Analyse Formelle de Concepts : Une approche pour fouiller des ensembles de données multi-relationnels, et quelques applications au génie logiciel

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Analyse Relationnelle de Concepts:
Une approche pour fouiller des ensembles de données multi-relationnels, et quelques applications au Génie Logiciel

Université de Montréal
Colloque du Département d'Informatique et de Recherche Opérationnelle

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Brief presentation of FCA – Formal Concept Analysis

A methodology for:

- data analysis, data mining
- knowledge representation
- unsupervised learning

Roots:

- lattice theory, Galois correspondences (Birkhoff, 1940; Barbut & Monjardet, 1970)
- concept lattices (Wille, 1982)

Brief presentation of FCA – Formal Concept Analysis

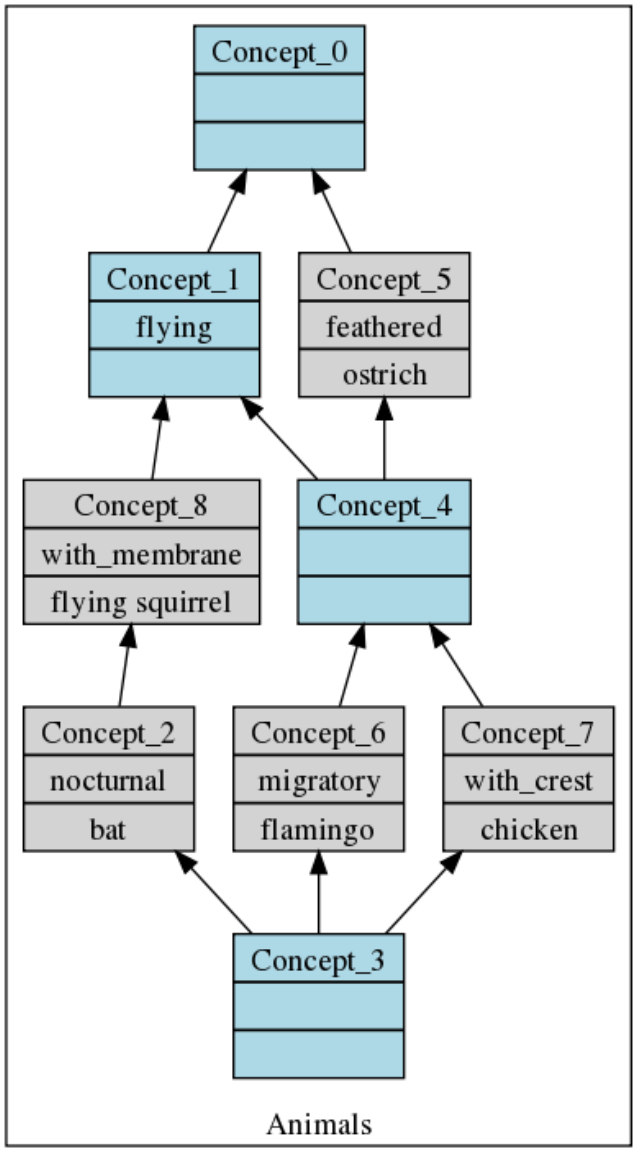
Contexts and concepts

- Handled data
  - entities with characteristics
  - provided with a Formal Context (a binary table)

	flying	nocturnal	feathered	migratory	with_crest	with_membrane
 flying squirrel	×					×
 bat	×	×				×
 ostrich			×			
 flamingo	×		×	×		
 chicken	×		×		×	

- Concept : maximal group of entities sharing characteristics
- Concept lattice : concepts with a partial order relation

Brief presentation of FCA – Formal Concept Analysis



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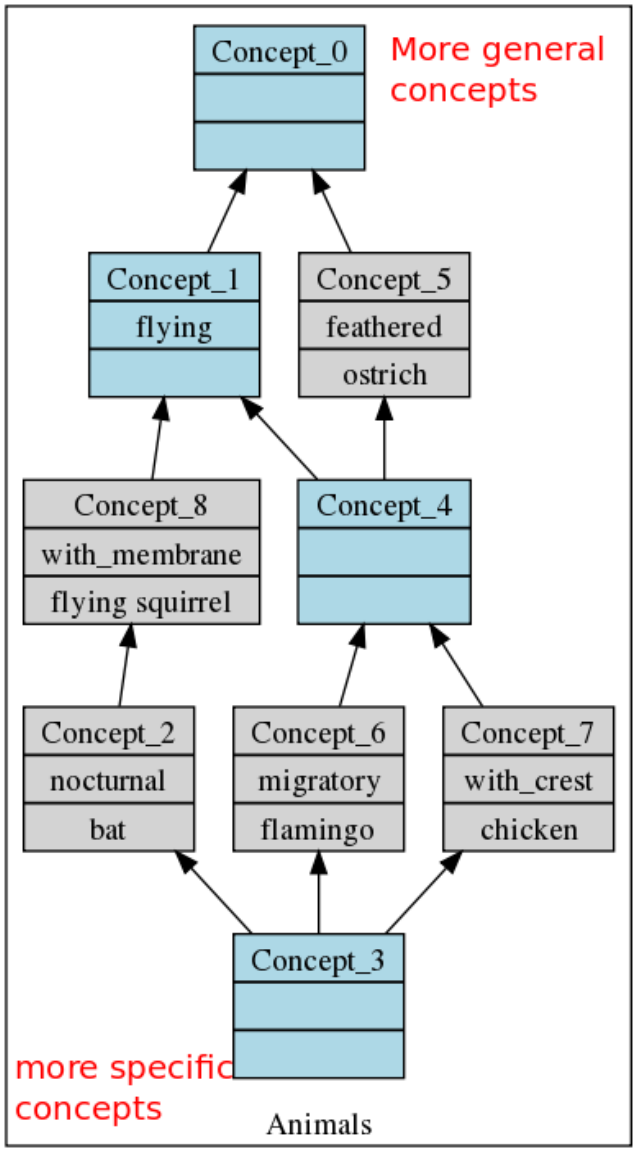
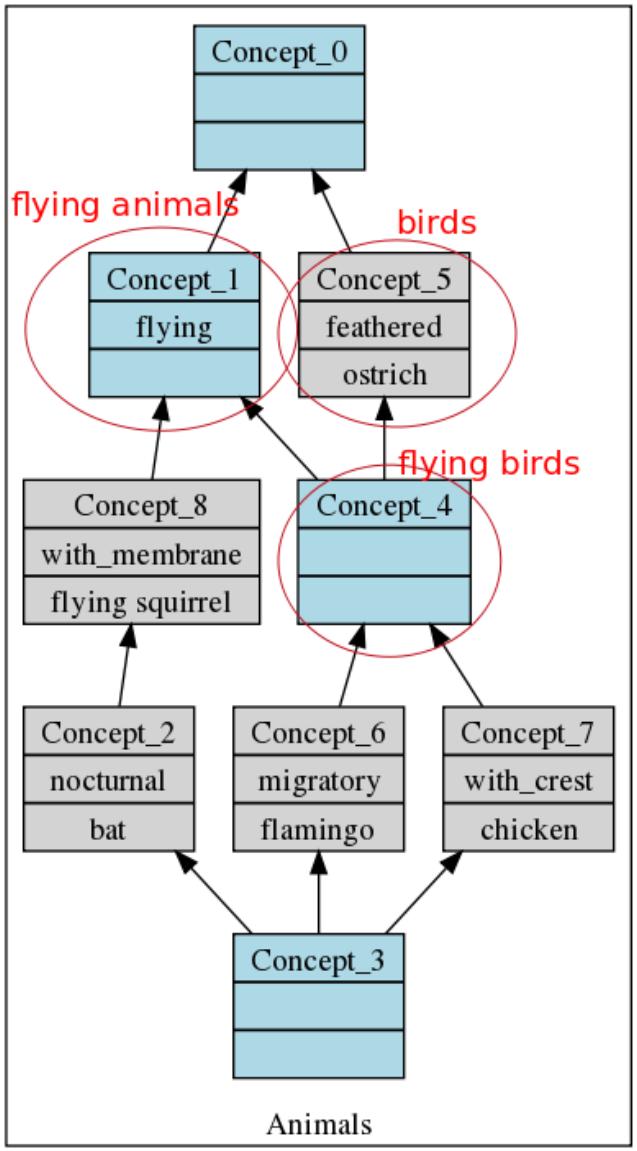
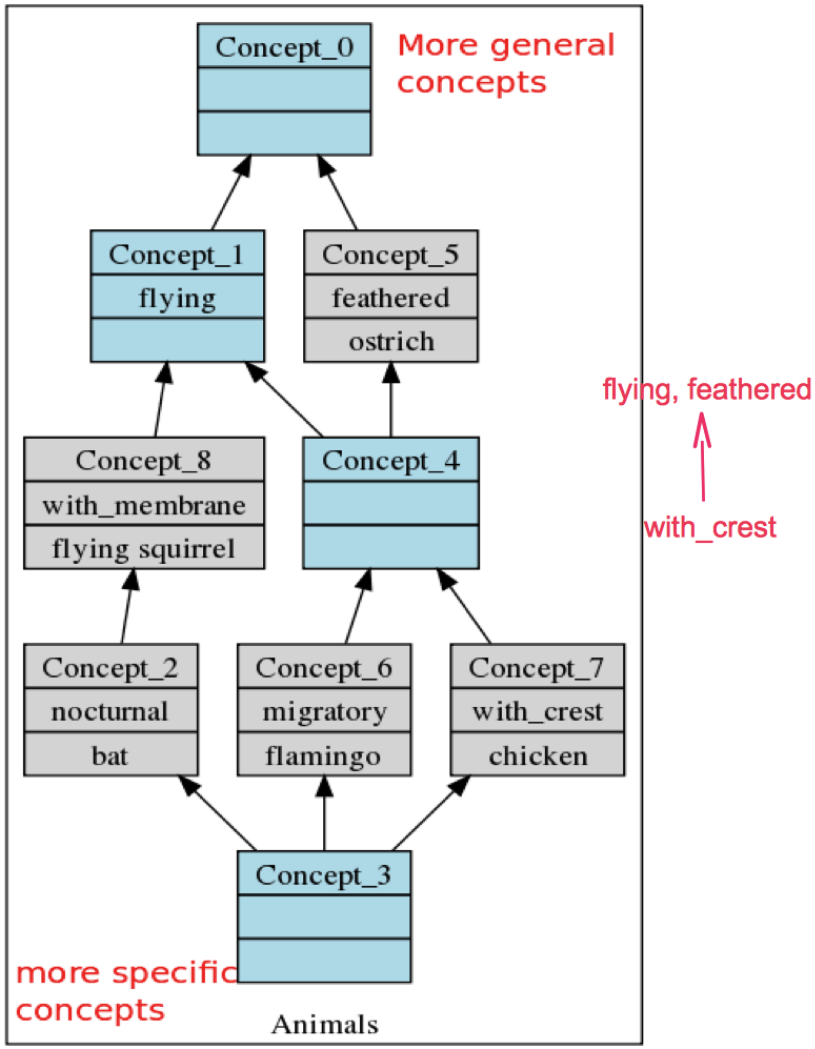


Brief presentation of FCA – Formal Concept Analysis



Brief presentation of FCA – Formal Concept Analysis



FCA and complex data

- many-valued contexts (integers, floats, terms, structures, symbolic objects, etc.) (Ganter et Wille, Polaillon, ...)
- fuzzy descriptions (Yahia et al., Belohlavek, ...)
- hierarchies on values (Godin et al., Carpineto et Romano, ...)
- logical description (Chaudron et al., Ferré et al., ...)
- graphs (Liquière, Prediger et Wille, ...)
- **linked objects** (Priss, Hacène-Rouane et al., ...)
- etc.

Relational Concept Analysis (RCA)

- Extends the purpose of FCA for taking into account object categories and links between objects
- Main principles:
  - a relational model based on the entity-relationship model
  - integrate relations between objects as *relational* attributes
  - iterative process
- RCA provides a set of interconnected lattices
- Produced structures can be represented as ontology concepts within a knowledge representation formalism such as description logics (DLs).

Joint work with:
A. Napoli, C. Roume, M. Rouane-Hacène, P. Valtchev

Relational Context Family (RCF)

A simple entity-relationship model to introduce RCA

Relational Context Family

- **object-attribute contexts**
  - Pizza
  - Ingredient
- **object-object context**
  - has-topping $\subseteq$ Pizza $\times$ Ingredient

Relational Context Family (RCF)

A RCF $\mathcal{F}$ is a pair (K, R) with:

- K is a set of object-attribute contexts $K_i = (O_i, A_i, I_i)$
- R is a set of object-object contexts $R_j = (O_k, O_l, I_j)$,
  - (O_k, O_l) are the object sets of formal contexts $(K_k, K_l) \in K^2$
  - $I_j \subseteq O_k \times O_l$
  - K_k is the *source/domain context*, K_l is the *target/range context*.
  - we may have $K_k = K_l$.

Relational Context Family (RCF) / object-attributes contexts

Pizza	thin	thick	calzone
okonomi			×
alberginia		×	
margherita	×		
languedoc	×		
four-cheeses	×		
three-cheeses	×		
frutti-di-mare	×		
quebec		×	
regina	×		
hawai		×	
lorraine	×		
kebab			×

Ingredient	fruit-vegetable	meat	fish	dairy	cereal-leguminous	veg-oil
tomato-sauce	×					
cream				×		
tomato	×					
basilic	×					
olive	×					
olive oil						×
soy	×					
mushroom	×					
eggplant	×					
onion	×					
pepper	×					
ananas	×					
mozza				×		
goat-cheese				×		
emmental				×		
fourme-ambert				×		
squid			×			
shrimp			×			
mussels			×			
ham		×				

Relational Context Family (RCF) / object-object context / part 1

has-topping	tomato-sauce	cream	tomato	basilic	olive	olive oil	soy	mushroom	eggplant	onion	pepper	ananas
okonomi	×					×	×	×				
alberginia	×					×	×		×	×		
margherita	×		×	×	×	×						
languedoc	×		×	×	×	×				×	×	
four-cheeses		×										
three-cheeses		×										
frutti-di-mare	×				×	×						
quebec	×											
regina	×							×				
hawai	×											×
lorraine		×								×		
kebab	×		×		×					×		

Relational Context Family (RCF) / object-object context / part 2

has-topping	mozza	goat-cheese	emmental	fourme-ambert	squid	shrimp	mussels	ham	bacon	chicken	maple-sirup	corn
okonomi												
alberginia												
margherita	×											
languedoc	×											
four-cheeses	×	×	×	×								
three-cheeses	×	×	×									
frutti-di-mare	×				×	×	×					
quebec	×							×			×	×
regina	×								×			
hawai	×							×				
lorraine			×						×			
kebab			×							×		

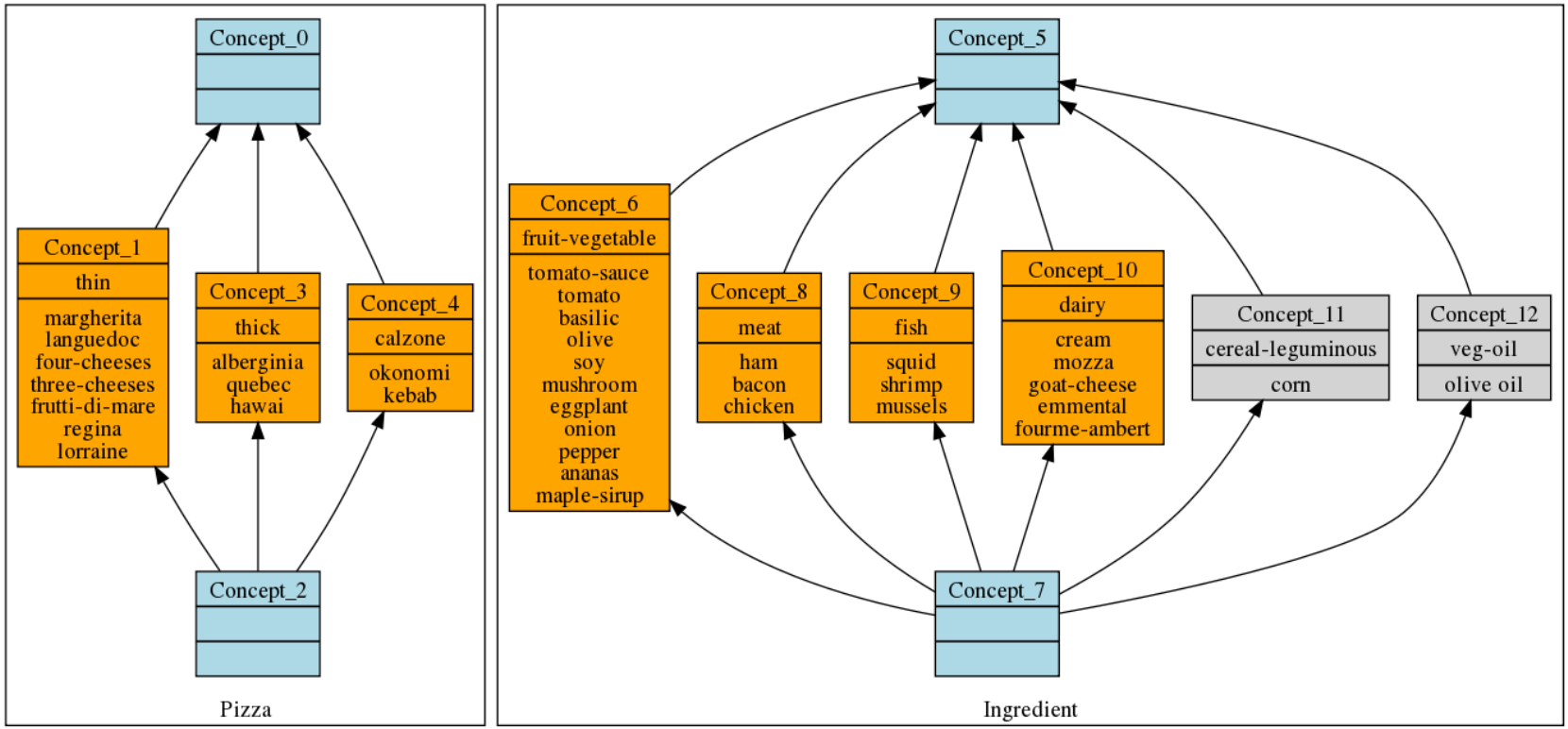
Data patterns we would like to extract

Using a classification on ingredients by their categories of topping (fruit-vegetable, dairy, etc.)

- All pizzas, even different, except four-cheese and three-cheese, contain at least one topping which is a vegetable
- Two pizzas (four-cheese and three-cheese) have all their topping in dairy ingredients
- For pizzas: have meat $\Rightarrow$ have dairy
- For pizzas: being thin $\Rightarrow$ have at least dairy
- For pizzas: have only dairy $\Rightarrow$ being thin

RCA - Initial Lattice building

At the beginning, only the object-attribute contexts are used to build the foundation of the concept lattice family



RCA - Introducing relations as relational attributes

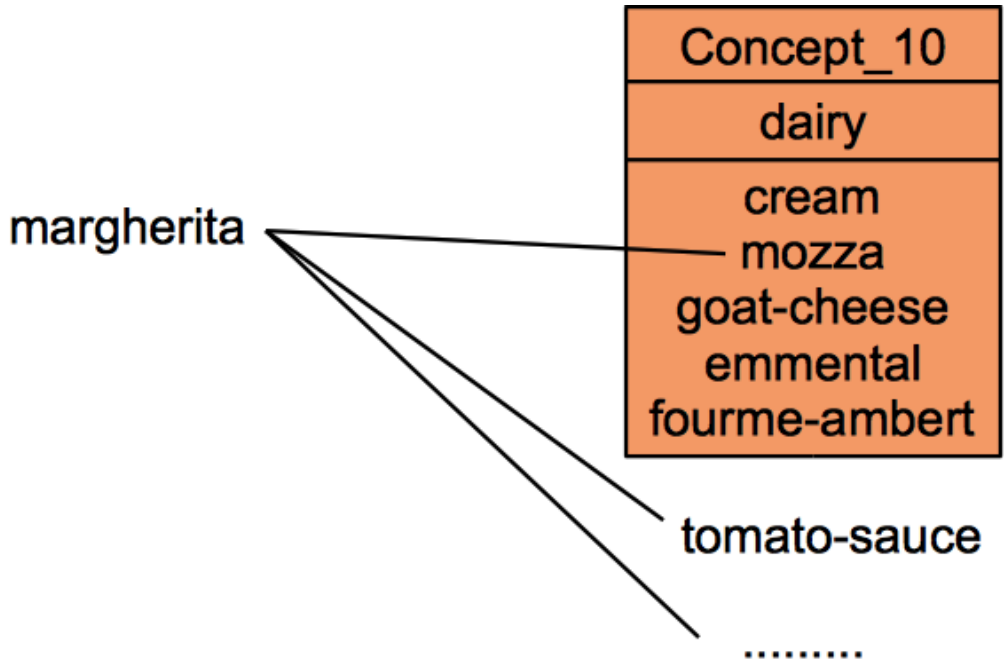
Given an object-object context $R_j = (O_k, O_l, I_j)$,
There are different notable schemas between an object of domain O_k and concepts formed on O_l.

E. g.

- **Existential**: an object is linked (by R_j) to at least one object of the extent of a concept
- **Universal**: an object is linked (by R_j) only to objects of the extent of a concept

RCA - Existential relational attributes

margherita has one topping in Concept_10 extent: **mozza**.
It has other links to other concept extents.



$\exists$**has-topping.Concept_10** is assigned to **margherita**

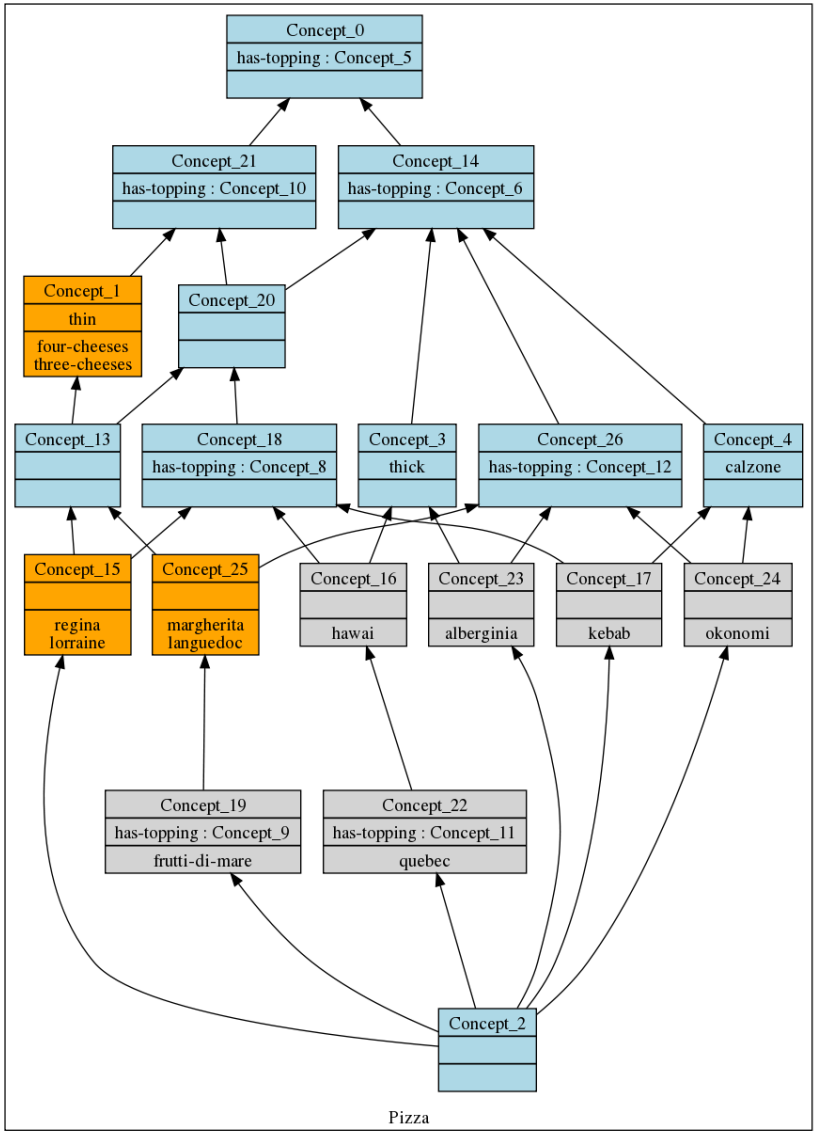
RCA - Existential relational attributes

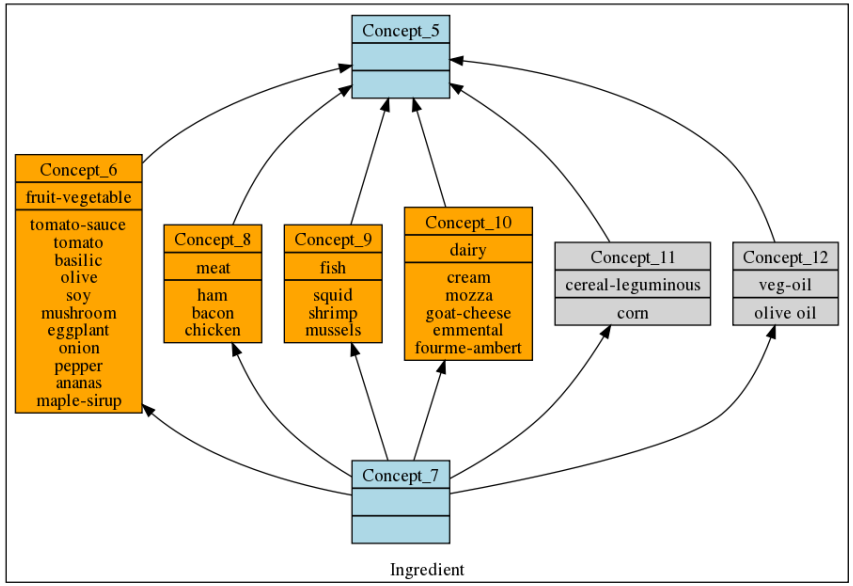
Scaled relations with domain O_i are concatenated to K_i, the object-attribute context on O_i

Pizza	thin	thick	calzone
okonomi			×
alberginia		×	
margherita	×		
languedoc	×		
four-cheeses	×		
three-cheeses	×		
frutti-di-mare	×		
quebec		×	
regina	×		
hawai		×	
lorraine	×		
kebab			×

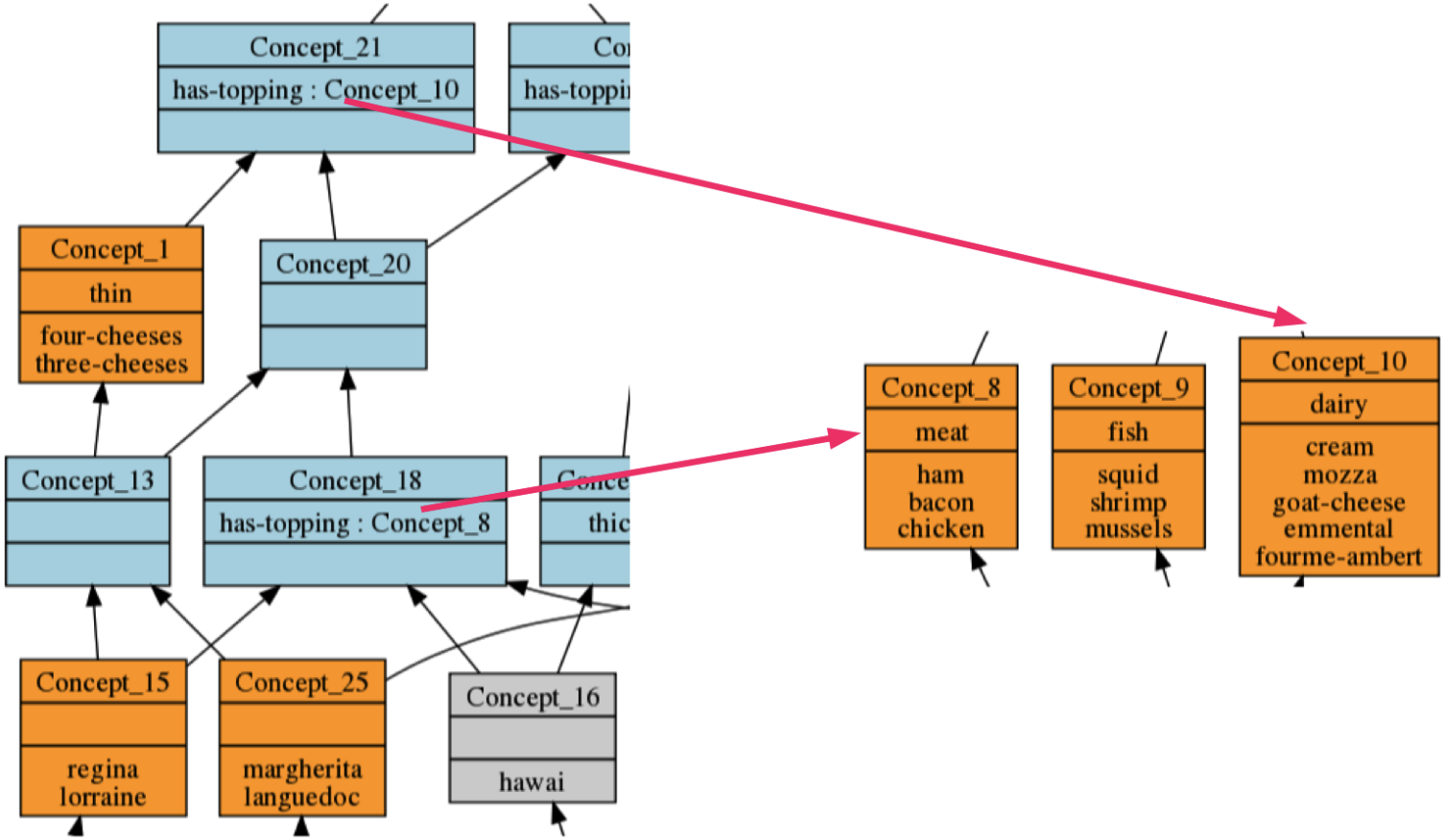
has-topping	∃has-topping. Concept_7	∃has-topping. Concept_5	∃has-topping. Concept_6	∃has-topping. Concept_8	∃has-topping. Concept_9	∃has-topping. Concept_10	∃has-topping. Concept_11	∃has-topping. Concept_12
okonomi		×	×					×
alberginia		×	×					×
margherita		×	×			×		×
languedoc		×	×			×		×
four-cheeses		×				×		
three-cheeses		×				×		
frutti-di-mare		×	×		×	×		×
quebec		×	×	×		×	×	
regina		×	×	×		×		
hawai		×	×	×		×		
lorraine		×	×	×		×		
kebab		×	×	×		×		

Relational Concept Family / exists





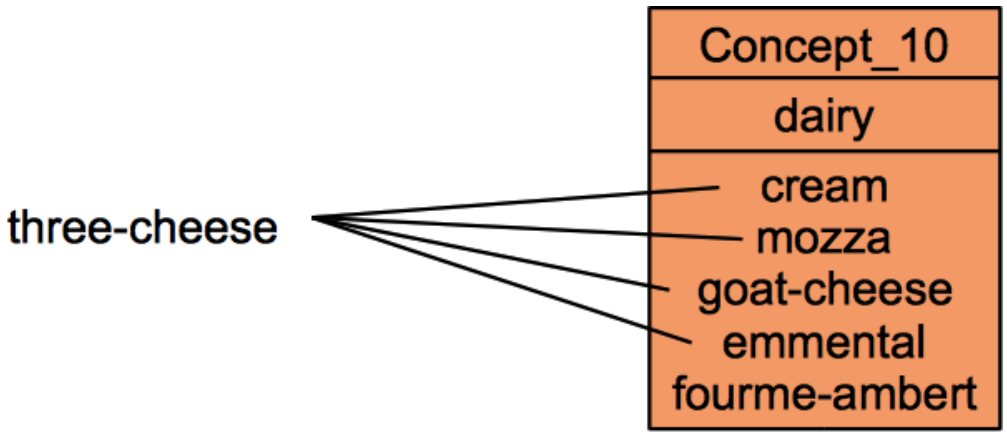
Relational Concept Family / exists



Concept_21: pizzas with at least one topping in dairy
Concept_18: pizzas with at least one topping in meat
have at least one meat topping $\Rightarrow$ have at least one dairy topping

RCA - Universal relational attributes

three-cheese has topping in and only in Concept_10 extent.



$\forall\exists$**has-topping.Concept_10** is assigned to **three-cheese**

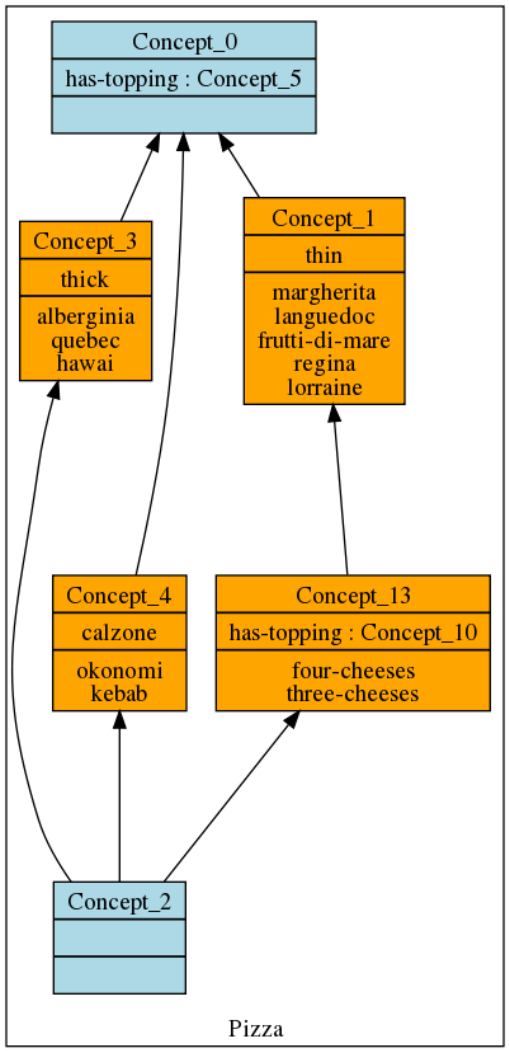
RCA - Universal relational attributes

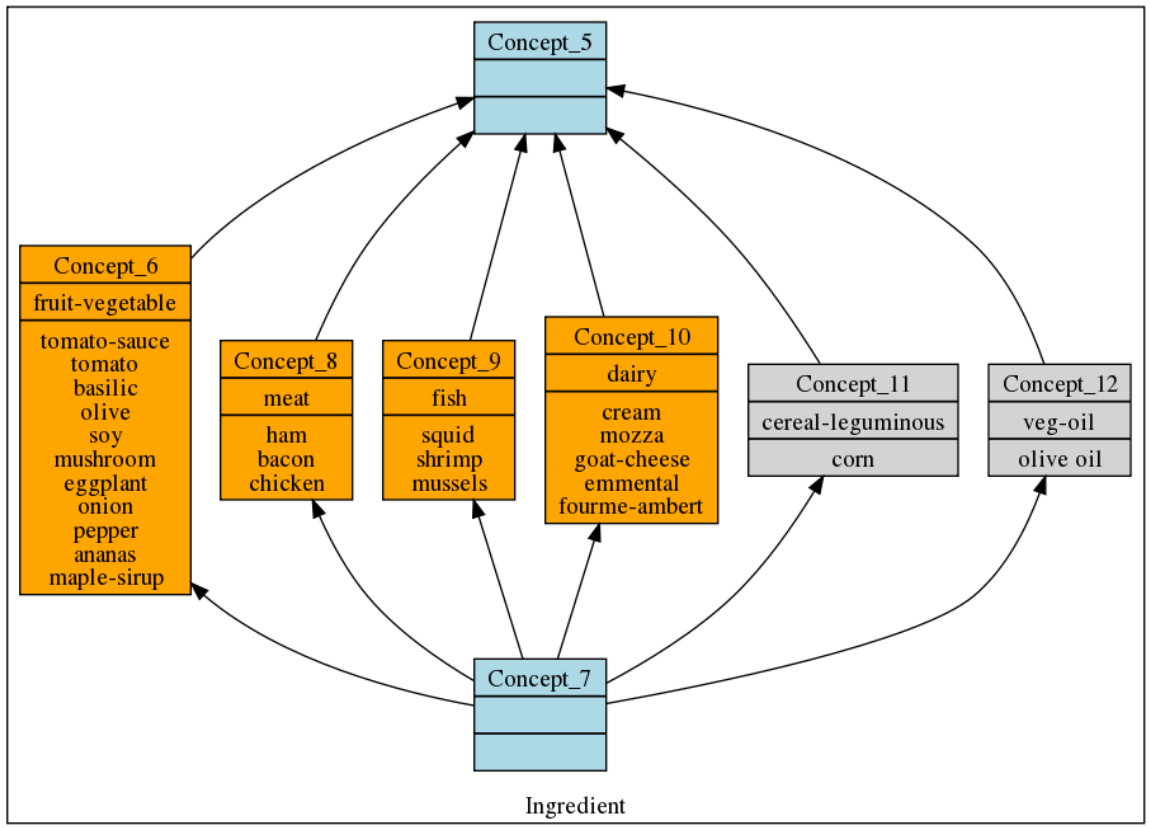
Scaled relations with domain O_i are concatenated to K_i, the object-attribute context on O_i

Pizza	thin	thick	calzone
okonomi			×
alberginia		×	
margherita	×		
languedoc	×		
four-cheeses	×		
three-cheeses	×		
frutti-di-mare	×		
quebec		×	
regina	×		
hawai		×	
lorraine	×		
kebab			×

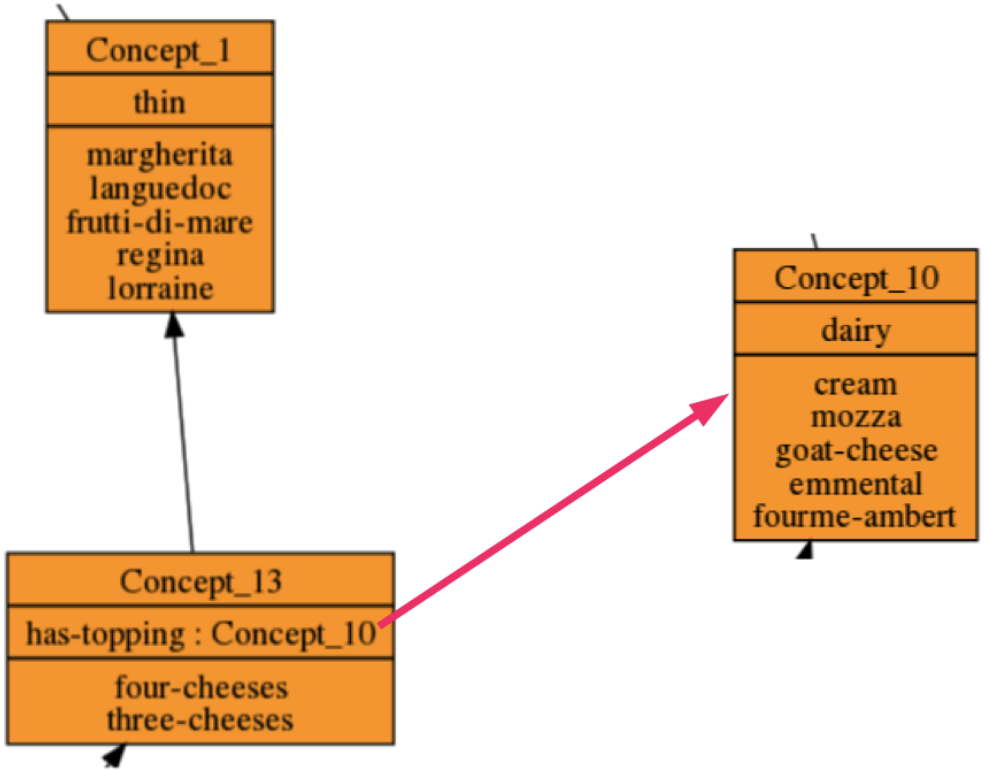
has-topping	∀∃has-topping. Concept_7	∀∃has-topping. Concept_5	∀∃has-topping. Concept_6	∀∃has-topping. Concept_8	∀∃has-topping. Concept_9	∀∃has-topping. Concept_10	∀∃has-topping. Concept_11	∀∃has-topping. Concept_12
okonomi		×						
alberginia		×						
margherita		×						
languedoc		×						
four-cheeses		×				×		
three-cheeses		×				×		
frutti-di-mare		×						
quebec		×						
regina		×						
hawai		×						
lorraine		×						
kebab		×						

Relational Concept Family / forall





Relational Concept Family / forall



Concept_13: pizzas with only dairy topping
Concept_1: thin pizzas
have only dairy topping $\Rightarrow$ thin

RCA - Introducing relations as relational attributes

Relational scaling is the process by which links are established between objects and concepts.

For each relational context $R_j = (O_k, O_l, I_j)$, a scaled context $R_j^* = (O_k, A, I_j)$ is created.

- A is a set of relational attributes $a = S\ R_j.C$, where C is in the concept set of a lattice built on objects of O_l, denoted by $\mathcal{L}_l^n$
- I_j contains (o, a) iff $S(R_j(o), Extent(C))$ is true.

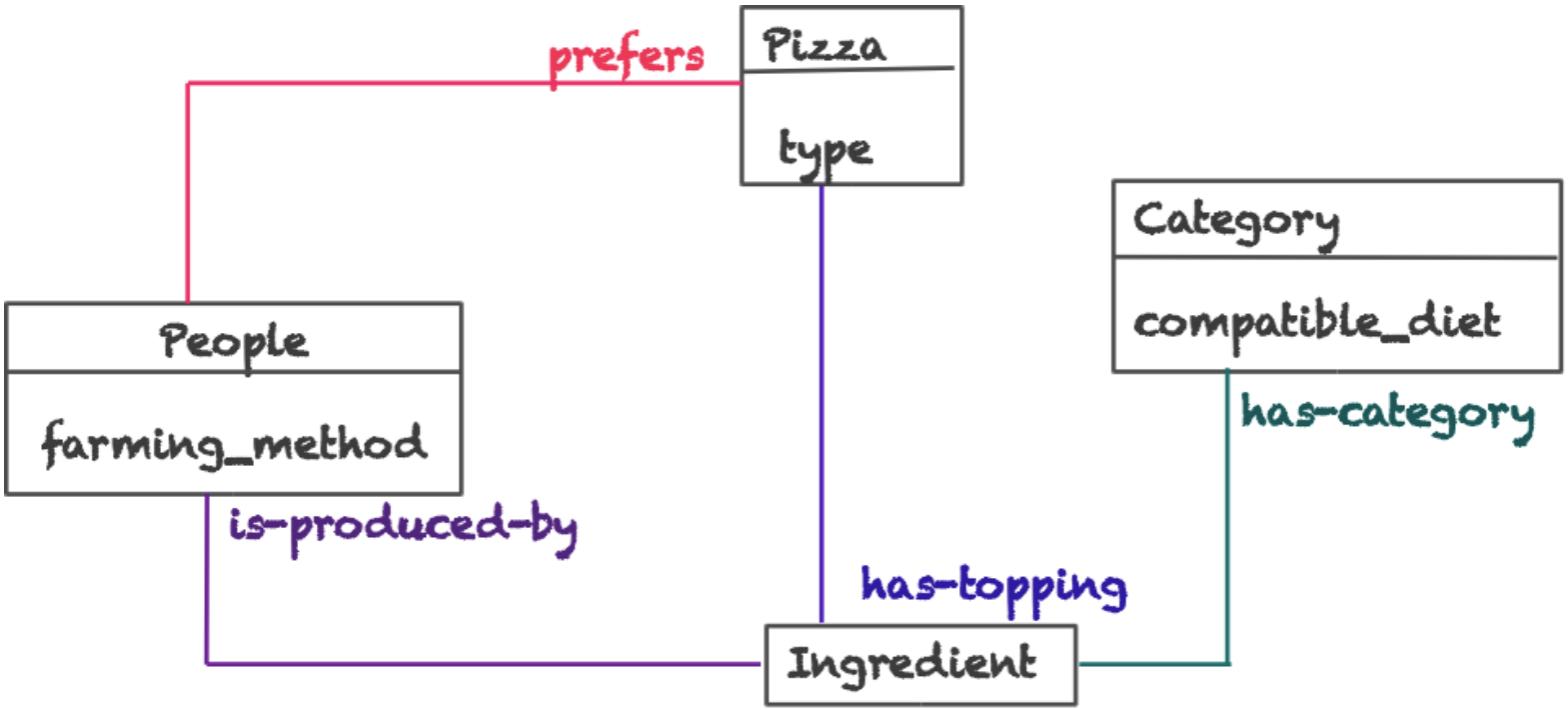
S is a *scaling* operator, the most used are:

- $S_\exists(R_j(o), Extent(C))$ is true iff $R_j(o) \cap Extent(C) \neq \emptyset$.
- $S_{\forall\exists}(R_j(o), Extent(C))$ is true iff $R_j(o) \subseteq Extent(C) \wedge \exists x \in R_j(o), x \in Extent(C)$

Scaling operators

Operator	Attribute form	Condition
Universal (narrow)	$\forall\ r.c$	$r(o) \subseteq Ext(c)$
Covers	$\supseteq\ r.c$	$r(o) \supseteq Ext(c)$
Existential (wide)	$\exists\ r.c$	$r(o) \cap Ext(c) \neq \emptyset$
Universal strict	$\forall\exists\ r.c$	$r(o) \subseteq Ext(c)$ *and* $r(o) \neq \emptyset$
Qualified cardinality restriction	$\geq\ n\ r.c$	$r(o) \subseteq Ext(c)$ *and* $\lvert r(o)\rvert \geq\ n$
Cardinality restriction	$\geq\ n\ r.\top_{\mathcal{L}}$	$\lvert r(o)\rvert \geq\ n$

General Entity-Relationship diagram



General ER diagram may present cycle/circuits between classes/objects

The RCA schema

Input

$RCF = (K, R)$: n object-attribute contexts, m object-object contexts

Initialization step

build, for i in $1..n$, $\mathbf{L}^0[i]$ the concept lattice of the context $\mathcal{K}_i$

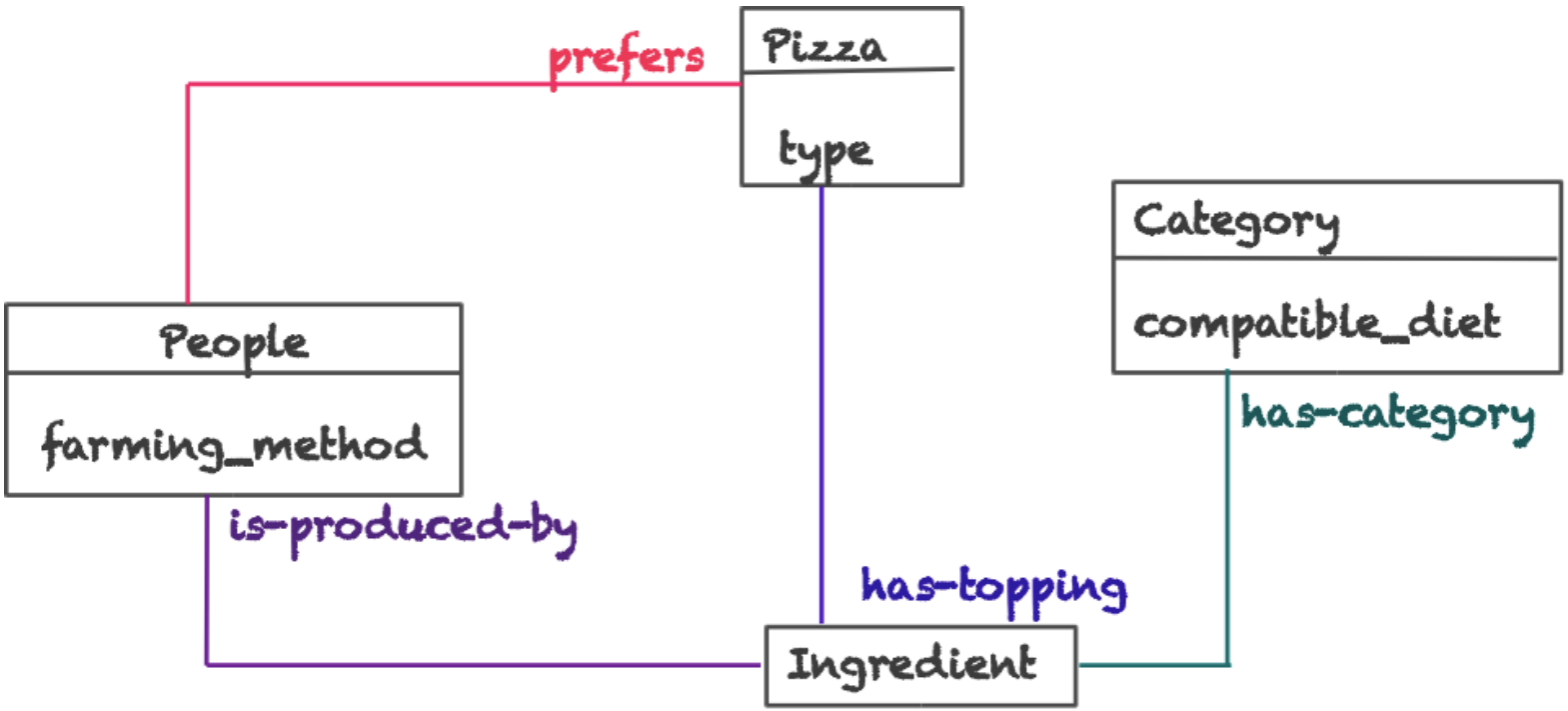
Step p

▷ Apply relational scaling to all object-object contexts R_j, using the lattices of step $p-1$ and the chosen scaling operator
▷ concatenate $\mathcal{K}_i$ with the scaled R_j^* whose domain is O_i
▷ update lattices of step $p-1$ to build, for i in $1..n$, the lattice $\mathbf{L}^p[i]$ for the context $\mathcal{K}_i$ concatenated as explained previously

Output (fix point)

The concept lattice family obtained when no new concepts are added

Analysis of pizza data



$\exists$ prefers $\forall\exists$ has-topping $\forall\exists$ has-category $\forall\exists$ is-produced-by

Analysis of pizza data - object-atribute contexts

Pizza	thin	thick	calzone
forest		×	
occitane			×
three-cheese	×		
four-cheese	×		
lorraine	×		
arctic		×	

People	organic-farmer	conventional-farmer
Amedeo	×	
Amine	×	
Cyril		×
Marianne	×	
Petko		×

Ingredient
tomato-sauce
cream
onion
bacon
salmon
soy-cream
mozza
goat-cheese
emmental
fourme-ambert
eggplant
mushroom

Category	mediterranean	vegan	vegetarian
fruit-vegetable	×	×	×
meat			
fish	×		
dairy	×		×

Analysis of pizza data - object-object contexts

prefers	forest	occitane	three-cheese	four-cheese	lorraine	arctic
Amedeo	×					
Amine		×				
Cyril				×	×	
Marianne			×			×
Petko						×

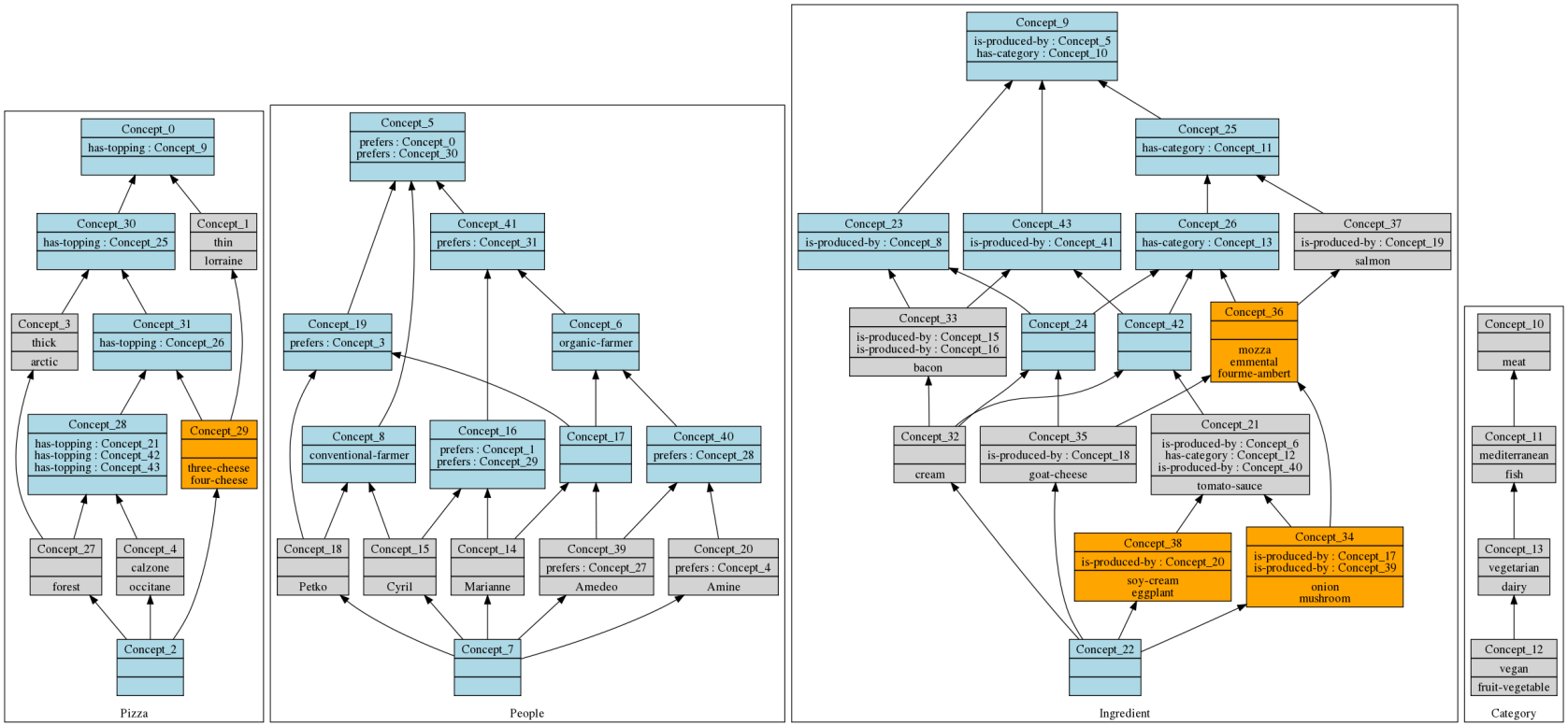
has-topping	tomato-sauce	cream	onion	bacon	salmon	soy-cream	mozza	goat-cheese	emmental	fourme-ambert	eggplant	mushroom
forest						×						×
occitane	×		×								×	
three-cheese	×						×	×	×			
four-cheese	×	×					×	×	×	×		
lorraine		×	×	×			×					
arctic	×	×			×		×					

Analysis of pizza data - object-object contexts

is-produced-by	Amedeo	Amine	Cyril	Marianne	Petko
tomato-sauce	×	×			
cream			×		
onion	×				
bacon			×		
salmon				×	×
soy-cream		×			
mozza				×	×
goat-cheese					×
emmental				×	×
fourme-ambert				×	×
eggplant		×			
mushroom	×				

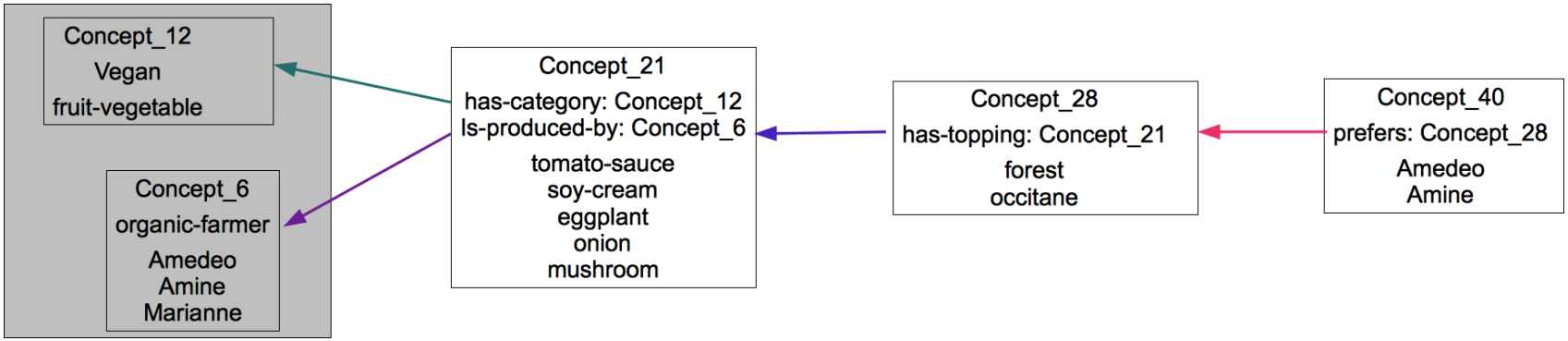
has-category	fruit-vegetable	meat	fish	dairy
tomato-sauce	×			
cream				×
onion	×			
bacon		×		
salmon			×	
soy-cream	×			
mozza				×
goat-cheese				×
emmental				×
fourme-ambert				×
eggplant	×			
mushroom	×			

Concept lattice family



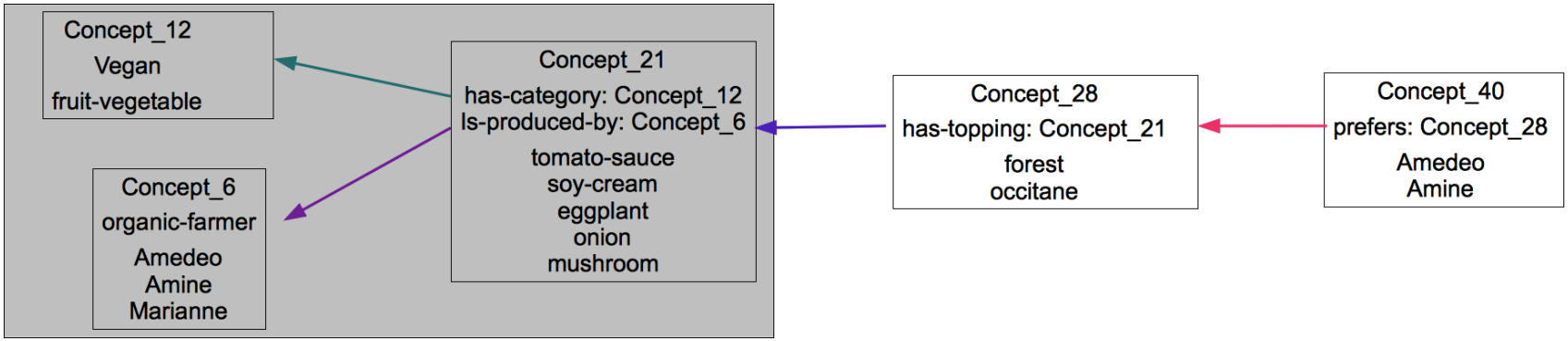
$\exists$ prefers $\forall\exists$ has-topping $\forall\exists$ has-category $\forall\exists$ is-produced-by

Concept lattice family



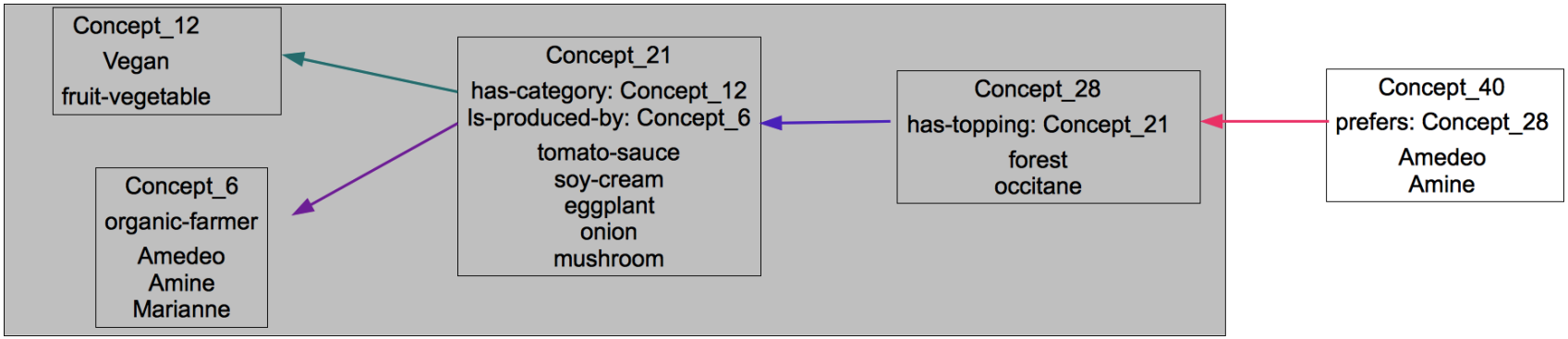
Step 0

Concept lattice family



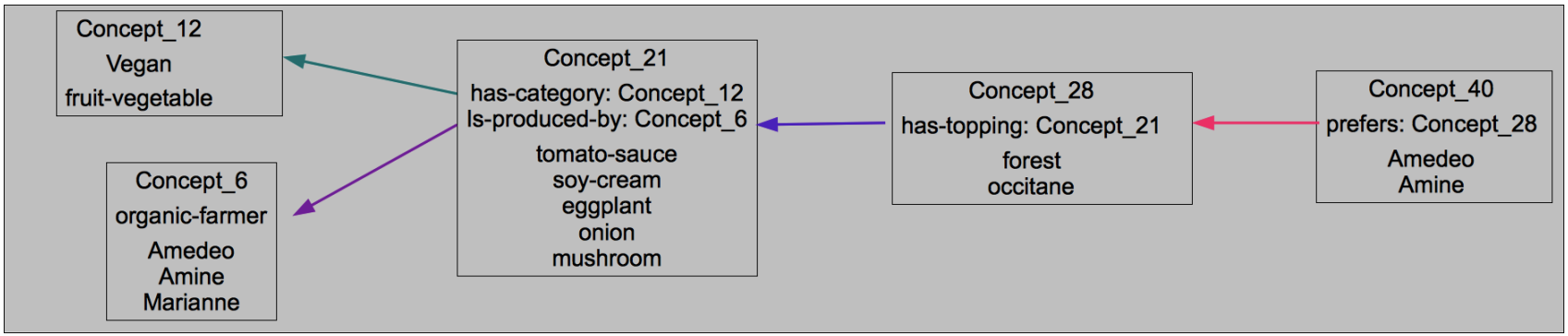
Step 1

Concept lattice family



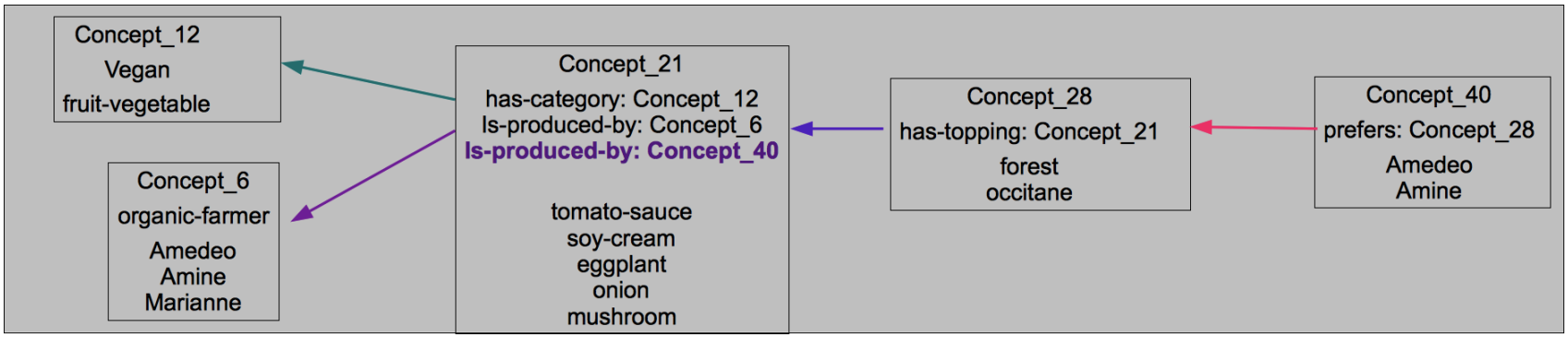
Step 2

Concept lattice family



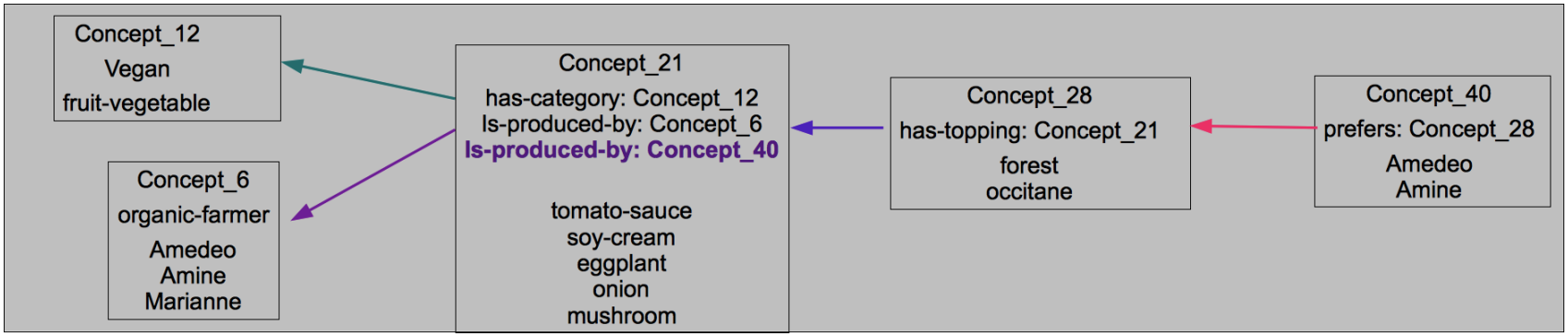
Step 3

Concept lattice family



Step 4

Concept lattice family



- People: $\exists$prefers.Concept_28 $\Rightarrow$ organic farmer
- Ingredient: $\forall\exists$has-category.Concept_12 $\Leftrightarrow$ $\forall\exists$is-produced-by.Concept_6 (organic farmers)
- Amedeo/Amine prefer at least one pizza with only vegan topping ingredients and produced only by organic farmers

A synthesis on RCA

- an iterative method to produce interconnected classifications
- converges after a number of iterations that depends on the structure
- variations on scaling can be done

Tools

- Galicia: http://galicia.sourceforge.net/
- eRCA: http://code.google.com/p/erca/

RCA - Current tracks

- Querying the concept lattice family
- Exploratory RCA: select, divide, step-by-step
- Metrics for guiding the process and filtering concepts
- Build Galois sub-hierarchy (AOC-poset)