Quantization of reconstruction error with an interval-based algorithm: an experimental comparison

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ABSTRACT

SPECT* image based diagnosis generally consists in comparing the reconstructed activities within two regions of interest. Due to noise in the measured activities, this comparison is subject to instability, mainly because both statistical nature and level of the noise in the reconstructed activities is unknown. In this paper, we experimentally show that an interval valued extension of the classical MLEM algorithm is efficient to estimate this noise level. The experimental settings consist in simulating the acquisition of a phantom composed of three zones having the same shape but different levels of activity. The levels are chosen to simulate usual medical image conditions. We evaluate the ability of the interval-valued reconstruction to quantify the noise level by testing whether or not it allows the association of two zones having the same activity and the differentiation between two zones having different activities. Our experiment shows that the error quantification truly reflects the difficulty in differentiating two zones having very close activity level. Indeed, the method allows a reliable association of two zones having the same activity level, whatever the noise conditions. However, the possibility of differentiating two zones having different levels of activity depends on the signal-to-noise ratio.

Keywords: Error quantization, Interval-valued reconstruction, Experimental setup

1. INTRODUCTION

SPECT is a technique used in nuclear medicine for detecting a certain number of degenerative diseases. Usually, the SPECT image based diagnosis consists in comparing the reconstructed activities in two regions of interest. However, the reliability of this comparison is reduced due to noise in the measures. In fact, due to the reconstruction process, the impact of measurment noise in the reconstructed activities is unknown. Several solutions have been proposed for quantifying the noise in the reconstructed images. Much of the literature (see e.g. Refs. 1, 2 and 3) discuss noise and resolution properties as a function of specific algorithms and iterations. For example, the method proposed by Fessler in Ref. 3 focuses on the noise and resolution properties of an estimator that maximizes a penalized likelihood objective function using an algorithm that have been iterated to a nearly converged solution.

What we propose in this paper is an experimental setting allowing to highlight the ability of an interval-based extension of the conventional MLEM algorithm to achieve a reliable quantification of the reconstruction error due to random variations in the measures.

2. INTERVAL BASED PROJECTION OPERATOR

In Ref. 4, Rico et al. propose an interesting alternative for modeling the projection operators used in the classical reconstruction algorithms. This modeling is based on replacing the discrete Radon transform by a Hough transform. This replacement allows a reliable representation of the uncertainty induced by a poor knowledge in the impulse response of the detectors. It also allows to truly represent the fact that the measurement space is discrete. This representation is obtained by considering a coherent set of sampling models instead of one single sampling model. As shown in Ref. 4, the bounds of the interval valued output projected vector $[S] = [\underline{S}, \overline{S}]$

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*Single Photon Emission Computed Tomography can be easily computed by mean of a Choquet integral. This new projection operator is denoted $\overline{\underline{\mathcal{P}}}(I)$ and the projection operation is written:

$$[S] = \overline{\underline{\mathcal{P}}}(I) = [\underline{S}, \overline{S}] \tag{1}$$

where I is the activity vector. $\underline{S}$ and $\overline{S}$ are respectively the lower and upper values of $[S]$. The back-projection operator $\overline{\underline{\mathcal{B}}}$ is deduced from the same geometric model. Both projection and back-projection operators could be easily extended to interval-valued input vectors. As shown in Ref. 5, the statistical variations of the input variable has an impact on the width of the output interval. Moreover, it has been shown that the length of the output intervals are strongly correlated with the variance of the noise in the projected image. This property is explained in Ref. 6.

3. A NON-ADDITIVE INTERVAL BASED EXPECTATION MAXIMIZATION (NIBEM)

The goal is to preserve the quantification properties obtained by the projection and back-projection operators by extending the reconstruction technique MLEM. This technique uses a probabilistic formulation of the reconstructed problem taking into account for a Poisson statistics. At each iteration, the MLEM algorithm updates an image to produce another image whose agreement with the measurement is increased. This process is obtained by computing a ratio between the measured and the estimated projections. The ratio is then back-projected and multiplied by the image reconstructed at the previous iteration. NIBEM is the interval based extension of MLEM method when considering the interval-valued operators $\overline{\underline{\mathcal{P}}}$ and $\overline{\underline{\mathcal{B}}}$. It consists in computing an interval-valued image that is in agreement with the measured activity values. This process is very similar to MLEM algorithm but accounts for the fact that the projection and back-projection operators are intervals-valued. In fact, the arithmetics operators are replaced by their interval-valued counterpart.

Let $[A] = [\underline{A}, \overline{A}]$ and $[B] = [\underline{B}, \overline{B}]$ be two positive real intervals-valued vectors. The extensions of multiplication and division to intervals can be achieved in different ways as described in Ref. 7:

- The Minkowski multiplication and division are defined respectively by $[\underline{A}, \overline{A}] \otimes [\underline{B}, \overline{B}] = [\underline{A} \times \underline{B}, \overline{A} \times \overline{B}]$ and $[\underline{A}, \overline{A}] \oslash [\underline{B}, \overline{B}] = [\underline{A} \div \underline{B}, \overline{A} \div \overline{B}]$,
- The dual multiplication is defined by $[\underline{A}, \overline{A}] \boxtimes [\underline{B}, \overline{B}] = [\min(\underline{A} \times \overline{B}, \overline{A} \times \underline{B}), \max(\underline{A} \times \overline{B}, \overline{A} \times \underline{B})]$

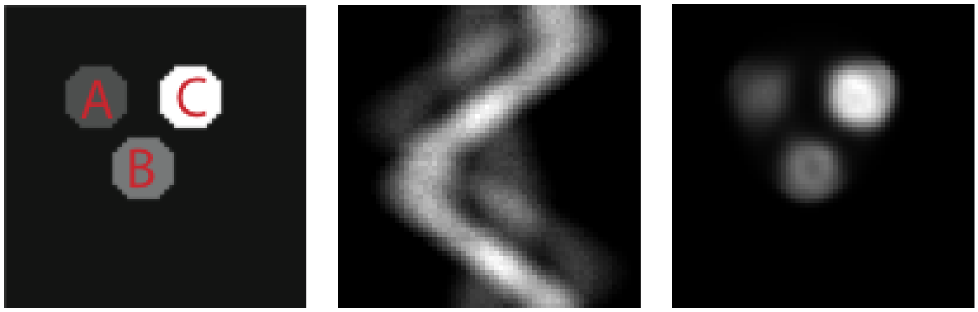
Where $\times$, $\div$ and min are term by term operators. The NIBEM algorithm is given by:

$$[I^{i+1}] = [I^i] \boxtimes \overline{\underline{\mathcal{B}}}(S \oslash \overline{\underline{\mathcal{P}}}([I^i])) \tag{2}$$

where i is the iteration number and $[I^i]$ is the reconstructed interval-valued at the iteration i.

4. EXPERIMENT

The goal of this experiment is to evaluate the ability of the NIBEM algorithm to estimate the noise in the reconstructed activity. This experiment consists in simulating the acquisition (see Fig. 1(b)) of a phantom composed of three zones (A, B and C) (see Fig. 1(a)) having the same shape but different levels of activity. The activity within each zone is uniformly distributed. The background has no activity. Two experiments are achieved by choosing different activity ratio between each zone. In the first experiment, we choose a ratio corresponding to acquisitions leading to visual difficulties in the diagnosis (the activity levels are too close to be differentiated). For this first experiment, the activity ratios are chosen to be $\frac{5}{4}$ between A and B and $\frac{3}{2}$ between A and C. For the second experiment, the ratio are chosen to be closer to acceptable experimental conditions, i.e. leading to a non ambiguous diagnosis. Within this second experiment, the ratios are $\frac{7}{3}$ between A and B and 5 between A and C. A is the reference level that defines the signal to noise ratio since each acquisition is corrupted by a Poisson noise. Three reference levels of activities (low, medium and high) have been chosen to simulate real medical settings. For each simulated acquisition, the noise level is characterized by a coefficient variation (CV) computed as the inverse of the square root of the mean count rate in projection data that differs from zero. The noise level in the acquisitions correspond to a CV ranging between 4-57%. Thirty acquisitions have been simulated for each activity level. Every acquisition is reconstructed with the iteration reconstruction process till the convergence of the median image (in Euclidian norm)(see Fig. 1(c)) to the original image. The median image is computed as being the real valued image whose pixel activity is the median of the interval valued activity of



(a) (b) (c)

Figure 1. Shows the original image that consists of three zones A, B and C with different levels of activity (a), computed noisy projection (b) and reconstructed median image with NIBEM algorithm (c).

the corresponding pixel in the reconstructed image.

Due to the phantom, the reconstructed interval-valued activity of each pixel within a zone can be considered as a realization of the same random variable. An estimate of this activity can be represented by simply averaging every interval-valued activity in the considered zone. Then, comparing two interval-valued activities can be reduced to detect the intersection of the two mean intervals. When the intersection between two interval-valued activities is empty, the activities are considered different. If the intersection is not empty, the fact that the activities are equal cannot be rejected. To implement the comparison, we manually select the three zones in the reconstructed images. For the image j, the zones A, B and C are denoted respectively A_j, B_j and C_j. $\underline{\overline{F}}(A_j) = [\underline{F}(A_j), \overline{F}(A_j)]$ is the mean interval-valued activity of the zone A_j. The same acts for zones B and C. Two different scores S_a and S_d have been used for evaluating the ability of the interval-valued reconstruction to quantify the noise level by testing whether or not it allows to associate two zones having the same activity and to differentiate two zones having different activity. S_a rates the correct associations between two similar zones of activities and S_d rates the correct differentiations between two different zones of activities.

Table 1 and Table 2 show the percentage that corresponds to scores S_d and S_a obtained for different levels of activity. The obtained scores in Table 1 correspond to the chosen ratio between each zone that leads to a visual difficulty in the diagnosis. As in Table 2, the obtained scores correspond to ratios that leads to a non ambiguous diagnosis. As shown in Table 1, the differentiation between zones having very close activity levels is quite difficult. As in Table 2, a total differentiation between the reconstructed activities has been obtained and the differentiation score decreases slightly for a high noise level. In both experiments, the score S_a shows a reliable association of two zones having the same activity level regardless of the noise conditions.

Table 1. S_d and S_a versus CV

	$CV = 4\%$	$CV = 13\%$	$CV = 40\%$
S_d	33%	33%	0%
S_a	100%	100%	100%

Table 2. S_d and S_a versus CV

	$CV = 7\%$	$CV = 17\%$	$CV = 57\%$
S_d	100%	100%	98%
S_a	100%	100%	100%

5. CONCLUSION

This paper has proposed an experimental settings for evaluating the ability of associating and differentiating different levels of activity. From these experiments, it appears that the error quantification reflects the difficulty of reliably differentiating two zones having very close activity levels. Indeed, the method allows a reliable association of two zones having the same activity level, whatever the noise conditions are. However, the possibility of differentiating two zones having different levels of activity depends on the signal-to-noise ratio.
Future work should focus on comparing the NIBEM algorithm and a reference technique developed by Fessler et al. in Ref. 3. For this comparison to be as reliable as possible, it will be applied on a real realization of the proposed phantom.

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