Vision-based Modeling and Control of Large-Dimension Cable-Driven Parallel Robots
Tej Dallej, Marc Gouttefarde, Nicolas Andreff, Redwan Dahmouche, Philippe Martinet

► To cite this version:

HAL Id: lirmm-00737658
https://hal-lirmm.ccsd.cnrs.fr/lirmm-00737658
Submitted on 14 Feb 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Vision-based Modeling and Control of Large-Dimension Cable-Driven Parallel Robots

Tej Dallej *, Marc Gouttefarde °, Nicolas Andreff °°, Redwan Dahmouche * and Philippe Martinet *

Email: tej.dallej@lasmea.univ-bpclermont.fr

Abstract—This paper is dedicated to vision-based modeling and control of large-dimension parallel robots driven by inextensible cables of non-negligible mass. An instantaneous inverse kinematic model devoted to vision is introduced. This model relies on the specificities of a parabolic profile hefty cable modeling and on the resulting simplified static analysis. By means of a kinematic visual servoing method, computer vision is used in the feedback loop for easier control. According to the modeling derived in this paper, measurements that allow the implementation of this visual servoing consist of the mobile platform pose, the directions of the tangents to the cable curves at their drawing points and the cable tensions. The presented method is validated on a broad class of parallel robots.

I. INTRODUCTION

Parallel cable-driven robots are a particular type of parallel kinematic machines in which cables connect the base to the mobile platform [1], [2]. A 6-DOF parallel robot driven by six cables can be thought as a Gough-Stewart platform turned upside down with cables instead of prismatic actuators. When the cables are tensed and considered massless and inextensible, the kinematic modeling of Gough-Stewart platforms and of parallel cable-driven robots are very similar.

Visual servoing techniques [3], [4], [5] were applied to parallel robotics [6], [7], [8]. These applications rely on 3D visual servoing where the mobile platform pose is indirectly measured and used for regulation. To make control robust with respect to modeling errors, it was proposed to servo the leg edges [9] which improved the practical robustness by servoing the legs in the image. This method has been validated on a broad class of parallel robots [10].

Using a 3D pose kinematic visual servoing method, in which the mobile platform pose is used for regulation, the work reported in [11] confirmed that visual servoing techniques are a good alternative for the control of parallel cable robots. However, in [11], the cables are supposed to be massless. This assumption is invalid in the case of large-dimension cable robots [12] since the cables sag under their own weight. Assuming that the cable is inextensible and the sag relatively small, a simplified hefty cable modeling can be considered [13]. This simplified modeling describes the cable profile explicitly as a parabolic curve. Compared to the use of the elastic catenary [14], it leads in turn to a simplified (quasi) static analysis of large-dimension parallel cable robots [15].

The contribution of this paper is to extend the vision-based control proposed in [11] to large-dimension parallel cable robots. To this end, based on the aforementioned simplified hefty cable modeling, the instantaneous inverse kinematic model of large-dimension parallel cable robots is determined. This model turns out to be dependent on the pose of the mobile platform but also on the directions of the tangents to the cables at their drawing points and on the cable tensions.

The cable tensions can be obtained by means of force sensors. To estimate the mobile platform pose and the cable tangent directions, one can rely on vision. In the case of a large-dimension cable robot, it is obviously not conceivable to measure all the needed variables using a single perspective camera in front of the robot. Instead, we plan to use a multi-camera setup together with force sensors.

Using a 3D pose kinematic visual servoing method, in which the mobile platform pose is used for regulation, the work reported in [11] confirmed that visual servoing techniques are a good alternative for the control of parallel cable robots. However, in [11], the cables are supposed to be massless. This assumption is invalid in the case of large-dimension cable robots [12] since the cables sag under their own weight. Assuming that the cable is inextensible and the sag relatively small, a simplified hefty cable modeling can be considered [13]. This simplified modeling describes the cable profile explicitly as a parabolic curve. Compared to the use of the elastic catenary [14], it leads in turn to a simplified (quasi) static analysis of large-dimension parallel cable robots [15].

The contribution of this paper is to extend the vision-based control proposed in [11] to large-dimension parallel cable robots. To this end, based on the aforementioned simplified hefty cable modeling, the instantaneous inverse kinematic model of large-dimension parallel cable robots is determined. This model turns out to be dependent on the pose of the mobile platform but also on the directions of the tangents to the cables at their drawing points and on the cable tensions.

The cable tensions can be obtained by means of force sensors. To estimate the mobile platform pose and the cable tangent directions, one can rely on vision. In the case of a large-dimension cable robot, it is obviously not conceivable to measure all the needed variables using a single perspective camera in front of the robot. Instead, we plan to use a multi-camera setup together with force sensors.

Using a 3D pose kinematic visual servoing method, in which the mobile platform pose is used for regulation, the work reported in [11] confirmed that visual servoing techniques are a good alternative for the control of parallel cable robots. However, in [11], the cables are supposed to be massless. This assumption is invalid in the case of large-dimension cable robots [12] since the cables sag under their own weight. Assuming that the cable is inextensible and the sag relatively small, a simplified hefty cable modeling can be considered [13]. This simplified modeling describes the cable profile explicitly as a parabolic curve. Compared to the use of the elastic catenary [14], it leads in turn to a simplified (quasi) static analysis of large-dimension parallel cable robots [15].

The contribution of this paper is to extend the vision-based control proposed in [11] to large-dimension parallel cable robots. To this end, based on the aforementioned simplified hefty cable modeling, the instantaneous inverse kinematic model of large-dimension parallel cable robots is determined. This model turns out to be dependent on the pose of the mobile platform but also on the directions of the tangents to the cables at their drawing points and on the cable tensions.

The cable tensions can be obtained by means of force sensors. To estimate the mobile platform pose and the cable tangent directions, one can rely on vision. In the case of a large-dimension cable robot, it is obviously not conceivable to measure all the needed variables using a single perspective camera in front of the robot. Instead, we plan to use a multi-camera setup together with force sensors.
on the simulator of the parallel CoGiRo cable robot in Section V. Conclusions are finally drawn in Section VI.

II. MODEL OF CABLE WITH NON-NEGREGIBLE MASS DEVOTED TO VISION

Throughout the paper, the notations given in Table I will be used.

- $i = 1...k$ denotes the driving cables, where $k$ is the number of cables.
- Boldface characters denote vectors. Unit vectors are underlined.
- $\mathcal{F}_A = (A_x, A_y, A_z)$, $\mathcal{F}_B = (B_x, B_y, B_z)$ and $\mathcal{F}_C = (C_x, C_y, C_z)$ denote the cable $i$, base, and end-effector reference frames, respectively.
- $\mathcal{F}_1 = (O_{1}, O_{2}, O_{3}, O_{4})$ is a fixed reference frame attached to the base of the robot. It can be a camera reference frame (eye-to-hand) or a pattern reference frame (eye-in-hand).
- $\mathcal{F}_0 = (O, X_r, Y_r, Z_r)$ is a mobile reference frame attached to the mobile platform.
- $\mathcal{F}_2 = (O, X_, Y_, Z_)$ is the reference frame attached to the pattern.
- In the case of multiple cameras, $\mathcal{F}_{ij} = (O_{ij}, X_{ij}, Y_{ij}, Z_{ij})$ is the reference frame attached to the camera $j$, $j = 1...r, r$ is the number of cameras.
- $\hat{\mathbf{v}}$ is vector $\mathbf{v}$ expressed in $\mathcal{F}_1$.
- $\mathbf{F}_{ij}$ is the desired position and orientation of $\mathcal{F}_j$.
- $g$ is the norm of the gravity acceleration vector.
- $q_i$ defines the motorized joint angle $i$.
- $\rho_i$ is the linear mass density of cable $i$.
- $L_i$ is the length of cable $i$ supposed to be massless (straight line segment).
- $l_i$ is the length of inextensible cable $i$ with non-negligible mass (with the parabolic profile modeling).
- $\mathbf{A}_i = (A_{ix}, A_{iy}, A_{iz})$ and $\mathbf{B}_i = (B_{ix}, B_{iy}, B_{iz})^T$ are the two extremities of the sagging part of cable $i$.
- $u_i = \left( u_{ix}, u_{iy}, u_{iz} \right)^T$ is the unit vector pointing from $A_i$ to $B_i$.
- $\bar{\mathbf{A}}_i = \bar{\mathbf{A}}_i = \left( \bar{A}_{ix}, \bar{A}_{iy}, \bar{A}_{iz} \right)$ is the force in the cable at point $A_i$.
- $\bar{\mathbf{B}}_i = \left( \bar{B}_{ix}, \bar{B}_{iy}, \bar{B}_{iz} \right)^T$ is the force in the cable at point $B_i$.
- $\bar{\mathbf{F}}_i = \left( \bar{F}_{ix}, \bar{F}_{iy}, \bar{F}_{iz} \right)$ is the force in the cable at $i$.
- $\mathbf{M}$ is the mass of the cable.
- $\mathbf{M}^+$ is the pseudo-inverse of $\mathbf{M}$.
- $\mathbf{M}_x$ is the cross-product matrix associated with vector $\mathbf{a}$.
- $\hat{\mathbf{M}}$ is the estimation of $\mathbf{M}$ based on measurements.

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTATIONS USED THROUGHOUT THE PAPER</td>
</tr>
</tbody>
</table>

Fig. 2 shows the profile of a cable with non-negligible mass. In static loading conditions, it lies in the vertical plane $\Pi_i$ containing $A_i$ and $B_i$. The reference frame $\mathcal{F}_A$ attached to $\Pi_i$ is obtained from frame $\mathcal{F}_B$ by a rotation of angle $\gamma_i$ around the $z$ axis ($Z_0 = Z_{Ai}$ being vertical).

Given $\mathbf{u}_i = \left( b_{iux}, b_{iuy}, b_{izu} \right)^T$, the angle $\gamma_i$ can be computed as:

$$\gamma_i = \tan^{-1} \left( \frac{b_{iuy}}{b_{iux}} \right)$$  \hspace{1cm} (1)

A simplified modeling of an inextensible hefty cable [13] is used in this paper. This model is obtained from the elastic catenary modeling [14], [15]. It results in a parabolic cable profile equation, expressed in frame $\mathcal{F}_A$ i, which is given by:

$$z_i = -\frac{\rho_i g L_i}{2 A_i B_{ix}} x_i (A_{iB_{ix}} - x_i) + x_i \tan(\beta_{0i})$$  \hspace{1cm} (2)

where $x_i$ and $z_i$ are the coordinates in frame $\mathcal{F}_A$ of cable point $S_i$ (see Fig. 2).

The angle $\beta_{0i}$ between $\mathbf{u}_i$ and $\mathbf{A}_i$ (Fig. 2) can be written as:

$$\beta_{0i} = \tan^{-1} \left( \frac{A_{iB_{ix}}}{A_{iB_{ix}}} \right)$$  \hspace{1cm} (3)

A. Tangents to the cable profile

The slope of the tangents to the cable profile (Eq. (2)) at points $B_i$ and $A_i$ (see Fig. 2) can be computed using

$$\frac{dz}{dx} \left( x_i = B_{ix} \right) = \tan(\beta_i) \quad \text{and} \quad \frac{dz}{dx} \left( x_i = 0 \right) = \tan(\alpha_i),$$

According to (2), one can write:

$$\left\{ \begin{array}{l}
\tan(\beta_i) = \frac{A_{iB_{ix}}}{\cos(\alpha_i)} = \tan(\beta_{0i}) + \frac{\rho_i g L_i}{A_{iB_{ix}}} \\
\tan(\alpha_i) = \frac{A_{iB_{ix}}}{\cos(\beta_{0i})} = \tan(\beta_{0i}) - \frac{\rho_i g L_i}{A_{iB_{ix}}} \end{array} \right.$$  \hspace{1cm} (4)

Since $A_{iB_{ix}}$ is the only external loading applied to the cable between $A_i$ and $B_i$ (the cable weight) and introducing $A_{iB_{ix}} = \left( \cos(\beta_{0i}) 0 \sin(\beta_{0i}) \right)^T$, one can compute:

$$\left\{ \begin{array}{l}
A_{iB_{ix}} = A_{iB_{ix}}^T A_{iB_{ix}} \mathbf{u}_i + \frac{\rho_i g L_i}{2} A_{iB_{ix}} Z_{Ai} \\
A_{iB_{ix}} = -\frac{\rho_i g L_i}{2} A_{iB_{ix}} Z_{Ai} \end{array} \right.$$  \hspace{1cm} (5)

It is important to note that the last equation can be expressed in any Euclidean reference frame. In the general case, we can write:

$$\left\{ \begin{array}{l}
\mathbf{B}_i = \mathbf{B}_i = \mathbf{u}_i + u_{pi} \\
\mathbf{A}_i = \mathbf{A}_i = -\mathbf{u}_i + u_{pi} \end{array} \right.$$  \hspace{1cm} (6)

where $u_i = \frac{A_{iB_{ix}}}{\cos(\beta_{0i})} \mathbf{u}_i$, $u_{pi} = \frac{\rho_i g L_i}{2} Z_{Ai}$.
B. Inverse kinematics

The inverse kinematics gives the length of each cable for a given pose of the platform. Using the cable profile of Eq. (2), one can compute the cable length by integrating a cable length element [15]:

\[
i_i = \int_{0}^{A_i B_i} \sqrt{1 + \left(\frac{dz}{dx}\right)^2} \, dx = A_i B_i \sqrt{c_1^2 k_1^2 - c_2^2 k_2^2 + 1} / \sqrt{c_1^2 k_1^2 + c_2^2 k_2^2}\]

(7)

where \( r_i = \frac{\rho L_i}{L_i} \), \( k_i = \tan(\beta_i) + \frac{z_i}{r_i} \), \( c_1 = \sqrt{1 + k_1^2} \) and \( c_2 = \sqrt{1 + k_2^2} \).

III. VISION-BASED KINEMATICS

The instantaneous inverse kinematic model of a parallel robot driven by cables relates the Cartesian velocity \( \mathbf{\tau}_c \) of the mobile platform to the time derivative \( \dot{\mathbf{l}} = (\dot{l}_1 \ldots \dot{l}_k)^T \) of cable length vector. The Cartesian velocity of the mobile platform expressed in a fixed reference frame \( \mathbf{F} \) is \( \dot{\mathbf{F}}_e = \dot{\mathbf{F}}_{e/f} \dot{\mathbf{e}}_f \) for a pseudo-static case, the time derivative of (20) leads to:

\[
\sum_{i=1}^{k} (\dot{f} \dot{\mathbf{l}}_i) = 0 \quad \text{(21)}
\]

A. Expression of \( D_{Li} \)

According, e.g., to [2], [11], the expressions of the time derivatives of \( \dot{L}_i \) and \( \dot{\mathbf{l}}_i \) (case of an inextensible and massless cable) are:

\[
\dot{L}_i = D_{Li} \dot{f} \mathbf{\tau}_c \quad \text{(11)}
\]

where \( D_{Li} = \mathbf{f} \mathbf{u}_i^T \left( \mathbf{I}_3 - [^f \mathbf{R}_{e} \mathbf{B}_i]_x \right) \), and:

\[
\dot{f} \mathbf{u}_i = D_{ui} \dot{f} \mathbf{\tau}_c \quad \text{(12)}
\]

where \( D_{ui} = \frac{1}{A_i} \left( \mathbf{I}_3 - \mathbf{u}_i \mathbf{u}_i^T \right) \left( \mathbf{I}_3 - [^f \mathbf{R}_{e} \mathbf{B}_i]_x \right) \).

B. Expression of \( D_{Bi} \)

1) Angular velocity of \( \mathbf{F} \) expressed in \( \mathbf{F} \):

The time derivative of (1) gives:

\[
\dot{\gamma}_i = D_{0i} \dot{\mathbf{u}}_i = D_{0i} \mathbf{R}_f \dot{\mathbf{u}}_i \quad \text{(13)}
\]

where \( D_{0i} = \frac{1}{u_{iy}^2 + u_{ix}^2} \left( -b_{uiy} b_{uix} 0 \right) \).

Taking into account the fact that \( \dot{\mathbf{z}}_0 = \dot{\mathbf{z}}_A \) and according to (12), the angular velocity of \( \mathbf{F} \) is defined by:

\[
\dot{\mathbf{\Omega}}_A = D_{\gamma_i} \dot{f} \mathbf{\tau}_c \quad \text{(14)}
\]

where \( D_{\gamma_i} = \dot{\mathbf{z}}_A \mathbf{D}_{0i} \mathbf{R}_f \mathbf{D}_{0i} \).

2) Inverse kinematic model associated with \( f \mathbf{B}_i \):

The time derivative of \( \mathbf{B}_i \), with respect to the fixed reference frame \( \mathbf{F} \), is:

\[
\dot{f} \mathbf{B}_i = D_{f\mathbf{B}_i} \dot{f} \mathbf{\tau}_c \quad \text{(15)}
\]

where \( D_{f\mathbf{B}_i} = \dot{\mathbf{z}}_A \mathbf{D}_{0i} \mathbf{R}_f \mathbf{D}_{0i} \).

C. Expression of \( D_{si} \)

1) Mobile platform static equilibrium:

One can write the force applied at point \( \mathbf{B}_i \) by the cable to the platform as (see Eq. (6)):

\[
\dot{f} \mathbf{f}_{\mathbf{B}_i} = -\dot{f} \mathbf{\theta}_{\mathbf{B}_i} = -\dot{f} \mathbf{u}_{\mathbf{B}_i} \quad \text{(18)}
\]

where \( \dot{\mathbf{\theta}}_{\mathbf{B}_i} \geq 0 \) since a cable can only pull on the platform.

Assuming that the moment of \( \dot{f} \mathbf{f}_{\mathbf{B}_i} \) at point \( \mathbf{B}_i \) is \( \dot{\mathbf{\eta}}_{\mathbf{B}_i} = 0 \), the wrench applied by cable \( i \) at the reference point \( \mathbf{E} \), expressed in \( \mathbf{F} \), is:

\[
\dot{f} \mathbf{\gamma}_i = \begin{pmatrix} \dot{f} \mathbf{f}_{\mathbf{B}_i} \\ f_\mathbf{EB}_i \end{pmatrix} \quad \text{(19)}
\]

According to the mobile platform equilibrium, one can write the relationship between the external wrenches \( \dot{f} \mathbf{\gamma}_{\mathbf{F}_e} \) and cable wrenches as:

\[
\dot{f} \mathbf{\gamma}_{\mathbf{F}_e} + \sum_{i=1}^{k} (\dot{f} \mathbf{\gamma}_i) = 0 \quad \text{(20)}
\]

Assuming that \( \dot{f} \mathbf{\gamma}_{\mathbf{F}_e} = 0 \) (a pseudo-static case), the time derivative of (20) leads to:

\[
\sum_{i=1}^{k} (\dot{f} \mathbf{\gamma}_i) = 0 \quad \text{(21)}
\]
At this time, we can write:

$$f \mathbf{E} f = \mathbf{D}_{E B} f \boldsymbol{\tau}_e$$

(23)

where $$\mathbf{D}_{E B i} = (\mathbf{0}_3 - [f \mathbf{R}_c \mathbf{B}_i] \times )$$. Therefore (22) becomes:

$$\mathbf{u}_{B i} = -\left( f \mathbf{u}_{B i} - [f \mathbf{u}_{B i}] \times \mathbf{D}_{E B i} \frac{f}{f} \mathbf{e} \right)$$

(24)

2) Time derivative of $$f \mathbf{u}_{B i}$$:

The time derivative of (6), expressed in the fixed frame $$\mathcal{F}_f$$, gives:

$$f \dot{\mathbf{u}}_{B i} = f \dot{\mathbf{u}}_{i} + f \dot{\mathbf{u}}_{p i}$$

(25)

Using (3), the time derivative of $$\dot{\beta}_{0i}$$ can be computed as:

$$\ddot{\beta}_{0i} = \frac{\cos(\beta_{0i})^2}{A_i B_{ix}} ( -A_i B_{iz} 0 A_i B_{ix} ) A_i \dot{\mathbf{B}}_i$$

(26)

Thus, according to (26) and to the expression of $$\mathbf{u}_i$$ given in Section II-A, the time derivative of $$f \mathbf{u}_i$$ is:

$$f \dot{\mathbf{u}}_{i} = D_{f i} \dot{\mathbf{B}}_i + \frac{f \mathbf{u}_{i}}{\cos(\beta_{0i})} A_i \dot{\mathbf{B}}_{ix} + A_i \dot{\mathbf{p}}_{pi} f \dot{\mathbf{u}}_{p i}$$

(27)

By substituting (12) and (17) in (27), one can deduce:

$$f \dot{\mathbf{u}}_{i} = D_{u i 1} f \boldsymbol{\tau}_e + D_{u i 2} A_i \dot{\mathbf{B}}_{ix}$$

(28)

where

- $$D_{u i 1} = (D_{f i} D_{B i} + A_i \frac{\dot{\mathbf{B}}_{ix}}{\cos(\beta_{0i})} D_{u i})$$
- $$D_{f i} = \frac{1}{A_i B_{ix}} \frac{\dot{\mathbf{B}}_{ix}}{\cos(\beta_{0i})}$$
- $$D_{u i 2} = \frac{f \mathbf{u}_{i}}{\cos(\beta_{0i})}$$

Then, using (11), the time derivative of $$f \mathbf{u}_{p i} = \frac{\rho \mathbf{g} t_{m}}{2} f \mathbf{Z}_{i t_{m}}$$ is:

$$f \dot{\mathbf{u}}_{p i} = D_{u p i} f \mathbf{\tau}_e$$

(29)

where

- $$D_{u p i} = \frac{\rho \mathbf{g}}{2} f \mathbf{Z}_{i t_{m}}$$

Finally, the time derivative of the non-unit vector $$f \mathbf{u}_{B i}$$ is:

$$f \dot{\mathbf{u}}_{B i} = (D_{u i 1} + D_{u p i}) f \mathbf{\tau}_e + D_{u i 2} A_i \dot{\mathbf{B}}_{ix}$$

(30)

3) Expression of $$A_i \dot{\mathbf{B}}_{ix}$$:

Using (30) and (24), one can deduce:

$$f \dot{\mathbf{u}}_{i} = -(D_{f i} f \mathbf{\tau}_e + D_{\dot{\beta}_{0i}} A_i \dot{\mathbf{B}}_{ix})$$

(31)

where

- $$D_{f i} = \left( D_{u i 1} + D_{u p i} \right)$$
- $$D_{\dot{\beta}_{0i}} = \left( \frac{f \mathbf{u}_{i}}{\cos(\beta_{0i})} \times (D_{u p i} + D_{u i 1}) \right)$$

Eq. (21) becomes:

$$D_{f i} + D_{\dot{\beta}_{0i}} = 0$$

(32)

where

- $$D_{f i} = (D_{f i} D_{B i} + A_i \frac{\dot{\mathbf{B}}_{ix}}{\cos(\beta_{0i})} D_{u i})$$
- $$D_{\dot{\beta}_{0i}} = \frac{f \mathbf{u}_{i}}{\cos(\beta_{0i})}$$

Finally, one can compute $$A_i \dot{\mathbf{B}}_{ix}$$ as:

$$A_i \dot{\mathbf{B}}_{ix} = \mathbf{D}_s f \mathbf{\tau}_e$$

(33)

where $$\mathbf{D}_s = -D_{\dot{\beta}} \sum_{i=1}^{k} \mathbf{D}_{s i}$$. Let us note that the use of the pseudo-inverse to compute $$\mathbf{D}_s$$ may not be the best choice. This potential issue is however out of the scope of this paper.

Hence, $$A_i \dot{\mathbf{B}}_{ix}$$ can be found by:

$$A_i \dot{\mathbf{B}}_{ix} = \mathbf{D}_s f \mathbf{\tau}_e$$

(34)

where $$\mathbf{D}_s$$ is the ith row of matrix $$\mathbf{D}_s$$.

IV. VISION-BASED CONTROL

A. Control

Visual servoing is based on the so-called interaction matrix $$\mathbf{L}_s$$ which relates the instantaneous relative Cartesian motion $$\mathbf{\tau}$$ between the mobile platform and the scene to the time derivative of the vector $$\mathbf{s}$$ of the visual primitives used for regulation [17] ($$\dot{s} = \mathbf{L}_s \mathbf{\tau}$$). According to the nature of the visual primitives, there exist many visual servoing techniques ranging from position-based visual servoing [18], [19] to image-based visual servoing [4], [20], most of them being based on point features. One can also find other visual primitives such as lines [21] or image moments [22].

As the instantaneous inverse kinematic model (10) depends on the platform pose, we choose position-based visual servoing in the 3D pose form [18], [19], [23]: $$\mathbf{s} = (s_t \ s_w)^T$$.

Consider $$\mathbf{F}_m$$, and $$\mathbf{F}_{m^*}$$ the current and the desired mobile frame locations, respectively. $$\mathbf{s}_t = m \mathbf{t}_{m^*}$$ is the translation error between $$\mathbf{F}_m$$ and $$\mathbf{F}_{m^*}$$ and $$\mathbf{s}_w = \mathbf{u} \theta$$, where $$\mathbf{u}$$ is the axis and $$\theta$$ is the angle of the rotation matrix $$m \mathbf{R}_{m^*}$$.

The interaction matrix associated to the pose can be written as [23], [24]:

$$\mathbf{L}_s = \begin{pmatrix} -I_3 & [m \mathbf{t}_{m^*}]_x & -L_w \end{pmatrix}$$

(35)

where

- $$\mathbf{L}_w = I_3 - \frac{\dot{\theta}}{\theta} \mathbf{u} \times + \left(1 - \frac{\sin(\theta)}{\sin(\frac{\theta}{2})}\right) \frac{\mathbf{u} \times}{\theta}$$
- $$\sin(\theta) = \frac{\sin(\theta)}{\theta}$$ and $$m \mathbf{t}_{m} = m \mathbf{t}_{m^*} = m \mathbf{t}_{m}/f$$

Fig. 3. Visual servoing of a cable-driven parallel robot

To regulate the error between the current primitive vector $$\mathbf{s}$$ and the desired one $$\mathbf{s}^* = \mathbf{0}$$, one can consider the exponential decay $$\dot{s} = -\lambda \mathbf{s}$$. The vision-based control (Fig. 3) can then be expressed as:

$$m \mathbf{t}_{m} = -\lambda \mathbf{L}_s \dot{\mathbf{s}}$$

(36)
With $\tau_e = D_\ell m \tau_m$ and using (36), (10) becomes:

$$\dot{q} = mD_m m \tau_m = -\lambda_m D_m \hat{L}_s + s (37)$$

where $D_\ell = \left( \ell R_m \ell R_m[^m] + \cdots \right)_{\text{error vector norm}}$. These accuracy results are deemed satisfactory enough for such a large-dimension parallel robot.

B. Examples of measured variables and parameters to identify

Note that the instantaneous inverse kinematic model (10) depends on $T_m$, the rigid transformation between the mobile and the fixed frame which defines the pose of the platform. It depends also on the tangents $u_{Ai}$ to the cables at their drawing points, which we plan to measure by vision, and on some constant (calibration) parameters ($A_i$, $B_i$, the transformations $T_{m_i}$ and $\ell_{B_i}$). Let us also note that only information from the vision sensor are used to define the interaction matrix given in (35), using $mT_{m_i} = mT_f / \ell_{m_i}$.

We plan to use 12 cameras. 4 cameras are used to measure the pattern position (one at the top of each pair). Additionally, 4 stereo pairs should allow us to measure the tangent direction at the cable drawing points. This particular setup is not the only possible one but has been selected based on practical constraints. All kinematic parameters are based on practical constraints. All kinematic parameters are expressed in a reference frame $F_f = F_{c_j}$ attached to the base frame.

![Fig. 4. A simulated CoGiRo robot](image)

In the instantaneous inverse kinematic model (10), we need to define the rotation $A_i R_b$ which depends on the angle $\gamma_i$ as shown in (1). This angle can also be computed using $\gamma_i = \tan^{-1} (u_{Ai} / u_{A_j})$, where $u_{Ai} = \ell R_m u_{Ai}$. We need also the angle $\beta_0_i$ defined in (3). This angle can be computed using the following expressions of the attachment point $B_i$:

$$A_i B_i = A_i R_b (\ell R_m B_i + \ell t_m) + A_i t_b$$

$$f B_i = f R_m m B_i + f t_m$$

where $\ell R_m$ and $\ell t_m$ can be measured by vision and $mB_i$ is a constant parameter.

The instantaneous inverse kinematic model (10) depends also on the length $L_i = || f A_i B_i ||$ of cable $i$ supposed to be massless (straight line) and on the component $A_i g_{Bix}$ of the force applied to the cable at its attachment point $B_i$.

Using (6), the non-unit vector $f u_{B_i}$ can be computed as:

$$f u_{B_i} = f \theta_{B_i} = -A_i g_{Bix} f u_{Ai} + \rho_0 g L_i f Z_{Ai}$$

where $f Z_{Ai} = f R_m Z_{Ai}$ and $A_i g_{Bix}$ is given by a tension sensor (possibly indirectly).

Consequently, using $A_i \theta_{B_i} = A_i R_b \ell f \theta_{B_i}$, one can compute $A_i g_{Bix} = A_i f X_{Ai} A_i \theta_{B_i}$.

V. Simulation results

The vision-based control strategy introduced in the previous section is validated by means of a CoGiRo cable-driven parallel robot simulation (Fig. 4). CoGiRo is a 6-DOF large-dimension parallel cable-driven robot. It has a moving platform (end-effector) connected to a fixed base by 8 driving cables of varying lengths $l_i$, $i \in 1..8$ (k = 8). Each cable (Fig. 4) is attached to the moving platform at point $B_i$ and extends from the base at point $A_i$.

In the simulation, $b E = (0, 0, 0)^T m$ and $(\theta_x, \theta_y, \theta_z)^T = (0, 0, 0)^T$ defines the initial configuration of the CoGiRo platform simulation. The desired one (Fig. 5) is $b E^* = (2, 1)^T m$ and $(\theta_x^*, \theta_y^*, \theta_z^*)^T = (0, 10°, 20°)^T$.

In a first simulation, we choose three geometric characteristic cases (dimensions: width x length x height) of the CoGiRo robot. Tab. II presents the angles $d \beta_i = \beta_i - \beta_{0i}$ (See Fig. 2 for the definition of these angles), in the desired position. $d \beta_i$ increases according to the dimensions of the robot which confirms that the cables cannot be considered as a straight lines segments, in the case of a large-dimension parallel cable-driven robots.

![Fig. 5. Desired (left) and initial (right) position of the CoGiRo robot.](image)

In a second simulation, we choose the dimensions of the real CoGiRo robot (10.8 x 14 x 5.5 m (width x length x height)). To show the robustness of the proposed vision-based control, we added noise to the estimation of the Cartesian pose $f T_m$. We define at each time a sample random rotation axis and a positive rotation unit vector in each direction. Consequently, we define at each time a sample random rotation axis and a positive rotation unit vector in each direction. Consequently, we define at each time a sample random rotation axis and a positive rotation unit vector in each direction. Consequently, we define at each time a sample random rotation axis and a positive rotation unit vector in each direction.
Fig. 6. Evolution of the Cartesian errors.

Fig. 7. Evolution of the mobile platform reference point trajectory.

| TABLE III | Mean values and standard deviations of final Cartesian errors |
| Translation error (mm) | 8.2 | 6.5 |
| Orientation error (°)   | 0.051 | 0.032 |

VI. CONCLUSION

In this paper, we have introduced an instantaneous inverse kinematic model of large-dimension cable-driven parallel robots. This model relies on a simplified model of an inextensible hefty cable in which the cable profile is considered to be a parabolic curve. It can be used to compute an explicit expression of each cable tangent and to compute a partially decoupled inverse kinematic problem. We have shown that the proposed instantaneous inverse kinematic model depends on the mobile platform pose, the cable tangent directions and the cable tensions. A multi-camera setup should allow us to measure the pose of a visual target attached to the mobile platform and the direction of the cable tangents. The cable tensions are measured by means of force sensors. Computer vision can then be used in the feedback loop by using of a 3D pose kinematic visual servoing method. This latter has been validated on the simulator of a large parallel cable robot.

The next step in our work is thus to implement the vision-based control strategy proposed in this paper on a real parallel CoGiRo cable robot.

VII. ACKNOWLEDGMENT

This work was supported by the ANR (grant 2009 SEGI 018, CoGiRo project).

REFERENCES


