# Relational Concept Analysis: a synthesis and open questions

Marianne Huchard

▶ To cite this version:

Marianne Huchard. Relational Concept Analysis: a synthesis and open questions. 2012. lirmm-00727002

HAL Id: lirmm-00727002
https://hal-lirmm.ccsd.cnrs.fr/lirmm-00727002v1

Submitted on 31 Aug 2012

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Relational Concept Analysis: a synthesis and open questions

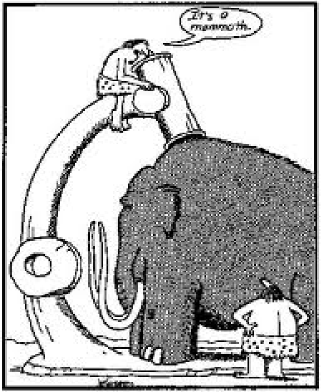
Marianne Huchard

*LIRMM, CNRS & Université Montpellier 2, France*

FCA4AI - 28 august 2012

# Outline

# Relational Concept Analysis (RCA)



Early microscope

http://www.mrnorton.com/Chemistry/Cartoons/FarSideMicroscope.gif

# Relational Concept Analysis (RCA)

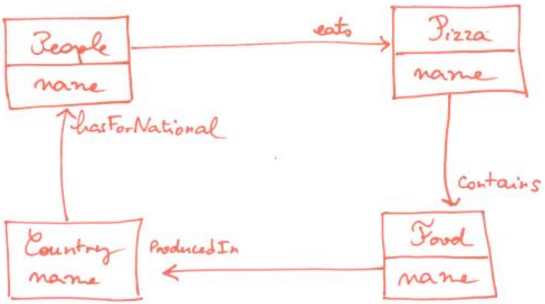
- Extends the purpose of FCA for taking into account links between objects
- Main principles:
  - a relational model based on the entity-relationship model
  - integrate relations between objects as relational attributes
  - iterative process
- RCA provides a *concept lattice family*
- Produced structures can be represented as ontology concepts within a knowledge representation formalism such as description logics (DLs).

Work with A. Napoli, M.A. Rouane-Hacène, C. Roume, P. Valtchev

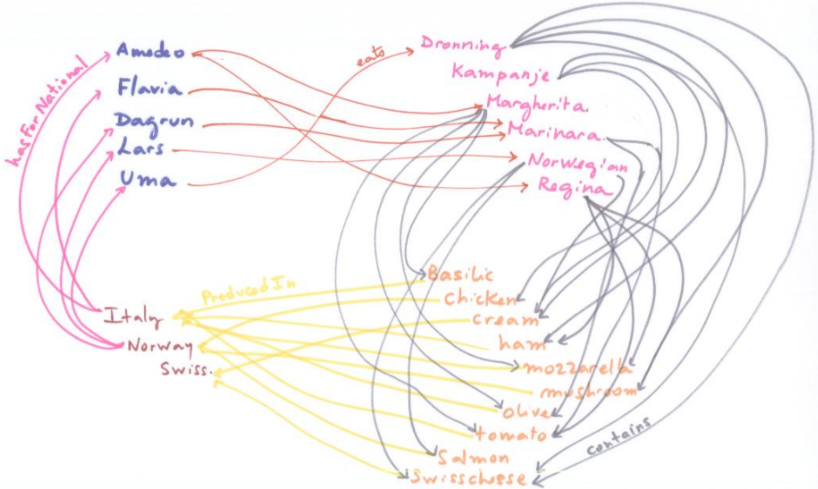
# Relational Concept Analysis (RCA)

A relational model based on the entity-relationship model

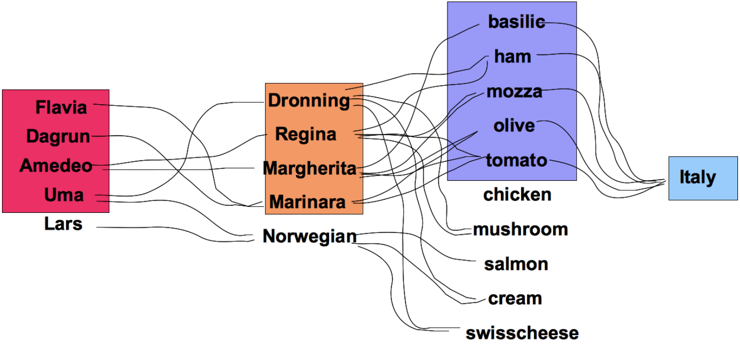
## Pizza story



# Objects and links



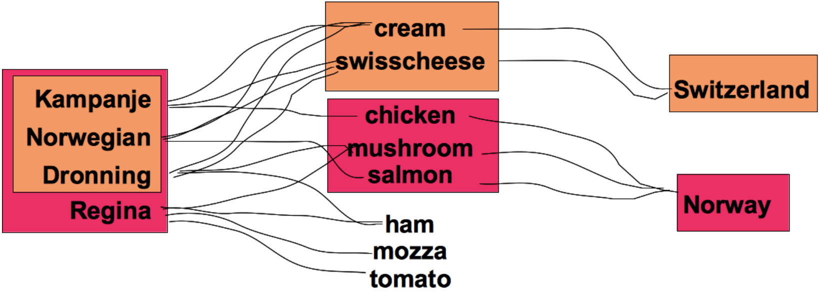
# Relevant groups of objects




*People who eat at least one pizza containing at least one ingredient produced in Italy*

The group is formed using relation compositions and objects far from initial objects

# Relevant implications




In Pizzas:
contains at least an ingredient produced in Switzerland
$\Rightarrow$ contains at least an ingredient produced in Norway

# Relational Context Family (RCF)

A RCF $\mathcal{F}$ is a pair $(K, R)$ with:

- $K$ is a set of object-attribute contexts $K_i = (O_i, A_i, I_i)$
- $R$ is a set of object-object contexts $R_j = (O_k, O_l, I_j)$,
  - $(O_k, O_l)$ are the object sets of formal contexts $(K_k, K_l) \in K^2$
  - $I_j \subseteq O_k \times O_l$
  - $K_k$ is the *source/domain context*, $K_l$ is the *target/range context*.
  - we may have $K_k = K_l$.

## Pizza RCF

$K = K_{People}, K_{Pizza}, K_{Ingredient}, K_{Country}$

$R = R_{eats}, R_{contains}, R_{producedIn}, R_{hasCitizen}$

# Example of object-attribute context $K_{Ingredient}$

$K_{Ingredient} = (O_{Ingredient}, A_{Ingredient}, I_{Ingredient})$ Here object (rows) are described by identifiers (columns), more relevant attributes can be used

| $I_{Ingredient}$ | basilic | chicken | cream | ham | mozzarella | mushroom | olive | tomato | salmon | swisscheese |
|---|---|---|---|---|---|---|---|---|---|---|
| **basilic** | × | | | | | | | | | |
| **chicken** | | × | | | | | | | | |
| **cream** | | | × | | | | | | | |
| **ham** | | | | × | | | | | | |
| **mozzarella** | | | | | × | | | | | |
| **mushroom** | | | | | | × | | | | |
| **olive** | | | | | | | × | | | |
| **tomato** | | | | | | | | × | | |
| **salmon** | | | | | | | | | × | |
| **swisscheese** | | | | | | | | | | × |

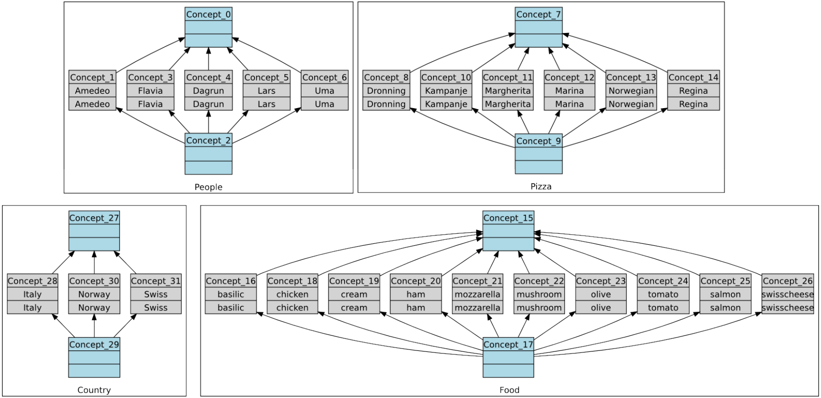
# Example of object-object context $R_{contains}$

$$R_{contains} = (O_{Pizza}, O_{Ingredient}, I_{contains})$$

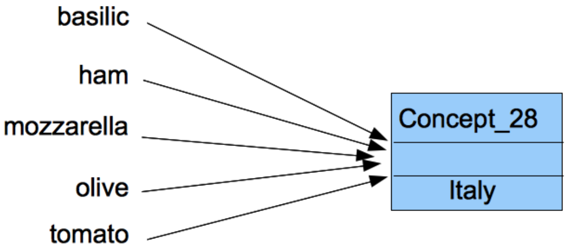
| $I_{contains}$ | basilic | chicken | cream | ham | mozzarella | mushroom | olive | tomato | salmon | swisscheese |
|---|---|---|---|---|---|---|---|---|---|---|
| **Dronning** | | | × | × | | × | | | | × |
| **Kampanje** | | × | × | | | | | | | × |
| **Margherita** | × | | | | × | | × | × | | |
| **Marina** | | | | | | | × | × | | |
| **Norwegian** | | | × | | | | | | × | × |
| **Regina** | | | | × | × | × | | × | | |

# RCA - Initial Lattice building

At the beginning, only the object-attribute contexts are used to build the foundation of the concept lattice family



# RCA - Introducing relations as relational attributes

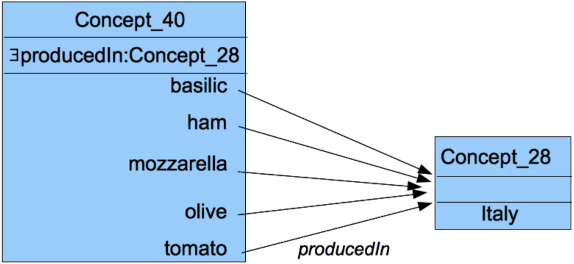



Because basilic, ham, mozzarella, olive and tomato are connected via *producedIn* to Concept_28 extent

basilic, ham, mozzarella, olive, tomato **own** the relational attribute $\exists producedIn.Concept\_28$

# RCA - Introducing relations as relational attributes

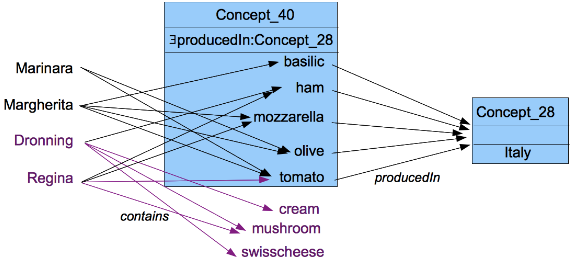
At a further step ...




basilic, ham, mozzarella, olive, tomato grouped because they **own** the relational attribute $\exists producedIn.Concept\_28$

# RCA - Introducing relations as relational attributes

At a further step ...

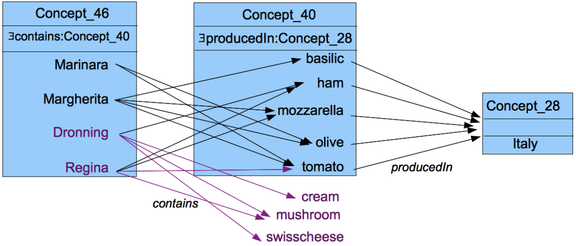



Dronning, Regina, Margherita and Marinara are connected via *contains* to Concept_40 extent

**several connection strengths:** All are connected to at least one italian ingredient; Margherita and Marinara are besides connected only to italian ingredients; Margherita and Regina are connected to more than 3 italian ingredients, etc.

# RCA - Introducing relations as relational attributes

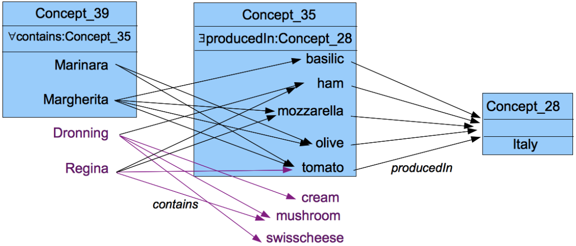
At a further step ...




Dronning, Regina, Margherita and Marinara are grouped, they own $\exists contains.Concept\_40$

# RCA - Introducing relations as relational attributes

Alternatively ...




Margherita and Marinara are grouped, they own $\forall contains.Concept\_35$
(in the whole process the concept numbers change, this is explained later)

# RCA - Introducing relations as relational attributes

Relational scaling is the process by which links are established between objects and concepts.

For each relational context $R_j = (O_k, O_l, I_j)$, a scaled context $R_j^* = (O_k, A, I_j)$ is created.

- $A$ is a set of relational attributes $a = S\ R_j.C$, where C is in the concept set of a lattice built on objects of $O_l$, denoted by $\mathcal{L}_l^n$
- $I_j$ contains $(o, a)$ iff $S(R_j(o), Extent(C))$ is true.

*S* is a *scaling* operator, the most used are:

- $S_\exists(R_j(o), Extent(C))$ is true iff $R_j(o) \cap Extent(C) \neq \emptyset$.
- $S_{\forall\exists}(R_j(o), Extent(C))$ is true iff $R_j(o) \subseteq Extent(C) \wedge \exists x \in R_j(o), x \in Extent(C)$

# Scaling operators

| Operator | Attribute form | Condition |
|---|---|---|
| Universal (narrow) | $\forall\ r : c$ | $r(o) \subseteq Ext(c)$ |
| Covers | $\supseteq\ r : c$ | $r(o) \supseteq Ext(c)$ |
| Existential (wide) | $\exists\ r : c$ | $r(o) \cap Ext(c) \neq \emptyset$ |
| Universal strict | $\forall\exists\ r : c$ | $r(o) \subseteq Ext(c)$ *and* $r(o) \neq \emptyset$ |
| Qualified cardinality restriction | $\geq\ n\ r : c$ | $r(o) \subseteq Ext(c)$ *and* $\lvert r(o)\rvert \geq\ n$ |
| Cardinality restriction | $\geq\ n\ r : \top_{\mathcal{L}}$ | $\lvert r(o)\rvert \geq\ n$ |

# Relational scaling

Some properties of relational scaling:

- The homogeneity of concept descriptions is kept: all attributes are considered as binary (even relational attributes).
- Standard algorithms for building concept lattices can be directly reused.

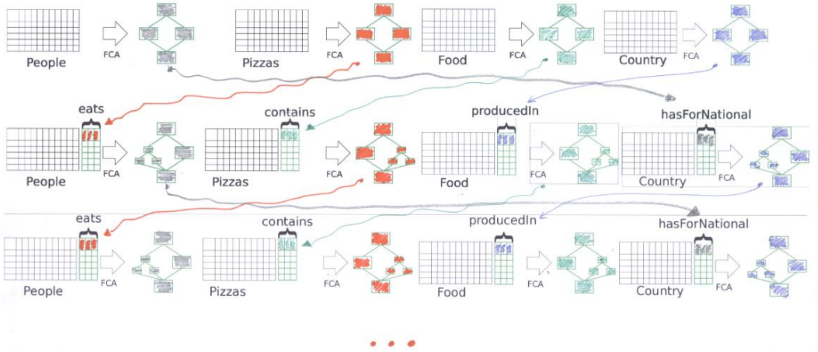
# Relational scaling

After the relational scaling, the scaled relations are concatenated to the appropriate object-attribute relation (same object domain). This forms new relations (one per object set) which describe the objects by their classical attributes and their relational attributes. Concept lattices are built using these relations.

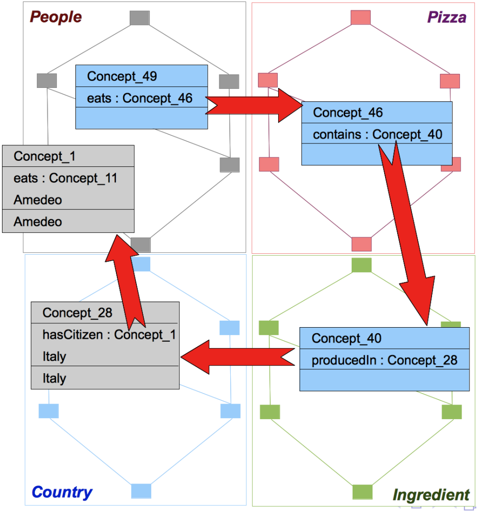
| | producedIn : Concept_27 | producedIn : Concept_28 | producedIn : Concept_30 | producedIn : Concept_31 |
|---|---|---|---|---|
| basilic | x | x | | |
| chicken | x | | x | |
| cream | x | | | x |
| ham | x | x | | |
| mozzarella | x | x | | |
| mushroom | x | | x | |
| olive | x | x | | |
| tomato | x | x | | |
| salmon | x | | x | |
| swisscheese | x | | | x |

# Outlining the iterative algorithm

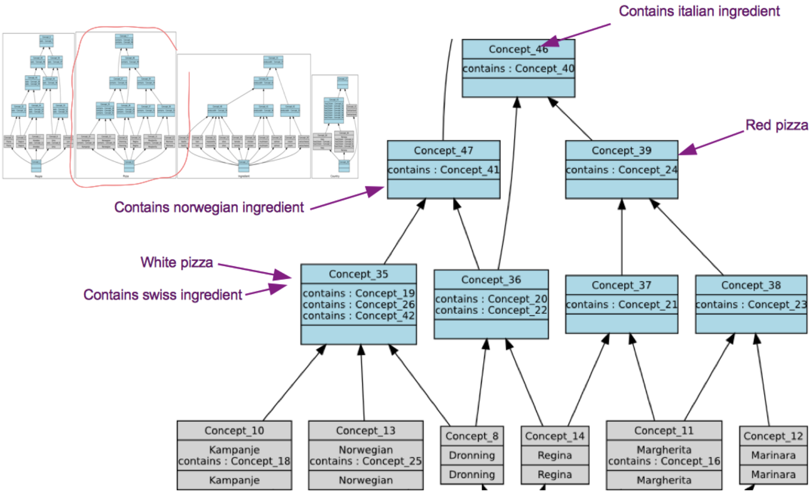
Learned concepts are used in the next steps to learn more



# RCA view on data: A connected set of lattices

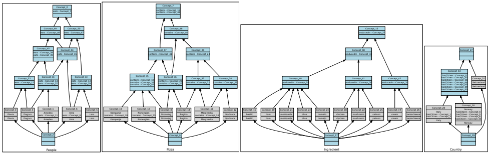


# Reading classifications and implications through links

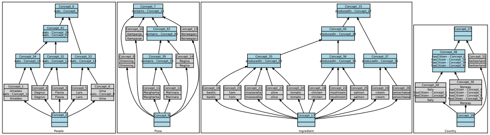


# Alternative views by varying the scaling operators

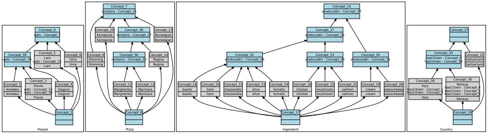
$\exists eats \exists contains \exists producedIn \exists hasCitizen$




$\exists eats \forall contains \exists producedIn \exists hasCitizen$




$\forall eats \forall contains \exists producedIn \exists hasCitizen$



# RCA engineering

## Applications

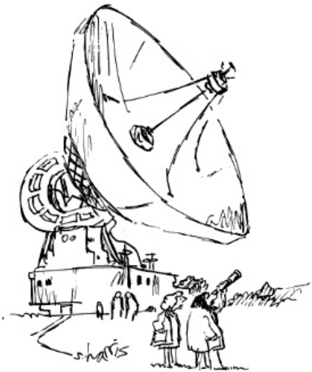
- Software engineering including: class model constructions, use case model reengineering, model transformations learning, analyzing and fixing bad smells in software, component architecture extraction, Web services classification ...
- Artificial Intelligence including: ontology constructions, data mining

## Making RCA practical

- We can play with the scaling operators
- On specific (small parts of the) contexts, we can add queries
- We can apply RCA on part of the RCF, and stop at a given step
- We can observe the growth of (inferred) knowledge as an indicator about data

# Outline

# Relational queries

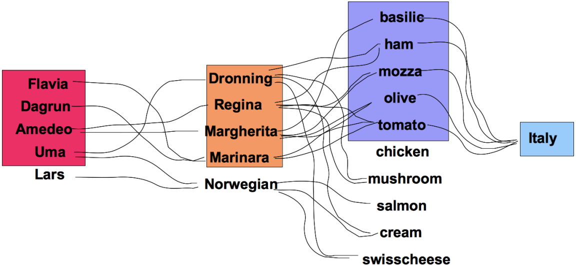



"Just checking."

http://astronomy.swin.edu.au/ gmackie/humour.html

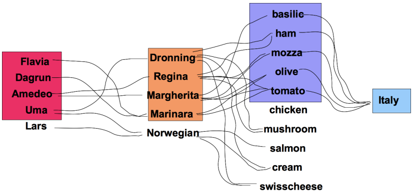
# Relational queries

Consider this query: *We search for people, pizzas, ingredients and a country s.t. the country is Italy, people eat the pizzas, which contain ingredients produced in the country*




The answers are the tuples we can read from the schema, e.g.
< Flavia, Marinara, Olive, Italy >
< Dagrun, Marinara, Olive, Italy >
< Amedeo, Regina, Mozza, Italy > (etc.)

# Relational queries




With FCA/RCA, we may add two interesting views:

- **classify the answers** (group answers concerning Flavia and Dagrun because they are based on the marinara pizza)
- **classify all objects to understand their relations w.r.t. the query**
  - Lars' preferences are not very far away from those of Uma, both like the Norwegian pizza; besides the Dronning pizza is not very different from the Norwegian pizza
  - most of the pizzas which are answers contain tomato

# Relational queries

## Approach

- Expressing the relational query to navigate the lattices
- A companion guiding algorithm with user interaction

## Two main implementations

- By looking in each lattice for the searched relational attributes
- By classifying variables of the query

# Relational query

**Unary predicates associated with a formal context**
We associate with a formal context $K$ several unary predicates
$P_K$: to express that an object $o$ belongs to the formal object set $O$
$P_a$: to express that an object $o$ owns a specific attribute $a \in A$

E.g. $P_{K_{People}} P_{Italy}$

**Binary predicates associated with a Relational Context Family**
Let us consider a RCF $(\mathbb{K}, R)$.
To every $r_{ij}$ relation in $R$, we associate a binary predicate
$P_{r_{ij}}$: to express that a pair of objects is connected via the relation

E.g. $P_{eats}$

# Relational query

## Definition (Relational query)

Let $\mathcal{V}$ denote a finite set of variables. A *relational query atom* is:

- either an expression $P(v_1)$ where $v_1$ is a variable from $\mathcal{V}$ and $P$ a unary predicate associated with a formal context in $\mathbb{K}$,
- or $P_{r_{ij}}(v_1, v_2)$, where $v_1$ and $v_2$ are two variables from $\mathcal{V}$ or a variable and an object of $O_i$, $i \in \{1...n\}$, and $P_{r_{ij}}$ is a binary predicate associated with a relation $r_{ij}$ in $R$.

A *relational query* with variables $\overrightarrow{v}$ is an expression $\varphi(\overrightarrow{v})$, which is a conjunction of relational query atoms involving variables of $\mathcal{V}$ and objects in $O_i$, where $i \in \{1...n\}$.

# Relational query

We search for people, pizzas, ingredients and a country s.t. the country is Italy, people eat the pizzas, which contain ingredients produced in the country

$$P_{K_{People}}(q_{people}) \wedge P_{K_{Pizza}}(q_{pizza}) \wedge P_{K_{Ingredient}}(q_{ingredient}) \wedge P_{K_{Country}}(q_{country}) \wedge P_{Italy}(q_{country}) \wedge P_{eats}(q_{people}, q_{pizza}) \wedge P_{contains}(q_{pizza}, q_{ingredient}) \wedge P_{producedIn}(q_{ingredient}, q_{country})$$

# Classifying variables of the query

## An example of a modified object-attribute context

Variables are added as new objects, here q-person is added in $K_{People}$.
It may have attributes depending from the query.

| $I_{People}$ | **Amedeo** | **Flavia** | **Dagrun** | **Lars** | **Uma** |
|---|---|---|---|---|---|
| Amedeo | × | | | | |
| Flavia | | × | | | |
| Dagrun | | | × | | |
| Lars | | | | × | |
| Uma | | | | | × |
| **q-person** | | | | | |

# Classifying variables of the query

## An example of a modified object-object context

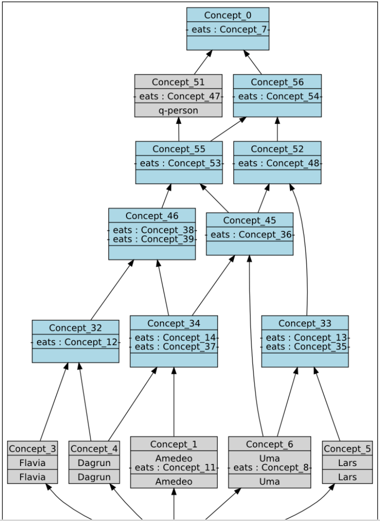
We look for people (q-people) connected to pizzas (q-pizzas)

| | Dronning | Kampanje | Margherita | Marinara | Norwegian | Regina | **q-pizza** |
|---|---|---|---|---|---|---|---|
| Amedeo | | | × | | | × | |
| Flavia | | | | × | | | |
| Dagrun | | | | × | | × | |
| Lars | | | | | × | | |
| Uma | × | | | | × | | |
| **q-person** | | | | | | | **X** |

# Reading answers by concept lattice family navigation

Answers are below the concept introducing q-people.

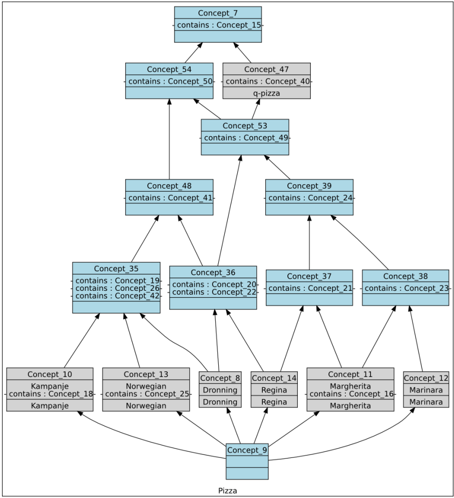
We see patterns in answers (e.g. similar consumer profiles Lars-Uma; Flavia-Dagrun)



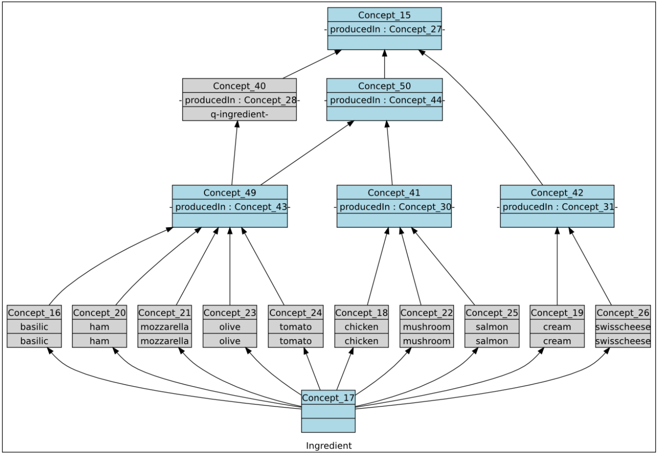
# Reading answers by concept lattice family navigation

Answers for pizzas are below the concept introducing q-pizza.

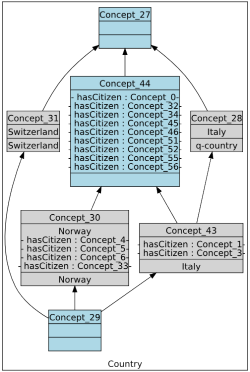
*Concept_54* has been introduced to factorize something that all initial objects (but not the object associated with the query) have: containing ingredients produced in a country which has citizens in the studied context.



# Reading answers by concept lattice family navigation



# Reading answers by concept lattice family navigation



# Relational queries

FCA/RCA is interesting for discovering unanticipated knowledge

## Why/When considering queries?

- to understand how existing objects correspond / don't correspond to the query
- to discover unanticipated knowledge about answers and the other objects

## Discussion

- When the query has circuits, define a navigation order
- Defining queries with variants (using other scaling operators)
- Based on a navigation algorithm, propose a tool to help the user navigating

# Specific relational schemes

In a sense, when we choose a RCF, we choose a relational scheme allowing to search data with a specific idea in mind.

| Relation | Step 1 | Step 2 | Step 3 | ... |
|---|---|---|---|---|
| eats | $\forall\ r : c$ | $\exists\ r : c$ | $\exists\ r : c$ | $\exists\ r : c$ |
| contains | $\geq\ n\, r : c$ | $\forall\ r : c$ | $\exists\ r : c$ | $\exists\ r : c$ |
| producedIn | $\geq\ n\, r : c$ | $\forall\ r : c$ | $\exists\ r : c$ | $\exists\ r : c$ |
| hasForNational | $\exists\ r : c$ | $\forall\ r : c$ | $\exists\ r : c$ | $\exists\ r : c$ |
| ... | ... | ... | ... | |

To discover for example:

- People that eat only pizza containing at least n ingredients produced in Norway
- Countries where at least one national eats at least one pizza that contains at least one food produced in Italy

Note: at one step, several scaling operators can be applied to the same relational context (giving several scaled contexts based on a same relational context)

# Outline

# Erratic RCA


http://www.cafepress.com/+labyrinth+greeting_cards

## Variations on the RCA algorithm

- to focus step-by-step on part of the data
- for scalability
- to launch the process in specific cases

An example for giving some insight
See Xavier Dolques' talk for more details

# Strange animals

| $I_{Motion}$ | |
|---|---|
| fly | |
| walk | |

| $I_{Animal}$ | |
|---|---|
| flamingo | |
| flying squirrel | |
| geek | |

| $I_{Places}$ | |
|---|---|
| lake | |
| forest | |
| city | |

| $I_{Food}$ | |
|---|---|
| artemia | |
| lichen | |
| pizza | |

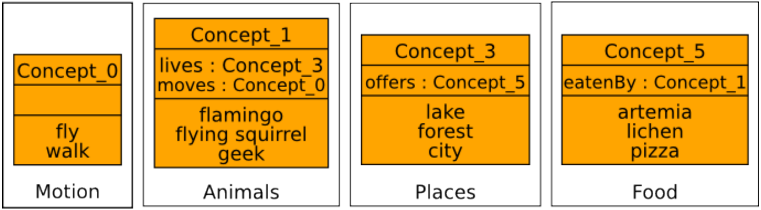
| $I_{lives}$ | lake | forest | city |
|---|---|---|---|
| flamingo | × | | |
| flying squirrel | | × | |
| geek | | | × |

| $I_{moves}$ | fly | walk |
|---|---|---|
| flamingo | × | |
| flying squirrel | × | |
| geek | | × |

| $I_{offers}$ | artemia | lichen | pizza |
|---|---|---|---|
| lake | × | | |
| forest | | × | |
| city | | | × |

| $I_{eatenBy}$ | flamingo | flying squirrel | geek |
|---|---|---|---|
| artemia | × | | |
| lichen | | × | |
| pizza | | | × |

# Classical RCA algorithm




For the rest of the example, these lattices are denoted by TMotion_T1form, TAnimals_T1form, TPlaces_T1form, TFood_T1form

# Erratic RCA

## An example of approach

- Successively choose a set of objects, and introduce the relations this set of objects is a domain
- When no lattice exists on a set of objects: introduce relations **without** relational scaling
- When a lattice exists on a set of objects: introduce relations **with** relational scaling

# Erratic RCA

## Introduce relation without relational scaling

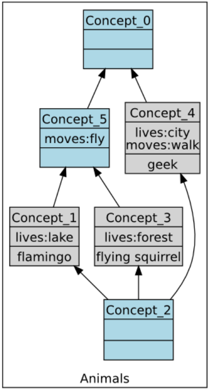
| $I_{Animals} + I_{lives} + I_{moves}$ | lives:lake | lives:forest | lives:city | moves:fly | moves:walk |
|---|---|---|---|---|---|
| **flamingo** | × | | | × | |
| **flying squirrel** | | × | | × | |
| **geek** | | | × | | × |

We will denote this kind of data by:
Animals+lives(Places)+moves(Motions)

# Erratic RCA

## Building a lattice at a step, ex. 1

| $I_{Animals} + I_{lives} + I_{moves}$ | lives:lake | lives:forest | lives:city | moves:fly | moves:walk |
|---|---|---|---|---|---|
| **flamingo** | × | | | × | |
| **flying squirrel** | | × | | × | |
| **geek** | | | × | | × |




We denote this step by:
Animals+lives(Places)+moves(Motions) –> TAnimals_T6form

# Erratic RCA

## Introduce relation with relational scaling

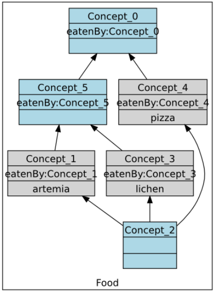
| $I_{Food} + I_{eatenBy(TAnimals\_T6form)}$ | ∃ eatenBy:Concept_0 | ∃ eatenBy:Concept_1 | ∃ eatenBy:Concept_3 | ∃ eatenBy:Concept_4 | ∃ eatenBy:Concept_5 | |
|---|---|---|---|---|---|---|
| **artemia** | × | × | | | × | |
| **lichen** | × | | × | | × | |
| **pizza** | × | | | × | | |

We will denote this kind of data by:
Food+eatenBy(TAnimals_T6form)

# Erratic RCA

## Building a lattice at a step, ex. 2

| $I_{Food} + I_{eatenBy(TAnimals\_T6form)}$ | ∃ eatenBy:Concept_0 | ∃ eatenBy:Concept_1 | ∃ eatenBy:Concept_3 | ∃ eatenBy:Concept_4 |
|---|---|---|---|---|
| **artemia** | × | × | | |
| **lichen** | × | | × | |
| **pizza** | × | | | × |

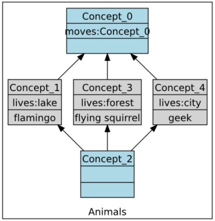



We denote this step by:
Food+eatenBy(TAnimals_T6form) –> TFood_T6form

# Erratic RCA

## Alternatively, using the lattice Motion reduced to a concept, ex. 3

| $I_{Animals} + I_{lives} + I_{moves(TMotion)}$ | lives:lake | lives:forest | lives:city | ∃ moves:Concept_0 |
|---|---|---|---|---|
| **flamingo** | × | | | × |
| **flying squirrel** | | × | | × |
| **geek** | | | × | × |




We denote this step by:
Animals+lives(Places)+moves(TMotion_T1form) –> TAnimals_T3form

# Erratic RCA

1. Animals+lives(Places)+moves(motion)–> TAnimals_T6form
2. Places+offers(Food) –> TPlaces_T3form
3. Food+eatenBy(TAnimals_T6form) –> TFood_T6form
4. Motion –> TMotion_T1form
5. Animals+lives(TPlaces_T3form)+moves(Tmotion_T1form)–> TAnimals _T3form
6. Places+offers(TFood_T6form) –> TPlaces_T6form
7. Food+eatenBy(TAnimals_T3form) –> TFood_T3form
8. Animals+lives(TPlaces_T6form)+moves(Tmotion_T1form)–> TAnimals_T6form
9. Places+offers(TFood_T3form) –> TPlaces_T3form
10. ...

A possibly diverging path!

# Erratic RCA

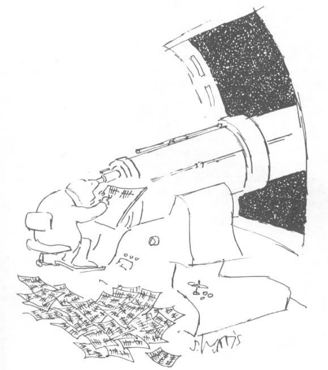
Lessons we can draw:

- concepts emerge step-by-step, only a partial view is given, which is interesting in an interactive process
- we may converge more rapidly (on other examples)
- we have to pay attention to possible on non-monotonic lattice construction

Applications, interpretation of the lattices and perspectives: see Xavier's Talk

# Outline

# Observing the concept lattice growth in RCA



http://astronomy.swin.edu.au/ gmackie/humour.html

# Observing the concept lattice growth in RCA

- to understand how data are connected and concept formation propagates
- applied to evolving datasets, to understand evolution of the concept formation
- to pre-cluster data before RCA

work inspired by experiments in IS applications done with: B. Amar, A. Osman Guedi, A. Miralles, C. Nebut, T. Libourel

# Concept lattice family evolution

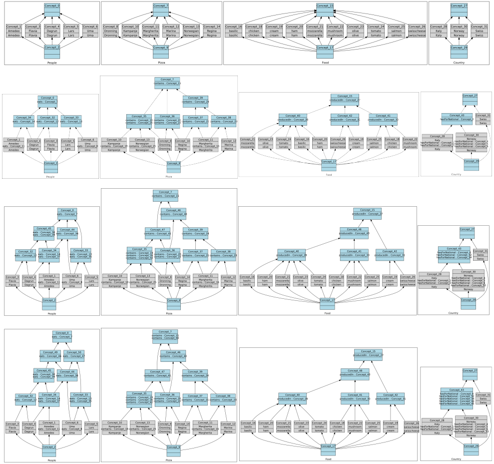
On the pizza example, successive steps show the growth of conceptual knowledge

step 0.

step 1.

step 2.

step 3.



# Pre-clustering a dataset for scalability

An example of a unique dataset (object-attribute context)

| | | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **z1** | | × | | | |
| **z2** | | × | | | |
| **z3** | | × | | | |
| **a1** | | | × | | |
| **a2** | | | × | | |
| **a3** | | | × | | |
| **b1** | | | | × | |
| **b2** | | | | × | |
| **b3** | | | | × | |
| **c1** | | | | | × |
| **c2** | | | | | × |
| **c3** | | | | | × |
| **c4** | | | | | × |
| **c5** | | | | | × |
| **c6** | | | | | × |
| **d1** | × | × | | | |
| **d2** | × | | × | | |
| **d3** | × | | | × | |

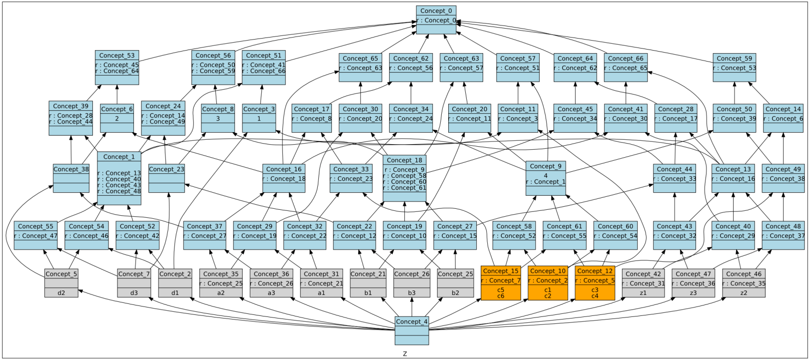
# Pre-clustering a dataset for scalability

An example of a unique dataset (object-object context)

| | z1 | z2 | z3 | a1 | a2 | a3 | b1 | b2 | b3 | c1 | c2 | c3 | c4 | c5 | c6 | d1 | d2 | d3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| z1 | | | | × | | | | | | | | | | | | | | |
| z2 | | | | | × | | | | | | | | | | | | | |
| z3 | | | | | | × | | | | | | | | | | | | |
| a1 | | | | | | | × | | | | | | | | | | | |
| a2 | | | | | | | | × | | | | | | | | | | |
| a3 | | | | | | | | | × | | | | | | | | | |
| b1 | | | | | | | | | | × | | × | | | | | | |
| b2 | | | | | | | | | | | × | | | × | | | | |
| b3 | | | | | | | | | | | | | × | | × | | | |
| c1 | | | | | | | | | | | | | | | | × | | |
| c2 | | | | | | | | | | | | | | | | × | | |
| c3 | | | | | | | | | | | | | | | | | × | |
| c4 | | | | | | | | | | | | | | | | | × | |
| c5 | | | | | | | | | | | | | | | | | | × |
| c6 | | | | | | | | | | | | | | | | | | × |
| d1 | × | × | | | | | | | | | | | | | | | | |
| d2 | | × | × | | | | | | | | | | | | | | | |
| d3 | × | | × | | | | | | | | | | | | | | | |

# Pre-clustering a dataset for scalability

The concept lattice family (one lattice)



# Pre-clustering a dataset for scalability

Alternative view on same data, by dividing the dataset and the links

| | 1 |
|---|---|
| z1 | × |
| z2 | × |
| z3 | × |

| | 2 |
|---|---|
| a1 | × |
| a2 | × |
| a3 | × |

| | 3 |
|---|---|
| b1 | × |
| b2 | × |
| b3 | × |

| | 4 |
|---|---|
| c1 | × |
| c2 | × |
| c3 | × |
| c4 | × |
| c5 | × |
| c6 | × |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| d1 | × | × | | |
| d2 | × | | × | |
| d3 | × | | | × |

| $I_{r1}$ | a1 | a2 | a3 |
|---|---|---|---|
| z1 | × | | |
| z2 | | × | |
| z3 | | | × |

| $I_{r2}$ | b1 | b2 | b3 |
|---|---|---|---|
| a1 | × | | |
| a2 | | × | |
| a3 | | | × |

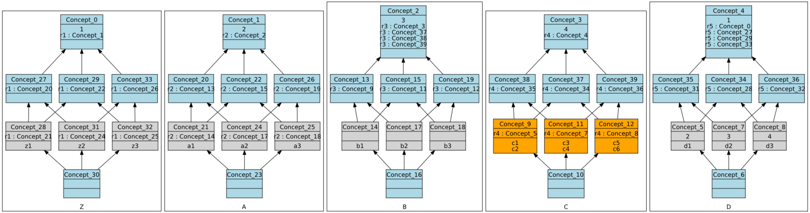
| $I_{r3}$ | c1 | c2 | c3 | c4 | c5 | c6 |
|---|---|---|---|---|---|---|
| b1 | × | | × | | | |
| b2 | | × | | | × | |
| b3 | | | | × | | × |

| $I_{r4}$ | d1 | d2 | d3 |
|---|---|---|---|
| c1 | × | | |
| c2 | × | | |
| c3 | | × | |
| c4 | | × | |
| c5 | | | × |
| c6 | | | × |

| $I_{r5}$ | z1 | z2 | z3 |
|---|---|---|---|
| d1 | × | × | |
| d2 | | × | × |
| d3 | × | | × |

# Pre-clustering a dataset for scalability

The obtained concept lattice family



# Outline

# Conclusion / Perspectives

## Conclusion

- RCA gives a view of data via a set of connected lattices
- Past and ongoing applications on various data

## Perspectives

- RCA engineering
  - Extracting classifications and implications
  - Queries and RCA
  - Understanding concept lattices growth
  - Erratic RCA
  - What about Metrics: e.g. a version of stability for RCA?