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Existential Rules: A Graph-Based View

Marie-Laure Mugnier

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Existential Rules: A Graph-based View

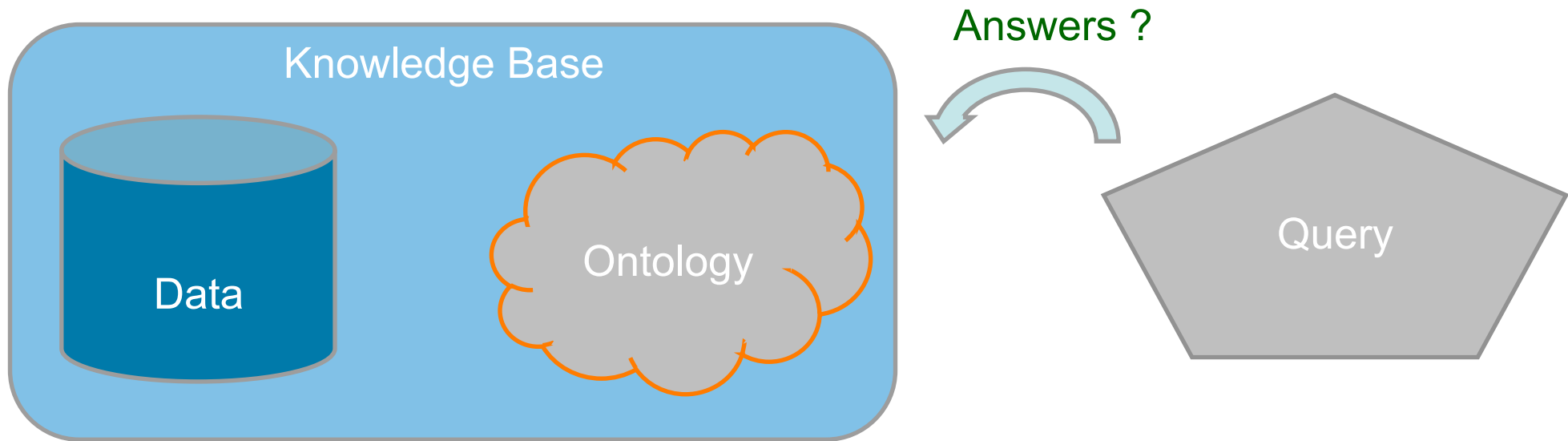
Marie-Laure Mugnier

University of Montpellier



Datalog 2.0, Vienna, 2012

Ontology-based Data Access (OBDA)



Adding an ontological layer:

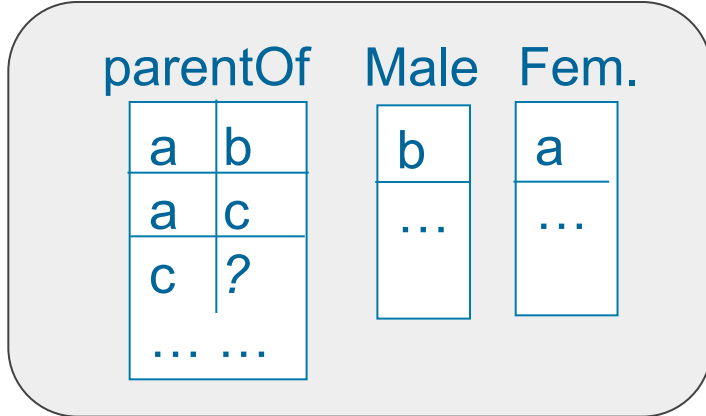
- to **abstract** from a specific database schema
- to provide a **unified view** of multiple sources
- to infer new facts, thus allowing for **data incompleteness**

Outline

- Existential rules: a logic- and graph-based framework
- Decidability and algorithmic issues
 - Focus on:
 - tree-shaped saturation in forward chaining
 - piece-based unification in backward chaining
- A (graph) tool for combining decidable classes of rules

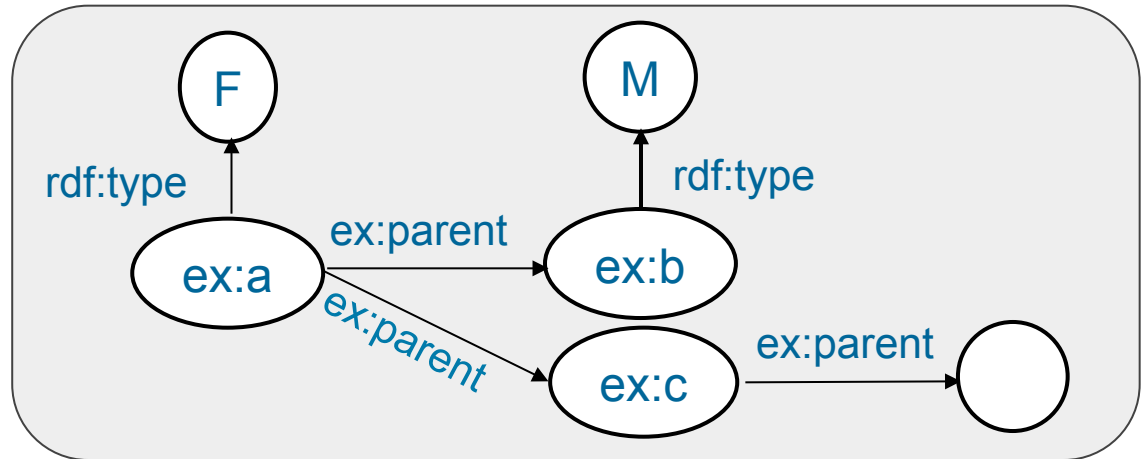
Data / Facts

Relational Database



RDF (Semantic Web)

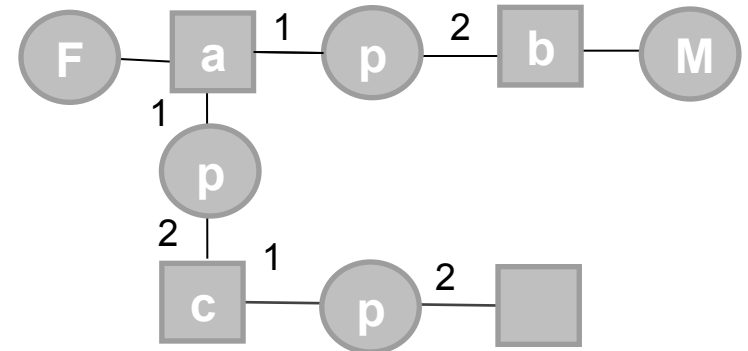
Etc.



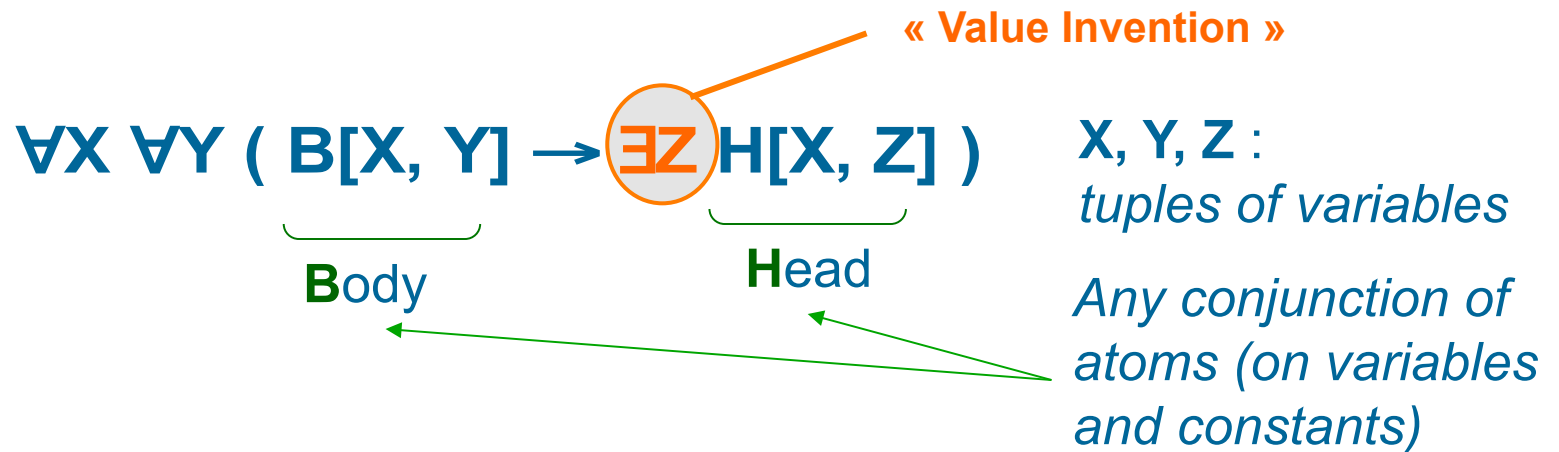
Abstraction in first-order logic

$\exists x(\text{parentOf}(a,b) \wedge \text{parentOf}(a,c) \wedge \text{parentOf}(c,x) \wedge F(a) \wedge M(b))$

Or in graphs / hypergraphs



Ontology: Existential Rules



$\forall x \forall y (\text{siblingOf}(x,y) \rightarrow \exists z (\text{parentOf}(z,x) \wedge \text{parentOf}(z,y)))$

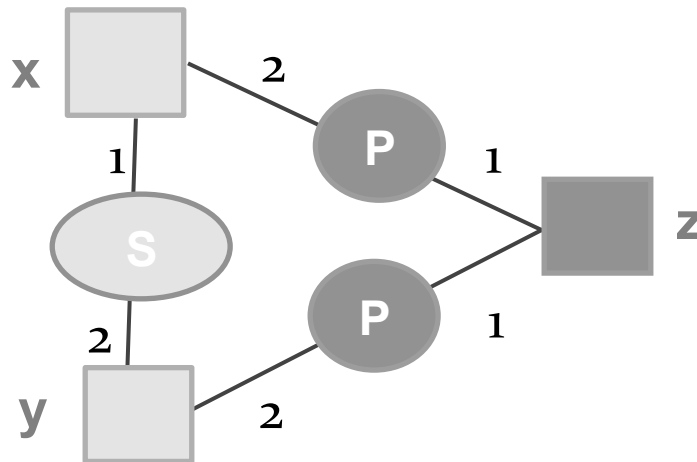
Simplified form: $\text{siblingOf}(x,y) \rightarrow \text{parentOf}(z,x) \wedge \text{parentOf}(z,y)$

- Same as Tuple Generating Dependencies (TGDs)
- See also Datalog+/-
- Same as the logical translation of Conceptual Graph rules
- Generalize Description Logics used for OBDA (DL-Lite, \mathcal{EL})

Ontology: Existential Rules

$$\forall X \forall Y (\underbrace{B[X, Y]}_{\text{graph}} \rightarrow \exists Z \underbrace{H[X, Z]}_{\text{graph}})$$

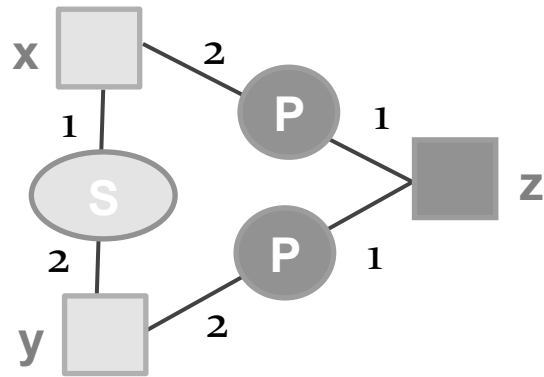
$$\forall x \forall y (\text{siblingOf}(x,y) \rightarrow \exists z (\text{parentOf}(z,x) \wedge \text{parentOf}(z,y)))$$



Value Invention

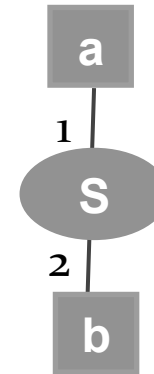
$$R = \forall x \forall y (\text{siblingOf}(x,y) \rightarrow \exists z (\text{parentOf}(z,x) \wedge \text{parentOf}(z,y)))$$

$$F = \text{siblingOf}(a,b)$$



$$h: \text{body} \rightarrow F$$

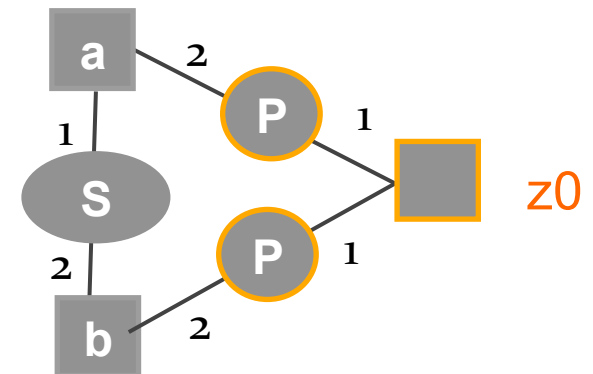
$$h = \{(x,a), (y,b)\}$$



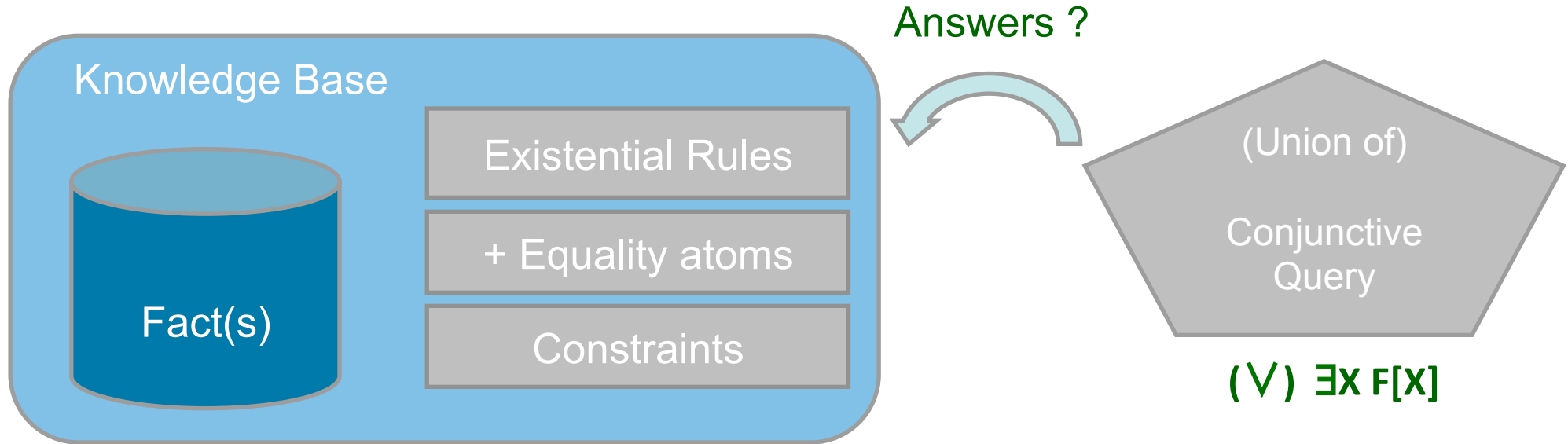
A rule $\text{body} \rightarrow \text{head}$ is applicable to a fact F if there is a homomorphism $h: \text{body} \rightarrow F$

Then $h(\text{head})$ can be « added » to F with renaming existential variables of head

$$F' = \exists z_0 (\text{siblingOf}(a,b) \wedge \text{parentOf}(z_0,a) \wedge \text{parentOf}(z_0,b))$$



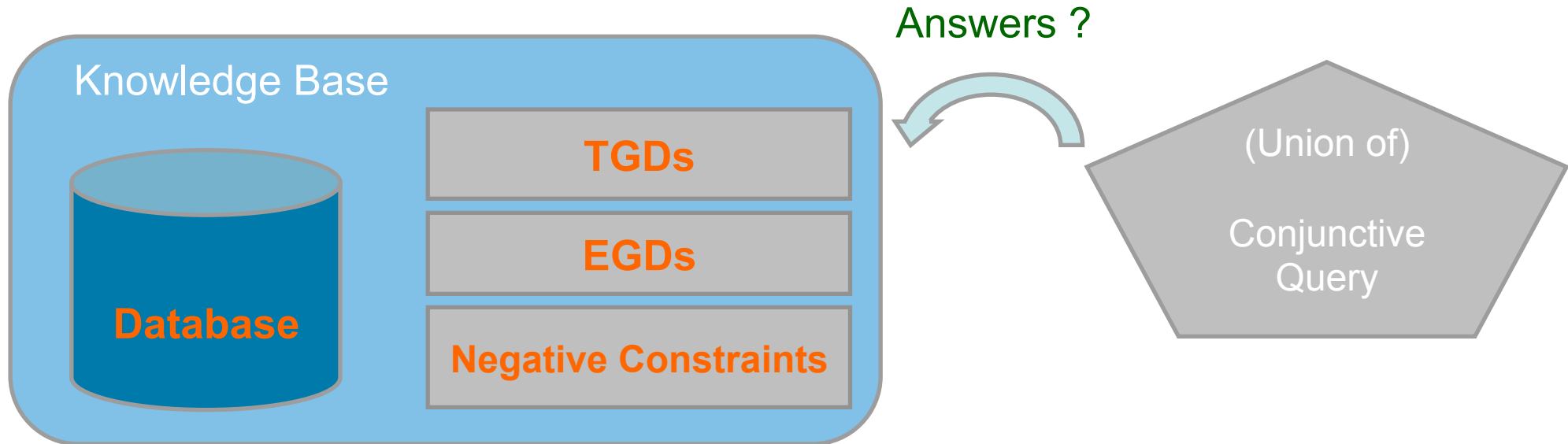
Logical /Graphical Framework



Negative constraint: $\neg (\exists X B[X])$ or $\forall X (B[X] \rightarrow \perp)$
« B[X] must not be found »

Positive constraint: $\forall X \forall Y (B[X, Y] \rightarrow \exists Z H[X, Z])$
« if B[X,Y] is found then H[X,Z] must also be found »

Similar Framework: Datalog +/-



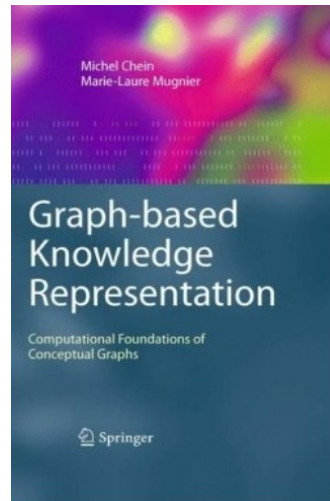
[Cali Gottlob Lukasiewicz PODS 2009]

Tuple Generating Dependency = (pure) existential rule

Equality Generating Dependency: $\forall X (B[X] \rightarrow x = e)$

The Conceptual Graph Origins

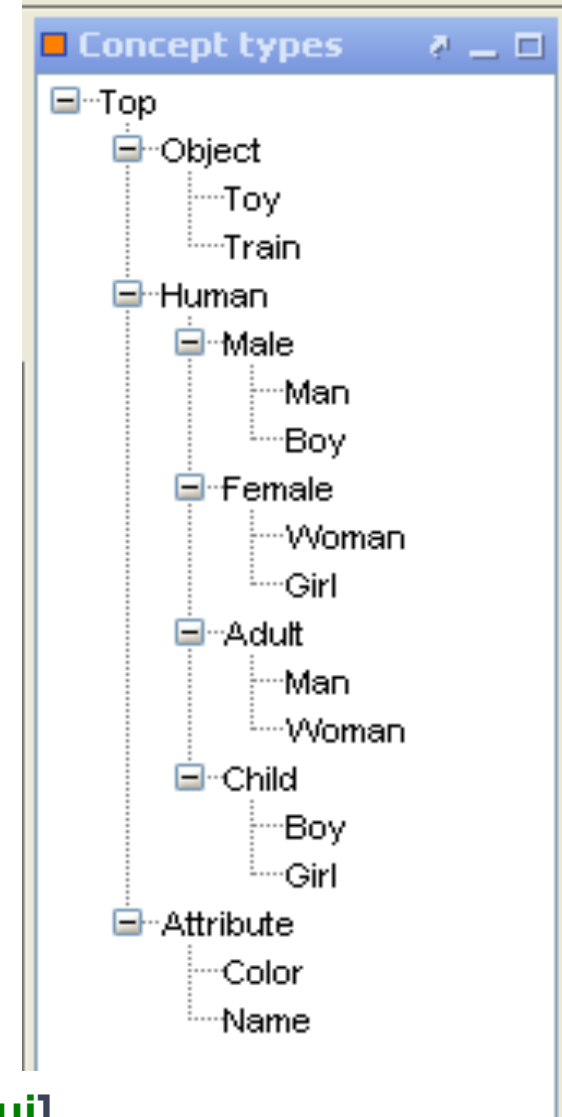
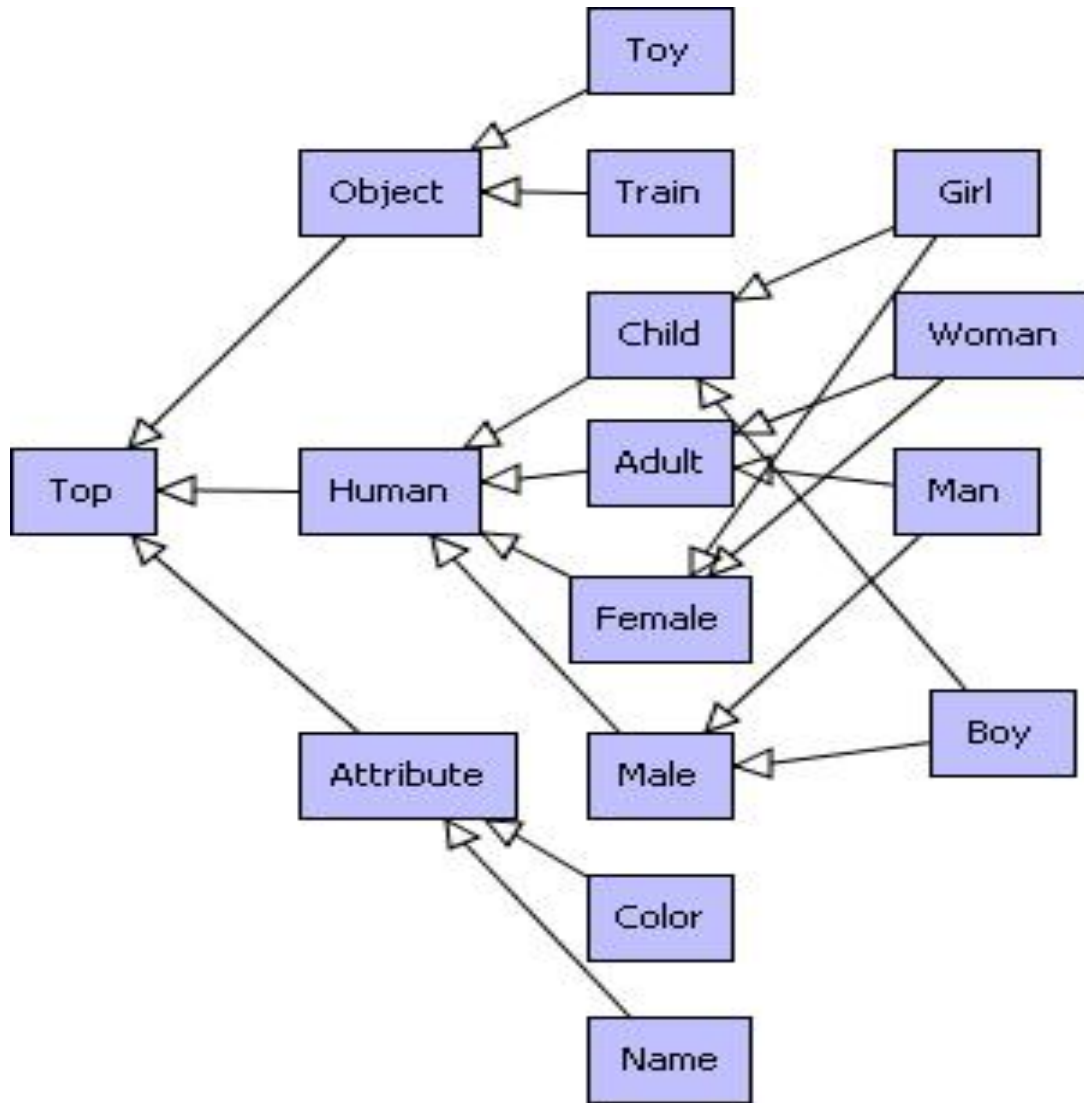
- Conceptual graphs introduced in [Sowa 76] [Sowa 84]
- Specific research line by Montpellier's group since 1992
 - « Graph-based » knowledge representation and reasoning



« Graph-Based Knowledge Representation: Computational Foundations of Conceptual Graphs », Chein & M..., Springer, 2009

Conceptual Graph Vocabulary:

1. partially (pre-)ordered set of **concepts**

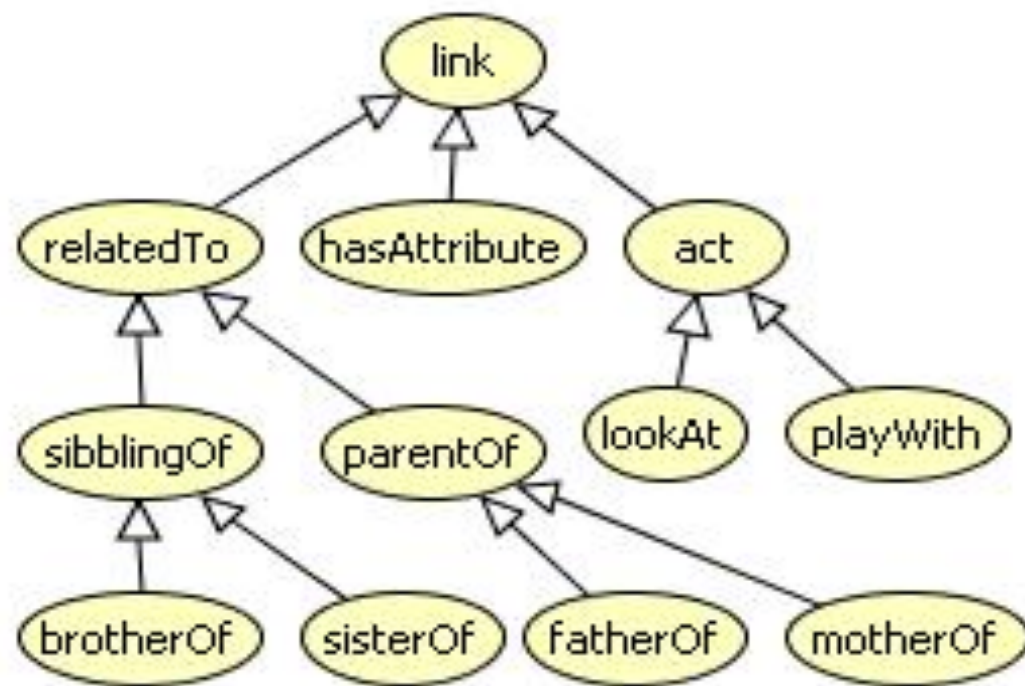


[screenshots from CoGui, <http://www.lirmm.fr/cogui>]



Conceptual Graph Vocabulary:

- partially (pre-)ordered set of **relations** with their **signature**
[any relation arity allowed]



link(Top ,Top)

- relatedTo(Human ,Human)
 - siblingOf(Human ,Human)
 - brotherOf(Male ,Human)
 - sisterOf(Female ,Human)
 - parentOf(Adult ,Human)
 - fatherOf(Man ,Human)
 - motherOf(Woman ,Human)
- act(Human ,Top)
 - lookAt(Human ,Top)
 - playWith(Human ,Object)
 - hasAttribute(Top ,Attribute)

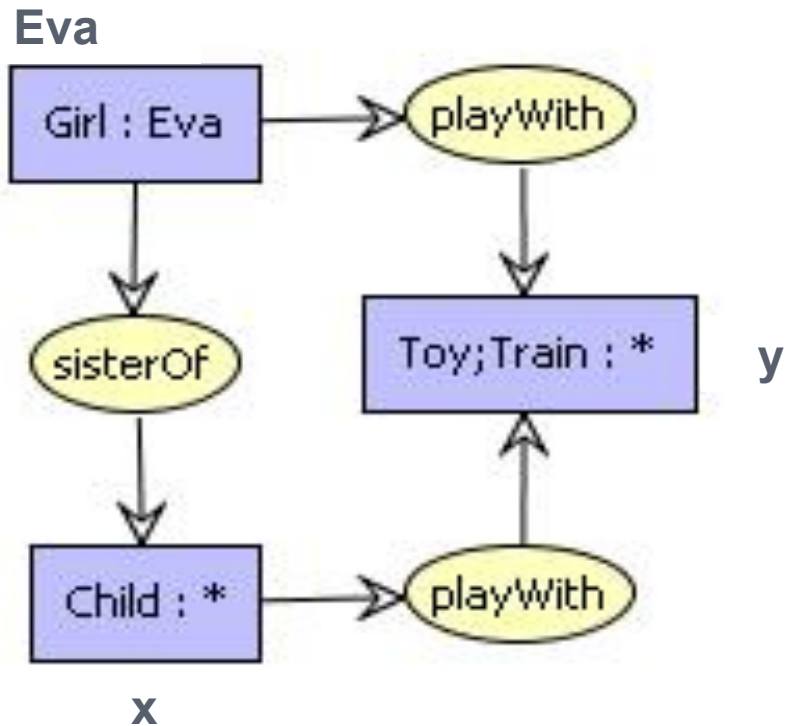
Logical translation (Φ) of the vocabulary: very simple rules

$$p < q \quad \forall x_1 \dots x_k (p(x_1 \dots x_k) \rightarrow q(x_1 \dots x_k))$$

Signature of r $\forall x_1 \dots x_k (p(x_1 \dots x_k) \rightarrow t_{i_1}(x_1) \dots t_{i_k}(x_k))$



Basic Conceptual Graph



[total order on the edges incident to a relation node]

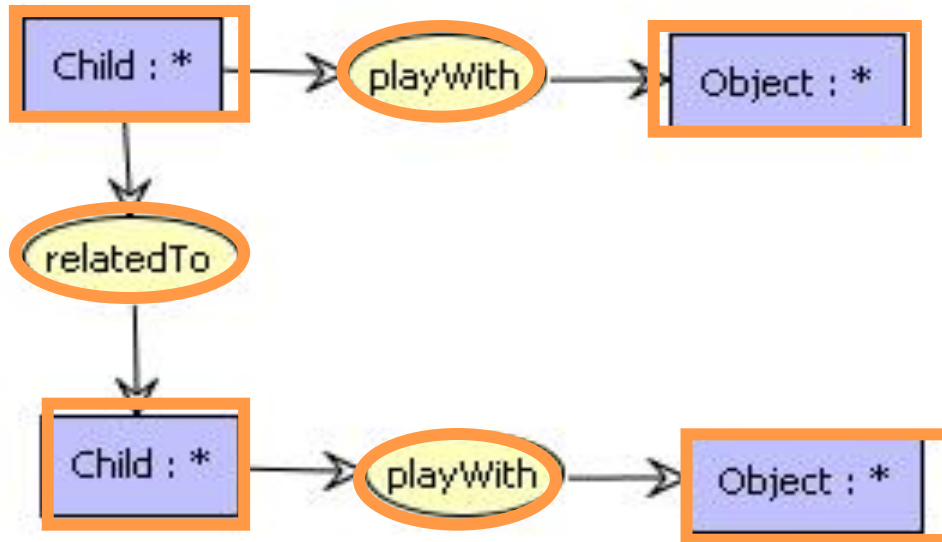
Logical translation (Φ): existentially closed conjunction of atoms

$$\exists x \exists y (\text{Girl}(\text{Eva}) \wedge \text{Child}(x) \wedge \text{Toy}(y) \wedge \text{Train}(y) \\ \wedge \text{sisterOf}(\text{Eva}, x) \wedge \text{playWith}(\text{Eva}, y) \wedge \text{playWith}(x, y))$$

Allows to represent **facts** and **conjunctive queries**

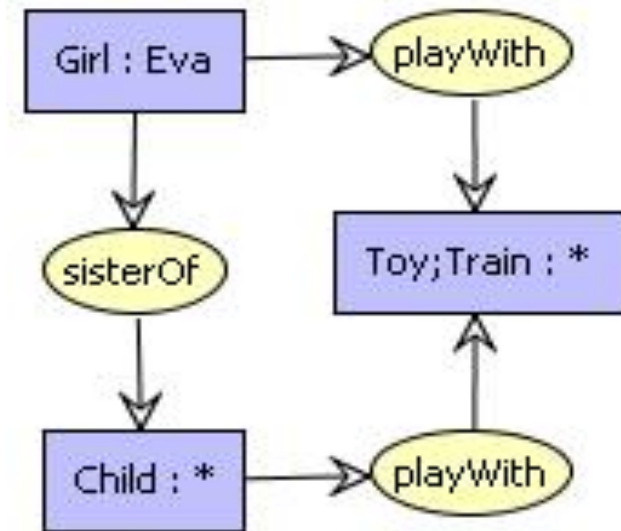


Homomorphism (with concept/relation preorders integrated)



Query Q

Fact F



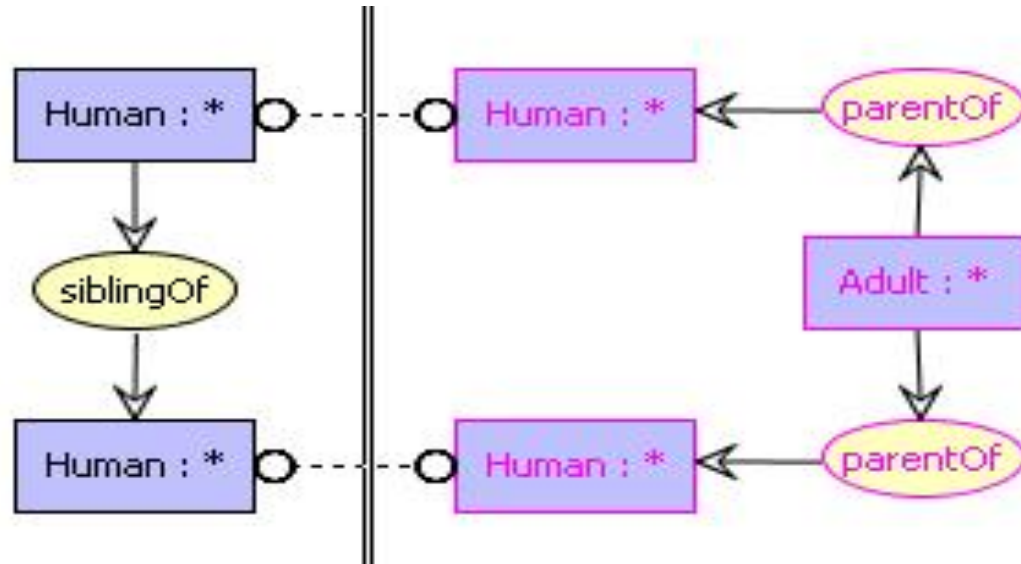
Logical **soundness** [Sowa 84] and **completeness** [Chein M... 92]:
there is a homomorphism from Q to F iff
 $\Phi(Q)$ is entailed by $\Phi(F)$ and $\Phi(\text{vocabulary})$

The Basic CG fragment restricted to binary relations
is equivalent to **RDFS** [Baget ISWC' 05] [Baget+ ICCS' 10]



Richer Fragments (nested graphs, rules, constraints, + negation, ...)

- Rule: pair of basic conceptual graphs



$$\forall x \forall y (\text{Human}(x) \wedge \text{Human}(y) \wedge \text{siblingOf}(x,y) \\ \rightarrow \exists z (\text{Adult}(z) \wedge \text{parentOf}(z,x) \wedge \text{parentOf}(z,y)))$$

- Sound and complete forward chaining and backward chaining [Salvat M... 1996]
- Several ways of combining rules and constraints [Baget M... JAIR 2002]

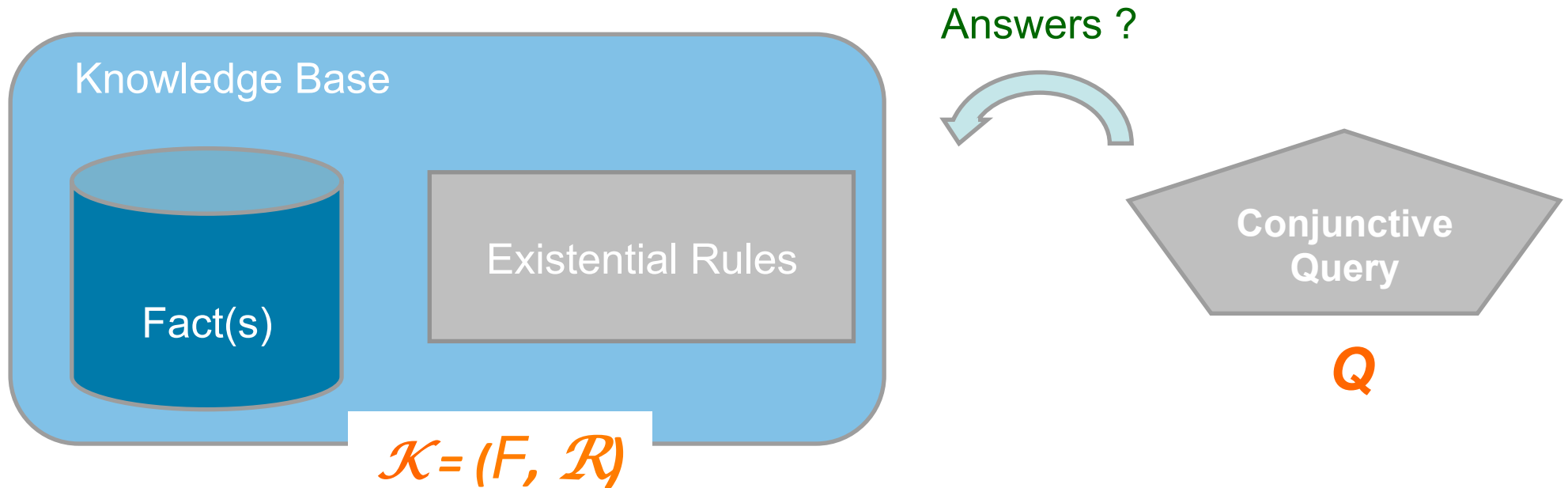
The existential rule framework can be seen as a fragment of CGs with a flat vocabulary



Outline

- Existential rules: a logic- and graph-based framework
- Decidability and algorithmic issues
 - Focus on:
 - tree-shaped saturation in forward chaining
 - piece-based unification in backward chaining
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Basic Problem

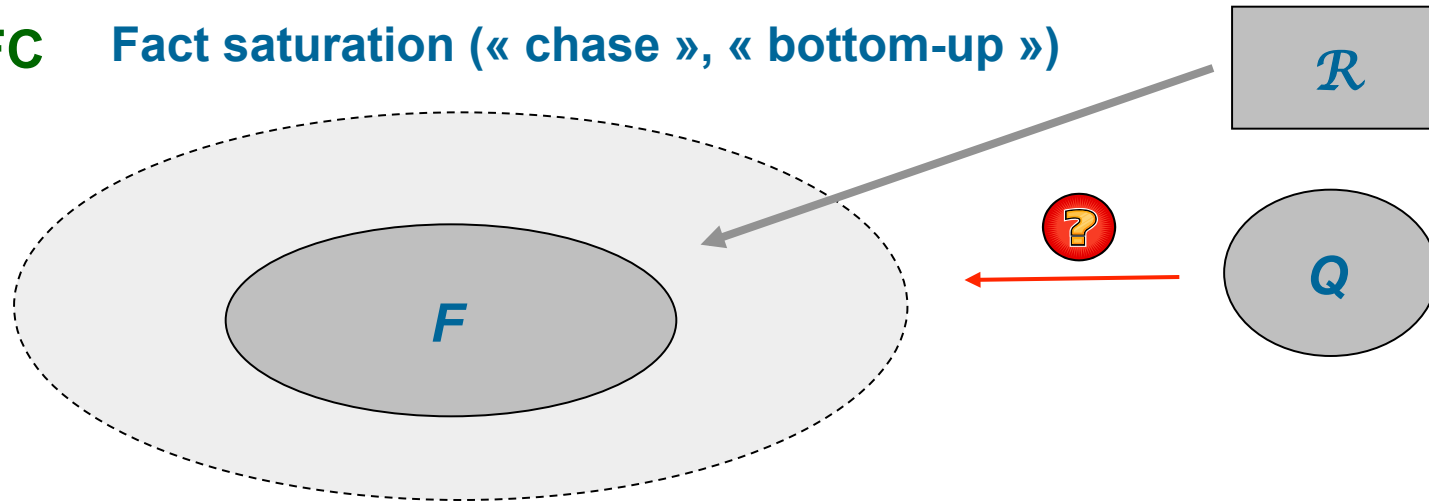


■ Conjunctive Query Entailment

Given a KB $\mathcal{K} = (F, \mathcal{R})$ and a (Boolean) conjunctive query Q ,
is Q entailed by \mathcal{K} ?

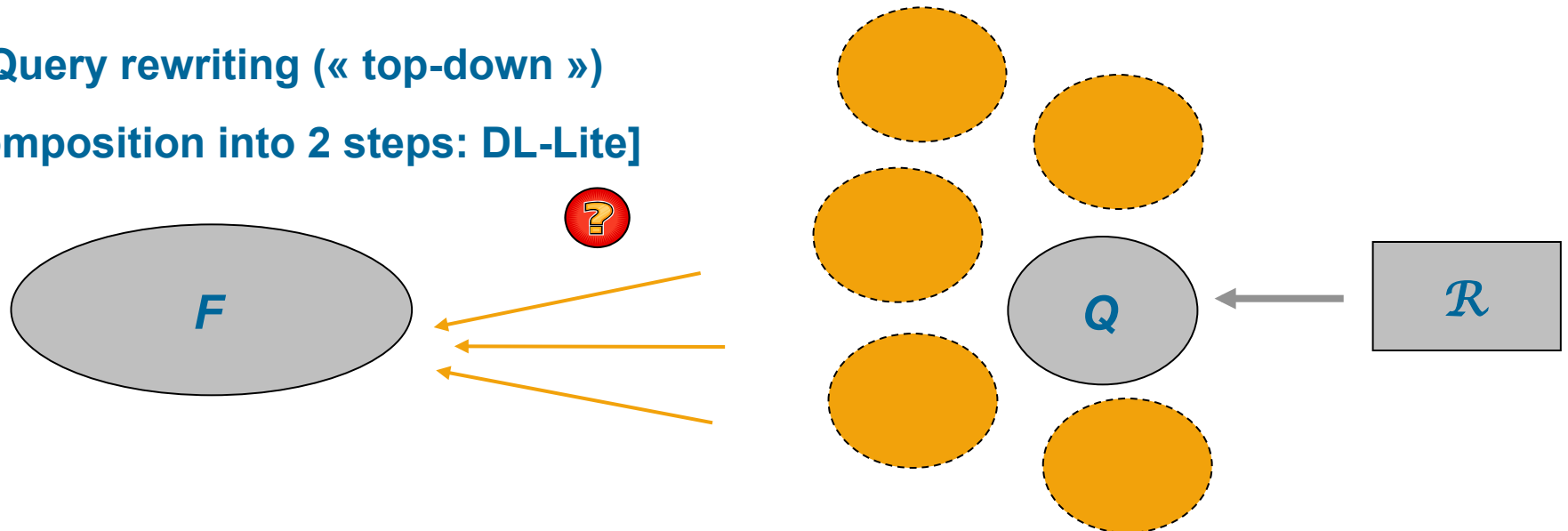
Forward vs Backward Chaining

FC Fact saturation (« chase », « bottom-up »)



BC Query rewriting (« top-down »)

[Decomposition into 2 steps: DL-Lite]



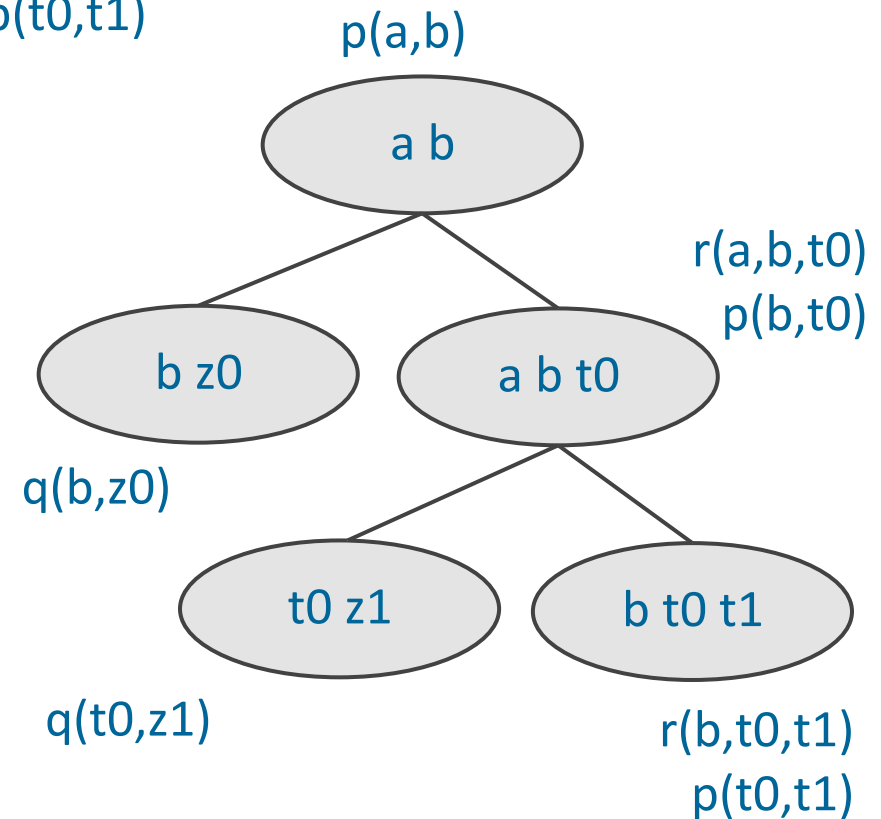
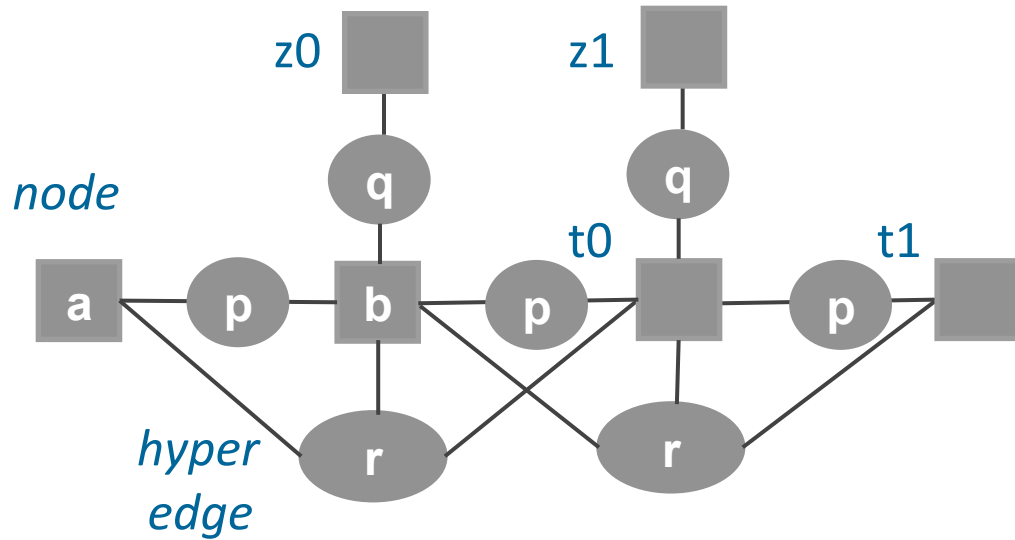
Decidability Issues

- Entailment is not decidable
- Many decidable classes exhibited in databases and KR
- Three generic kinds of properties ensuring decidability:
 - Saturation by Forward Chaining halts (« finite expansion set », *fes*)
 - Query rewriting by Backward Chaining halts (« finite unification set », *fus*)
 - Saturation by Forward Chaining may not halt *but* the generated facts have a tree-like structure (« bounded treewidth set », *bts*)

None of these properties is *recognizable* [Baget+ KR 10] but they provide *generic* algorithms

Decomposition Tree / Treewidth

$p(a,b)$ $q(b,z_0)$ $r(a,b,t_0)$ $p(b,t_0)$ $q(t_0,z_1)$ $r(b,t_0,t_1)$ $p(t_0,t_1)$



Decomposition tree:

- 1) each node (*term*) appears in a bag
- 2) each hyperedge (*atom*) has all its nodes in a bag
- 3) for each node x , the subgraph induced by the bags containing x is connected

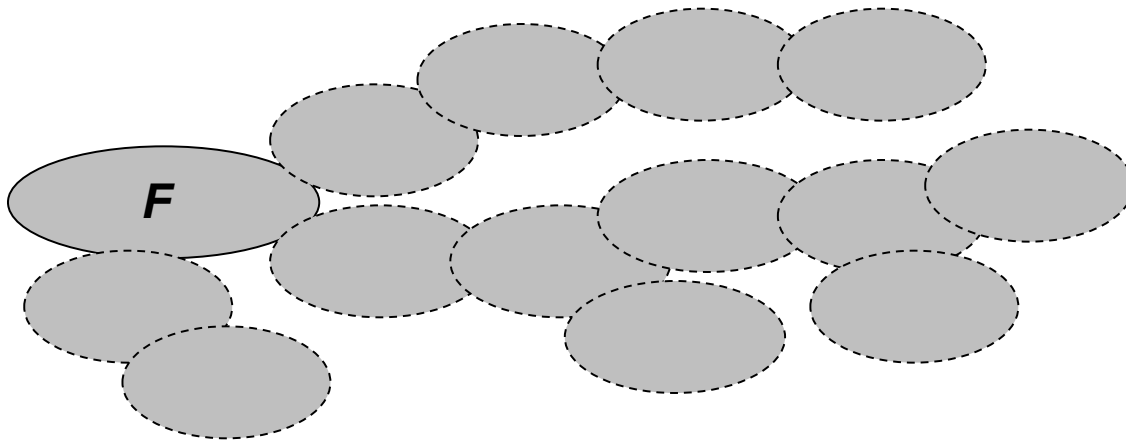
Width of a tree decomposition = *max* number of nodes in a bag (minus 1)

Treewidth of a graph = *min* width over all decomposition trees of this graph

Bounded Treewidth of the Derived Facts (*bts*)

Essentially [Cali Gottlob Kifer KR'08]

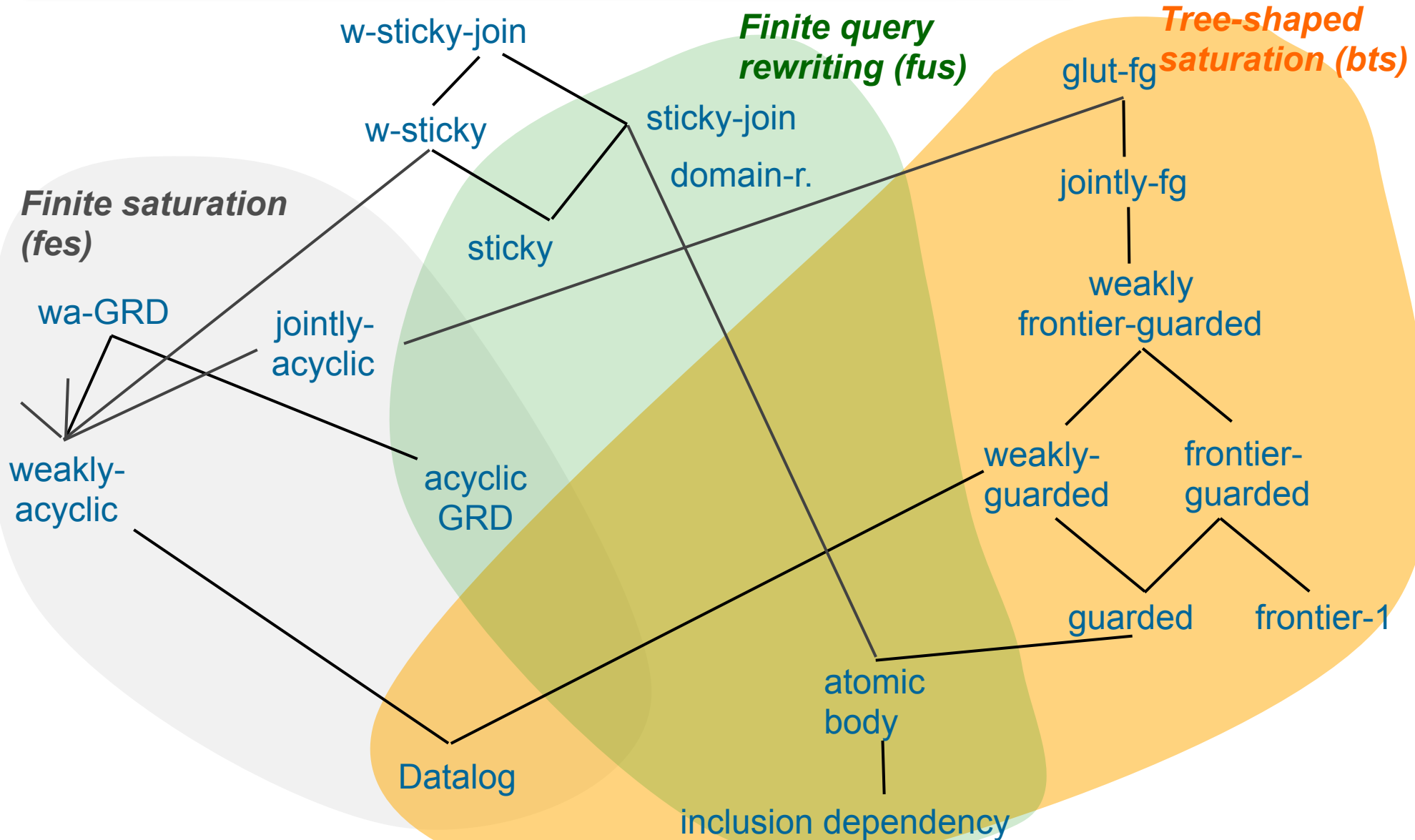
\mathcal{R} is *bts* if FC with \mathcal{R} generates facts with **bounded treewidth**
i.e., for any fact F , there is an integer b s.t.
any fact \mathcal{R} -derived from F has treewidth bounded by b



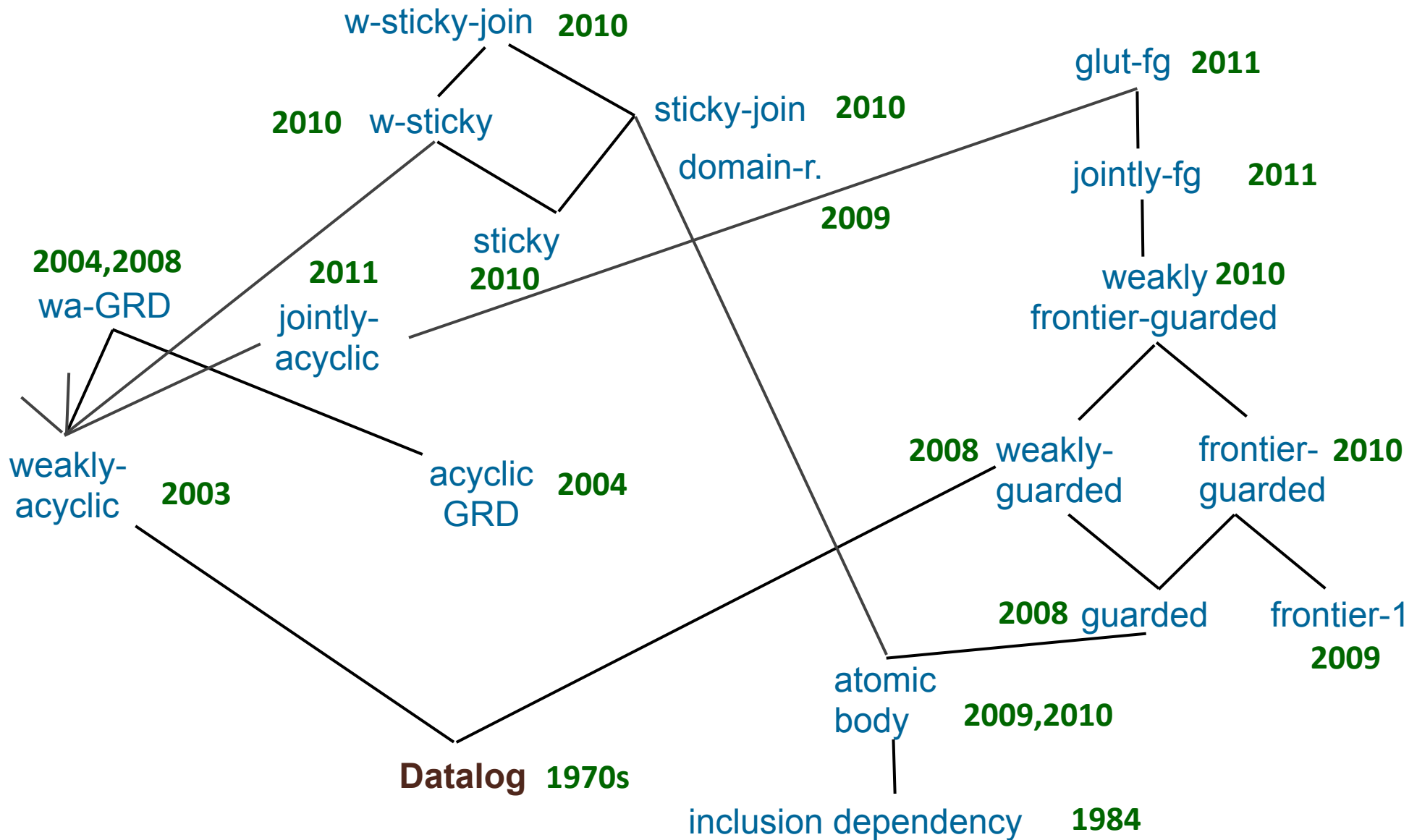
fes (*finite saturation*) is included in *bts*
(bound given by the number of terms in the finite « saturated fact »)

The decidability proof does not provide a halting algorithm
(relies on the bounded treewidth model property [Courcelle 90])

(Partial) Inclusion Map of Decidable Classes



(Partial) Inclusion Map of Decidable Classes



Some Recognizable bts (and not fes) Classes of Rules

Frontier: variables shared by the body and the head

Guard only *affected* variables from the *frontier*

[Baget+ KR' 10]

Guard only the *frontier*

[Baget+ KR' 10]

$r(x,y) \wedge r(y,z) \rightarrow r(y,u) \wedge r(z,u)$

The *frontier* has size 1

[Baget+ IJCAI' 09]

Guard only *affected* variables (i.e. possibly mapped to new existentials)

[Cali+ KR' 08]

datalog



$r(x,y) \wedge r(y,z) \wedge r(x,z) \rightarrow r(z,u)$

$r(x,y) \wedge r(y,z) \wedge s(x,y,z) \rightarrow r(y,u) \wedge r(z,u)$

An atom in the body *guards* all the body variables

[Cali+ KR' 08]

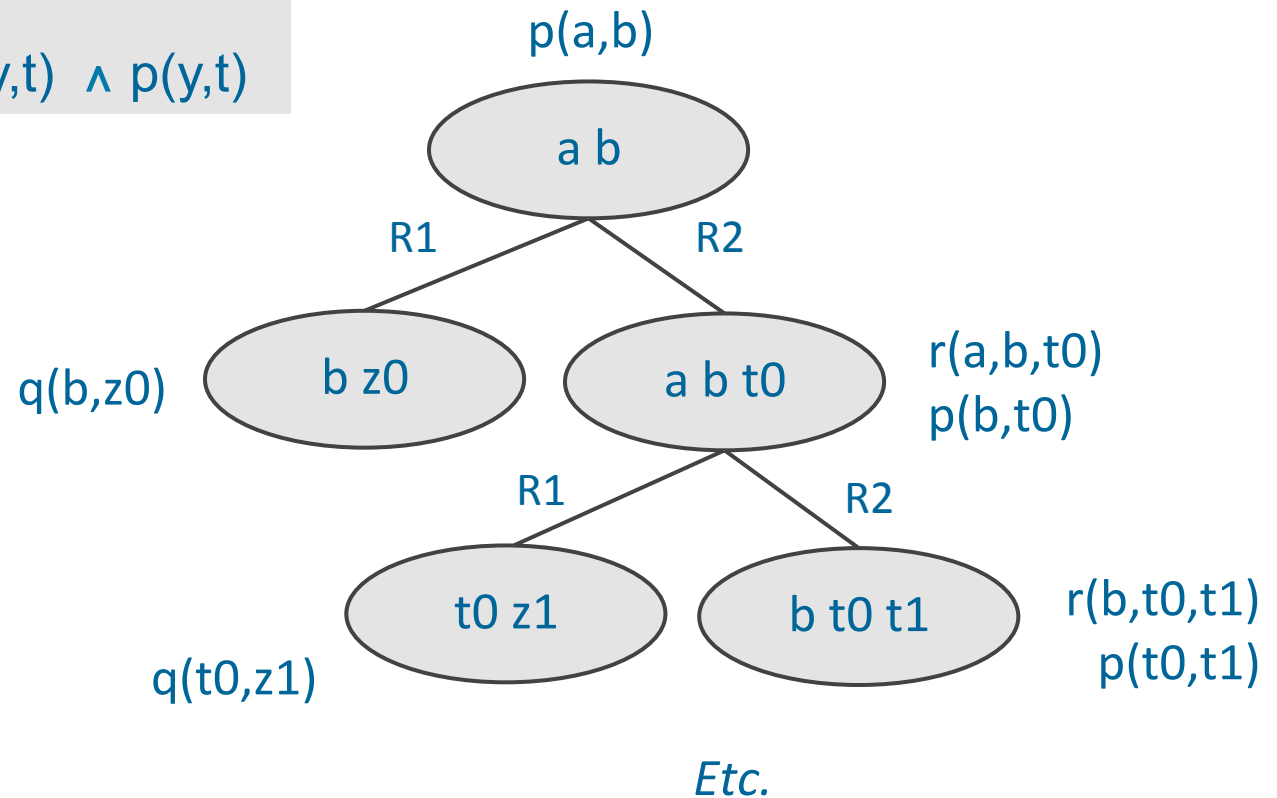
These classes are moreover « **greedy bts** » => a halting algorithm [Baget+ IJCAI' 11]

Greedy *bts*

$$R1 = p(x,y) \rightarrow p(y,z)$$

$$R2 = p(x,y) \wedge q(x,z) \rightarrow r(x,y,t) \wedge p(y,t)$$

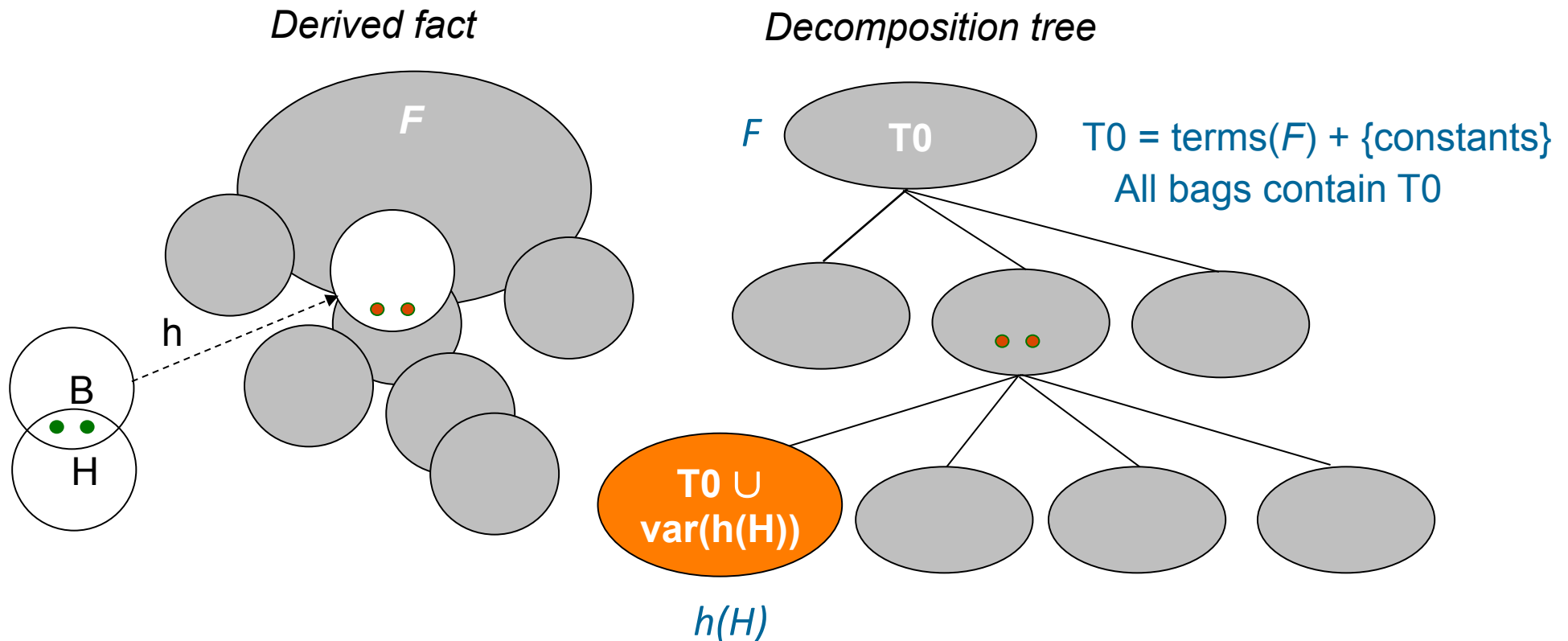
$$F = p(a,b)$$



Greedy construction of a **decomposition tree** of the derived fact
with bounded width

The « Greedy bts » Property [Baget+ IJCAI' 11]

For any fact, for each rule application,
frontier variables not being mapped to initial terms are *jointly* mapped to variables occurring in atoms added by a single previous rule application



Main Ideas of the Algorithm for *gbts* (1)

Build a **finite** decomposition tree that encodes a potentially infinite fact

1. **Bag pattern** = { *homomorphisms from part of a rule body to « current fact » that use some terms of the bag* }

→ A rule is applicable to the current fact *iff* a bag pattern contains its body

→ FC can be performed on the decorated tree

2. **Equivalence relation** on bags

Only one bag per equivalence class is developed

The other nodes are *blocked*

Bounded number of equivalence classes → finite « full blocked tree » T^*

Main Ideas of the Algorithm for *gbts* (2)

Query this finite decomposition tree

[Baget+ IJCAI 2011] Q seen as a rule « $Q \rightarrow match$ »

Q is entailed iff it occurs in a bag pattern

i.e. Q maps by homomorphism to $atoms(T^*)$

[Thomazo+ KR 2012] offline /online separation

(1) compilation: tree T^* built independently from *any* query

(2) querying: *any* Q is entailed iff it maps by **-homomorphism* to T^*

i.e. Q maps by homomorphism to a bounded « development » of T^*

Backward Chaining: Unification Step

$$R = r(x) \rightarrow p(x,y)$$

$$Q = p(u,v) \wedge p(u,w) \wedge p(v,w)$$

Atomic unification:

$$u \rightarrow x \quad v \rightarrow y$$

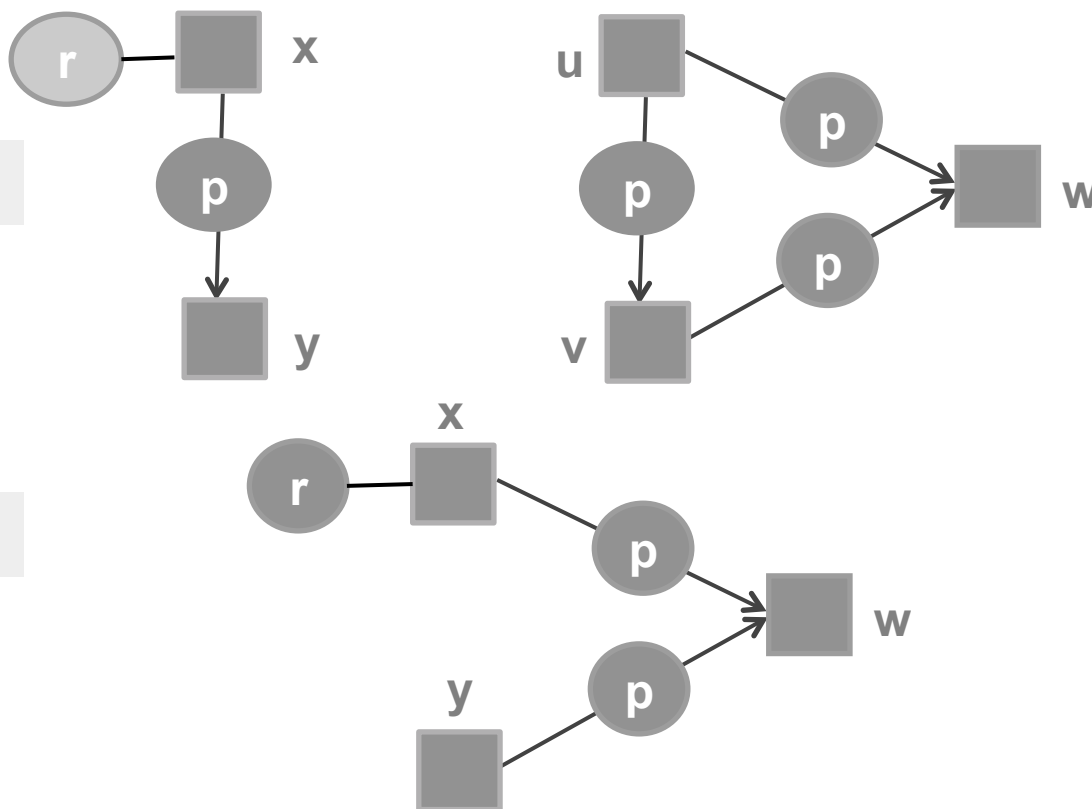
$$Q1 = r(x) \wedge p(x,w) \wedge p(y,w)$$

Soundness lost!

Indeed let $F = Q1$

$\text{saturation}(F,R) \cong F$

Q does not map to F



Existentials in rule heads produce a **structure** that must be taken into account

Key Notion: « Piece »

- Given a subset T of its variables, a set of atoms is partitioned into pieces.

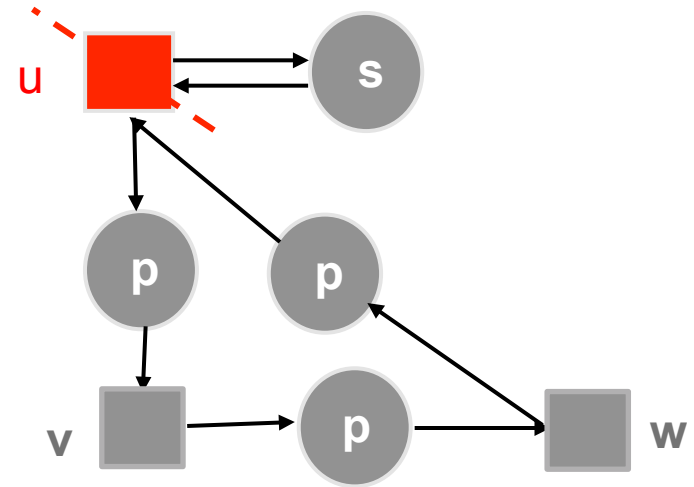
A piece = all atoms linked by a « path » of variables not belonging to T

$p(u,v)$ $p(v,w)$ $p(w,u)$ $s(u,u)$

$T = \{ u \}$

Piece 1 = $\{ p(u,v)$ $p(v,w)$ $p(w,u) \}$

Piece 2 = $\{ s(u,u) \}$

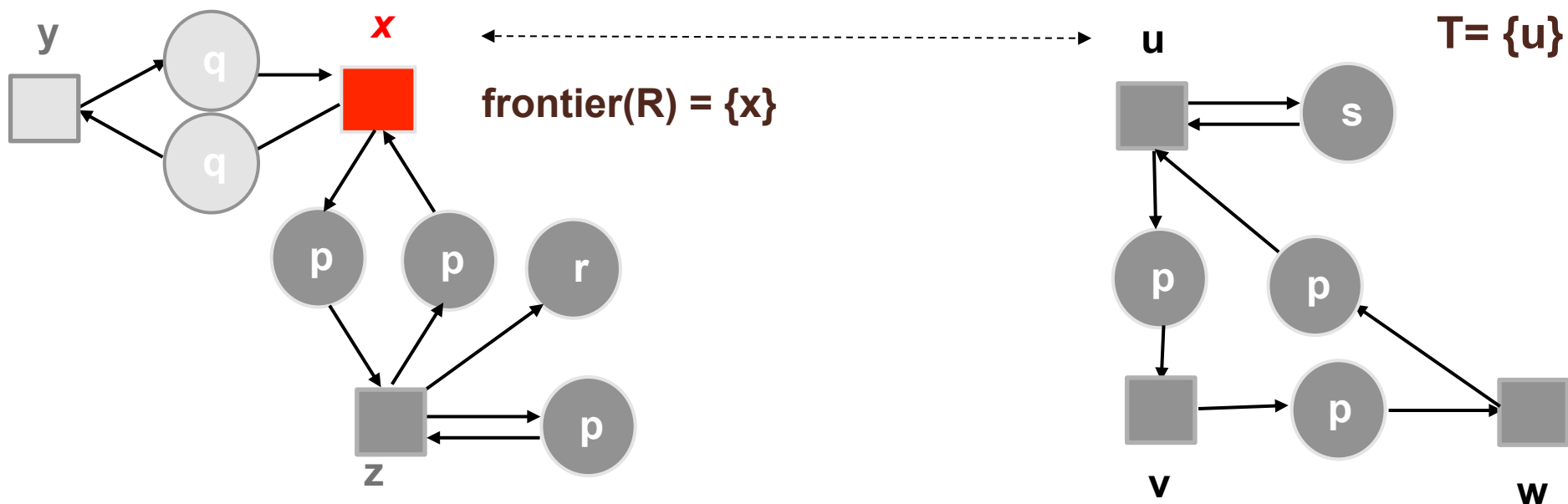


Basic notion for unification in backward chaining, dependency between rules, decomposition of a rule into equivalent rules, ...

Piece-Unification (1)

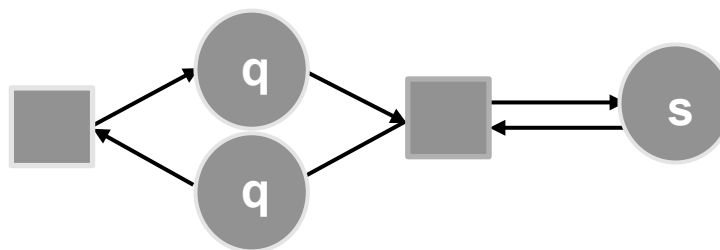
$$R = q(x,y) \wedge q(y,x) \rightarrow p(x,z) \wedge p(z,x) \wedge p(z,z) \wedge r(z)$$

$$Q = p(u,v) \wedge p(v,w) \wedge p(w,u) \wedge s(u,u)$$



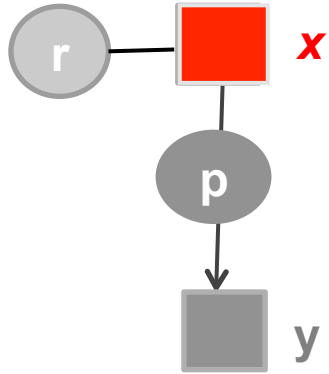
A piece-unifier has to map **at least one piece** of the query to the rule head

New query

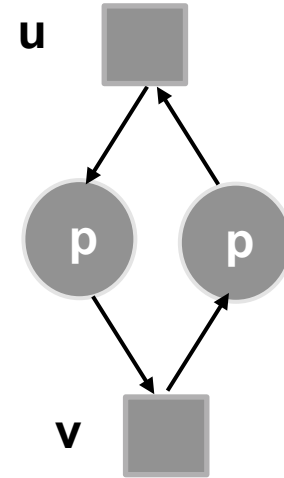


Piece-Unification (2)

$$R = r(x) \rightarrow p(x,y)$$



$$Q = p(u,v) \wedge p(v,u)$$



A piece-unifier has to map **at least one piece** of the query to the rule head

→ *failure*

Piece-Unification (3)

Initially [Salvat M... ICCS 1996] on conceptual graphs

Piece-unifier of a query Q with a rule R :

- a substitution s of $\text{frontier}(R)$ by $\text{frontier}(R) + \text{constants}(Q + \text{head}(R))$
- a homomorphism h from $Q' \subseteq Q$ to $s(\text{head}(R))$
s.t. Q' is a set of *pieces* according to s and h

◆ [Salvat M... 1996]

$F, \mathcal{R} \models Q$ iff there is a sequence of piece-unifications that empties Q
(considering facts as rules with an empty body)

◆ [Baget+ IJCAI 2009] for *fus* existential rules

$F, \mathcal{R} \models Q$ iff one of the piece-based rewritings of Q maps to F

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Union of Decidable Sets of Rules

- Next question:

is the union of two decidable sets of rules still decidable ?

practically:

- can we safely merge several decidable ontologies ?

- can we build a decidable hybrid language from two languages whose semantics can be expressed by decidable subsets of rules ?

- Bad news:

Almost all classes are pairwise incompatible

- Next question:

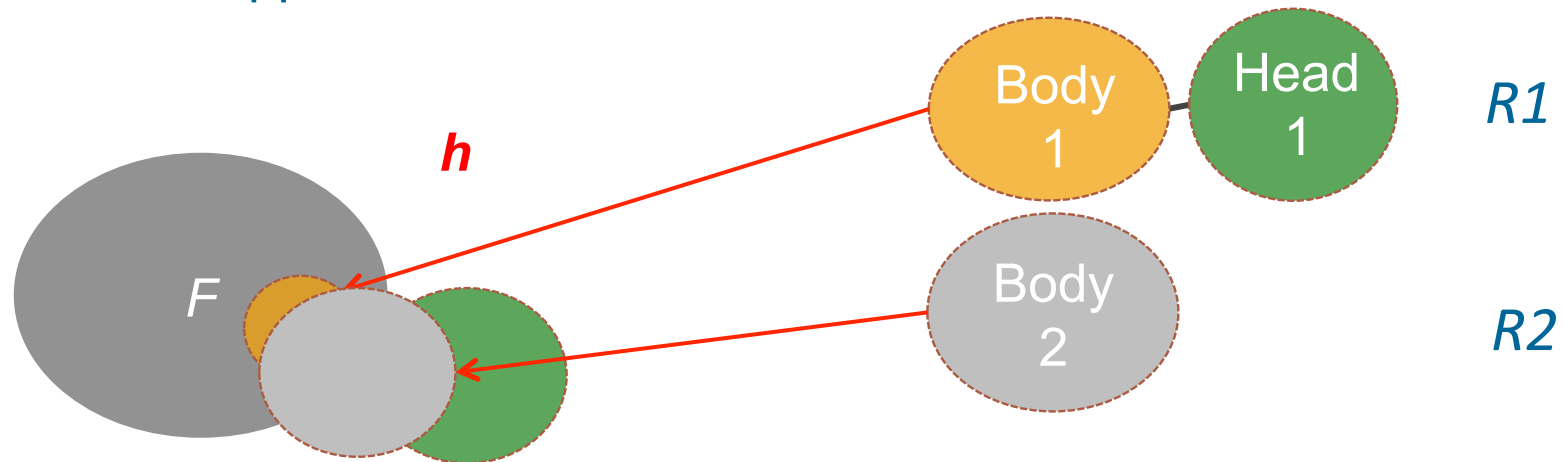
which conditions on the interactions between rules ensure compatibility ?

A tool : the Graph of Rule Dependencies

[Baget KR 2004, Baget+ IJCAI 2009, AIJ 2011]

$R2$ depends on $R1$ if applying $R1$ may trigger a new application of $R2$

i.e., there exists a fact F s.t. $R1$ is applicable to F but $R2$ is not
and there is an application of $R1$ to F leading to F'
s.t. $R2$ is applicable to F'



Effective computation of dependencies with a **piece-unification** test:

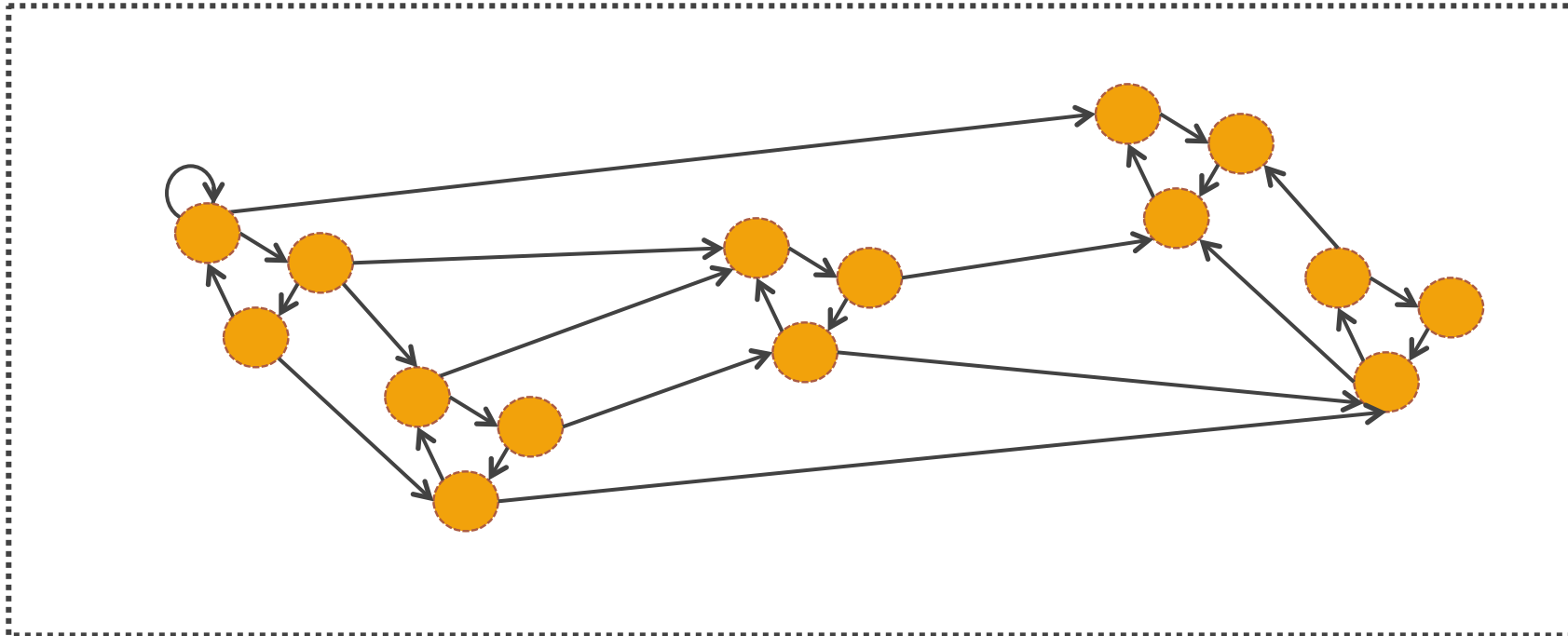
$R2$ depends on $R1$ iff there is a « **piece-unifier** » of $\text{body}(R2)$ with $\text{head}(R1)$

Combining Decidable Classes with the Graph of Rule Dependencies



Rules

$R1 \longrightarrow R2 : R1 \ll \text{may trigger} \gg R2$ ($R2$ depends on $R1$)



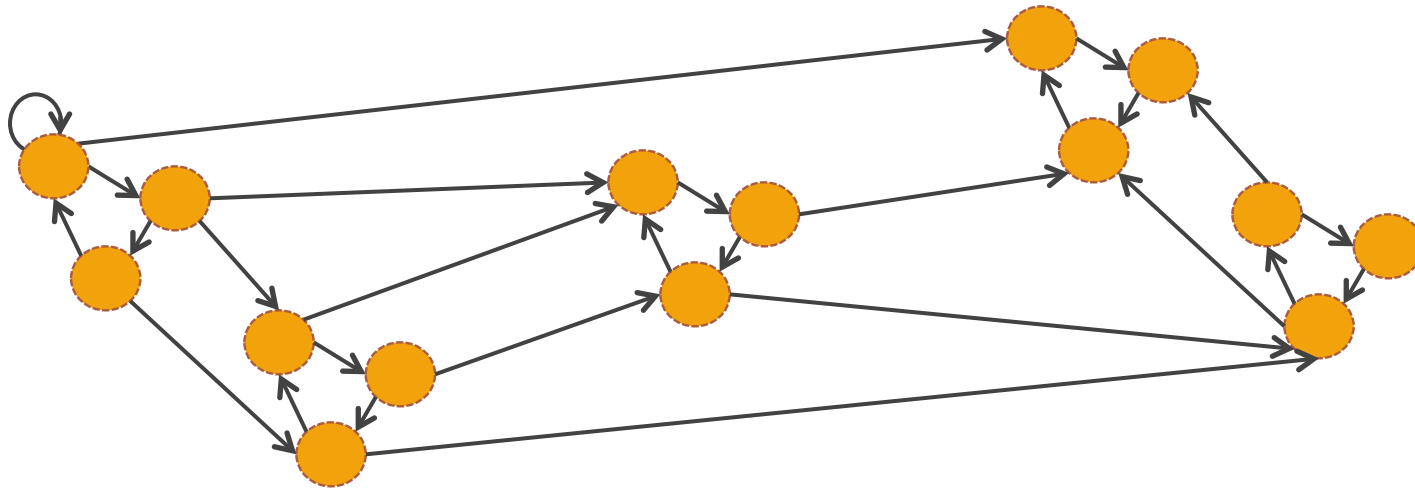
Combining Decidable Classes with the Graph of Rule Dependencies

If $\text{GRD}(\mathcal{R})$ is **without circuit** then \mathcal{R} is both *fes* (thus *bts*) and *fus*

fes = finite fact saturation

fus = finite query rewriting

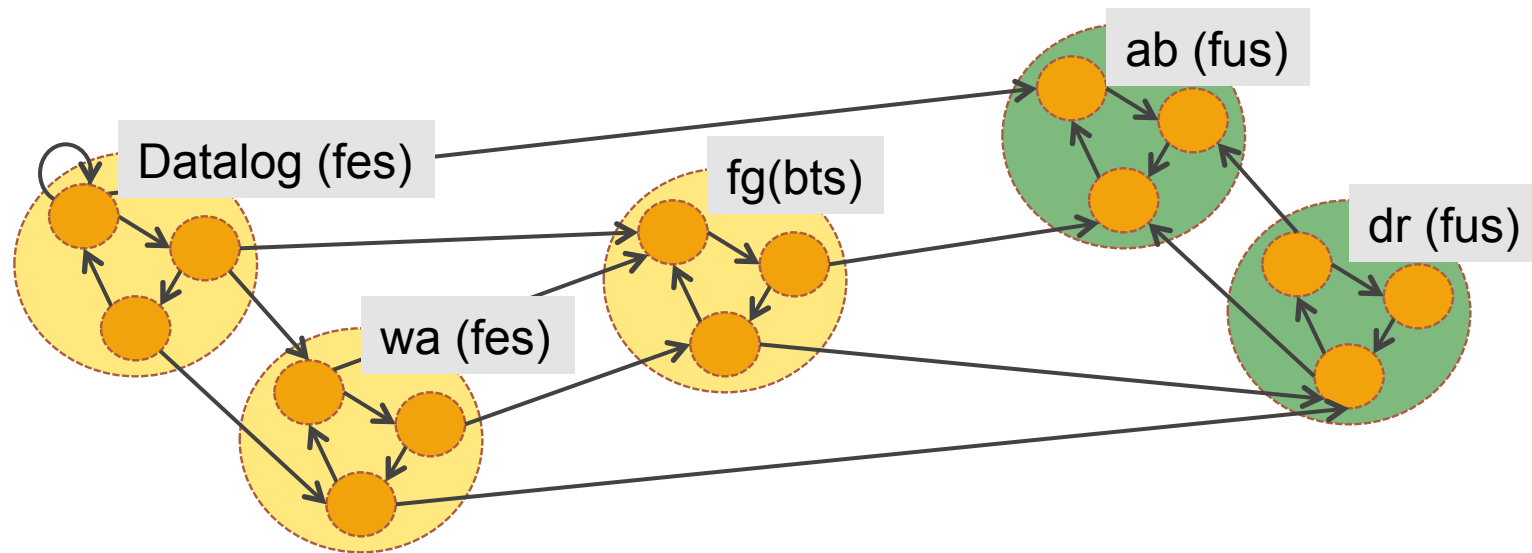
bts = (possibly infinite) tree-shaped saturation



Combining Decidable Classes with the Graph of Rule Dependencies

If all strongly connected components of $\text{GRD}(\mathcal{R})$ are *fes* then \mathcal{R} is *fes* [Baget 2004]

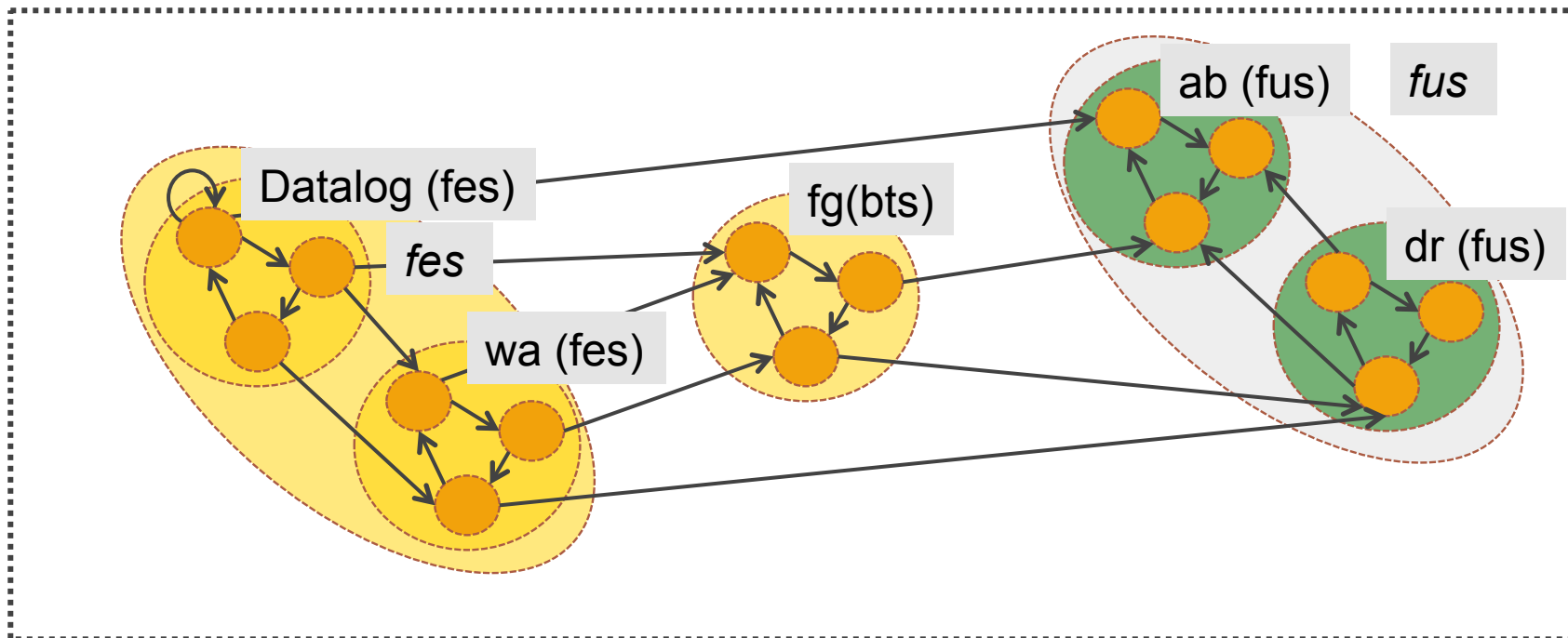
The same holds for *fus* (but not for *bts*)



Combining Decidable Classes with the Graph of Rule Dependencies

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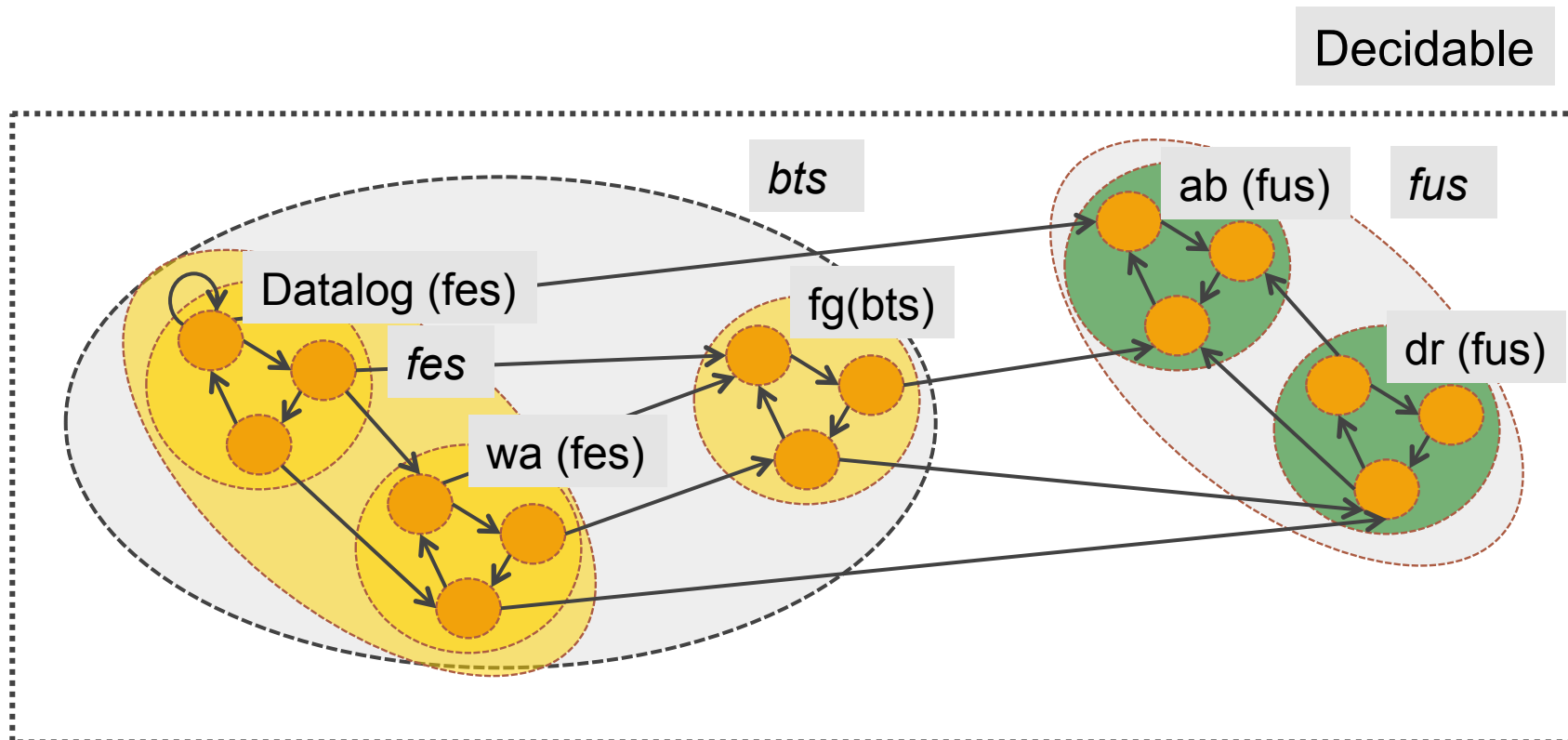
The same holds for *fus* (but not for *bts*)



Combining Decidable Classes with the Graph of Rule Dependencies

Let $\mathcal{R}_1 \setminus \mathcal{R}_2$ be a partition of \mathcal{R} s.t. no rule of \mathcal{R}_1 depends on a rule of \mathcal{R}_2

- If \mathcal{R}_1 is *fes* and \mathcal{R}_2 is *bts*, then \mathcal{R} is *bts*
- If \mathcal{R}_1 is *bts* and \mathcal{R}_2 is *fus*, then \mathcal{R} is decidable



Conclusion

- An emerging rule-based framework for OBDA
 - simple
 - expressive
 - flexible
- Logic-based and Graph-based
- Currently:
 - A quite clear picture of decidable classes and their complexities
 - First implementations – often for very specific subclasses
- Next challenge: scalability