Corrigendum to ”Min-domain retroactive ordering for Asynchronous Backtracking”

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Abstract

The asynchronous backtracking algorithm with dynamic ordering (ABT_DO), proposed in [ZM06], allows changing the order of agents during distributed asynchronous complete search. In a later study [ZZA09], retroactive heuristics which allowed more flexibility in the selection of new orders were introduced, resulting in the ABT_DO-Retro algorithm, and a relation between the success of heuristics and the min-domain property was identified. Unfortunately, the description of the time-stampping protocol used to compare orders in ABT_DO-Retro in [ZZA09] is confusing and may lead to an implementation in which ABT_DO-Retro may not terminate. In this corrigendum, we demonstrate the possible undesired outcome and give a detailed and formal description of the correct method for comparing time-stamps in ABT_DO-Retro.

1 Introduction

The ABT_DO algorithm allows the use of dynamic ordering heuristics in asynchronous search algorithms for solving DisCSPs. This algorithm was the main theme of two recent publications in the *Constraints* journal, [ZM06] and [ZZA09]. The algorithm proposed in the first among them ([ZM06]) allows the use of heuristics where agents can propose order changes when they replace the value assignment to their variables. This change can include only agents that are ordered after (with lower priority) the agent that replaced its assignment. In the second ([ZZA09]), more flexible heuristics that allow changing the order of agents that come before the agent that replaces its assignment (retroactive heuristics) are introduced to the algorithm.

The most successful ordering heuristic found in [ZM06] was the *nogood-triggered* heuristic in which an agent that receives a nogood moves the nogood generator to be right after it in the order. The heuristic investigation in [ZZA09] demonstrates the relation between the success of the nogood-triggered heuristic and the min-domain property. This relation was exploited in the retroactive version of this heuristic by moving a nogood generator to the highest position in the order that does not causes values previously removed to be reentered into the variable's current domain (and increase its size).

Recent attempts to implement the ABT_DO-Retro algorithm proposed in [ZZA09] have revealed a specific detail of the algorithm that was vaguely described and can lead to an interpretation that affects the correctness of the algorithm. In this corrigendum we address this vague description by describing the undesired outcome and propose an alternative deterministic description that ensures the outcome expected in [ZZA09].

2 Background

The degree of flexibility of the retroactive heuristics mentioned above depends on a parameter K. K defines the level of flexibility of the heuristic with respect to the amount of information an agent can store in its memory. Agents that detect a dead end move themselves to a higher priority position in the order. If the length of the nogood created is not larger than K then the agent can move to any position it desires (even to the highest priority position) and all agents that are included in the nogood are required to add the nogood to their set of constraints and hold it until the algorithm terminates. If the size of the created nogood is larger than K, the agent that created the nogood can move up to the place that is right after the second last agent in the nogood. Since agents must store nogoods that are smaller than or equal to K, the space complexity of agents is exponential in K.

The best retroactive heuristic introduced in [ZZA09] is called ABT_DO-Retro-MinDom. This heuristic does not require any additional storage (i.e., $K = 0$). In this heuristic, the agent that generates a nogood is placed in the new order between the last and the second last agents in the generated nogood. However, the generator of the nogood moves to a higher priority position than the backtracking target (the agent the nogood was sent to) only if its domain is smaller than that of the agents it passes on the way up. Otherwise, the generator of the nogood is placed right after the last agent with a smaller domain between the last and the second last agents in the nogood.

In asynchronous backtracking algorithms with dynamic ordering, agents propose new orders asynchronously. Hence, one must enable agents to coherently decide which of two different orders is more up-to-date. To this end, as it has been explained in [ZM06] and recalled in [ZZA09], each agent in ABT_DO holds a *counter vector* (one counter attached to each position in the order). The counter vector and the indexes of the agents currently in these positions form a time-stamp. Initially, all counters are set to zero and all agents are aware of the initial order. Each agent that proposes a new order increments the counter attached to its position in the current order and sets to zero counters of all lower priority positions (the counters of higher priority positions are not modified). The most up-to-date order is determined by a lexicographic comparison of counter vectors combined with the agent indexes. However, the rules for reordering agents in ABT_DO imply that the most up-to-date order is always the one for which the first different counter is larger.

Regarding the procedure by which orders are compared in ABT_DO-Retro, the description given by the authors was vague and was limited to two sentences: *"The most relevant order is determined lexicographically. Ties which could not have been generated in standard ABT_DO, are broken using the agents indexes"*[quoted from [ZZA09], page 190, Theorem 1].

The natural understanding of this description is that the most up-to-date order is the one associated with the lexicographically greater counter vector, and when the counter vectors are equal the lexicographic order on the indexes of agents breaks the tie by preferring the one with smaller vector of indexes. We will refer to this interpretation as method m_1. Let us illustrate method m_1 via an example. Consider two orders o_1=$[A_1, A_3, A_2, A_4, A_5]$ and o_2=$[A_1, A_2, A_3, A_4, A_5]$ where the counter vector associated with o_1 equals $[2, 4, 2, 2, 0]$ and the counter vector associated with o_2 equals $[2, 4, 2, 1, 0]$.

Since in m_1 the most up-to-date order is determined by comparing lexicographically the counter vectors, in this example o_1 is considered more up-to-date than o_2. In Section 3 of this corrigendum, we show that method m_1 may lead ABT_DO-Retro to fall in an infinite loop when $K = 0$.

The right way to compare orders is to compare their counter vectors, one position at a time from left to right, until they differ on a position (preferring the order with greater counter) or they are equal on that position but the indexes of the agents in that position differ (preferring the smaller index). We will refer to this method as m_2. Consider again the two orders o_1 and o_2 and associated counter vectors defined above. The counter at the first position equals 2 on both counter vectors and the index of the first agent in o_1 (i.e., A_1) is the same as in o_2, the counter at the second position equals 4 on both counter vectors, however the index of the second agent in o_2 (i.e., A_2) is smaller than the index of the second agent in o_1 (i.e., A_3). Hence, in this case o_2 is considered more up-to-date than o_1. (Note that according to m_1, o_1 is more up-to-date than o_2.) In Section 4 of this corrigendum, we give the proof that method m_2 for comparing orders is correct.

3 ABT_DO-Retro May Not Terminate

In this section we show that ABT_DO-Retro may not terminate when using m_1 and when $K = 0$. We illustrate this on ABT_DO-Retro-MinDom as described in [ZZA09] as it is an example of ABT_DO-Retro where $K = 0$. Consider a DisCSP with 5 agents $\{A_1, A_2, A_3, A_4, A_5\}$ and domains $D(x_1) = D(x_5) = \{1, 2, 3, 4, 5\}$, $D(x_2) = D(x_3) = D(x_4) = \{6, 7\}$. We assume that, initially, all agents store the same order $o_1 = [A_1, A_5, A_4, A_2, A_3]$ with associated counter vector $s_1 = [0, 0, 0, 0, 0]$. The constraints are:
$c_{12} : (x_1, x_2) \notin \{(1, 6), (1, 7)\}$;
$c_{13} : (x_1, x_3) \notin \{(2, 6), (2, 7)\}$;
$c_{14} : (x_1, x_4) \notin \{(1, 6), (1, 7)\}$;
$c_{24} : (x_2, x_4) \notin \{(6, 6), (7, 7)\}$.
$c_{35} : (x_3, x_5) \notin \{(7, 5)\}$.

In the following we give a possible execution of ABT_DO-Retro-MinDom.

t_0: All agents assign the first value in their domains to their variables and send ***ok?*** messages to their neighbors.

t_1: A_4 receives the first ***ok?*** ($x_1 = 1$) message sent by A_1 and generates a nogood $ng_1 : \neg(x_1 = 1)$. Then, it proposes a new order $o_2 = [A_4, A_1, A_5, A_2, A_3]$ with $s_2 = [1, 0, 0, 0, 0]$. Afterwards, it assigns the value 6 to its variable and sends ***ok?*** ($x_4 = 6$) message to all its neighbors (including A_2).

t_2: A_3 receives $o_2 = [A_4, A_1, A_5, A_2, A_3]$ and deletes o_1 since o_2 is more up-to-date; A_1 receives the nogood sent by A_4, it replaces its assignment to 2 and sends an ***ok?*** ($x_1 = 2$) message to all its neighbors.

t_3: A_2 has not yet received o_2 and the new assignment of A_1. A_2 generates a new nogood $ng_2 : \neg(x_1 = 1)$ and proposes a new order $o_3 = [A_2, A_1, A_5, A_4, A_3]$ with $s_3 = [1, 0, 0, 0, 0]$; Afterwards, it assigns the value 6 to its variable and sends ***ok?*** ($x_2 = 6$) message to all its neighbors (including A_4).

$o_1 = [\, A_1 \,, A_5 \,, A_4 \,, A_2 \,, A_3 \,] \quad s_1 = [\, 0 \,, 0 \,, 0 \,, 0 \,, 0 \,]$

$o_2 = [\, A_4 \,, A_1 \,, A_5 \,, A_2 \,, A_3 \,] \quad s_2 = [\, 1 \,, 0 \,, 0 \,, 0 \,, 0 \,]$

$o_3 = [\, A_2 \,, A_1 \,, A_5 \,, A_4 \,, A_3 \,] \quad s_3 = [\, 1 \,, 0 \,, 0 \,, 0 \,, 0 \,]$

$o_4 = [\, A_4 \,, A_3 \,, A_1 \,, A_5 \,, A_2 \,] \quad s_4 = [\, 1 \,, 1 \,, 0 \,, 0 \,, 0 \,]$

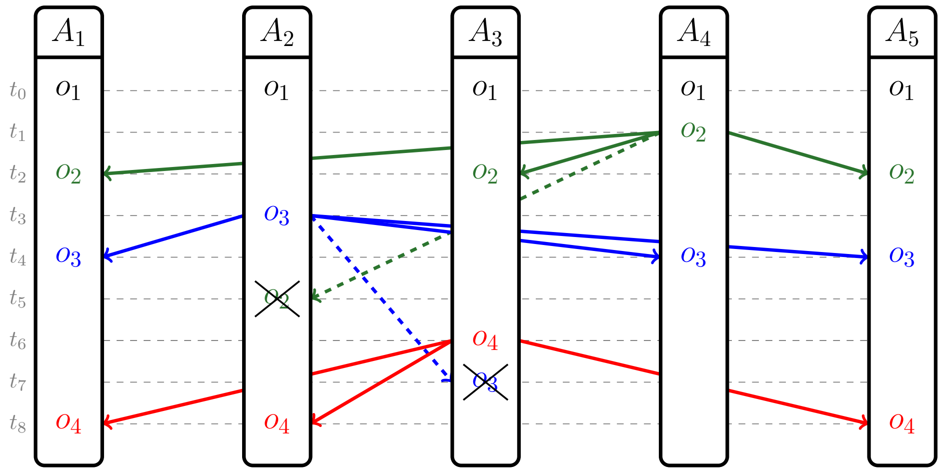

Figure 1: The schema of exchanging ***order*** messages by ABT_DO-Retro

t_4: A_4 receives the new assignment of A_2 (i.e., $x_2 = 6$) and $o_3 = [A_2, A_1, A_5, A_4, A_3]$. Afterwards, it discards o_2 since o_3 is more up-to-date; Then, A_4 tries to satisfy c_{24} because A_2 has a higher priority according to o_3. Hence, A_4 replaces its current assignment (i.e., $x_4 = 6$) by $x_4 = 7$ and sends an ***ok?*** $(x_4 = 7)$ message to all its neighbors (including A_2).

t_5: When receiving o_2, A_2 discards it because its current order is more up-to-date;

t_6: After receiving the new assignment of A_1 (i.e., $x_1 = 2$) and before receiving $o_3 = [A_2, A_1, A_5, A_4, A_3]$, A_3 generates a nogood $ng_3 : \neg(x_1 = 2)$ and proposes a new order $o_4 = [A_4, A_3, A_1, A_5, A_2]$ with $s_4 = [1, 1, 0, 0, 0]$; The order o_4 is more up-to-date according to m_1 than o_3. Since in ABT_DO, an agent sends the new order only to lower priority agents, A_3 will not send o_4 to A_4 because it is a higher priority agent.

t_7: A_3 receives o_3 and then discards it because it is obsolete;

t_8: A_2 receives o_4 but it has not yet received the new assignment of A_4. Then, it tries to satisfy c_{24} because A_4 has a higher priority according to its current order o_4. Hence, A_2 replaces its current assignment (i.e., $x_2 = 6$) by $x_2 = 7$ and sends an ***ok?*** $(x_2 = 7)$ message to all its neighbors (including A_4).

t_9: A_2 receives the ***ok?*** $(x_4 = 7)$ message sent by A_4 in t_4 and changes its current value (i.e., $x_2 = 7$) by $x_2 = 6$. Then, A_2 sends an ***ok?*** $(x_2 = 6)$ message to all its neighbors (including A_4). At the same time, A_4 receives ***ok?*** $(x_2 = 7)$ sent by A_2 in t_8. A_4 changes its current value (i.e., $x_4 = 7$) by $x_4 = 6$. Then, A_4 sends an ***ok?*** $(x_4 = 6)$ message to all its neighbors (including A_2).

t_{10}: A_2 receives the ***ok?*** ($x_4 = 6$) message sent by A_4 in t_9 and changes its current value (i.e., $x_2 = 6$) by $x_2 = 7$. Then, A_2 sends an ***ok?*** ($x_2 = 7$) message to all its neighbors (including A_4). At the same moment, A_4 receives ***ok?*** ($x_2 = 6$) sent by A_2 in t_9. A_4 changes its current value (i.e., $x_4 = 6$) by $x_4 = 7$. Then, A_4 sends an ***ok?*** ($x_4 = 7$) message to all its neighbors (including A_2).

t_{11}: We come back to the situation we were facing at time t_9, and therefore ABT_DO-Retro-MinDom may fall in an infinite loop when using method m_1.

4 The Right Way to Compare Orders

Let us formally define the second method, m_2, for comparing orders in which we compare the indexes of agents as soon as the counters in a position are equal on both counter vectors associated with the orders being compared. Given any order o, we denote by $o(i)$ the index of the agent located in the *ith* position in o and by $s(i)$ the counter in the *ith* position in the counter vector s. An order o_1 with counter vector s_1 is more up-to-date than an order o_2 with counter vector s_2 if and only if there exists a position $i, 1 \leq i \leq n$, such that for all $1 \leq j < i$, $s_1(j) = s_2(j)$ and $o_1(j) = o_2(j)$, and $s_1(i) > s_2(i)$ or $s_1(i) = s_2(i)$ and $o_1(i) < o_2(i)$.

In our correctness proof for the use of m_2 in ABT_DO-Retro we use the following notations: When an agent proposes a new order where the position of the highest priority agent has been changed, the new order will be denoted by a capital O. The initial order known by all agents is denoted by O_0. Each agent stores a current order with an associated counter vector. Each counter vector consists of n counters $ct_1, \ldots, ct_n$. We denote by $ct_1(o)$ the value of the first counter of the counter vector associated with o. In a similar way, we denote by $ct_1(A_i)$ the value of the first counter in the counter vector stored by the agent A_i. We define ct_{max} to be equal to $max(ct_1(A_i) \mid i \in 1..n)$. The value of ct_{max} evolves during the search so that it always corresponds to the value of the largest counter among all the first counters stored by agents.

Let K be the parameter defining the degree of flexibility of the retroactive heuristics (see Section 1). Next we show that the ABT_DO-Retro algorithm is correct when using m_2 and with $K = 0$. The proof that the algorithm is correct when $K \neq 0$ can be found in [ZZA09].

To prove the correctness of ABT_DO-Retro we use induction on the number of agents. For a single agent the order is static therefore the correctness of standard ABT implies the correctness of ABT_DO-Retro. Assume ABT_DO-Retro is correct for every DisCSP with $n - 1$ agents. We show in the following that ABT_DO-Retro is correct for every DisCSP with n agents. To this end we first prove the following lemmas.

Lemma 1 *Given enough time, if the value of ct_{max} does not change, the highest priority agent in all orders stored by all agents will be the same.*

Proof. Assume the system reaches a state σ where the value of ct_{max} no longer increases. Let h be the value of ct_{max}. Let O_1 be the order that, when generated, caused the system to enter state σ. Inevitably, we have $ct_1(O_1) = h$. Assume that $O_1 \neq O_0$ and let A_i be the agent that generated O_1. The agent A_i is necessarily the highest priority agent in the new order O_1 because, the only possibility for the generator of a new order to change the position of the highest priority agent is to put itself in the first position in the new order. Thus, O_1 is sent by A_i to all other agents because A_i must send O_1 to all agents that have a lower priority than itself. So after a finite time all agents will be aware of O_1. This is also true if $O_1 = O_0$. Now, by assumption the value of ct_{max} no longer increases. As a result, the only way for another agent to generate an order O' such that the highest priority agents in O_1 and O' are different (i.e., $O'(1) \neq O_1(1)$) is to put itself in first position in O' and to do that *before* it has received O_1 (otherwise O' would increase ct_{max}). Therefore, the time passed from the moment the system entered state σ until a new order O' was generated is finite. Let O_2 be the most up-to-date such order and let A_j be the agent that generated O_2. That is, A_j is the agent with smallest index among those who generated such an order O'. The agent A_j will send O_2 to all other agents and O_2 will be accepted by all other agents after a finite amount of time. Once an agent has accepted O_2, all orders that may be generated by this agent do not reorder the highest priority agent otherwise ct_{max} would increase. □

Lemma 2 *If the algorithm is correct for $n-1$ agents then it terminates for n agents.*

Proof. If during the search ct_{max} continues to increase, this means that some of the agents continue to send new orders in which they put themselves in first position. Hence, the nogoods they generate when proposing the new orders are necessarily unary (i.e., they have an empty left-hand side) because in ABT_DO-Retro, when the parameter K is zero the nogood sender cannot put itself in a higher priority position than the second last in the nogood. Suppose $ng_0 = \neg(x_i = v_i)$ is one of these nogoods, sent by an agent A_j. After a finite amount of time, agent A_i, the owner of x_i, will receive ng_0. Three cases can occur. First case, A_i still has value v_i in its domain. So the value v_i is pruned once and for all from $D(x_i)$ thanks to ng_0. Second case, A_i has already received a nogood equivalent to ng_0 from another agent. Here, v_i no longer belongs to $D(x_i)$. When changing its value, A_i has sent an ***ok?*** message with its new value v'_i. If A_i and A_j were neighbors, this ***ok?*** message has been sent to A_j. If A_i and A_j were not neighbors when A_i changed its value to v'_i, this ***ok?*** message was sent by A_i to A_j after A_j requested to add a link between them at the moment it generated ng_0. Thanks to the assumption that messages are always delivered in a finite amount of time, we know that A_j will receive the ***ok?*** message containing v'_i a finite amount of time after it sent ng_0. Thus, A_j will not be able to send forever nogoods about a value v_i pruned from $D(x_i)$. Third case, A_i already stores a nogood with a non empty left-hand side discarding v_i. Notice that although A_j moves to the highest priority position, A_i may be of lower priority, i.e., there can be agents with higher priority than A_i according to the current order that are not included in ng_0. Thanks to the standard *highest possible lowest variable involved* [HY00, BMBM05] heuristic for selecting nogoods in ABT algorithms, we are guaranteed that the nogood with empty left-hand side ng_0 will replace the other existing nogood and v_i will be permanently pruned from $D(x_i)$. Thus, in all three cases, every time ct_{max} increases, we know that an agent has moved to the first position in the order, and a value was definitively pruned a finite amount of time before or after. There is a bounded number of values in the network. Thus, ct_{max} cannot increase forever. Now, if ct_{max} stops increasing, then after a finite amount of time the highest priority agent in all orders stored by all agents will be the same (Lemma 1). Since the algorithm is correct for $n-1$ agents, after each assignment of the highest priority agent, the rest of the agents will either reach an idle state,[1] generate an empty nogood indicating that there is no solution, or generate a unary nogood, which is sent to the highest priority agent. Since the number of values in the system is finite, the third option, which is the only one that does not imply immediate termination, cannot occur forever. □

Lemma 3 *If the algorithm is correct for $n-1$ agents then it is sound for n agents.*

[1]As proved in Lemma 3, this indicates that a solution was found.

Proof. Let o be the most up-to-date order generated before reaching the state of quiescence and let O be the most-up-to-date order generated such that $ct_1(O) = ct_1(o)$ (and such that O has changed the position of the first agent –assuming $O \neq O_0$). Given the rules for reordering agents, the agent that generated O has necessarily put himself first because it has modified ct_1 and thus also the position of the highest agent. So it has sent O to all other agents. When reaching the state of quiescence, we know that no order O_2 with $O_2(1) \neq O(1)$ has been generated because this would break the assumption that O is the most-up-to-date order where the position of the first agent has been changed. Hence, at the state of quiescence, every agent A_i stores an order o_i such that $o_i(1) = O(1)$. (This is also true if $O = O_0$.) Let us consider the DisCSP P composed of the $n-1$ lower priority agents according to O. Since the algorithm is correct for $n-1$ agents, the state of quiescence means that a solution was found for P. Also, since all agents in P are aware that $O(1)$ is the agent with the highest priority, the state of quiescence also implies that all constraints that involve $O(1)$ have been successfully tested by agents in P, otherwise at least one agent in P would try to change its value and send an ***ok?*** or nogood message. Therefore, the state of quiescence implies that a solution was found. □

Lemma 4 *The algorithm is complete*

Proof. All nogoods are generated by logical inferences from existing constraints. Thus, an empty nogood cannot be inferred if a solution exists. □

Following Lemmas 2, 3 and 4 we obtain the correctness of the main theorem in this corrigendum.

Theorem 1 *The ABT_DO-Retro algorithm with $K = 0$ is correct when using the m_2 method for selecting the most up-to-date order.*

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