Cost Bounds and Approximation Ratios of Multicast Light-Trees in WDM Networks

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***Abstract*—The construction of light-trees is one of the principal subproblems for multicast routing in sparse splitting Wavelength Division Multiplexing (WDM) networks. Due to the light splitting constraint and the absence of wavelength converters, several light-trees may be required to establish a multicast session. However, the computation of the cost-optimal multicast light-trees is NP-hard. In this paper, first we study the cost bounds of the light-trees built for a multicast session in unweighted WDM networks. Then, partially based on this result, the approximation ratios of some classical multicast light-tree computation algorithms, i.e., Reroute-to-Source (R2S) and Member-Only (MO) algorithms are derived in both unweighted and non-equally weighted WDM networks. Moreover, integer linear programming (ILP) formulations are introduced and carried out to search the optimal light-trees for multicast routing. The cost bounds and approximation ratios of R2S and MO algorithms in all-optical backbone networks are examined through simulations.**

***Index Terms*—Cost Bound; Approximation Ratio; Light-tree, All-Optical Multicast Routing (AOMR); WDM Network; Sparse Splitting.**

I. Introduction

ALL-optical multicast routing (AOMR) [1] is to determine a set of lightpaths from a source to the multicast members of the same session in a WDM network. The light-tree concept is introduced in [2] to minimize the number of wavelength channels and transceivers for all-optical multicasting. Branching nodes in a light-tree should be equipped with light splitters to support multicasting. However, in sparse splitting [3] WDM networks, there are two kinds of nodes: multicast capable nodes (MC [3], i.e. the nodes equipped with light splitters) and multicast incapable nodes (MI [3], i.e. the nodes without light splitters). An MC node is able to replicate the data packets in the optical domain via light splitting and send the split light beam to all the outgoing ports. While an MI node cannot split but generally has the *Tap-and-Continue* (TaC [7]) capability. The TaC permits to tap a small amount of optical power from the incoming light beam for local usage and forward the rest to only one outgoing port. Although one tree is sufficient to span all the multicast destinations in a network without constraints, minimizing the cost of the multicast tree is already a Steiner-Problem which is proven to be NP-complete. Due to sparse splitting, lack of wavelength converters, as well as continuous wavelength and distinct wavelength constraints [8], one light-tree may not be able to cover the entire multicast group members while several ones may be required, i.e., a light-forest [6]. As a result, it is even harder to optimize the total wavelength channel cost for a multicast session.

Although many light-tree computation heuristics have been proposed recently [6], [12], [13], [14], none of them has addressed the cost bound of multicast light-trees in sparse splitting WDM networks, let alone the approximation ratios [1] of the heuristic algorithms. Since the wavelength channel cost is a very important metric for the selection of the multicast light-trees, it is very critical to know at least the cost bound of the light-trees, which could be referenced when designing a WDM network. In [14], a heuristic is proposed to construct multicast light-trees with QoS guarantee and the cost upper bound of the light-trees is given. However, in [14] it is supposed that all the network nodes are equipped with costly light splitters, while it is not realistic in large WDM mesh networks due to the high cost and complex architecture of light splitters. Literature [15] also gives a cost upper bound of $\frac{N^2}{4}$ for the multicast light-trees, where N denotes the number of nodes in the network. However, the cost bound in [15] has the following two shortcomings. First it is derived on the hypothesis that the set of multicast light-trees computed for a multicast session still retain a tree structure in the IP layer (i.e., when all these light-trees are merged together). In fact, this hypothesis is not always held as demonstrated in the

Part of this work has been presented in [4] on IFIP Networking 2010 conference and in [5] on IEEE GLOBECOM 2010 conference.

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[1]A heuristic algorithm has an approximation ratio of ρ in network G, if it can be guaranteed that for all possible multicast sessions in G the total cost of the multicast light-forest computed by the heuristic algorithm is at most ρ times worse than the total cost of the optimal solution.

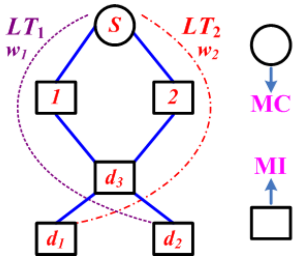


Fig. 1. An example sparse splitting WDM network

following example. A multicast session with source s and destinations d_1, d_2 and d_3 is required in a sparse splitting optical network shown in Fig. 1 with solid line. Since node d_3 is an MI node, two light-trees (i.e., LT_1 (dotted line) and LT_2 (dashed line)) on two different wavelengths may be computed. As we can see the IP layer of the merged LT_1 and LT_2 are drawn in Fig. 1 with solid line, which is the same as the network topology. Obviously, it is not a tree but a cycle. Second, the bound $\frac{N^2}{4}$ in [15] seems to be too large for small size multicast sessions, e.g., a multicast session with a source and only two destinations.

For the above reasons, in this paper we give a more accurate bound for wavelength channel cost of multicast light-trees. It is valid for most of the multicast routing algorithms under sparse splitting constraint, even if the IP layer of the set of multicast light-trees does not retain the tree structure (e.g, the iterative multicast routing algorithms as Member-Only [6]). Costly and complex wavelength converters are supposed to be unavailable, and an equal cost of 1 *unit hop-count cost* is assumed over all the fiber links in the network. We prove that the total cost of a multicast session is upper bounded to (1) $K(N-K)$, when $K < \frac{N}{2}$; (2) $\lfloor \frac{N^2}{4} \rfloor$, when $K \geq \frac{N}{2}$, where K is the number of destinations in the multicast session and N is the number of nodes in the network. Besides, the wavelength channel cost is lower limited to K. Moreover, in unweighted WDM rings the optimal multicast light-tree has a total cost inferior to $N - \lceil \frac{N}{K+1} \rceil$.

Solving the Steiner problem, the Shortest Path Tree algorithm approximates the optimal solution with a ratio of K, which is the number of destinations to be covered. A better heuristic algorithm named Minimum Path Heuristic [9] guarantees the result cost with a ratio of $2(1 - \frac{1}{K+1})$ [10]. Solving the multicast routing problem in sparse splitting WDM networks, the Reroute-to-Source (R2S) and Member-Only (MO) algorithms are proposed in [6]. These two heuristics are the variant algorithms of the Shortest Path Tree and Minimum Path Heuristic in WDM networks. Will they retain the same approximation ratios as for solving the Steiner problem? We investigate their approximation ratios in both equally weighted and non-equally weighted WDM networks. Reroute-to-Source algorithm (R2S) [6] achieves an approximation ratio $\rho(R2S)$ equal to K in non-equally weighted WDM networks, while in equally weighted WDM networks $\rho(R2S)$ is inferior to (1) K, when $1 \leq K < \frac{N}{2}$; (2) $\frac{N^2}{4K}$, when $\frac{N}{2} \leq K < N$. Member-Only algorithm (MO) [6] approaches the optimal solution with a ratio $\rho(MO)$ inferior to $(K^2+3K)/4$ for any WDM networks. More specially in equally weighted WDM networks, $\rho(MO)$ is no bigger than (1) $(K^2+3K)/4$, when $1 \leq K < \frac{\sqrt{16N+49}-7}{2}$; (2) $N-K$, when $\frac{\sqrt{16N+49}-7}{2} \leq K < \frac{N}{2}$; (3) $\frac{\lfloor \frac{N^2}{4} \rfloor}{K}$, when $\frac{N}{2} \leq K < N$.

Moreover, cost bounds and approximation ratios of multicast light-trees in some candidate all-optical backbone networks are examined through simulations. Integer Linear Programming (ILP) formulations are proposed to find the optimal multicast light-trees. Member-Only and Reroute-to-Source [6] algorithms are also implemented in the simulation.

The rest of this paper is organized as follows. System model is given and the multicast routing problem is formulated in Section II. Then the cost bound of multicast light-trees in WDM mesh network is discussed in Section III. After that, the cost bound of multicast light-trees in WDM rings is investigated in Section IV. Furthermore, the approximation ratios of two classical multicast routing algorithms are derived in Section V. To search the optimal solution for sparse splitting multicast routing, the ILP formulations are introduced in Section VI-A. The proposed cost bounds and approximation ratios are evaluated in Section VII by extensive simulations. Finally, we conclude the paper in Section VIII.

II. Multicast Routing with Sparse Splitting

A. *Multicast Routing Problem*

Multicast routing involves a source and a set of destinations. In sparse splitting WDM networks, a set of light-trees is employed to distribute messages from the source to all the group members simultaneously. The objective of studying multicast routing in WDM networks is to minimize the wavelength channel cost while fulfilling a multicast session. The computation of light-trees for a multicast session generally has the following principles.

1) Due to sparse splitting and absence of wavelength conversion, in a light-tree, the degree of an MI node cannot exceed two. In consequence some destinations cannot be included in the same light-tree. Thus, several light-trees on different wavelengths may be required for one multicast session.
2) Among the light-trees built for a multicast session, one destination may be spanned (used to forward the incoming light beam to other destination nodes) by several light-trees, but it should be served (used to receive messages from the source) by only one light-tree. (e.g., d_3 in Fig. 1

is spanned by both LT_1 and LT_2 to forward the incoming light beam to d_2 and d_1 respectively. Thus, it must tap the light beam only once for recovering multicast messages either in LT_1 or in LT_2).
3) Since the number of wavelengths supported per fiber link is limited, the maximum number of wavelengths required and the traffic congestion in a fiber link should be taken into account during the selection of multicast light-trees. Thus, if a set of destinations D have been spanned by a light-tree LT_1, $D \subseteq LT_1$, it is entirely useless to construct another light-tree LT_2 to serve and only serve the destinations in subset D_i, with $D_i \subseteq D$. This is because that destinations in D_i could be served directly in LT_1. For instance, three light-trees LT_1, LT_2 and LT_3 are computed to serve d_1, d_2, d_3 respectively, where LT_1 only contains d_1, d_2, LT_2 only contains d_2, d_3 and LT_3 only contains d_3, d_1. However, LT_3, for instance, should be eliminated since d_3 is spanned in LT_2 and can be served directly in LT_2 instead of using the tree LT_3.

B. System Model

A sparse splitting WDM network can be modeled by an undirected graph $G(V, E, c)$. V represents the vertex-set of G, $|V| = N$. Each node $v \in V$ is either an MI or an MC node. E represents the edge-set of G, which corresponds to the fiber links between the nodes in the network. Each edge $e \in E$ is consisted of two optical fibers for opposite direction communications. And e is associated with a cost function $c(e)$. Function c is additive over the links of a lightpath $LP(u, v)$ between two nodes u and v, i.e.,

$$c\big(LP(u,v)\big) = \sum_{e \in LP(u,v)} c(e) \tag{1}$$

We consider a multicast session $ms(s, D)$, which requests for setting up a light distribution structure (i.e., light-forest) under optical constraint (i.e., wavelength continuity, distinct wavelength, sparse splitting and lack of wavelength conversion constraints) from the source s to a group of destinations D. Let K be the number of destinations, $K = |D|$. Without loss of generality, it is assumed that k light-trees $LT_i(s, D_i)$ are required to span all the destinations involved in a multicast session $ms(s, D)$, where $i \in [1, k]$. It holds true that

$$1 \le k \le K \le N - 1 \tag{2}$$

Although the i^{th} light-tree $LT_i(s, D_i)$ may span some destinations already spanned in the previous light-trees, D_i is used to denote exclusively the set of newly served destinations in $LT_i(s, D_i)$. Since all the destinations in D are served by k light-trees and each destination should be served only once, we obtain

$$D = \bigcup_{i=1}^{k} D_i \tag{3}$$

These k sets of destinations D_i are disjoint, i.e.,

$$\forall i, j \in [1, k] \ and \ i \neq j, \ \ D_i \cap D_j = \emptyset \tag{4}$$

Let a positive integer $K_i = |D_i|$ denote the size of the subset D_i, then we have

$$\sum_{i=1}^{k} K_i = |D| = K \tag{5}$$

The total cost of a multicast session $ms(s, D)$ is defined as the wavelength channel cost of the light-trees built to serve all the destinations in set D. It can be calculated by

$$\begin{aligned} c\big(ms(s,D)\big) &= \sum_{i=1}^{k} c\big[LT_i(s, D_i)\big] \\ &= \sum_{i=1}^{k} \sum_{e \in LT_i(s,D_i)} c(e) \end{aligned} \tag{6}$$

III. Cost Bounds of Multicast Light-Trees in WDM Mesh Networks

In this section, we will study the cost bounds of light-trees in unweighted WDM networks with two different light splitting configurations: full light splitting and sparse splitting. Let $SR = N_{MC}/N$ be the ratio of MC nodes in the network. For the full light splitting case $SR = 1$, and for the sparse splitting case $0 \le SR < 1$. In addition, we only investigate the cost bounds in link equally-weighted WDM networks. It is assumed that all links have the same cost function

$$c(e) = 1 \ unit \ \text{hop-count-cost} \tag{7}$$

Thus,

$$c\big(ms(s,D)\big) = \sum_{i=1}^{k} \sum_{e \in LT_i(s,D_i)} 1 \tag{8}$$

A. Full Light Splitting WDM Networks

In the case that all network nodes are equipped with light splitters, each node could act as a branching node in a light-tree. Hence, one light-tree is sufficient to span all the multicast members. It is a Steiner-problem which tries to find a minimum partial spanning tree covering the source and all the multicast members. In a light-tree, there are at most N nodes when all the network nodes are spanned (i.e., when $\{v|v \in LT\} = V$), and at least $K+1$ nodes if and only if the light-tree just contains the source and the multicast members (i.e. when $\{v|v \in LT\}$ = $\{s\} \cup D$). So, the cost of the multicast light-tree is bounded to

$$K \le c\big(ms(s,D)\big) \le N - 1 \tag{9}$$

To minimize the total cost in full light splitting case, the Minimum Path heuristic [9] and the Distance Network heuristic [11] can be good choices, since they are guaranteed to get a light-tree with a total wavelength channel cost no more than $2(1-\frac{1}{K+1})$ times that of the optimal Steiner tree [10], [11]. i.e.,

$$c\big(ms(s,D)\big) \leq 2\big(1-\frac{1}{K+1}\big) \times C_{Opt} \qquad (10)$$

where C_{Opt} denotes the wavelength channel cost of the Steiner tree.

B. Sparse Splitting WDM Networks

In the case of sparse splitting, only a subset of nodes can act as branching nodes in a light-trees. One light-tree may not be sufficient to accommodate all the group members simultaneously. Generally, several light-trees should be employed.

Lemma 1: $\forall j \in [1,k]$, the cost of the j^{th} light-tree holds

$$K_j = |D_j| \leq c\big(LT_j(s,D_j)\big) \leq N-k \qquad (11)$$

Proof: According to Eq. (4), all the k subsets of destinations D_i, $i \in [1,k]$, are disjoint. Based on the third assumption in subsection II-A, at leat $k-1$ destinations are not included in a light-tree. The number of nodes in a light-tree is consequently no more than $N-(k-1)$. Furthermore, if no other nodes are included in the j^{th} light-tree except the source s and the destinations in D_j (i.e. $\{v|v \in LT_j(s,D_j)\} = \{s\} \cup D_j$), then the number of nodes in the j^{th} light-tree is minimal and equals K_j+1. Hence, the cost bounds of a light-tree can be obtained as

$$K_j \leq c\big(LT_j(s,D_j)\big) \leq N-k \qquad (12)$$

■

Theorem 1: In sparse splitting WDM networks, the total cost of the light-trees built for the multicast session $ms(s,D)$ satisfies

$$K \leq c\big(ms(s,D)\big) \leq \begin{cases} K(N-K), & K < \frac{N}{2} \\ \lfloor \frac{N^2}{4} \rfloor, & K \geq \frac{N}{2} \end{cases} \qquad (13)$$

Proof: According to $Lemma$ 1 and Eq. (6), the total cost of the light-trees built for a multicast session $ms(s,D)$ holds

$$\begin{aligned} c\big(ms(s,D)\big) &\leq \sum_{i=1}^{k}(N-k) \\ &\leq k(N-k) \\ &\leq -(k-\frac{N}{2})^2 + \frac{N^2}{4} \end{aligned} \qquad (14)$$

Regarding k is an integer and $1 \leq k \leq K$, we obtain

$$c\big(ms(s,D)\big) \leq \begin{cases} K(N-K), & K < \frac{N}{2} \\ \frac{N^2}{4}, & K \geq \frac{N}{2} \text{ and } N \text{ is even} \\ \frac{N^2-1}{4}, & K \geq \frac{N}{2} \text{ and } N \text{ is odd} \end{cases} \qquad (15)$$

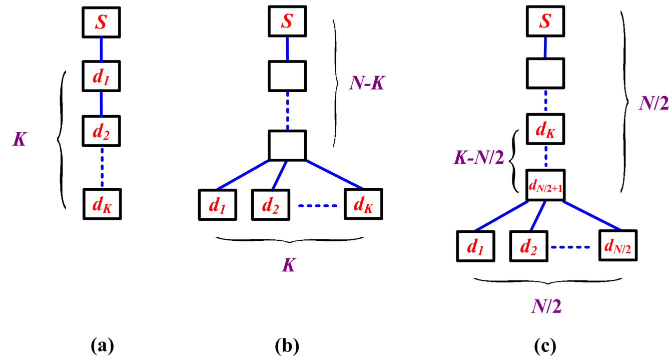


Fig. 2. (a) The best case; (b) The worst case when $K < \frac{N}{2}$; (c) The worst case when $K \geq \frac{N}{2}$

Moreover, according to $Lemma$ 1, it is also true that

$$c\big(ms(s,D)\big) \geq \sum_{i=1}^{k} K_i = K \qquad (16)$$

■

In fact the cost bounds given in $Theorem$ 1 are tight. In the following we give two examples to show their accuracy. It is not difficult to imagine that the case with the minimal cost appears when all and only all the destinations are involved in the light-tree computed for multicast session $ms(s,D)$, as shown in Fig. 2(a). That is to say $\{v|v \in LT\} = \{s\} \cup D$. It is obvious that the lower bound K is tight.

The worst case depends on the relationship between K and N. In case that $K < \frac{N}{2}$, the worst case may happen when the network topology is like that in Fig. 2(b), where K lightpaths on different wavelengths are needed to serve K destinations to the source. Here, it is observed that the cost of the optimal light-trees equals $K(N-K)$. When $K \geq \frac{N}{2}$, the worst case may take place in the topology of Fig. 2(c). In this topology, $\lfloor \frac{N}{2} \rfloor$ lightpaths from the source to each of the destinations at the bottom are required to serve all the group members. The $K - \lfloor \frac{N}{2} \rfloor$ destinations in the middle can be served in any one of them. As each lightpath has a cost of $\lceil \frac{N}{2} \rceil$, an exact total cost of $\lfloor \frac{N^2}{4} \rfloor$ should be consumed to establish the multicast session $ms(s,D)$. This example verifies the accuracy of the upper bound given in $Theorem$ 1.

IV. Cost Bound of Multicast Light-Trees in Unweighted WDM Rings

A. Multicast Light-tree in unweighted WDM Rings

In WDM rings, all the nodes are mandatorily equipped with TaC [7] capability, one light-tree is able to span all the multicast members. The multicast light-tree in a WDM ring consists of either a lightpath or two edge disjoint lightpaths originating from the same source. In an N-node WDM ring, the cost of the

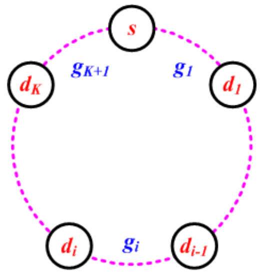


Fig. 3. The gaps in a WDM ring

multicast light-tree for multicast session $ms(s, D)$ is subject to

$$K \le c\big(ms(s, D)\big) \le N - 1 \tag{17}$$

B. Optimal Multicast Light-tree in unweighted WDM Rings

Different from WDM mesh networks, minimizing the cost of the multicast light-tree in a WDM ring is very simple. The minimum spanning tree for the multicast members is the optimal solution. Here, we use the concept gap introduced in [16], [17]. A gap is a path between two adjacent multicast members in $\{s\} \cup D$ so that no other members are involved in this path. The optimal multicast light-tree can be obtained by removing the biggest gap from the ring [16].

Theorem 2: In a WDM ring, the cost of the optimal light-tree for multicast session $ms(s, D)$ complies

$$K \le c\big(ms(s, D)\big) \le N - \lceil \frac{N}{K+1} \rceil \tag{18}$$

Proof: Beginning from the source node s, we index the destination nodes from d_1 to d_K in a clockwise manner. Let g_1 denote the length of the gap between the source s and d_1, g_i be the length of the i^{th} gap, i.e., the gap between d_{i-1} and d_i, and g_{K+1} be the gap between source s and d_K as shown in Fig. 3. In a WDM ring of N nodes, we obtain

$$\sum_{i=1}^{K+1} g_i = N \tag{19}$$

The cost of the optimal multicast light-tree for multicast session $ms(s, D)$ can be determined by

$$c\big(ms(s, D)\big) = N - \max_{1 \le i \le K+1} g_i \tag{20}$$

In order to obtain the cost bound of the light-tree, we have to determine the value range of $\max_{1 \le i \le K+1} g_i$. Note that all g_i are positive integers and satisfy Eq. (19). We obtain the following inequality

$$\max_{1 \le i \le K+1} g_i \ge \lceil \frac{N}{K+1} \rceil \tag{21}$$

This result corresponds to the case that multicast members are evenly distributed in a WDM ring. Thus we obtain

$$c\big(ms(s, D)\big) \le N - \lceil \frac{N}{K+1} \rceil \tag{22}$$

Besides, if all the multicast group members stick together one by one, the optimal light-tree thus only consists of the source and the destinations. Then, we can obtain the lower bound

$$c\big(ms(s, D)\big) \ge K. \tag{23}$$

■

V. Approximation Ratios of the Heuristic Algorithms for Sparse Splitting Multicast Routing

Like the Steiner problem, it is NP-hard to find the light-trees with the optimal cost for multicast routing in sparse splitting WDM networks. This is why many heuristic algorithms have been proposed to solve this problem in polynomial time. In order to guarantee the quality of the resultant light-trees, it is imperative to determine the cost approximation ratios of the proposed heuristic solutions. The approximation ratio $\rho(H)$ of a heuristic algorithm H in WDM network G can be defined as follows: for any possible multicast session $ms(s, D)$ in G, let $c(H)$ be the total cost of the multicast light-forest computed by H and let C_{Opt} be the total cost of the optimal solution (the solution with the minimized cost), $\rho(H)$ is the tight upper bound of the equation below

$$1 \le \frac{c(H)}{C_{Opt}} \le \rho(H), \ \forall ms(s, D) \ in \ G \tag{24}$$

Nevertheless, the approximation ratios of heuristic algorithms have not been investigated before. In this section, we try to deduce the approximation ratios of two classical light-trees computation heuristics namely Reroute-to-Source (R2S) and Member-Only (MO) [6]. Define C_{Opt} as the optimal cost of the light-trees fulfilling the multicast session $ms(s, D)$, and let $\rho(\cdot)$ denote the cost approximation ratio of a heuristic solution. Specially, we discuss the approximation ratios of these algorithms in two types of WDM networks $G(V, E)$: non-equally-weighted one and unweighted one. In the first case, the link cost can be an arbitrary positive number. While in the latter case, all the link costs are set to be $1 \ unit$ hop-count-cost as shown in Eq. (7). At first, we study the approximation ratios in unweitghted WDM networks and some special network topologies.

Theorem 3: Given that the WDM network $G(V, E)$ is unweighted, if an all-optical multicast routing algorithm $AOMR$ follows the assumptions in II-A then its approximation ratio holds

$$\rho(AOMR) \le \begin{cases} N-K & 1 \le K < \frac{N}{2} \\ \frac{\lfloor \frac{N^2}{4} \rfloor}{K} & \frac{N}{2} \le K \le N \end{cases} \tag{25}$$

Proof: If $G(V, E)$ is unweighted i.e., Eq. (7) is valid. As demonstrated in subsection III-B, the light-forest computed by the multicast routing algorithm following the assumptions in II-A has both a lower bound and an upper bound. Obviously, the optimal cost of light-forest should also be no less than the lower bound. Hence, the approximation ratio of the algorithm can not be greater than the value of the upper bound divided by the lower bound. According to $Theorem$ 1, we obtain $Theorem$ 3. It is obviously also valid for both Reroute-to-Source and Member-Only algorithms, since they respect the sparse splitting constraint and follow the aforementioned assumptions. ■

It should be also noted that any heuristic algorithm is capable of finding the cost-optimal light-forest in some special topologies, for instance the tree network, the line network, and ect.

Lemma 2: Given the WDM network G in which there is one and only one path between each pair of nodes, the approximation ratios of any heuristic algorithms are equal to 1.

Proof: As there is only one path between each pair of nodes, any solution will find the identical light-forest to realize a multicast session. ■

A. Reroute-to-Source Algorithm

Reroute-to-Source algorithm constructs the shortest path tree rooted at the source, then it checks the splitting capacity of the branching nodes. If a branching node is an MI node, the algorithm cuts all but one downstream branch. The affected leaf destinations rejoin the light-tree along a shortest path to the source on another wavelength.

Theorem 4: Given that the WDM network $G(V, E)$ is non-equally-weighted, the Reroute-to-Source algorithm [6] provides an approximation ratio of $\rho(R2S) = K$ for multicast routing with sparse splitting constraint.

Proof: Let r_{max} be the cost of the shortest path from the furthest destination to the source s, i.e.

$$r_{max} = \max_{d_i \in D} c\big[SP(s, d_i)\big] \tag{26}$$

Obviously, we have

$$C_{Opt} \geq r_{max} \tag{27}$$

Hence, we can obtain

$$\begin{aligned} \rho(R2S) &= c(R2S)/C_{Opt} \\ &\leq \sum_{d_i \in D} c\big(SP(s, d_i)\big)/C_{Opt} \\ &\leq |D| \cdot r_{max}/r_{max} \\ &\leq K \end{aligned} \tag{28}$$

Next, we will show that $\rho(R2S)$ may tend to be K in a non-equally-weighted topology like Fig. 4, where r is a positive integer denoting the distance from s to d_1 and δ is a very small non-negative number. We can see the optimal solution for multicast communication $ms(s, d_1 - d_K)$ is the lightpath $s \rightarrow d_1 \rightarrow d_2 ... \rightarrow d_K$, while the shortest path tree is the set of direct paths from s to each destination. Then,

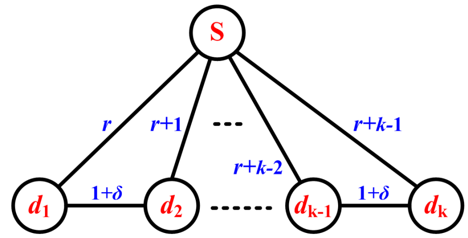


Fig. 4. Illustration of $Theorem$ 4

$$c(R2S) = K\big(r + \frac{K-1}{2}\big) \tag{29}$$

$$C_{Opt} = r + (K-1)(1+\delta) \tag{30}$$

Thus, the approximation ratio of R2S algorithm is

$$\rho(R2S) = K\left(1 - \frac{1}{\frac{2r}{(K-1)(1+2\delta)} + \frac{2(1+\delta)}{1+2\delta}}\right) \tag{31}$$

Since $G(V, E)$ is non-equally-weighted and K is inferior to N, r can be arbitrarily large and independent of K and N. Thus, for any $K \in (1, N)$, when $\frac{r}{N} \rightarrow \infty$ and $\delta \rightarrow 0$, we obtain $\rho(R2S) = K$. ■

$Discussion$:
Obviously, $Theorem$ 4, i.e. $\rho(R2S) \leq K$ is true for both unweighted and non-equally-weighted networks G. However, it should be noticed that $\rho(R2S) = K$ is not valid for all possible $1 < K < N$ in unweighted WDM networks, especially when K is very close to N. Take the same example in Fig. 4, if G is unweighted, r is always below $N - K$ and $\delta = 0$, thus $\frac{r}{K} \leq \frac{N-K}{K}$ will never reach ∞ when K is close to N. As a result, Eq. (31) can not tend to K any more, and a better ratio should be found in this case.

Theorem 5: Given that WDM network $G(V, E)$ is unweighted,

$$\rho(R2S) \leq \begin{cases} K & 1 \leq K < \frac{N}{2} \\ \frac{\lfloor \frac{N^2}{4} \rfloor}{K} & \frac{N}{2} \leq K \leq N-1 \end{cases} \tag{32}$$

Proof: As proved in $Theorem$ 4 that $\rho(R2S) \leq K$ is always true for any WDM networks. In addition, $Theorem$ 3 is also valid for Reroute-to-Source algorithm in unweighted graphs. By combining these two results, the proof follows. ■

B. Member-Only Algorithm

According to Member-Only (MO) algorithm [6], the shortest path between each pair of nodes is precalculated and stored in a table. Then, the computation of the light-trees for a multicast request is done iteratively as shown in Algorithm 1.

MC_SET: includes source node, MC nodes and the leaf MI nodes. They may be used to span the light-tree LT and, thus are also called connector nodes in LT.

Algorithm 1 Member-Only Algorithm

Input: A graph $G(V, E, c, W)$ and a multicast session $ms(s, D_0)$.
Output: A set of Light-trees $LT_k(s, D_k)$ each on a different wavelength w_k for $ms(s, D_0)$.

1: $k \leftarrow 1$ {k is the serial number of a light-tree}
2: $D \leftarrow D_0$
3: **while** $(D \neq \emptyset)$ **do**
4: $LT_k \leftarrow \{s\}$, $MC\_SET \leftarrow \{s\}$, $MI\_SET \leftarrow \emptyset$
5: **for** $(d \in D$ and $c \in MC\_SET)$ **do**
6: Try to find the shortest path $SP(\mathbf{d}, \mathbf{c})$ which doest not involve any node in MI_SET.
7: **end for**
8: **if** Such a path $SP(d, c)$ is found **then**
9: $LT_k \leftarrow LT_k \cup SP(\mathbf{d}, \mathbf{c})$
10: $MC\_SET \leftarrow MC\_SET \cup$ {MC in $SP(\mathbf{d}, \mathbf{c})\} \cup \{\mathbf{d}\}$
11: $MI\_SET \leftarrow MI\_SET \cup$ {non-leaf MI in $SP(\mathbf{d}, \mathbf{c})\}$
12: **if** ($\mathbf{c}$ is an MI node) **then**
13: $MC\_SET \leftarrow MC\_SET \setminus \{\mathbf{c}\}$
14: $MI\_SET \leftarrow MI\_SET \cup \{\mathbf{c}\}$
15: **end if**
16: $D \leftarrow D \setminus \{d\}$
17: goto step 5
18: **else if** No such path could be found **then**
19: Assign wavelength w_k to LT_k
20: $k \leftarrow k + 1$ and goto step 4 to begin a new light-tree LT_{k+1}
21: **end if**
22: **end while**

MI_SET: includes only the non-leaf MI nodes, whose splitting capability is exhausted. Hence, these nodes are not able to connect a new destination to the subtree LT.

D: includes unserved multicast members which are neither joined to the current light-tree LT nor to the previously constructed multicast light-trees.

At each step $i + 1$, try to find the shortest paths between the destinations $d \in D$ and the connector nodes $c \in MC\_SET$ of light-tree LT_i, such that they do not involve any TaC capability exhausted nodes in MI_SET. Among them, the constraint-satisfying shortest path $SP(d, c)$ with the smallest cost is selected. Then generate LT_{i+1} by adding $SP(d, c)$ to LT_i. In case that no such destination can be found, begin a new light-tree rooted at the source. Member-Only algorithm is an adjustment of the famous Minimum Path Heuristic (MPH) proposed for the Steiner problem. As mention in Section III, MPH is able to approximate the Steiner tree with a ratio smaller than 2. However, by adjusting MPH for multicast routing under sparse splitting constraint (i.e., Member-Only algorithm), it is difficult to determine the approximation ratio. Next, we introduce *Lemma* 3 before determining $\rho(MO)$.

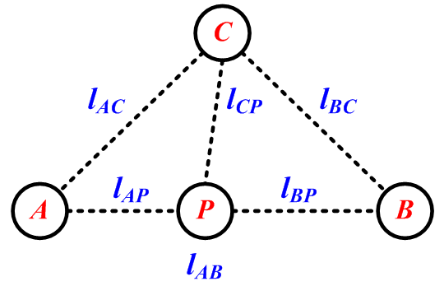


Fig. 5. Illustration of *Lemma* 3

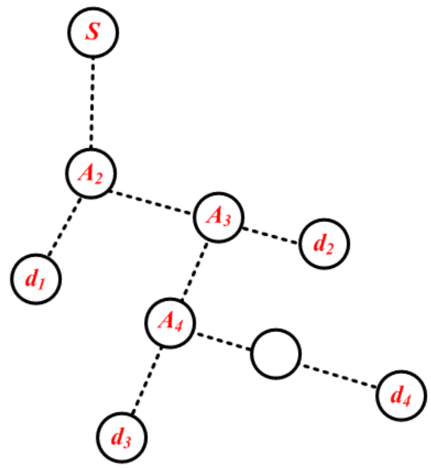


Fig. 6. Demonstration of the worst case of the Member-Only algorithm

Define l_{XY} as the cost of the shortest path $SP(X, Y)$.

Lemma 3: In Fig. 5, suppose P is a node in the shortest path $SP(A, B)$ from node A to node B, and C is connected to P by the shortest path. We obtain

$$l_{CP} \leq \frac{1}{2}(l_{AB} + l_{AC} + l_{BC}) \tag{33}$$

Proof: Since node P is in $SP(A, B)$, both paths AP and BP are the shortest paths, then

$$l_{AB} = l_{AP} + l_{BP} \tag{34}$$

As a result the graph in Fig. 5 is a distance network, where the triangle inequality is valid. Then,

$$l_{CP} \leq l_{AC} + l_{AP} \tag{35}$$

$$l_{CP} \leq l_{BC} + l_{BP} \tag{36}$$

Adding Eq. (35) to Eq. (36) gives

$$2l_{CP} \leq (l_{AP} + l_{BP}) + l_{AC} + l_{BC} \tag{37}$$

By substituting Eq. (34) into the above equation, *Lemma* 3 follows. ■

Theorem 6: Given any kind of WDM networks $G(V, E)$, the Member-Only algorithm provides a cost approximation ratio $\rho(MO) \leq \frac{K^2+3K}{4}$ for sparse splitting multicast routing.

Proof: We use the proof by induction. Let l_{max} be the cost of the shortest path between the furthest two members in a multicast session $ms(s,D)$, i.e.

$$l_{max} = \max_{m_i,m_j \in s\cup D} c\big[SP(m_i,m_j)\big] \tag{38}$$

Member-Only algorithm starts the multicast light-tree LT from the source s and spans the light-trees iteratively. Let l_i denote the cost of the shortest path that connects the destination d_i to the current LT, and l_i^m be its upper bound. In other words, the cost of LT increases by l_i after spanning d_i, and at most l_i^m. In the following, we are trying to determine the worst case of the upper bound l_i^m for each l_i by applying the triangle inequality in $Lemma$ 3. As shown in Fig. 6, the nearest destination node d_1 to the source s is first added to LT. Now, the cost of LT is $l_1 \leq l_{max}$ and $l_1^m = l_{max}$. Then in the second step, the nearest destination d_2 to LT is added using the shortest path. If d_2 is spanned via d_1 or s, then obviously $l_2 \leq l_{max}$. It should be noted that the worst case appears when d_2 is spanned via an intermediate node (say A_2) in $SP(s,d_1)$. If this happens to be the case, we obtain $l_2 \leq \frac{3}{2}l_{max}$ and $l_2^m = \frac{3}{2}l_{max}$ according to $Lemma$ 3. In the third step, the nearest destination d_3 is added using the shortest path. It is evident that l_3^m is the largest when d_3 is spanned via an intermediate node (say A_3) in $SP(A_2,d_2)$. This can be explained as follows. If d_3 is spanned via any member node (i.e., s, d_1 or d_2), then obviously $l_3 \leq l_{max}$. Otherwise, d_3 must be connected via an intermediate node in the shortest path $SP(s,d_1)$ or $SP(A_2,d_2)$. According to $Lemma$ 3, $l_3 \leq \frac{3}{2}l_{max}$ if d_3 connects to LT through a node in $SP(s,d_1)$. In case that d_3 connects to LT through a node in $SP(A_2,d_2)$, the cost of $SP(A_2,d_3)$ should be calculated before using the triangle inequality. Similar to $SP(A_2,d_2)$, $c\big[SP(A_2,d_3)\big] \leq l_2^m$. Then, go back to l_3, and we obtain:

$$\begin{aligned} l_3 &\leq \frac{1}{2}\Big(c\big[SP(A_2,d_3)\big] + c(SP(d_2,d_3)) + l_2\Big) \\ &\leq \frac{1}{2}(l_2^m + l_{max} + l_2) \\ &\leq l_2^m + \frac{1}{2}l_{max} \end{aligned} \tag{39}$$

Hence,

$$l_3^m = l_2^m + \frac{1}{2}l_{max} \tag{40}$$

Suppose that Eq. (41) is obtained by applying $Lemma$ 3

$$l_i^m = l_{i-1}^m + \frac{1}{2}l_{max} \tag{41}$$

Next, we try to prove that it is also true for the case of l_{i+1}^m. Since a Member-Only multicast light-tree is only consisted of the shortest paths, each node in the light-tree must be in the shortest path between two member nodes or between a destination and a joint node of two shortest paths. And, l_i^m is monotonically increasing. Consequently, the worst case of l_{i+1}^m occurs when d_{i+1} connects to LT through an intermediate node in the shortest path between d_i and a joint node A_i. According to $Lemma$ 3, $c\big[SP(A_i,d_{i+1})\big] \leq l_i^m$ also holds. Then, applying the triangle inequality again in the distance network of $G(A_i,d_i,d_j)$ leads to,

$$\begin{aligned} l_{i+1} &\leq \frac{1}{2}\Big(c\big[SP(A_i,d_{i+1})\big] + c(SP(d_i,d_{i+1})) + l_i\Big) \\ &= \frac{1}{2}(l_i^m + l_{max} + l_i) \\ &\leq l_i^m + \frac{1}{2}l_{max} \end{aligned} \tag{42}$$

So, it is always valid for all the steps during the span of a light-tree that $l_{i+1}^m = l_i^m + \frac{1}{2}l_{max}$. Hence, we have $l_i^m = \frac{i+1}{2}l_{max}$. Assuming k light-trees are constructed for multicast session $ms(s,D)$, and $|D_i|$ destinations are unique served in the i^{th} light-tree. This also means that $|D_i|$ steps are processed in the i^{th} light-tree. Thus, the total cost of the i^{th} light-tree is upper bounded by

$$\begin{aligned} c(LT_i) &= \sum_{i=1}^{|D_i|} l_i \\ &\leq \sum_{i=1}^{|D_i|} l_i^m \\ &\leq \frac{1}{4}\big(|D_i|^2 + 3|D_i|\big)l_{max} \end{aligned} \tag{43}$$

Then, the total cost consumed by $ms(s,D)$ using Member-Only algorithm is

$$\begin{aligned} c(MO) &= \sum_{i=1}^{k} c(LT_i) \\ &\leq \sum_{i=1}^{k} \frac{1}{4}(|D_i|^2 + 3|D_i|)l_{max} \\ &\leq \frac{1}{4}\big(3|D| + \sum_{i=1}^{k}|D_i|^2\big)l_{max} \\ &\leq \frac{1}{4}\big(3|D| + |D|^2\big)l_{max} \end{aligned} \tag{44}$$

As $C_{Opt} \geq l_{max}$, the following inequality can be obtained

$$\begin{aligned} \rho(MO) &= c(MO)/C_{Opt} \\ &\leq c(MO)/l_{max} \\ &\leq \frac{1}{4}\big(3K + K^2\big) \end{aligned} \tag{45}$$

■

Theorem 7: Given that the WDM network $G(V,E)$ is unweighted, then

$$\rho(MO) \leq \begin{cases} \frac{1}{4}(K^2+3K) & 1 \leq K < \frac{\sqrt{16N+49}-7}{2} \\ N-K & \frac{\sqrt{16N+49}-7}{2} \leq K < \frac{N}{2} \\ \frac{\lfloor \frac{N^2}{4} \rfloor}{K} & \frac{N}{2} \leq K \leq N-1 \end{cases} \tag{46}$$

Proof: If $G(V,E)$ is unweighted, $Theorem$ 3 is valid for the Member-Only algorithm. By merging two approximation ratios in $Theorems$ 3 and 7, the proof follows. ■

VI. ILP Formulation

Since minimizing the total cost of the light-forest for a multicast session is NP-hard, the integer linear programming (ILP) method is applied to search the optimal solution.

Notations and Variables:

W	: The wavelengths supported per fiber.
λ	: A wavelength $\lambda \in W$.
V	: The set of nodes.
$In(m)$	: The set of nodes leading an edge to node m.
$Out(m)$	: The set of nodes to which m is connected.
$Deg(m)$	: The degree of node m.
$link(m,n)$	: The directed link from node m to node n.
$L_{m,n}(\lambda)$	: Equals to 1 if multicast request $ms(s,D)$ uses wavelength λ on $link(m,n)$, equals to 0 otherwise.
$U^d_{m,n}(\lambda)$	: Equals to 1 if $link(m,n)$ is used on wavelength λ in the lightpath from d to the source s, equals to 0, otherwise.

A. ILP Formulation

The objective of the studied sparse splitting multicast routing problem is to minimize the wavelength channel cost of the light-trees built for a multicast session $ms(s,D)$. It can be formulated as follows:

$$Minimize: \sum_{\lambda \in W} \sum_{m \in V} \sum_{n \in In(m)} L_{n,m}(\lambda) \tag{47}$$

The objective function is subject to a set of constraints, which are listed below:

1) Multicast Light-tree Constraints: Source Constraints:

$$\sum_{\lambda \in W} \sum_{n \in In(s)} L_{n,s}(\lambda) = 0 \tag{48}$$

$$1 \leq \sum_{\lambda \in W} \sum_{n \in Out(s)} L_{s,n}(\lambda) \leq |D| \tag{49}$$

Constraints (48) and (49) ensure that the light-trees for multicast session $ms(s,D)$ are rooted at the source node s. In a light-tree, s must not have any input link, but should have at least one output link. And the number of outgoing links from s should not go beyond the number of sink nodes, i.e., $|D|$.

Destinations Constraints:

$$1 \leq \sum_{\lambda \in W} \sum_{n \in In(d)} L_{n,d}(\lambda) \leq |D|, \quad \forall d \in D \tag{50}$$

Constraint (50) guarantees that each destination node sinks at least one incoming light beam. Since some destinations, which act an intermediate node in a light-tree, will forward the incoming light beam to successor destinations, a destination node d can receive at most $|D|$ light beams on all the wavelength layers. However, this constraint cannot ensure that destination d is reachable from the source s, which will be illustrated later.

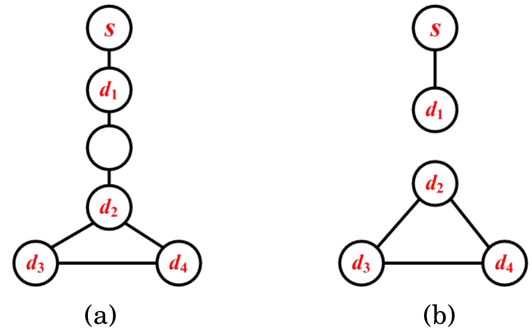


Fig. 7. A contradict example with a loop in the result light-tree: (a) The network topology; (b) The result

Input Constraint:

$$\sum_{n \in In(m)} L_{n,m}(\lambda) \leq 1, \quad \forall \lambda \in W, \text{ and } \forall m \in V \tag{51}$$

Equation (51) indicates that each node (except the source s) in a light-tree has and only has one predecessor. Nevertheless, this constraint can not guarantee that the resultant structure is a set of light-trees, due to the fact that loops can not be avoided (refer to Fig. 7).

Leaf Nodes Constraint:

$$\sum_{n \in Out(m)} L_{m,n}(\lambda) \geq \sum_{n \in In(m)} L_{n,m}(\lambda) \tag{52}$$
$$\forall \lambda \in W, \forall m \in V \text{ and } m \notin D$$

Constraint (52) ensures that only the destination nodes can be leaf nodes in a light-tree while the non-member nodes can not.

Sparse Splitting Constraints:

$$\sum_{n \in Out(m)} L_{m,n}(\lambda) \leq R \times \sum_{n \in In(m)} L_{n,m}(\lambda) \tag{53}$$
$$\forall \lambda \in W, \forall m \in V \text{ and } m \neq s$$

where

$$\begin{cases} R = 1, & \text{if } m \text{ is an MI node} \\ R = Deg(m) - 1, & \text{if } m \text{ is an MC node} \end{cases} \tag{54}$$

Constraint (53) together with constraint (52) indicates the splitting capabilities of the nodes. If a node m is spanned in a light-tree, then the number of outgoing links from m is equal to 1 for an MI node and less than $Deg(m) - 1$ for an MC node. Otherwise, it must be 0.

Only with the light-tree structure constraints developed above [18], [19], one can not guarantee that each light-tree of the resultant light-forest should be connected and loop free. An contradictory example is given next. Suppose we just employ the light-tree constraints formulation to find the light-trees for a multicast session $ms(s,(d_1 - d_4))$ in topology Fig. 7(a). The result in

Fig. 7(b) uses some wavelength λ_1, where $L_{s,d_1}(\lambda_1) = 1$, $L_{d_2,d_3}(\lambda_1) = 1$, $L_{d_3,d_4}(\lambda_1) = 1$, $L_{d_4,d_2}(\lambda_1) = 1$ and all the other variables $L_{m,n}(\lambda)$ are zero. It is true that all the constraints from (48) to (53) are satisfied in this result. Besides, the wavelength channel cost of the result is optimal. Unfortunately, this result has a loop $d_2 - d_3 - d_4 - d_2$ and three destinations are separated from the source node s. Thereby, the proposed light-trees constraints are not sufficient to guarantee the resultant light-tree structure. This is why next the destinations reachability constraints are introduced to solve these problems.

2) Destination Nodes Reachability Constraints:
Source node:

$$\sum_{n \in In(s)} U^d_{n,s}(\lambda) = 0, \quad \forall \lambda \in W, \text{ and } \forall d \in D \tag{55}$$

$$1 \leq \sum_{\lambda \in W} \sum_{n \in Out(s)} U^d_{s,n}(\lambda) \leq |D|, \quad \forall d \in D \tag{56}$$

Similar to constraint (48), Eq. (55) gives the constraint that no link leading to the source will be employed to serve destinations in the light-trees.

Equation (56) ensures that all the destination nodes could be reached from the source node s in the light-trees. By combining Eqs. (51) and (56), the loops can be avoided. Still refer to the contradictory example aforementioned, the result in Fig.7(b) does not satisfy constraint (56), since destination nodes $d_2 - d_4$ can not be reached from the source node s.

Destination nodes autocorrelation:

$$\sum_{n \in Out(d)} U^d_{d,n}(\lambda) = 0, \quad \forall \lambda \in W, \text{ and } \forall d \in D \tag{57}$$

$$\sum_{n \in In(d)} U^d_{n,d}(\lambda) \leq 1, \quad \forall \lambda \in W, \text{ and } \forall d \in D \tag{58}$$

$$1 \leq \sum_{\lambda \in W} \sum_{n \in In(d)} U^d_{n,d}(\lambda) \leq |D| - 1, \quad \forall d \in D \tag{59}$$

Constraint (57) avoids the loops of destinations, such as that in Fig.7(b). Constraints (58) and (59) make sure that each destination has one and only one input link in a light-tree, which are equivalent to constraints (51) and (50) respectively.

Non-member nodes and destination nodes cross correlation:

$$\sum_{n \in Out(m)} U^d_{m,n}(\lambda) = \sum_{n \in In(m)} U^d_{n,m}(\lambda) \leq 1 \tag{60}$$
$$\forall \lambda \in W, \forall d \in D, \forall m \in V \text{ and } m \neq s, d$$

$$\sum_{\lambda \in W} \sum_{n \in Out(m)} U^d_{m,n}(\lambda) \leq |D| \tag{61}$$
$$\forall \lambda \in W, \forall d \in D, \forall m \in V \text{ and } m \neq s, d$$

The distinct wavelength constraint is illustrated by Eq. (60). It ensures that one link can be used at most once on one wavelength, and will be used at most $|D|$ times to establish multicast session $ms(s, D)$ on all the wavelengths which is expressed by Eq. (61).

3) Relationship between $L_{m,n}(\lambda)$ and $U^d_{m,n}(\lambda)$: In order to avoid loops in the resultant light-trees, variable $U^d_{m,n}(\lambda)$ is employed to restrict variable $L_{m,n}(\lambda)$. Their relations are shown in Eqs. (62) and (63).

$$L_{m,n}(\lambda) \leq \sum_{d \in D} U^d_{m,n}(\lambda), \forall \lambda \in W, \text{ and } \forall m, n \in V \tag{62}$$

$$U^d_{m,n}(\lambda) \leq L_{m,n}(\lambda), \forall \lambda \in W, \forall m, n \in V, \text{ and } \forall d \in D \tag{63}$$

VII. Simulation and Numerical Results

In this section, simulations are conducted to compute the multicast light-trees in sparse splitting WDM mesh networks. ILP formulations are implemented by Cplex [20], while Member-Only and Reroute-to-Source are conducted in C++ with LEDA package [21]. Since the proposed cost bounds and the approximation ratios of Member-Only and Reroute-to-Source algorithms only correspond to the worst or extreme cases, they may only appear in special topologies with special configurations. Hence, here we do not mean to verify the accuracy of the proposed bounds and approximation ratios. Instead the numerical results are obtained to just show the quality of the resultant light-trees when applying the Member-Only and Reroute-to-Source algorithms in some popular candidate WDM backbone networks like 14 nodes NSF network and 28 nodes USA Longhaul network.

A. Cost Bounds of Multicast Light-trees

Member-Only (MO) and Reroute-to-Source (R2S) algorithms are conducted in unweighted NSF network and unweighted USA Longhaul network. All the links are associated an identical cost of 1 $hop - count\ cost$. Since the worst case of the cost bound occurs when there is no light splitters in the network, we configure the network without light splitters. The source and multicast members are assumed to be distributed uniformly over the topology. The cost bounds of the multicast light-trees computed by MO and R2S heuristics are demonstrated in Fig. 8 when the multicast group size (counting the source node) $K + 1$ varies from 2 (Unicast) to the nodes number of the network (Broadcast). 5000 multicast sessions are randomly generated for a given multicast group size, meanwhile, Member-Only and Reroute-to-Source algorithms are employed to compute the multicast light-forest for each session. Among 5000 light-forests, the biggest cost of the light-forests (denoted by R2S-Max and MO-Max) and smallest cost of the light-forests (denoted by R2S-Min and MO-Min) are figured out and plotted in Fig. 8. The lower bound and the upper bound provided in $Theorem$ 1 are compared with the simulation result. According to the figure, it is observed that the proposed lower bound is covered by MO-Min since they are almost the same. The lower bound is also very near to R2S-Min. Meanwhile, we can also find that the upper

TABLE I
COMPARISON OF COST BOUNDS IN NSF NETWORK

$\|D\| = K$	LB	ILP	MO	R2S	UB
2	2	3.2	3.2	3.6	24
3	3	4.5	4.6	5.2	33
4	4	5.7	5.7	6.7	40
5	5	6.7	6.9	8.2	45
6	6	8.2	8.5	9.1	48
7	7	8.3	8.5	10.9	49
8	8	8.7	9.3	11.7	49
9	9	9.6	10.1	12.3	49
10	10	10.8	11.1	15	49
11	11	11.3	11.7	17.3	48
12	12	12	12	17.3	49
13	13	13	13.1	18.9	49

TABLE II
COMPARISON OF APPROXIMATION RATIOS IN NSF NETWORK

$\|D\| = K$	ρ'(MO)	ρ(MO)	ρ' (R2S)	ρ(R2S)
2	2.50	1.00	2	1.13
3	4.5	1.03	3	1.16
4	7	1.00	4	1.18
5	9	1.03	5	1.23
6	8	1.04	6	1.11
7	7	1.03	7	1.32
8	6.13	1.07	6.13	1.35
9	5.44	1.06	5.44	1.29
10	4.9	1.03	4.9	1.39
11	4.45	1.04	4.45	1.54
12	4.08	1.00	4.08	1.45
13	3.77	1.01	3.77	1.46

bound is much bigger than the biggest costs obtained (MO-Max and R2S-Max) by the simulation. This can be explained by the fact thatthe simulation results depend on the simulation topology. The proposed upper bound is valid for all the algorithms which complies the three rules mentioned in section II. As discussed in subsection III-B, given the network topology in Fig. 2, both the lower bound and the upper bound are always tight.

B. Approximation Ratio of Multicast Light-trees

ILP formulations are carried out in C++ with Cplex library in the NSF network to search for the optimal light-trees for each multicast session. We set NSF network to be an equally weighted graph, where each link has the same cost of $1\ hop-count\ cost$. Provided a multicast group size, 20 random sessions are generated. Hence, each cost is the average of 20 sessions with the same group size. The cost bounds (**LB** and **UB**) and the approximation ratios of the Reroute-to-Source and Member-Only algorithms are compared in tables I and II. ρ'(MO) denotes the upper bound of the approximation ratio given in $Theorem$ 6 and ρ'(R2S) stands for the upper bound of the approximation ratio derived from $Theorem$ 5, while ρ(MO) and ρ(R2S) indicate the approximation ratios obtained by $c(MO)/c(ILP)$ and $c(R2S)/c(ILP)$ respectively in the simulations. In addition, |D| $= K$ is the number of destinations in the session. As shown in table I, Member-Only algorithm achieves a very near cost to the result of ILP solution. In table II, it is observed that Member-Only algorithm has a better approximation ratio than Reroute-to-Source algorithm in the simulation. However, the approximation ratio gotten from the simulations is much smaller than that derived from the proof. This result can be explained as follows. First, the approximation ratio derived from the proof is the ratio of the worst case. Second, similar to the cost bound, the approximation ratio depends also on the network topology. Finally, the approximation ratios given in $Theorems$ 4 and 6 are not tight enough.

TABLE III
NEW APPROXIMATION RATIOS OF R2S AND MO IN NSF NETWORK

$\|D\| = K$	2	3	4	5	6	8	9	10	11	12	13
$\overline{\rho}$(MO)	2.5	3	3	3	3	3	3	3	3	3	3
$\overline{\rho}$(R2S)	2	3	3	3	3	3	3	3	3	3	3

In fact, another important impact is the characteristic of unweighted NSF network, which plays an important role in helping Member-Only and Reroute-to-Source to get good performances. This can be explained by the following $Lemma$ 4.

Lemma 4: Given that the WDM network G is unweighted, the approximation ratios of both Member-Only and Reroute-to-Source are inferior to the diameter of network $Diam(G)$.

Proof: It is trivial. Any shortest path $SP_G(\cdot)$ in the network G is always $SP_G(\cdot) \leq Diam(G)$. Both Reroute-to-Source and Member-Only algorithm exclusively make use of the shortest path in the network. Thus, the total cost $c(LF)$ of the resultant light-forest is

$$c(LF) \leq K \times Diam(G) \tag{64}$$

Besides, there are K destinations in session $ms(s, D)$ and G is unweighted, the optimal cost of multicast light-trees is always no less than K. Thus,

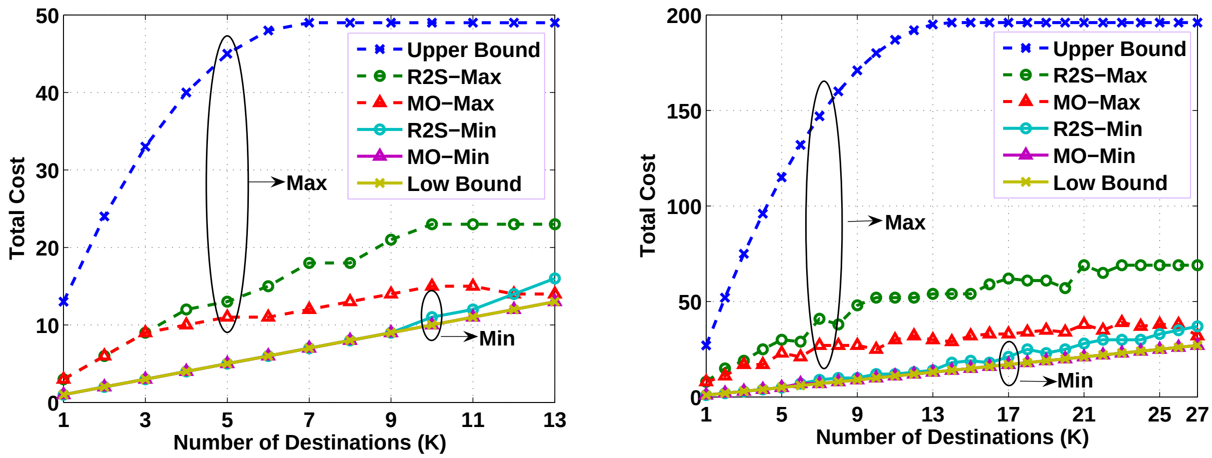
$$\rho(LF) \leq K \times Diam(G)/K = Diam(G) \tag{65}$$

■

It is not hard to find that the diameter of the unweighted NSF network is $Diam(NSF)$ =3. By taking $Theorems$ 6, 4 and $Lemma$ 4 into consideration concurrently, pretty better approximation ratios $\overline{\rho}(MO)$ and $\overline{\rho}(R2S)$ can be found in Table III.

VIII. CONCLUSION

Multicast routing in all-optical WDM mesh networks is an important but challenging problem. It is NP-complete to minimize the wavelength channel cost consumed per multicast session under the sparse splitting constraint. Although many papers have focused on the algorithms of multicast light-trees computation, neither the cost bounds of light-trees nor the approximation ratios of heuristic algorithms have been addressed. In this paper, we first investigate the bounds



(a) NSF Network (b) Longhaul Network

Fig. 8. The Cost Bound of multicast light-trees when the number of destinations K varies

of wavelength channel cost consumed by a multicast session in equally weighted WDM networks, where an equal cost of 1 *unit hop-count cost* is associated over all the fiber links. We find that it is tightly lower limited to the number of destinations K, and strictly upper bounded to (1) $K(N-K)$ when $K < \frac{N}{2}$; (2) $\lfloor \frac{N^2}{4} \rfloor$, when $K \geq \frac{N}{2}$, where K is the number of destinations in the multicast session and N is the number of nodes in the network. Source-oriented multicast light-trees computation heuristic algorithms like Reroute-to-Source [6] and Member-Only [6] follow this cost bounds, as they respect the three principles for light-trees computation mentioned in Section II. In a particular situation, where the network topology is a WDM ring, the optimal multicast light-tree can be determined by removing the biggest gap from the ring. We find that its cost is inferior to $N - \lceil \frac{N}{K+1} \rceil$.

Furthermore, some interesting results are found on the approximation ratios of some classical multicast light-trees computation algorithms in both unweighted and non-equally-weighted WDM networks. Reroute-to-Source algorithm (R2S) [6] achieves an approximation ratio $\rho(R2S)$ equal to K in non-equally-weighted WDM networks, while in unweighted WDM networks $\rho(R2S)$ is inferior to (1) K, when $1 \leq K < \frac{N}{2}$; (2) $\frac{\lfloor \frac{N^2}{4} \rfloor}{K}$, when $\frac{N}{2} \leq K < N$. Member-Only algorithm (MO) [6] approaches the optimal solution with a ratio $\rho(MO)$ inferior to $(K^2+3K)/4$ for any WDM networks. More specially in unweighted WDM networks, $\rho(MO)$ is no bigger than (1) $(K^2+3K)/4$, when $1 \leq K < \frac{\sqrt{16N+49}-7}{2}$; (2) $N-K$, when $\frac{\sqrt{16N+49}-7}{2} \leq K < \frac{N}{2}$; (3) $\frac{\lfloor \frac{N^2}{4} \rfloor}{K}$, when $\frac{N}{2} \leq K < N$. It is also reported that if WDM network is unweighted, the approximation ratios of R2S and MO are always inferior to the diameter of the network.

Simulation results illustrate that in popular candidate WDM backbone network topologies, the cost bounds and the approximation ratios of Member-Only and Reroute-to-Source heuristics are far from the worst case ones. This is due to the fact that unweighted NSF network has a very small diameter of three. In addition, the Member-Only algorithm achieves better cost than the Reroute-to-Source algorithm.

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