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On the Dynamics Modeling of Free-Floating-Base Articulated Mechanisms and Applications to Humanoid Whole-Body Dynamics and Control

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Abstract—We propose in this paper a general analytic scheme based on Gauss principle of least constraint for the derivation of the Lagrangian dynamics equation of motion of arbitrarily parameterized free-floating-base articulated mechanisms. The free-floating base of the mechanism is a non-actuated rigid object evolving in the 6D Lie group SE(3), the SO(3) component of which can be parameterized using arbitrary coordinate charts with equality constraints, for instance unit quaternions (also known as Euler parameters). This class of systems includes humanoid robots, and the presented formalism is particularly suitable for the whole-body dynamics modeling and control problem of such humanoid systems. Example motions of humanoid in arbitrary contact states with the environment demonstrate the originality of the approach.

I. INTRODUCTION

In [1] an elegant scheme for the derivation of the equation of motion of a humanoid robot is presented. This scheme allows for an analytic expression of the Lagrangian dynamics of humanoid robots, and more generally of any free-floating-base articulated mechanism (such as those studied in space robotics [2]–[5] before being applied to humanoids [6]), that contrasts with other algorithmically-oriented Newton-Euler recursive schemes, e.g. [7]. The analytic scheme directly considers the whole humanoid system, including its free-floating base, as one Lagrangian system with generalized coordinates that are simply time-differentiated once and twice in order to yield the equation of motion, rather than using specific kinematic quantities such as the angular velocity vector and its time derivative, that are not obtained through the time-differentiation of the generalized coordinates of the systems [8] [9] [10].

Advantages of this analytic approach are multiple. First it allows us to model the entire humanoid system without pre-specification of a particular contact state. The same model can thus be used for a humanoid in single-support stance on either foot, in double-support stance on both feet, or in any other stance on hands, knees, or elbows, with or without using the feet for support, see Fig. 1. It also allows for simple-to-implement yet general dynamics model algorithm provided that a sub-algorithm for the computation of basic geometric Jacobian matrices between two bodies of a kinematic chain is available, see [11] [12]. The dynamics model algorithm here just consists in the straightforward implementation of the presented analytic expressions. Finally, as opposed to classical treatments of Lagrangian dynamics, and as stressed by the author in [1], the proposed approach does not require the computation of the Christoffel symbols that need a powerful symbolic (algebraic) computation framework in order to be general enough to encompass more than one particular robot model.

![Fig. 1. Examples of fixed-base mechanism and free-floating-base mechanisms in various contact configurations.](image-url)
Jacobian matrices and push-forward mappings in Section II.
We then apply Gauss principle to derive the equation of
motion in Section III, and summarize our computations in
the algorithm that we present in Section IV. The dynamics
equation is then used in the control scheme of Section V, and
Section VI demonstrates some example humanoid motions
synthesized through this scheme. We conclude the paper in
Section VII. In the Appendix Section, a synthetic view of all
the notations of the paper is provided.

II. PRELIMINARY COMPUTATIONS
Let us consider an articulated mechanism made of \( n + 1 \)
rigid bodies (links) indexed by the variable \( k \) in \( \{ 0, \ldots , n \} \),
the body \( k = 0 \) being the free-floating base link of the
mechanism, and that are articulated through \( m \) revolute or
prismatic joints indexed by the variable \( j \) in \( \{ 1, \ldots , m \} \).
The general contact state of the mechanism is modeled as
follows. On each body \( k \in \{ 0, \ldots , n \} \) a set of \( n_k \) contact
forces \( f_k, 1, \ldots , f_k, n_k \) are applied at the respective body-
frame-expressed contact points \( a_k, 1, \ldots , a_k, n_k \) belonging to
the surface cover of the body \( (n_k = 0 \) if there is no contact
on body \( k \)). We emphasize here the fact that, in the case
of an anthropomorphic system, these contact forces are not
restricted to be applied on the foot bodies.

We then consider the following generalized coordinates of
the system (for convenience, we will use throughout the
paper the same notations as in [1] when applicable). The base
link of the system (for convenience, we will use throughout the
restricted to be applied on the foot bodies.

Finally, we can show that the following fundamental
relation holds, \( \forall u \in \mathbb{R}^3 \):

\[
\frac{\partial (R_0 u)}{\partial \theta_0} = -[(R_0 u) \times] J_{\omega_0} = -R_0 [u \times] R_0^T J_{\omega_0}.
\]

By applying eq. (5) on the canonical basis vectors of \( \mathbb{R}^3 \), we get, denoting \( R_{0,i} \) the \( i^{th} \) column of \( R_0 \) for \( i \in \{ 1, 2, 3 \} \),

\[
\frac{\partial R_{0,i}}{\partial \theta_0} = -[R_{0,i} \times] J_{\omega_0}.
\]

Let now \( (e_1, e_2, e_3) \) be any orthonormal basis of \( \mathbb{R}^3 \), from
the properties of the scalar triple product it follows that, for
any vector \( u \in \mathbb{R}^3 \),

\[
u = (u \times e_1 e_2) e_3 + (u \times e_2 e_3) e_1 + (u \times e_3 e_1) e_2.
\]

This identity is extendible to any \( 3 \times 3 \) matrix \( M \),

\[
M = -e_3 e_1^T[e_1 \times] M - e_2 e_1^T[e_2 \times] M - e_1 e_3^T[e_3 \times] M.
\]

Let \( R_0 \) being an orthogonal matrix, its columns form an
orthonormal basis of \( \mathbb{R}^3 \), thus we can apply eq.(8) to get

\[
J_{\omega_0} = -R_{0,3} R_{0,2}^T [R_{0,1} \times] J_{\omega_0} - R_{0,2} R_{0,1}^T [R_{0,3} \times] J_{\omega_0} - R_{0,1} R_{0,3}^T [R_{0,2} \times] J_{\omega_0}.
\]

Finally, replacing the corresponding expressions from eq.(6),
we get our final expression

\[
J_{\omega_0} = R_{0,3} R_{0,2}^T \frac{\partial R_{0,1}}{\partial \theta_0} + R_{0,2} R_{0,1}^T \frac{\partial R_{0,3}}{\partial \theta_0} + R_{0,1} R_{0,3}^T \frac{\partial R_{0,2}}{\partial \theta_0}.
\]

Note that the latter expression requires the computation of the
derivatives \( \frac{\partial R_{0,i}}{\partial \theta_0} \), which can be easily obtained from the
mapping \( \rho : \theta_0 \rightarrow R_0 \) that can be found in introductory
kinematics textbooks (writing a rotation matrix as a function
of the rotation parameterization). For instance, let us consider

\[
\hat{\mathbf{\omega}}_{\omega_0} = J_{\omega_0} \dot{\theta}_0 \quad {\text{and}} \quad \hat{\mathbf{\omega}}_{\omega_0} = J_{\omega_0} \dot{\theta}_0
\]
a unit quaternion parameterization, \( \theta_0 = (\alpha, \beta, \gamma, \delta) \), then the expression of the mapping \( \rho \) is given as \( R_0 = \rho(\theta_0) = \begin{bmatrix} 2(\alpha^2 + \beta^2) - 1 & 2(\beta\gamma - \alpha\delta) & 2(\beta\delta + \alpha\gamma) \\ 2(\beta\gamma + \alpha\delta) & 2(\alpha^2 + \gamma^2) - 1 & 2(\gamma\delta - \alpha\beta) \\ 2(\beta\delta - \alpha\gamma) & 2(\gamma\delta + \alpha\beta) & 2(\alpha^2 + \delta^2) - 1 \end{bmatrix} \). (11)

And the \( \frac{\partial R_0}{\partial \theta_0} \) can be derived as

\[
\begin{align*}
\frac{\partial R_{0,1}}{\partial \theta_0} &= \begin{bmatrix} 4\alpha & 4\beta & 0 & 0 \\
2\beta & 2\gamma & 2\beta & 2\alpha \\
-2\gamma & 2\delta & -2\alpha & 2\beta 
\end{bmatrix}, \\
\frac{\partial R_{0,2}}{\partial \theta_0} &= \begin{bmatrix} 4\alpha & 0 & 4\gamma & 0 \\
2\beta & 2\gamma & 2\beta & 2\alpha \\
-2\delta & -2\alpha & 2\delta & 2\gamma 
\end{bmatrix}, \\
\frac{\partial R_{0,3}}{\partial \theta_0} &= \begin{bmatrix} 4\alpha & 0 & 0 & 4\delta 
\end{bmatrix}.
\end{align*}
\]

Now that we have computed \( J_{\text{base}}^{\text{total}} \) in eq. (10), we have

\[
J_{\text{base}}^{\text{total}} = R_0^T J_{\text{base}}^{\text{total}}.
\]

To conclude this notation section, we denote \( c_k \) the body-frame-expressed center of mass of the body \( k \), \( m_k \) its mass, and \( I_k(p) \) its inertia matrix expressed at the body-frame-expressed point \( p \). We recall that

\[
\begin{align*}
J_k(c_k) &= I_k(0) - m_k \left( c_k^T c_k \right) 1_{3 \times 3} - c_k \otimes c_k, \\
I_k(p) &= J_k(b) - R_k^T R_k \left( (a-b) \times J_{\text{base}}^{\text{local}} \right).
\end{align*}
\]

This gives as an alternative scheme for computing the \( d \) middle columns in eq.(14).

We also have, for the rotational Jacobian matrices,

\[
\begin{align*}
J_{\text{base}}^{\text{total}} &= \begin{bmatrix} 0_{3 \times 3} & R_k^T R_0 J^{\text{total}}_{\text{base}} | \grave{\theta}_k^{\text{base}} \end{bmatrix} \\
J_{\text{base}}^{\text{total}} &= \begin{bmatrix} 0_{3 \times 3} & R_0 J^{\text{total}}_{\text{base}} | \grave{\theta}_k^{\text{base}} \end{bmatrix}
\end{align*}
\]

\[\text{Remark 1:} \] If we denote \( p^0 = R_k^T (x_k + R_k p - x_0) \) as the base-link-frame-expressed position of \( p \), which does not depend on \( \theta_0 \), then we can show that

\[
-(x_k + R_k p - x_0) \times R_0 J^{\text{total}}_{\text{base}} = -[R_0 p^0 \times] J^{\text{total}}_{\text{base}} = \begin{bmatrix} \frac{\partial R_0}{\partial \theta_0} |_{\theta_0} \end{bmatrix} p^0 |_{i=1, \ldots, d}
\]

(15)

\[\text{Remark 2:} \] Expression (17) is consistent with the composition rule of angular velocities

\[
\begin{align*}
\omega^{\text{local}}_k &= J^{\text{local}}_{\text{base}} \bar{\omega}_k, \\
\omega^{\text{local}}_k &= J^{\text{local}}_{\text{base}} \bar{\omega}_k + R_0 (\dot{\theta}_k^{\text{base}} \times), \\
\omega^{\text{local}}_k &= \bar{\omega}_0 + R_0 \omega^{\text{base}}_k,
\end{align*}
\]

where \( \omega^{\text{base}}_k \) is the base-link-frame-expressed rotational velocity of the body \( k \) with respect to the base link.

\[\text{Remark 3:} \] We can easily verify that the two following transport formulas are consistent with the above derivations of the translational and rotational Jacobian matrices:

\[
\begin{align*}
J_k(a) &= J_k(b) - R_k[(a-b) \times] J^{\text{local}}_{\text{base}}, \\
\dot{J}_k(a) &= \dot{J}_k(b) - R_k^T R_k [(a-b) \times] \dot{J}^{\text{local}}_{\text{base}}.
\end{align*}
\]

(19) (20)

III. GAUSS’ PRINCIPLE WITH MANIFOLD EQUALITY CONSTRAINTS

Following 1, we apply Gauss’ principle of least constraint 13 (also used in robotics for redundant robots in 14, or for simulation in 15) which states that at a given state of the system (given position and velocity), the acceleration of a constrained articulated system deviates the least from the acceleration it would have in the absence of any constraints induced by the presence of the joints linking the bodies. The deviation is measured through the following quantity

\[
D = \sum_{k=0}^{n} \frac{m_k}{2} (\ddot{x}_k - \ddot{\bar{x}}_k) \dot{x}_k (\ddot{x}_k - \ddot{\bar{x}}_k) + \sum_{k=0}^{n} \frac{m_k}{2} (\ddot{\dot{x}}_k - \ddot{\dot{\bar{x}}}_k) T \dot{x}_k (\ddot{x}_k - \ddot{\bar{x}}_k),
\]

(22)

where the underlined quantities are the accelerations of the bodies if they were not linked together.
Without the equality constraints that stem from the parameterization of the orientation of the base link $\theta_0$, this principle would translate into the following optimality condition

$$\frac{\partial D}{\partial \vec{q}} = 0,$$  \hspace{1cm} (23)

which can be rewritten in the form (see [1] for details of the derivation $^2$),

$$M(q)\ddot{q} + N(q, \dot{q})\dot{q} = M(q)\begin{pmatrix} g \\ 0_d \\ 0_m \end{pmatrix} + \begin{pmatrix} 0_d \\ 0_m \end{pmatrix} + \sum_{k=0}^{n} \sum_{i=1}^{n_q} J_{ik}(a_{k,i})^T f_{k,i},$$  \hspace{1cm} (24)

where the expressions of the matrices $M$ and $N$ are given as

$$M(q) = \sum_{k=0}^{n} J_{ik}(c_k)^T m_k J_{ik}(c_k) + J_{ik}^T I_k(c_k) J_{ik}^{\text{real}},$$  \hspace{1cm} (25)

$$N(q, \dot{q}) = \sum_{k=0}^{n} J_{ik}(c_k)^T m_k J_{ik}(c_k) + J_{ik}^T I_k(c_k) J_{ik}^{\text{real}} - J_{ik}^{\text{real}} (I_k(c_k) J_{ik}^{\text{real}} q_k).$$  \hspace{1cm} (26)

However, the choice of a particular manifold (algebraic variety) parameterization of the rotation group $SO(3)$ brings along a set of equality constraints

$$C(q) = 0,$$  \hspace{1cm} (27)

that act on the set of parameters $\theta_0, C : \mathbb{R}^{3+d+m} \rightarrow \mathbb{R}^{d-\dim(SO(3))}$ being a mapping of dimension $\dim(C) = d - \dim(SO(3)) = d - 3$, see Table II (note: in the last row we did not remove redundant (symmetric) constraints for the sake of notation, we also do not take into account $\det(\theta_0) = +1$ given that the initial state satisfies this latter condition and that we stay throughout the continuous motion in the same connected component of $O(3)$).

**TABLE II**

<table>
<thead>
<tr>
<th>$\theta_0$</th>
<th>$\dim(C)$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler quat. (roll-pitch-yaw)</td>
<td>0</td>
<td>$C(q) = \theta_0^T$ $\theta_0 - 1$</td>
</tr>
<tr>
<td>Unit quat. (Euler parameters)</td>
<td>1</td>
<td>$C(q) = \theta_0^T$ $\theta_0 - 1$</td>
</tr>
<tr>
<td>Rotation matrix</td>
<td>6</td>
<td>$C(q) = \theta_0^T$ $\theta_0 - 1_{3x3}$</td>
</tr>
</tbody>
</table>

For instance, let us consider the case of the unit quaternion parameterization. In that case $C$ takes the form

$$C(q) = q^T S^T S q - 1,$$  \hspace{1cm} (28)

where $S$ is a selection matrix that selects the $\theta_0$ component in $q$

$$S = \begin{pmatrix} 0_{d \times 3} \\ 1_{d \times d} \\ 0_{d \times m} \end{pmatrix}.$$  \hspace{1cm} (29)

We then time-differentiate eq.(27) twice to get a constraint on the variable $\ddot{q}$

$$\dddot{C}(q) = q^T S^T S \ddot{q} + q^T S^T S \dot{q} = 0.$$  \hspace{1cm} (30)

The optimality condition eq.(23) becomes now, taking into account the constraint eq.(30),

$$\begin{pmatrix} \frac{\partial D}{\partial \vec{q}} - \frac{\partial \dddot{C}(q)^T}{\partial \vec{q}} \end{pmatrix} \lambda = 0,$$  \hspace{1cm} (31)

where $\lambda \in \mathbb{R}^{d-3}$ is the Lagrange multiplier associated with the constraint eq.(30). The equation of motion eq.(24) thus becomes:

$$M(q)\ddot{q} + N(q, \dot{q})\dot{q} = M(q)\begin{pmatrix} g \\ 0_d \\ 0_m \end{pmatrix} + \begin{pmatrix} 0_d \\ 0_m \end{pmatrix} + \sum_{k=0}^{n} \sum_{i=1}^{n_q} J_{ik}(a_{k,i})^T f_{k,i} + S^T S \dot{q} \lambda,$$  \hspace{1cm} (32)

where the matrices $M$ and $N$ keep the exact same forms as in eq.(25) and eq.(26) respectively, and where we can solve for the Lagrange multiplier $\lambda$ by pre-multiplying eq.(32) by $q^T$ as follows [16]:

$$\lambda = q^T \left[ M(q)\ddot{q} + N(q, \dot{q})\dot{q} - M(q)\begin{pmatrix} g \\ 0_d \\ 0_m \end{pmatrix} + \begin{pmatrix} 0_d \\ 0_m \end{pmatrix} \right] - \sum_{k=0}^{n} \sum_{i=1}^{n_q} J_{ik}(a_{k,i})^T f_{k,i}. $$  \hspace{1cm} (33)

Let us define the following matrix

$$A = 1_{(3+d+m) \times (3+d+m)} - S^T S q q^T.$$  \hspace{1cm} (34)

By re-injecting eq.(33) into eq.(32) we get our final dynamics equation of motion expression

$$A[M(q)\ddot{q} + N(q, \dot{q})\dot{q}] = A \left[ M(q)\begin{pmatrix} g \\ 0_d \\ 0_m \end{pmatrix} + \begin{pmatrix} 0_d \\ 0_m \end{pmatrix} \right] + \sum_{k=0}^{n} \sum_{i=1}^{n_q} J_{ik}(a_{k,i})^T f_{k,i}. $$  \hspace{1cm} (35)

It is worth pointing out that in eq.(35), $\ddot{q} = (\dddot{x}_0, \dddot{\theta}_0, \dddot{\phi})$ is the actual second time-derivative of $q$. In particular, $\theta_0$ is simply obtained by time-differentiating twice the parameters in $\theta_0$.

**IV. DYNAMICS ALGORITHM**

The proposed algorithm here consists simply in carrying out the computations presented in the two previous sections in the order outlined in the following algorithm:

1: Compute eq.(10) then eq.(13)
Remark 4

We improved the motions demonstrated in [17] by applying
the new control scheme eq.(37) to the humanoid robot
HRP-2 [20] with unit-quaternion-parameterized waist link
(the base link) in simulation. Motions on different contact
configurations (feet/hands) were dynamically simulated with-
out needing to re-parameterize the configuration of the robot
depending on the current contact state. See the attached video
and Figs. 2, 3, and 4.

In this video, the sequence of static postures used as input
and displayed before each motion is autonomously planned
offline using the multi-contact stance planning algorithm
in [21] after the user have specified an initial and goal posture
or initial and goal set of contact locations. The tasks \( \tau \)
can be decided online by a finite-state machine as described in [17],
these tasks track the position of the CoM and the overall
posture of the robot in the next static posture whenever a
static posture of the sequence is reached, and an additional
task tracks the position of the limb being moved to a new
contact location.

As for computation time, the problem eq.(37) is solved at
every simulation step, of 1ms, in an average of approximately
7ms, on a 3.06GHz Intel Pentium IV system with 1Gb of
RAM memory.

VI. EXAMPLES

VII. CONCLUSION
same common formulation for the dynamics of the system made of the manipulator and the object.

**APPENDIX: NOTATION TABLE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n + 1 )</td>
<td>Number of bodies</td>
</tr>
<tr>
<td>( k = 0 )</td>
<td>Base link</td>
</tr>
<tr>
<td>( m )</td>
<td>Number of Joints</td>
</tr>
<tr>
<td>( \dot{q} )</td>
<td>Configuration vector</td>
</tr>
<tr>
<td>( \theta_0 )</td>
<td>Orientation of the base link</td>
</tr>
<tr>
<td>( d )</td>
<td>( \dim(\theta_0) )</td>
</tr>
<tr>
<td>( x_k )</td>
<td>Position of the body ( k )</td>
</tr>
<tr>
<td>( \omega_k )</td>
<td>Angular velocity of the body ( k )</td>
</tr>
<tr>
<td>( R_k )</td>
<td>Orientation matrix of the body ( k )</td>
</tr>
<tr>
<td>( \dot{\theta}_k )</td>
<td>Orientation matrix of the body ( k ) expressed in the body frame</td>
</tr>
<tr>
<td>( \Phi_{local} )</td>
<td>Oriented expressed in the base-link frame</td>
</tr>
<tr>
<td>( \Phi_{global} )</td>
<td>Oriented expressed in inertial (global) frame</td>
</tr>
<tr>
<td>( J_{\omega_k} )</td>
<td>Mapping from ( \theta_0 ) to ( \omega_k )</td>
</tr>
<tr>
<td>( J_{R_k} )</td>
<td>Trans. Jac. Mat. of the body ( k ) with resp. to ( q )</td>
</tr>
<tr>
<td>( J_{\dot{\theta}_k} )</td>
<td>Trans. Jac. Mat. of the body ( k ) with resp. to ( \dot{\theta}_k )</td>
</tr>
<tr>
<td>( J_{\dot{\theta}_k} )</td>
<td>Rot. Jac. Mat. of the body ( k ) with resp. to ( \dot{\theta}_k )</td>
</tr>
<tr>
<td>( f_{k,i} )</td>
<td>Contact forces applied on the body ( k )</td>
</tr>
<tr>
<td>( \sigma_{k,i} )</td>
<td>Application point of the contact force ( f_{k,i} )</td>
</tr>
<tr>
<td>( D )</td>
<td>Deviation from the unconstrained motion</td>
</tr>
<tr>
<td>( M, N )</td>
<td>Dynamics quantities</td>
</tr>
<tr>
<td>( C )</td>
<td>Constraint on ( q ) (on ( \theta_0 ))</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Lagrange multiplier</td>
</tr>
<tr>
<td>( S )</td>
<td>Selection matrix (of the ( \theta_0 ) component in ( q ))</td>
</tr>
<tr>
<td>( u )</td>
<td>Actuation torques (control input)</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravity vector</td>
</tr>
<tr>
<td>( \epsilon_k )</td>
<td>Local-frame expressed CoM of the body ( k )</td>
</tr>
<tr>
<td>( m_k )</td>
<td>Mass of the body ( k )</td>
</tr>
<tr>
<td>( I_k )</td>
<td>Inertia matrix of the body ( k )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Control task (objective, feature)</td>
</tr>
<tr>
<td>( U )</td>
<td>Torque limits</td>
</tr>
<tr>
<td>( K )</td>
<td>Linearized Coulomb friction cone</td>
</tr>
<tr>
<td>( A )</td>
<td>Matrix accounting for the equality constraint</td>
</tr>
<tr>
<td>( 0 )</td>
<td>Zero vector or matrix</td>
</tr>
</tbody>
</table>

**ACKNOWLEDGMENT**

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**REFERENCES**


Fig. 2. Motion on single-support and double-support stances on feet.

Fig. 3. Motion on single-support and double-support stances on hands.

Fig. 4. Motion on single-support and double-support stances on feet.