and coming back to the depot

## Meta-heuristics

- Fleishmann (1990) : first idea of multi-trip
- Tabu search : Taillard, Laporte and Gendreau (1996), Brandao and Mercer (1998)
- Genetic algorithm : Salhi and Petch (2004)
- Decomposition approach : Battarra, Monaci and Vigo (2009)

Exact methods for a variant where a limit duration is imposed on the trip

- Azi et al (2007 and 2010) and Macedo et al (2011)
  - Limit duration decrease the complexity that allows the use of a specific strategy

Image: Image:

ullet  $\Rightarrow$  In our problem there is no limit duration

 $\mathsf{MTVRPTW} \Rightarrow \mathsf{variant}$  of the vehicle routing problem with time windows (VRPTW)



- Visit all customers (graph covering)
- 1 demand and 1 service time per customer
- 1 time windows per customer
- 1 cost and 1 travel time between each customer

VRPTW	MTVRPTW
<ul> <li>unlimited fleet</li> </ul>	<ul> <li>limited number of vehicles</li> </ul>
• 1 vehicle = 1 route	• 1 vehicle = many trips

## Like VRPTW

- Linear relaxation of explicit formulation is very weak
- ullet  $\Rightarrow$  Formulation where variables represent trips

## $\mathsf{MTVRPTW} \neq \mathsf{VRPTW}$

- Temporal constraints appear between two trips
- ⇒ Trips must be located in time

### **MTVRPTW**

- Trips definition is extended
- $\Rightarrow$  Structure definition

## Structure definition

A structure is defined by:

- sequence of visited customers
- length / cost
- duration
- time window  $[\mathcal{A}, \mathcal{B}]$  where  $\mathcal{A}$  is the depot earliest departure time and  $\mathcal{B}$  is the depot latest arrival time for which this structure is valid and its duration is minimal (i.e., minimum waiting time)

## Trip definition

A trip is defined by a structure and:

start and end times

Many trips with different schedules can be derived from every structure

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## A set covering formulation for VRPTW

- $\Omega$  a set of feasible trips, fixed in time
- $\theta_k$  indicates the number of times where trip  $r_k$  is selected for covering,  $c_k$  cost of trip  $r_k$
- $a_{ik} = 1$  if the customer *i* is visited by  $r_k$ , 0 else

$$egin{aligned} & \textit{minimize} \sum_{r_k \in \Omega} c_k heta_k \ & \sum_{k \in \Omega} a_{ik} heta_k \geq 1 & (i \in V \setminus \{0\}) \ & heta_k \in \mathit{IN} & (r_k \in \Omega) \end{aligned}$$

How to model the temporal constraints ?

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• 1 vehicle with a capacity of 2

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1 vehicle with a capacity of 2
VRPTW : Solution cost 10, not feasible

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## A set covering formulation for MTVRPTW

- $\Omega$  a set of feasible trips, fixed in time
- θ<sub>k</sub> indicates the number of times where trip r<sub>k</sub> is selected for covering, c<sub>k</sub> cost of trip r<sub>k</sub>
- $a_{ik} = 1$  if the customer *i* is visited by  $r_k$ , 0 else
- $b_{tk} \in \{0,1\}$  indicates if the trip  $r_k$  includes the instant  $\delta_t$

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## One time constraint by instant ?

- ullet one time interval  $\delta_t$  by instant  $\Rightarrow b_{tk}$  are binary
- combinatorial explosion of constraint number related to temporal precision



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- time interval length = minimal duration of possible trips



## One time constraint by instant ?

- one time interval  $\delta_t$  by instant  $\Rightarrow$   $b_{tk}$  are binary
- combinatorial explosion of constraint number related to temporal precision
- time interval length = minimal duration of possible trips
- $\Rightarrow$   $b_{tk} \in [0,1]$  become the fraction of time interval  $\Delta_t$  occupied by trip  $r_k$



## A set covering formulation for MTVRPTW

- $\Omega$  a set of feasible trips, fixed in time
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$$\begin{array}{l} \text{minimize } \sum\limits_{r_k \in \Omega} c_k \theta_k \\ \sum\limits_{r_k \in \Omega} a_{ik} \theta_k \geq 1 \qquad (i \in V \setminus \{0\}) \\ \theta_k \in IN \qquad (r_k \in \Omega) \\ \sum\limits_{r_k \in \Omega} b_{tk} \theta_k \leq U \qquad (\forall \Delta_t) \end{array}$$

## Time interval length = minimal duration of possible trips

### • relaxation of the problem

## $\boldsymbol{\Omega}$ is a set of feasible trips, fixed in time

• too many variables

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relaxation of the problem

•  $\Rightarrow$  if the solution found is not feasible then the problem have to be solved with one time interval  $\delta_t$  by instant

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## $\boldsymbol{\Omega}$ is a set of feasible trips, fixed in time

too many variables

ullet  $\Rightarrow$  adressed by column generation

# What is the column generation ?

## Inspiration

Simplexe algorithm

• Only basic varaibles are interesting

How does a nonbasic variable becomes a basic variable ?

• In minimisation case : only if its reduced cost is negative

## Method

The column generation consists to solve iteratively two problems.

- the restricted master problem = problem restricted to a sub set of variables (basic and nonbasic)
- the pricing problem = Find new nonbasic variables that can become basic

Stop when the pricing problem don't find new nonbasic variables that can improve the solution.

### Problem decomposition

- Restricted master problem : Set covering problem where the integrity constraints are relaxed, reduced to a subset of variables  $\Omega_w$
- Subproblem : Find a negative reduced cost variable ⇒ Elementary shortest path problem with resource constraints (ESPPRC)

### Problem decomposition

- Restricted master problem : Set covering problem where the integrity constraints are relaxed, reduced to a subset of variables  $\Omega_w$
- Subproblem : Find a negative reduced cost variable ⇒ Elementary shortest path problem with resource constraints (ESPPRC)

### Reduced cost

$$c_k^r = c_k - \sum_{i \in V \setminus \{0\}} a_{ik} \lambda_i + \sum_{\Delta_t} b_{tk} \mu_t$$

- $\lambda_i$  dual value associated to customer *i*
- $\mu_t$  dual value associated to time interval  $\Delta_t$

## Dynamic programming :

- labels =  $L_{num} = (T_{num}^1, \cdots, T_{num}^n)$
- each node has a label list
- during label extension
  - create a new label and insert it in correponding node label list
  - set resource consumption
  - check that the resource constraints are meet
- stop when no more label can be extended

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### Problem

too many generated labels

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### Problem

too many generated labels

## Solution

 apply a dominance relation after each extension on corresponding label list

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## Objective 1

• take into account the loading times

## The loading times are at the depot before departure

 $\bullet$  add a customer to a trip  $\Rightarrow$  delay time of service for the previous customers

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## **Objective 2**

• generate a trip with minimal reduced cost

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## **Objective 2**

• generate a trip with minimal reduced cost

## avoid the time interval associated with a dual value $\mu_t$ not null

• take into account all possible departure time

## Objective 1

take into account the loading times

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- $\bullet$  add a customer to a trip  $\Rightarrow$  delay time of service for the previous customers
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## **Objective 2**

• generate a trip with minimal reduced cost

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- take into account all possible departure time
  - too many labels

## Objective 1

take into account the loading times

## The loading times are at the depot before departure

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## **Objective 2**

• generate a trip with minimal reduced cost

## avoid the time interval associated with a dual value $\mu_t$ not null

- take into account all possible departure time
  - too many labels
  - $\bullet \; \Rightarrow \; group \; labels \; that \; represent the same structure and select a representative$

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# Subproblem: Label groups and representative label

## Label group definition

A group of labels is a set of labels that represent the same partial path and whose arrival-to-destination dates belong to the same time interval

### How to select a representative label

two main rules

- it can be dominated by an other label if and only if all labels of its group are dominated by this label
- it must accept all extensions accepted by at least one label of its group

# Subproblem: Label groups and representative label

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### In our previous studies

To compare the time dependent reduced cost of two different labels L<sub>k</sub> and L<sub>k'</sub> during the dominance relation process we use this formula
 c<sup>r</sup><sub>k</sub> + Σ<sup>t=h<sub>k</sub></sup><sub>t>h<sub>t</sub></sub> μ<sub>t</sub> ≤ c<sup>r</sup><sub>k'</sub>

## Select relative to the reduced cost

Let  $L_2$  and  $L_3$  in the same group (their represent the same partial path)

• reduced cost formula :  $c_k^r + \sum_{t>h_{k'}}^{t=h_k} \mu_t \leq c_{k'}^r$ 

• 
$$c_1^r = 3$$
;  $c_3^r = 5$ ;  $c_2^r = c_3^r + 3.5 = 8.5$ 

comparaisons :



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• comparaisons :  $c_1^r + 4 < c_2^r$ 



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• 
$$c_1^r = 3$$
;  $c_3^r = 5$ ;  $c_2^r = c_3^r + 3.5 = 8.5$ 

• comparaisons : 3 + 4 < 8.5



## Select relative to the reduced cost

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$$c_1^r = 3$$
;  $c_3^r = 5$ ;  $c_2^r = c_3^r + 3.5 = 8.5$ 

• comparaisons :  $c_1^r + 4.5 > c_3^r$ 



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• 
$$c_1^r = 3$$
;  $c_3^r = 5$ ;  $c_2^r = c_3^r + 3.5 = 8.5$ 

• comparaisons : 3 + 4.5 > 5



## Select relative to the possible extensions

• it must accept all extensions accepted by at least one label of its group



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## Select relative to the possible extensions

- it must accept all extensions accepted by at least one label of its group
- ullet all labels of the group have to take into account  $\Rightarrow$  retardation



## Select relative to the possible extensions

- it must accept all extensions accepted by at least one label of its group
- ullet all labels of the group have to take into account  $\Rightarrow$  retardation



### Définition

A path p from j to 0 is represented by the label:  $L_p = (c_p^r, h_p, q_p, d_p, rd_p, V_p^1, \cdots, V_p^n)$ , where:

- $c_p^r$  is the reduced cost of p
- $h_p$  is the starting time of the service of j
- $q_p$  is the carried quantity
- d<sub>p</sub> is the arrival time to the depot
- $rd_p$  is the possible retardation of the arrival time of the depot
- $V_p^i = 1$  if the customer *i* is unreachable by *p*, 0 else

## Initialisation

• One label is created at the end of the planning time horizon and one label at the beginning of each time interval with a not null dual value

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## Label $L_1$ dominates label $L_2$ if and only if:

- $q_1 \leq q_2$  (carried quantity)
- All customers unreachable by  $L_1$  are not by  $L_2$  too
- $h_2 + rd_2 \leq h_1$  (starting time of the service)
- $c_1^r + \sum_{t>h_2}^{t=h_1} \mu_t \le c_2^r$

## Column generation scheme



## Recall

Branch and bound	Branch and Price		
Simplexe	Column generation		

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## Recall

Branch and bound	Branch and Price
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## Branching on arcs

- select an arc (i,j) with a fractional flow
- $x_{ij} = 1 \Rightarrow \theta_k = 0$  if trip k visits customer i or j without using arc (i, j)
- $x_{ij} = 0 \Rightarrow \theta_k = 0$  if trip k uses arc (i, j)

## Recall

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### Problem

• having all arcs with an integer flow  $(x_{ij} \in IN)$  does not imply that the solution is integer  $(\theta_k \in \{0, 1\})$ 

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## Recall

Branch and bound	Branch and Price		
Simplexe	Column generation		

## Branching on arcs

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### Problem

• having all arcs with an integer flow  $(x_{ij} \in IN)$  does not imply that the solution is integer  $(\theta_k \in \{0, 1\})$ 

$$ullet$$
  $\Rightarrow$  call a repair strategy

## Case when the flow matrix is integer and some variables fractional

- The flow matrix is such that every customer has an unique successor
- The flow matrix represents a set of structures
- In the actual fractional solution some structures are represented by several trips with different time positions

## We consider the following issue:

- Is it possible to assign a single time position to every structure?
- Equivalent to determining the existence of an integer solution supported by the integer flow matrix

## Solved using a VRPTW modeling

- Nodes: structures
- Arcs: feasible successions

Instance of VRPTW	Our branching problem
• customers	• structures
<ul> <li>service time</li> <li>demande</li> <li>time window</li> <li>arcs (i, j)</li> </ul>	<ul> <li>duration</li> <li>time window</li> <li>arcs (i, j)</li> </ul>
• travei time • cost	• cost of the structure <i>j</i>

Instance of VRPTW	Our branching problem
<ul> <li>customers</li> </ul>	• structures
<ul> <li>service time</li> <li>demande</li> <li>time window</li> <li>arcs (i, j)</li> <li>travel time</li> <li>cost</li> </ul>	<ul> <li>duration</li> <li>0</li> <li>time window</li> <li>arcs (i, j)</li> <li>0</li> <li>cost of the structure j</li> </ul>

Instance of VRPTW	Our branching problem
<ul> <li>customers</li> </ul>	<ul> <li>structures</li> </ul>
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• travel time • cost	<ul> <li>0</li> <li>cost of the structure j</li> </ul>

### We solve it with:

• Standard branch and price scheme

## if a solution of the VRPTW is found

- Update the upper bound
- prune the node

## If no VRPTW solution exists

- Select an arc not involved in previous branching constraints
- Branch on this arc

### Hardware and software

- Language: C++
- Solver: GLPK (open source)
- Computer: Intel Core2Duo E7300 2.66GHz, 3Gb RAM

## Benchmarks

- based on Solomon's instances: R2, RC2, C2
- 25 customers: 2 vehicles allowed
- 50 customers: 4 vehicles allowed
- computation time limited to 30h per instance

### Instances with 25 customers

Instances	% GAP		Time (sec)			
	Min	Max	Avg	Min	Max	Avg
С	0.55	4.85	2.27	12	371	170
R	0.44	4.70	2.41	22	3769	1006
RC	3.52	8.98	5.41	9	20038	12537

• 25/27 instances closed

• great variation of computation times and GAPs

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### Instances with 50 customers

Instances	% GAP	Time (sec)
C	-	-
R201-50	1.58	237
R202-50	2.42	78880
R205-50	2.39	24062
RC201-50	2.24	662
RC202-50	2.40	99346

- Only 5/27 instances closed
- great variation of computation times

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## Conclusion

- Master problem: time constraint between trips
  - time aggregation
  - all solution found with time aggregation are feasible that imply a good relaxation of time constraint
- Sub-problem: time dependent reduced cost
  - appropriate dynamic programming  $\Rightarrow$  representative labels

## Perspectives

- Problem with a great temporal dependence
  - mainly solved with a structural branching scheme
  - ullet  $\Rightarrow$  improve the branching scheme with temporal branching politic

# Thank you for your attention

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