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Optimal Pattern Generator For Dynamic Walking in humanoid Robotics

David Galdeano, Ahmed Chemori, Sebastien Krut and Philippe Fraisse
LIRMM, Université Montpellier 2
LIRMM - UMR 5506 - CC 477, 161 rue Ada, 34095 Montpellier Cedex 5, France.
Email: galdeano(chemori,krut,fraisse)@lirmm.fr

***Abstract*—This paper deals with an optimal ZMP based pattern generator for stable dynamic walking. The proposed method is based on a Three-Mass Linear Inverted Pendulum Model (3MLIPM), used as a simplified dynamics of the biped robot. The 3MLIPM simplifies the biped robot as a three point masses and two-link system. A ZMP based criterion is then used in an optimization problem whose solution gives the best values of the model's parameters w.r.t. dynamic walking stability. Numerical simulations are presented to show the effectiveness of the proposed optimal pattern generator.**

I. Introduction

A humanoid robot is a robot with an appearance based on that of the human body. Humanoid robot walking is a very challenging field of research due to the complexity of synthesizing stable walking gaits for these systems. Within this field, humanoid robot control needs sophisticated control schemes to deal with their complexity including:

- nonlinear and high-order dynamics,
- variable structure model according to the different phases of the walking cycle,
- contact constraints with the ground that should be managed,
- the hybrid character of the dynamics due to rigid impacts between the robot's foot and the ground,
- stability during walking that should be ensured.

Most of the proposed control schemes in the literature are based on the use of reference trajectories that should be followed in real time. This fact shows the importance of a pattern generator in humanoid walking control. Several types of pattern generators have then been proposed. However, those who guarantee *a-priori* stability are often based on one of the following stabilization criteria:

- The Center Of Mass (COM)
- The Zero Moment Point (ZMP)
- The Foot Rotation Indicator (FRI)

The COM [1] is the mean location of all masses of the robot links. It is usually used as a static stability criterion.

The ZMP [2], [3] is the point of junction between the vertical reaction forces and the ground. It is the most used dynamic stability criterion.

The FRI [4] is a point on the foot/ground-contact surface where the net ground-reaction force would have to act for the aim to keep the foot stationary. It is also used as a dynamic stability criterion.

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II. Related Work

In the literature several methods have been proposed for generation of walking trajectories in humanoid robotics.

One interesting method is the so called Inverted Pendulum Model (IPM) [5], [6] that considers the robot as a single point mass and massless legs. This method simplifies the dynamics of the robot to an inverted pendulum with a pointwise mass linked by a telescopic leg to a spherical ground/leg joint.

The Linear Inverted Pendulum Model (LIPM) [7], [8] is an extension of the IPM were the height of the torso is kept constant leading to a more natural movement.

The IPM considers the humanoid robot as an inverted pendulum with a pointwise mass and ignores the dynamics of the legs. This fact can involve a loss of walking stability since the considered model is not sufficiently accurate. For instance, this is the case when the robot legs are heavy or without a torso (case of our robot SHERPA). In order to obtain a more accurate model for these walking robots, it would be necessary to consider the dynamics of the legs by using a preview control to correct the error between the IPM and the real robot as in [9] or to use a multiple pointwise mass model instead of considering a one pointwise mass model.

For instance, a Two-Mass Inverted Pendulum Model (TMIPM) and Multiple Mass Inverted Pendulum Model (MMIPM) extending the IPM were studied in [10]. In the MMIPM, the considered model is composed by more than one mass. For instance these masses are located at the hip and along the swinging leg in the case of a two-mass model.

The Gravity-Compensated Inverted Pendulum Model (GCIPM) [11] uses one mass to represent the body of the robot and an additional mass to represent the swing leg.

A three-mass model using the concept of ZMP has been studied in [12], however it doesn't use any LIPM to generate trajectories and the torso of the biped robot moves up and down.

A generation of walking trajectories using a three pointwise mass model to compute reference torque instead of ZMP trajectories has been proposed in [13].

An other three-mass model using the inverted pendulum concept is that proposed in [14]. However, the geometrical parameters, such as position and location of masses, have been chosen arbitrarily. One good idea would be to tune this parameters at their best values in order to enhance the walking stability of the walking robot, and this is the scope of the present paper.

The proposed solution is an extension of the method proposed in [14] to deal with dynamic stability of walking in

the generated trajectories, as well as change of direction during walking. The robot's dynamics is approximated by a Three-Mass Linear Inverted Pendulum. The hip and feet trajectories are generated by the movement of masses. The walking movement of the robot is generated in the sagittal and frontal planes separately. The joint trajectories are computed using inverse kinematics of the biped robot. In order to ensure a stable dynamic walking, the parameters of the model must be well tuned. The best way is to perform this tuning through an optimization of the generated joints' trajectories to minimize the the ZMP excursion within the footprint. The second contribution of this paper is the online change of walking direction, where new optimal trajectories are generated to ensure *a-priori* stability during walking while turning.

This paper is organized as follows: in next section, the prototype of our demonstrator SHERPA is introduced. Section IV introduces the simplified model that will be used in the generation of reference trajectories, as well as the pattern generator proposed in [14]. In section V, an extension of this method is presented, where our contributions are highlighted. Numerical simulations are presented and discussed in Section VI where the effectiveness of the proposed method is shown and compared with the original method. The paper ends with some concluding remarks.

III. Description of Sherpa prototype

The SHERPA walking robot (cf. Fig. 1) is a French biped robot developed at LIRMM laboratory (`http://www.lirmm.fr`) within the framework of the national project ANR-BLANC SHERPA. This robot is built to carry loads while walking in a human environment [15].

Fig. 1. The SHERPA biped robot

The SHERPA biped robot is composed of one waist linking two legs together. Each leg has six degrees of freedom (dof), and the robot is equiped with 12 actuators, which is enough to reproduce a human walking.

These dofs are distributed on different articulations of the robot as follows: three dofs at the hip, one dof at the knee and two dofs at the ankle.

The geometric parameters of SHERPA are summarized in TABLE I.

The computation of the ZMP and CoM of the robot is based on the formalism presented in [16], in which the ZMP evaluation uses the angular momentum.

Parameter	Description	Value
L_{shin}	Length of shin	0.51 m
L_{thigh}	Length of thigh	0.5033 m
L_s	Step width	0.31 m
$mass$	Weight of the robot	47.1 kg

TABLE I
Geometric parameters of SHERPA biped robot

IV. Three-Mass Linear Inverted Pendulum Model

The three-mass Linear Inverted Pendulum Model (3MLIPM) as introduced in [14] simplifies the biped robot to a three links system (cf. Fig. 2) with a pointwise mass on each link. The three masses represent the torso and the two legs, unlike the single mass model (as in IPM or LIPM) where a unique mass is located at the hip of the robot. The links are connected together at the hip.

This model is more accurate than the LIPM, especially for a biped robot without torso, where the position of the CoM can be very different from the hip position.

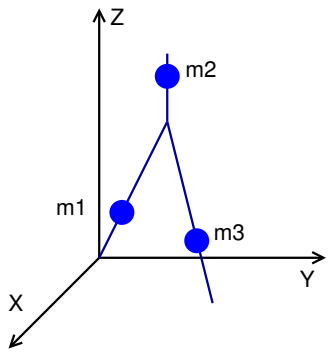

Fig. 2. Graphical representation of the Three-Mass Linear Inverted Pendulum Model

From Fig. 2, the equation of moment applied on the supporting ankle can be formulated as follows:

$$\tau_x = \sum_{i=1}^{3} m_i(g + \ddot{z}_i)y_i - \sum_{i=1}^{3} m_i\ddot{y}_i z_i \tag{1}$$

$$\tau_y = \sum_{i=1}^{3} m_i\ddot{x}_i z_i - \sum_{i=1}^{3} m_i(g + \ddot{z}_i)x_i \tag{2}$$

where τ_x and τ_y are the torques applied on the ankle, m_i represents the value of the ith pointwise mass, (x_i, y_i, z_i) are the Cartesian positions of the ith pointwise mass ($i = 1, 2$ or 3 represents respectively the stance leg, the torso and the swing leg).

For simplification, the height of the masses can be assumed constant and the torque applied on the supporting ankle is assumed to be zero [14]. With these assumptions, (1) and (2) can be simplified into:

$$\sum_{i=1}^{3} m_i\ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i \quad \text{and} \quad \sum_{i=1}^{3} m_i\ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i \tag{3}$$

These equations are decoupled, so the movement can be generated in the sagittal and lateral planes separately.

A. Assumptions and notations

The following assumptions are considered to simplify calculations [14]:

- The ground is flat and horizontal.
- The height variation of each mass can be neglected.
- The double support phase is considered instantaneous.
- There is no energy loss during impact.
- The standing foot has no overturn when touching and leaving the ground.
- The swing foot is parallel to the ground.
- The torso is upright.

The following notations are used:

- $2T$: The whole stepping cycle.
- $2D_s$: The steeping length.
- L_s: The width of a step.

B. Motion in sagittal plane

The movement in the sagittal plane and the associated notations are illustrated in Fig. 3.

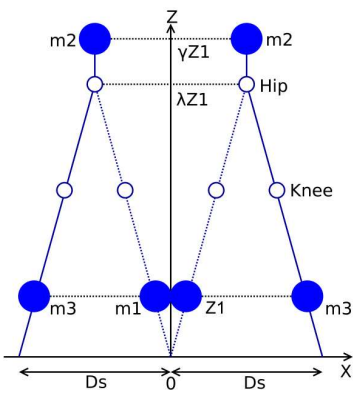

Fig. 3. Illustration of 3MLIPM in the sagittal plane

With the above assumptions, heights and masses can be normalized as follow:

$$z_2 = \gamma z_1 \quad z_3 = z_1 \quad x_2 = \lambda x_1 \quad m_2 = k m_1 \quad m_3 = m_1 \tag{4}$$

were γ, λ and k are parameters extracted from the robot's model for computing. With (4), (3) can be simplified into:

$$b\ddot{x}_1 + d\ddot{x}_3 = a x_1 + x_3 \tag{5}$$

$$a = k\lambda + 1 \quad , \quad b = \frac{z_1}{g}(1 + k\gamma\lambda) \quad , \quad d = \frac{z_1}{g} \tag{6}$$

In order to generate walking gaits, a trajectory for the swing foot (m_3) is necessary. If the trajectory of this last one is known, then those of the two other masses can be computed. According to [14], a sinusoidal function is chosen to represent the trajectory of the mass m_3, as follows:

$$x_3 = k\cos(\omega t + \phi) \tag{7}$$

were parameters ϕ and k are given in (11). Here, ω is not the step frequency, it's a parameter that must be chosen to keep the foot position and velocity positive in order to ensure a forward movement of the foot during the step.

Using (5) and (7), x_1 can be expressed by:

$$x_1 = C_1\cos(\omega t + \phi) + C_2 e^{\sqrt{\frac{a}{b}}t} + C_3 e^{-\sqrt{\frac{a}{b}}t} \tag{8}$$

With the above assumptions given in sectionIV-A, initial conditions can be fixed as:

$$x_1(0) = -\frac{D_s}{2\lambda} \quad , \quad x_3(0) = -\frac{D_s}{2\lambda}(2\lambda - 1) \tag{9}$$

$$x_1(\frac{T}{2}) = 0 \quad , \quad x_3(\frac{T}{2}) = 0 \tag{10}$$

The equations' coefficients can be found using initial conditions:

$$\phi = -\frac{\pi}{2} - \frac{T}{2}\omega \quad , \quad k = -\frac{D_s(2\lambda - 1)}{2\lambda\cos(\phi)} \tag{11}$$

$$C_1 = -\frac{k(1 + d\omega^2)}{a + b\omega^2} \qquad C_3 = -e^{\sqrt{\frac{a}{b}}t} C_2 \tag{12}$$

$$C_2 = \frac{1}{1 - e^{\sqrt{\frac{a}{b}}t}}(-\frac{D_s}{2\lambda} - C_1\cos(\phi)) \tag{13}$$

With all pointwise masses trajectories computed, the trajectories of hip and ankle can then be determined using geometrical restrictions from Fig. 3.

$$\begin{aligned} x_{st}(t) &= 0 \ \ for\ 0 \le t \le T, \ \ x_h(t) = \lambda x_1(t) \ \ for\ 0 \le t \le T \\ z_{st}(t) &= 0 \ \ for\ 0 \le t \le T, \ \ z_h(t) = \lambda z_1 \qquad for\ 0 \le t \le T \end{aligned} \tag{14}$$

where x_{st}, z_{st} are the positions of the stance foot along x and z axis respectively, and x_h, z_h are the positions of the hip along x and z axis respectively.

The vertical position of the swing foot needs to be higher than the floor during walking. A sinusoidal shape is then proposed for the swing foot trajectory along the z axis. Due to previous assumptions, the influence of this change of height is neglected in computations.

$$\begin{aligned} x_{sw}(t) &= \frac{\lambda x_3(t) - \lambda x_h(t)}{\lambda - 1} \qquad for\ 0 \le t \le T \\ z_{sw}(t) &= \frac{\sqrt{2}H_s}{2}(\sin(\frac{2\pi}{T}t - \frac{\pi}{2}) + 1)^{0.5} \quad for\ 0 \le t \le T \end{aligned} \tag{15}$$

where x_{sw}, z_{sw} are the positions of the swing foot along x and z axis respectively, and H_s is the maximal height of the swing foot during walking.

C. Motion in frontal plane

The movement in the frontal plane and the associated notations are illustrated in Fig. 4 below.

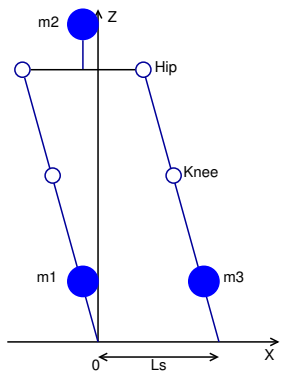

Fig. 4. Illustration of 3MLIPM in the frontal plane

The 3MLIPM equations are now applied in the frontal plane.

$$u\ddot{y}_1 - vy_1 = w$$
$$u = (2+k\lambda\gamma)\frac{z_1}{g} \ , \ v = (2+k\lambda) \ , \ w = (L_s + k\frac{L_s}{2}) \tag{16}$$

These equations are solved considering the following initial conditions:

$$y_1(0) = 0 \quad , \quad \dot{y}_1(\frac{T}{2}) = 0 \quad , \quad y_1(T) = 0 \tag{17}$$

The trajectory of the mass m_1 along the y axis is then computed and expressed by:

$$y_1(t) = C_1 e^{\sqrt{\frac{v}{u}}t} + C_2 e^{-\sqrt{\frac{v}{u}}t} - \frac{w}{v}$$
$$C_1 = \frac{w}{v}\frac{1-e^{\sqrt{\frac{v}{u}}T}}{1-e^{2\sqrt{\frac{v}{u}}T}} \quad , \quad C_2 = e^{\sqrt{\frac{v}{u}}t}C_1 \tag{18}$$

From the trajectory of m_1, the trajectories of m_2 and m_3 can be determined using geometrical restrictions illustrated on Fig. 4. From these trajectories and the inverse kinematics of the robot, the trajectories of joints can then easily be computed.

D. Adaptation of the general model for SHERPA robot

The SHERPA robot is a biped robot without a torso (cf. Fig. 1). In order to simplify the model, the torso pointwise mass is set on the hip. This change results in $\gamma = \lambda$.

The basic principle of proposed method in [14] and summarized above uses geometrical parameters of the robot but doesn't give a method to calculate z_1 and m_1. In order to take into account the dynamic stability of the resulting walking trajectories, these parameters are proposed to be tuned using optimization to enhance this walking performance.

V. Reference Trajectories Optimization

The proposed pattern generator in [14] computes joint reference trajectories using arbitrary defined parameters. This can be improved by choosing the best values of these parameters to ensure the dynamic walking stability. The position of the Zero Moment Point (ZMP) [2] is then used during the walking as an indicator of stability.

A. Optimization w.r.t. dynamic walking stability

The first main contribution of this paper is to improve the pattern generator proposed in [14] by using an optimization criterion in order to find the best values for z_1 and m_1 to enhance the dynamic walking stability.

In order to improve such a stability, the ZMP corresponding to the generated joint trajectories should be computed with a full robot dynamics and compared with the desired ZMP, set to the center of the stance foot. This chosen desired position corresponds to a maximum stability margins.

The following objective function is then proposed to be optimized w.r.t. parameters z_1 and m_1:

$$\begin{bmatrix}\hat{z}_1\\ \hat{m}_1\end{bmatrix} = \underset{\begin{bmatrix}z_1\\ m_1\end{bmatrix}}{ArgMinMax}\left(\sqrt{\frac{1}{\alpha}(x_{zmp}-x_{dzmp})^2+\frac{1}{\beta}(y_{zmp}-y_{dzmp})^2}\right) \tag{19}$$

where x_{zmp} and y_{zmp} are the positions of computed ZMP, x_{dzmp} and y_{dzmp} are those of the desired one. z_1 and m_1 are the optimization parameters, α and β are the length and the width of the foot (i.e. along x axis and y axis respectively, cf Fig. 5).

By using a normalized deviation of the ZMP with respect to the center of the footprint of the stance foot as cost function, the objective function computes the mass distribution to ensure the largest stability margins.

B. Change of direction and stability optimization

One classical feature of pattern generator is to allow the biped robot to change walking direction like in [17]. As proposed in [14], the 3MLIPM pattern generator is designed to make the robot walk in a straight line, our second contribution is then to modify the original 3MLIPM to allow a change of direction while the biped robot is walking. The change of direction is obtained by applying a rotation at the hip of the stance leg, as follows:

$$\Omega = -\frac{R}{2}\cos(\frac{\pi t}{T}) \tag{20}$$

with Ω is the angle of rotation and R is its amplitude.

The change of direction alters the stability of the biped robot. Again, the proposed optimization criterion (19) is used to improve dynamic walking stability. The solution of this optimization problem gives the best values for z_1 and m_1 which allow a better dynamic stability of the robot walking and turning.

C. Stability margins

The stability margins are defined as illustrated in Fig. 5.

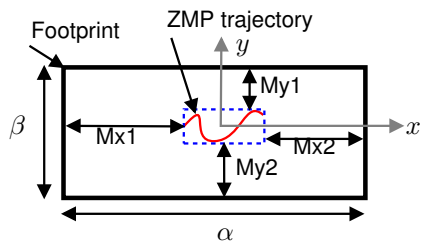

Fig. 5. ZMP displacement under the footprints and stability margins

The dashed interior rectangle explicit the boundaries of the ZMP displacements. Based on Fig. 5, the stability margins can be computed as follows:

$$Mx_1 = \frac{\alpha}{2} + Min(dZMP_x(t)), \ Mx_2 = \frac{\alpha}{2} - Max(dZMP_x(t))$$
$$My_1 = \frac{\beta}{2} + Min(dZMP_y(t)), \ My_2 = \frac{\beta}{2} - Max(dZMP_y(t))$$
$$for\ all \quad t \in [ts_i \ , \ ts_f]$$
$$Mx = Min(Mx_1, Mx_2), \ My = Min(My_1, My_2) \tag{21}$$

were $dZMP_x(t)$ and $dZMP_y(t)$ are the deviation of the ZMP trajectory with respect to the center of the stance foot along x axis and y axis respectively, ts_i and ts_f are times of landing and lift-off of the stance foot respectively.

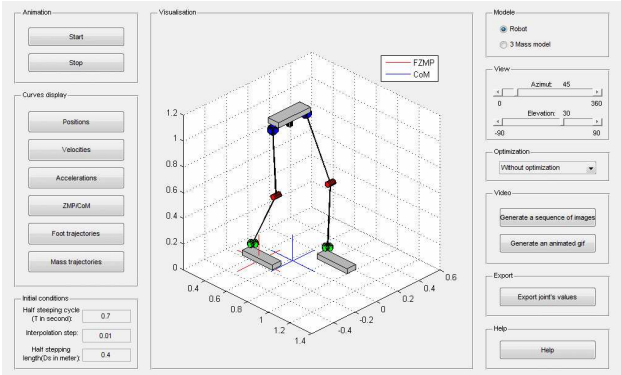

Fig. 6. Graphical User Interface of SHERPA biped robot simulator

VI. Simulation Result

A simulator for SHERPA biped robot was developed using the Graphical User Interface of Matlab^{TM}software. The graphical interface of this simulator is shown in Fig. 6.

This interface enables to tune some parameters of the robot as well as those of the optimization criterion. A graphical animation of the robot or the three mass model can also be displayed to show the obtained movements.

The parameters z_1 and m_1 are computed by optimization, the other parameters are constant and summarized in TABLE II.

Parameter	Description	Value
m_1	Weight of one leg	Calculated
m_2	Weight of the torso	$mass - 2m_1$
z_1	Height of the mass m_1	Calculated
z_2	Height of the hip	$(L_{shin} + L_{thigh}) \cdot 0.95$
T	Step time	0.7 s
D_s	Step length	0.4 m
ω	Calculus variable	$\pi/(5 \cdot T)$
$\lambda = \gamma$	Calculus variable	z_2/z_1

TABLE II
Parameters of SHERPA biped robot

Using the developed simulator, three simulation scenarios are proposed to validate the proposed optimal pattern generator, namely:

- optimal trajectories generation for straight walking,
- trajectories generation for walking with change of direction without optimisation,
- trajectories generation for walking with change of direction and optimization.

These simulations will be detailed in the following.

A. Simulation 1: Straight walking

The optimization criterion given in (19) is used to find the best values for z_1 and m_1 using the `fminsearch` algorithm proposed within Matlab software. The obtained optimal solution is summarized in table III.

Parameter	Without optimization	With optimization
z_1	0.6 m	0.2598 m
m_1	6 kg	0.4442 kg

TABLE III
Resulting optimal mass distribution

The obtained simulation results are illustrated through Figs. 7-10.

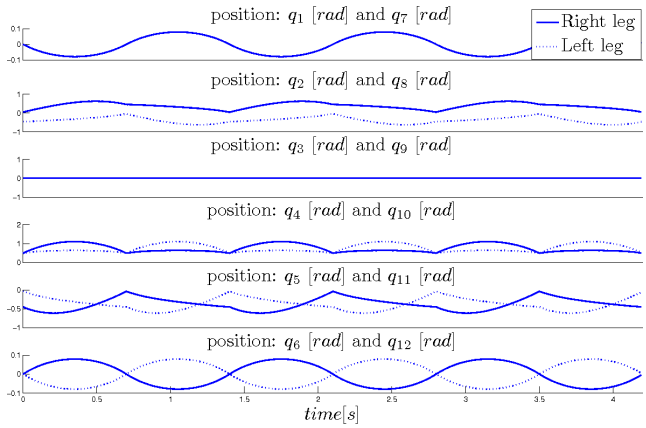

Fig. 7. Joints' positions with optimization

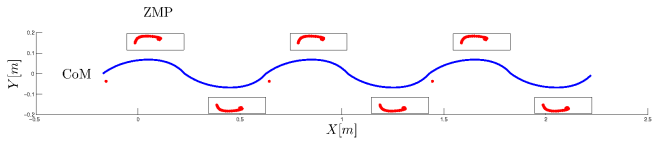

Fig. 8. Evolution of ZMP and CoM trajectories with the LIPM

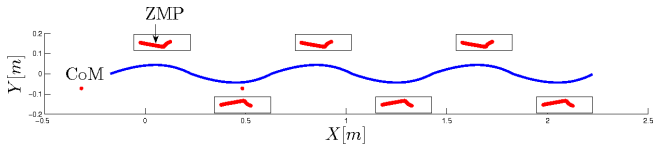

Fig. 9. Evolution of ZMP and CoM trajectories with original 3MLIPM

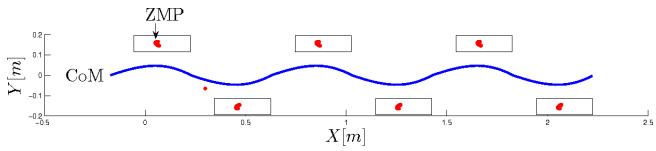

Fig. 10. Evolution of ZMP and CoM trajectories with optimal 3MLIPM

Figure 7 represents the evolution of the joints' positions, where it can clearly be seen that the obtained trajectories are periodic.

Figures 8, 9 and 10 represent the evolution of the ZMP position, and the CoM position with respect to footprints of the biped robot on the ground using respectively the linear inverted pendulum model (LIPM), the original (3MLIPM) pattern generator proposed in [14] and the optimal one proposed in this paper.

For the LIPM as well as the original 3MLIPM, the ZMP moves inside the footprint with a big variation, however when the optimal parameters are used, the ZMP is more concentrated in the center of the footprint. Therefore, the walking dynamic stability is clearly improved with the proposed optimal 3MLIPM pattern generator due to the increase of stability margins.

B. Simulation 2: Walking with change of direction

The objective of this simulation is to evaluate the stability margins of the pattern generator proposed in [14] with a change of direction during walking. It is worth to note that the change of direction during walking has not been yet considered

in the original 3MLIPM, therefore a simple simulation (cf. Fig. 11) shows the loss of stability during dynamic walking.

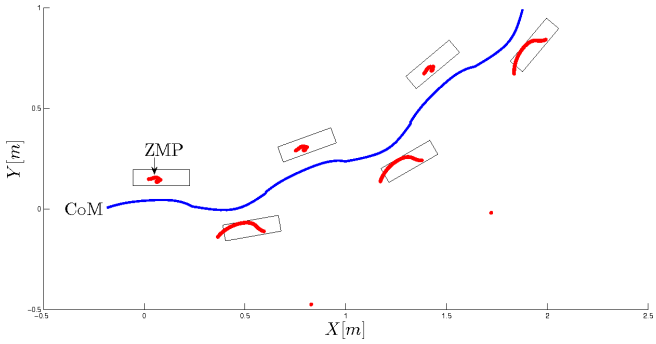

Fig. 11. Evolution of ZMP and CoM trajectories with change of direction

Indeed, when the robot changes the walking direction, the position of ZMP is moving within footprints, being sometimes outside of the stance foot, therefore the robot becomes unstable. A computation of optimal parameters in this scenario is necessary to improve the dynamic walking stability. This will be the objective of next simulation scenario.

C. Simulation 3: Change of direction with optimization

In the proposed optimal pattern generator, the optimization process computes the best values for parameters z_1 and m_1 and uses them to generate the joints' trajectories.

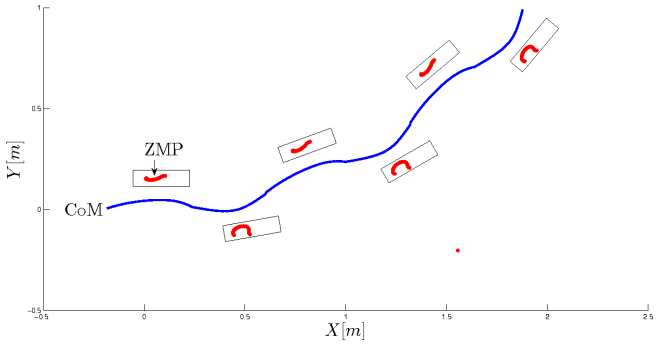

Fig. 12. Evolution of ZMP and CoM trajectories with change of direction and optimization

Figure 12 represents the evolution of the ZMP and the CoM positions as well as footprints of the biped robot on the ground level when turning with an angle of 20 degrees by step with the obtained optimal values of z_1 and m_1.

With this optimization, according to Fig. 12, the robot remains stable during walking while turning since it keeps the ZMP always inside the footprint of the supporting leg.

According to the obtained results, from simulation 1 and 3, it is clearly shown that the proposed optimal pattern generator has significantly improved the dynamic stability of the biped robot.

VII. Conclusions And Future Works

The objective of this work was to generate stable dynamic walking for SHERPA biped robot. To reach this objective, the original pattern generator proposed in [14] based on a Three-Mass Linear Inverted Pendulum Model has been extended with a ZMP optimization to improve dynamic walking stability. The case of turning during walking has also been studied and stability in this case has been enhanced through optimization. The next step of this work deals with real-time implementation of the proposed method on the prototype of SHERPA biped robot.

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