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Analyse Formelle de Concepts : Une approche pour fouiller des ensembles de données multi-relationnels, et quelques applications au génie logiciel

Marianne Huchard

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Marianne Huchard. Analyse Formelle de Concepts : Une approche pour fouiller des ensembles de données multi-relationnels, et quelques applications au génie logiciel. 2012. lirmm-00808686

HAL Id: lirmm-00808686

<https://hal-lirmm.ccsd.cnrs.fr/lirmm-00808686v1>

Submitted on 5 Jan 2024

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Analyse Relationnelle de Concepts:

Une approche pour fouiller des ensembles de données multi-relationnels, et quelques applications au Génie Logiciel

Université de Montréal
Colloque du Département d'Informatique et de Recherche Opérationnelle

Marianne Huchard

November 1, 2012

Brief presentation of FCA – Formal Concept Analysis

A methodology for:

- ▶ data analysis, data mining
- ▶ knowledge representation
- ▶ unsupervised learning






Roots:

- ▶ lattice theory, Galois correspondences (Birkhoff, 1940; Barbut & Monjardet, 1970)
- ▶ concept lattices (Wille, 1982)

Brief presentation of FCA – Formal Concept Analysis

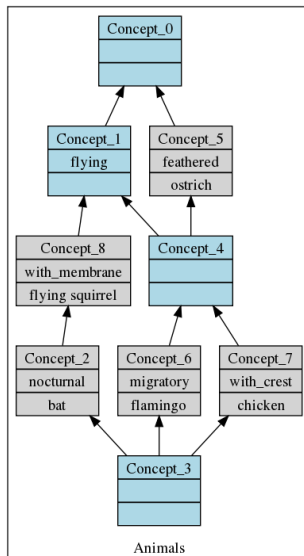
Contexts and concepts

- ▶ Handled data
 - ▶ entities with characteristics
 - ▶ provided with a Formal Context (a binary table)

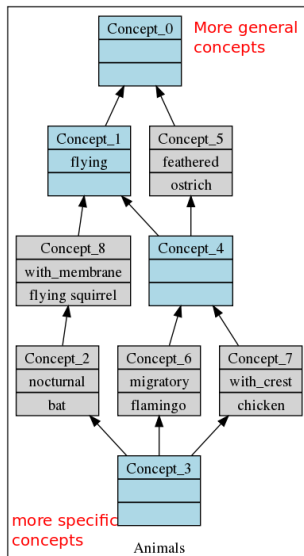
	flyng	nocturnal	feathered	migratory	with_crest	with_membrane
 flying squirrel	×					×
 bat	×	×				×
 ostrich			×			
 flamingo	×		×	×		
 chicken	×		×		×	

- ▶ Concept : maximal group of entities sharing characteristics
- ▶ Concept lattice : concepts with a partial order relation

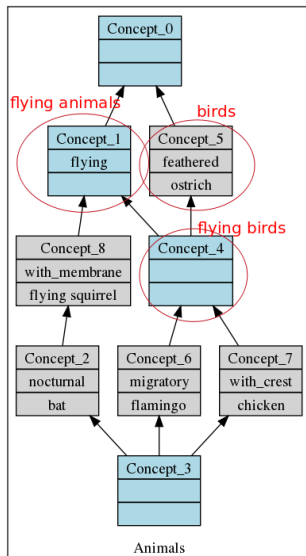
Brief presentation of FCA – Formal Concept Analysis



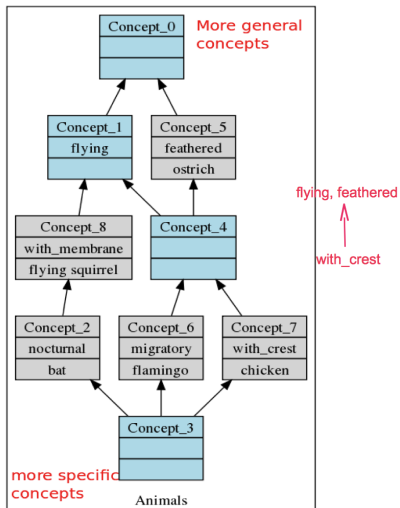
Brief presentation of FCA – Formal Concept Analysis



Brief presentation of FCA – Formal Concept Analysis



Brief presentation of FCA – Formal Concept Analysis



FCA and complex data

- ▶ many-valued contexts (integers, floats, terms, structures, symbolic objects, etc.) (Ganter et Wille, Polaillon, ...)
- ▶ fuzzy descriptions (Yahia et al., Belohlavek, ...)
- ▶ hierarchies on values (Godin et al., Carpineto et Romano, ...)
- ▶ logical description (Chaudron et al., Ferré et al., ...)
- ▶ graphs (Liquière, Prediger et Wille, ...)
- ▶ **linked objects** (Priss, Hacène-Rouane et al., ...)
- ▶ etc.

Relational Concept Analysis (RCA)

- ▶ Extends the purpose of FCA for taking into account object categories and links between objects
- ▶ Main principles:
 - ▶ a relational model based on the entity-relationship model
 - ▶ integrate relations between objects as *relational* attributes
 - ▶ iterative process
- ▶ RCA provides a set of interconnected lattices
- ▶ Produced structures can be represented as ontology concepts within a knowledge representation formalism such as description logics (DLs).

Joint work with:

A. Napoli, C. Roume, M. Rouane-Hacène, P. Valtchev

Relational Context Family (RCF)

A simple entity-relationship model to introduce RCA

Relational Context Family

- ▶ **object-attribute contexts**
 - ▶ Pizza
 - ▶ Ingredient
- ▶ **object-object context**
 - ▶ $\text{has-topping} \subseteq \text{Pizza} \times \text{Ingredient}$

Relational Context Family (RCF)

A RCF \mathcal{F} is a pair (K, R) with:

- ▶ K is a set of object-attribute contexts $K_i = (O_i, A_i, I_i)$
- ▶ R is a set of object-object contexts $R_j = (O_k, O_l, I_j)$,
 - ▶ (O_k, O_l) are the object sets of formal contexts $(K_k, K_l) \in K^2$
 - ▶ $I_j \subseteq O_k \times O_l$
 - ▶ K_k is the *source/domain context*, K_l is the *target/range context*.
 - ▶ we may have $K_k = K_l$.

Relational Context Family (RCF) / object-attributes contexts

Pizza	thin	thick	calzone
okonomi			×
alberginia		×	
margherita	×		
languedoc	×		
four-cheeses	×		
three-cheeses	×		
frutti-di-mare	×		
quebec		×	
regina	×		
hawai		×	
lorraine	×		
kebab			×

Ingredient	fruit-vegetable	meat	fish	dairy	cereal-leguminous	veg-oil
tomato-sauce	×					
cream				×		
tomato	×					
basilic	×					
olive	×					
olive oil						×
soy	×					
mushroom	×					
eggplant	×					
onion	×					
pepper	×					
anas	×					
mozza				×		
goat-cheese				×		
emmental				×		
fourme-ambert				×		
squid			×			
shrimp			×			
mussels			×			
ham		×				

Relational Context Family (RCF) / object-object context / part 1

	tomato-sauce	cream	tomato	basilic	olive	olive oil	soy	mushroom	eggplant	onion	pepper	ananas
has-topping												
okonomi	x					x	x	x				
alberginia	x					x	x		x	x		
margherita	x		x	x	x	x						
languedoc	x		x	x	x	x				x	x	
four-cheeses		x										
three-cheeses		x										
frutti-di-mare	x				x	x						
quebec	x											
regina	x							x				
hawai	x											x
lorraine		x								x		
kebab	x		x		x					x		

Relational Context Family (RCF) / object-object context / part 2

	mozza	goat-cheese	emmental	fourme-ambert	squid	shrimp	mussels	ham	bacon	chicken	maple-sirup	corn
has-topping												
okonomi												
alberginia												
margherita	×											
languedoc	×											
four-cheeses	×	×	×	×								
three-cheeses	×	×	×									
frutti-di-mare	×				×	×	×					
quebec	×							×			×	×
regina	×								×			
hawai	×							×				
lorraine			×						×			
kebab			×							×		

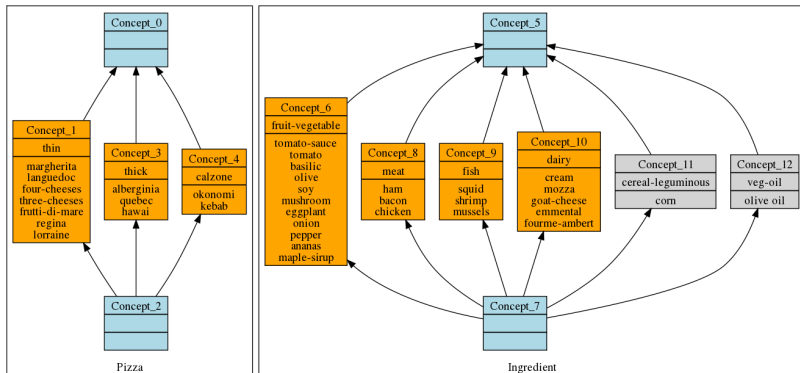
Data patterns we would like to extract

Using a classification on ingredients by their categories of topping (fruit-vegetable, dairy, etc.)

- ▶ All pizzas, even different, except four-cheese and three-cheese, contain at least one topping which is a vegetable
- ▶ Two pizzas (four-cheese and three-cheese) have all their topping in dairy ingredients
- ▶ For pizzas: have meat \Rightarrow have dairy
- ▶ For pizzas: being thin \Rightarrow have at least dairy
- ▶ For pizzas: have only dairy \Rightarrow being thin

RCA - Initial Lattice building

At the beginning, only the object-attribute contexts are used to build the foundation of the concept lattice family



RCA - Introducing relations as relational attributes

Given an object-object context $R_j = (O_k, O_l, I_j)$,

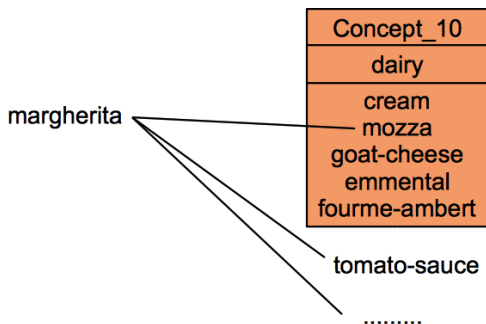
There are different notable schemas between an object of domain O_k and concepts formed on O_l .

E. g.

- ▶ **Existential**: an object is linked (by R_j) to at least one object of the extent of a concept
- ▶ **Universal**: an object is linked (by R_j) only to objects of the extent of a concept

RCA - Existential relational attributes

margherita has one topping in **Concept_10** extent: **mozza**.
It has other links to other concept extents.



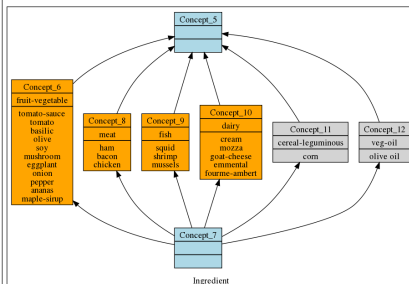
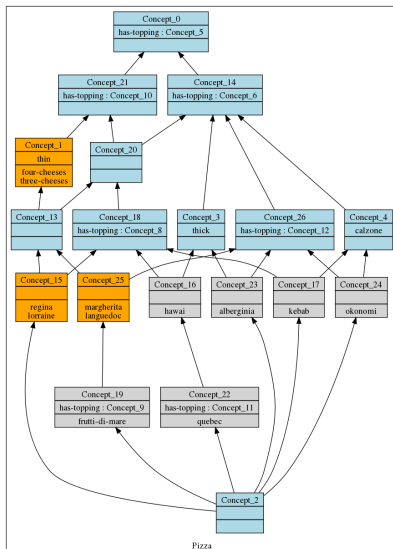
\exists has-topping.Concept_10 is assigned to **margherita**

RCA - Existential relational attributes

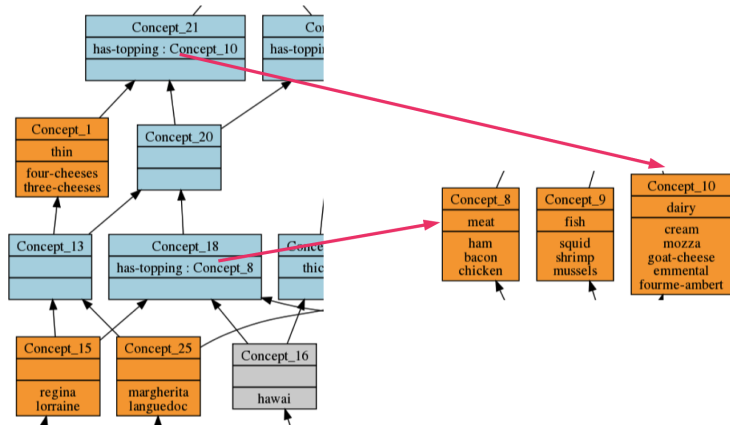
Scaled relations with domain O_i are concatenated to K_i , the object-attribute context on O_i

Pizza	thin	thick	calzone		has-topping. Concept_7	has-topping. Concept_5	has-topping. Concept_6	has-topping. Concept_8	has-topping. Concept_9	has-topping. Concept_10	has-topping. Concept_11	has-topping. Concept_12
okonomi			x	has-topping	\exists							
alberginia		x		okonomi		x	x					x
margherita	x			alberginia		x	x					x
languedoc	x			margherita		x	x			x		x
four-cheeses	x			languedoc		x	x			x		x
three-cheeses	x			four-cheeses		x				x		
frutti-di-mare	x			three-cheeses		x				x		
quebec		x		frutti-di-mare		x	x		x	x		x
regina	x			quebec		x	x	x		x	x	
hawai		x		regina		x	x	x		x		
lorraine	x			hawai		x	x	x		x		
kebab			x	lorraine		x	x	x		x		
				kebab		x	x	x		x		

Relational Concept Family / exists



Relational Concept Family / exists



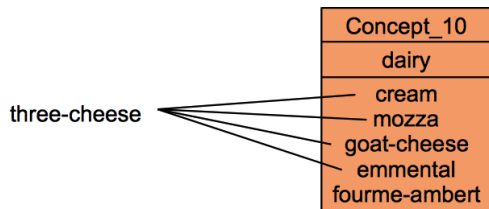
Concept_21: pizzas with at least one topping in dairy

Concept_18: pizzas with at least one topping in meat

have at least one meat topping \Rightarrow have at least one dairy topping

RCA - Universal relational attributes

three-cheese has topping in and only in **Concept_10** extent.



$\forall \exists$ has-topping. **Concept_10** is assigned to **three-cheese**

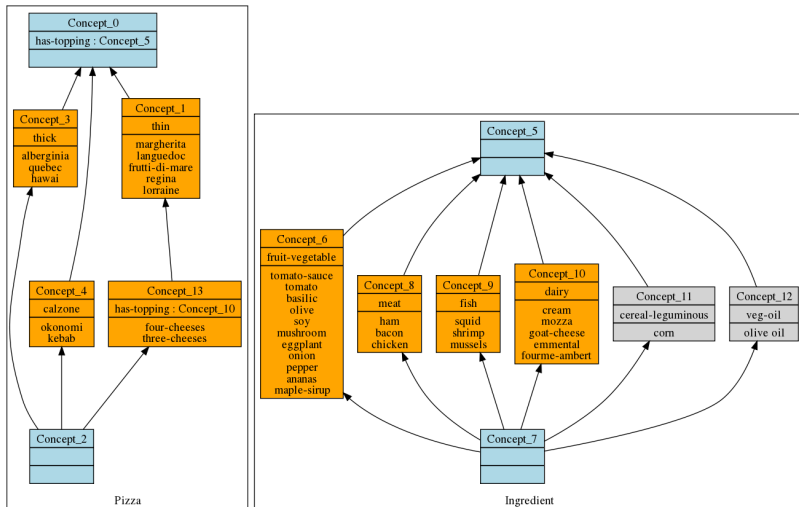
RCA - Universal relational attributes

Scaled relations with domain O_i are concatenated to K_i , the object-attribute context on O_i

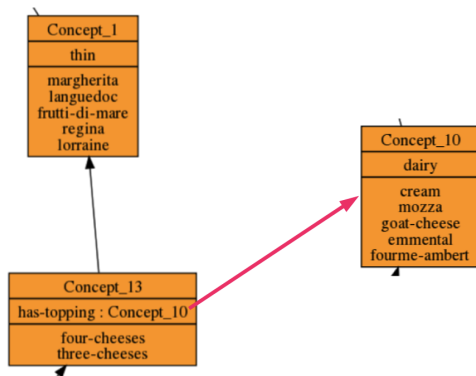
Pizza	thin	thick	calzone
okonomi			x
alberginia		x	
margherita	x		
languedoc	x		
four-cheeses	x		
three-cheeses	x		
frutti-di-mare	x		
quebec		x	
regina	x		
hawai		x	
lorraine	x		
kebab			x

has-topping	$\forall \exists$ has-topping. Concept_7	$\forall \exists \forall$ has-topping. Concept_5	$\forall \exists \forall$ has-topping. Concept_6	$\forall \exists \forall \forall$ has-topping. Concept_8	$\forall \exists \forall \forall \forall$ has-topping. Concept_9	$\forall \exists \forall \forall \forall \forall$ has-topping. Concept_10	$\forall \exists \forall \forall \forall \forall \forall$ has-topping. Concept_11	$\forall \exists \forall \forall \forall \forall \forall \forall$ has-topping. Concept_12
okonomi		x						
alberginia		x						
margherita		x						
languedoc		x						
four-cheeses		x						
three-cheeses		x				x		
frutti-di-mare		x					x	
quebec		x						
regina		x						
hawai		x						
lorraine		x						
kebab		x						

Relational Concept Family / forall



Relational Concept Family / forall



Concept_13: pizzas with only dairy topping

Concept_1: thin pizzas

have only dairy topping \Rightarrow thin

RCA - Introducing relations as relational attributes

Relational scaling is the process by which links are established between objects and concepts.

For each relational context $R_j = (O_k, O_l, I_j)$, a scaled context $R_j^* = (O_k, A, I_j)$ is created.

- ▶ A is a set of relational attributes $a = S R_j.C$, where C is in the concept set of a lattice built on objects of O_l , denoted by \mathcal{L}_l^n
- ▶ I_j contains (o, a) iff $S(R_j(o), \text{Extent}(C))$ is true.

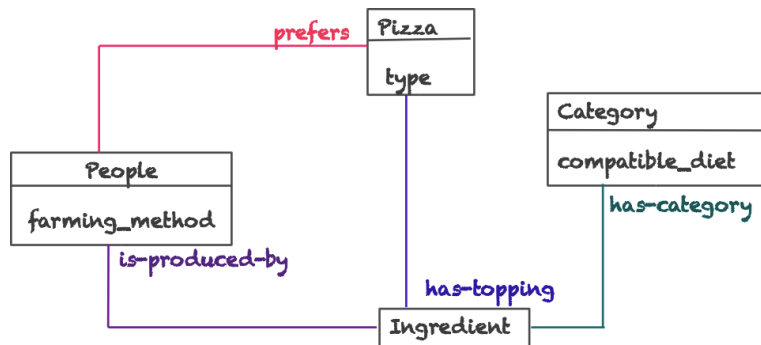
S is a *scaling* operator, the most used are:

- ▶ $S_{\exists}(R_j(o), \text{Extent}(C))$ is true iff $R_j(o) \cap \text{Extent}(C) \neq \emptyset$.
- ▶ $S_{\forall\exists}(R_j(o), \text{Extent}(C))$ is true iff $R_j(o) \subseteq \text{Extent}(C) \wedge \exists x \in R_j(o), x \in \text{Extent}(C)$

Scaling operators

Operator	Attribute form	Condition
Universal (narrow)	$\forall r.c$	$r(o) \subseteq \text{Ext}(c)$
Covers	$\supseteq r.c$	$r(o) \supseteq \text{Ext}(c)$
Existential (wide)	$\exists r.c$	$r(o) \cap \text{Ext}(c) \neq \emptyset$
Universal strict	$\forall \exists r.c$	$r(o) \subseteq \text{Ext}(c)$ and $r(o) \neq \emptyset$
Qualified cardinality restriction	$\geq n r.c$	$r(o) \subseteq \text{Ext}(c)$ and $ r(o) \geq n$
Cardinality restriction	$\geq n r.\top_{\mathcal{L}}$	$ r(o) \geq n$

General Entity-Relationship diagram



General ER diagram may present cycle/circuits between classes/objects

The RCA schema

Input

$RCF = (K, R)$: n object-attribute contexts, m object-object contexts

Initialization step

build, for i in $1..n$, $\mathbf{L}^0[i]$ the concept lattice of the context \mathcal{K}_i

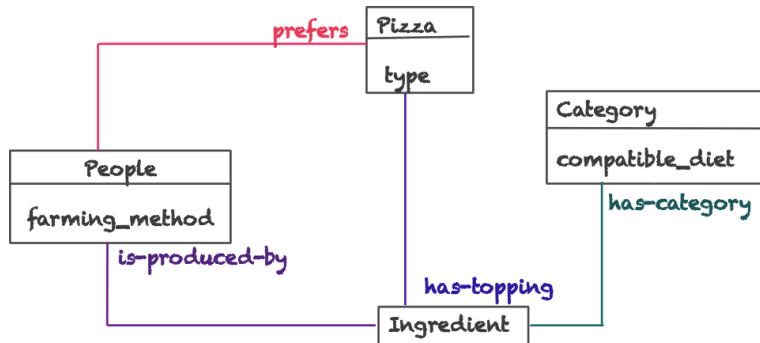
Step p

- ▷ Apply relational scaling to all object-object contexts R_j , using the lattices of step $p - 1$ and the chosen scaling operator
- ▷ concatenate \mathcal{K}_i with the scaled R_j^* whose domain is O_j
- ▷ update lattices of step $p - 1$ to build, for i in $1..n$, the lattice $\mathbf{L}^p[i]$ for the context \mathcal{K}_i concatenated as explained previously

Output (fix point)

The concept lattice family obtained when no new concepts are added

Analysis of pizza data



\exists prefers $\forall \exists$ has-topping $\forall \exists$ has-category $\forall \exists$ is-produced-by

Analysis of pizza data - object-attribute contexts

Pizza	thin	thick	calzone
forest		×	
occitane			×
three-cheese	×		
four-cheese	×		
lorraine	×		
arctic		×	

People	organic-farmer	conventional-farmer
Amedeo	×	
Amine	×	
Cyril		×
Marianne	×	
Petko		×

Ingredient
tomato-sauce
cream
onion
bacon
salmon
soy-cream
mozza
goat-cheese
emmental
fourme-ambert
eggplant
mushroom

Category	mediterranean	vegan	vegetarian
fruit-vegetable	×	×	×
meat			
fish	×		
dairy	×		×

Analysis of pizza data - object-object contexts

prefers	forest	occitane	three-cheese	four-cheese	lorraine	arctic
Amedeo	×					
Amine		×				
Cyril				×	×	
Marianne			×			×
Petko						×

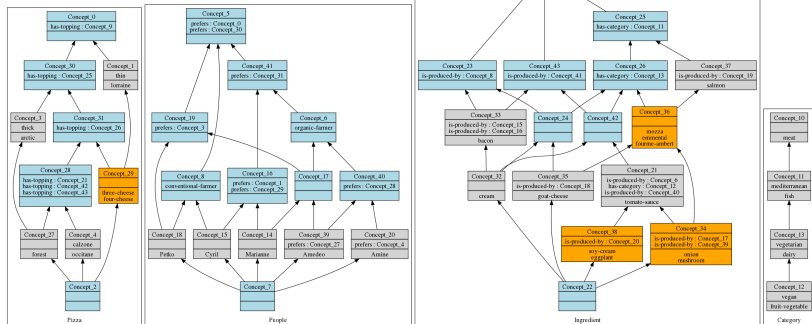
	tomato-sauce	cream	onion	bacon	salmon	soy-cream	mozza	goat-cheese	emmental	fourme-ambert	eggplant	mushroom
has-topping												
forest						×						×
occitane	×		×								×	
three-cheese	×						×	×	×			
four-cheese	×	×					×	×	×	×		
lorraine		×	×	×			×					
arctic	×	×			×		×					

Analysis of pizza data - object-object contexts

is-produced-by	Amedeo	Amine	Cyril	Marianne	Petko
tomato-sauce	×	×			
cream			×		
onion	×				
bacon			×		
salmon				×	×
soy-cream		×			
mozza				×	×
goat-cheese					×
emmental				×	×
fourme-ambert				×	×
eggplant		×			
mushroom	×				

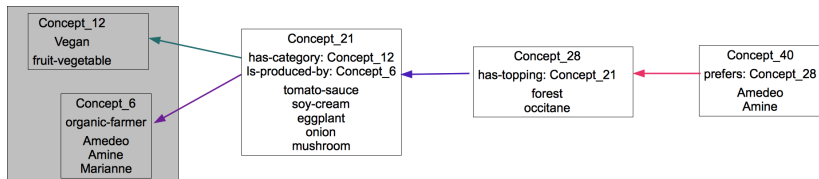
has-category	fruit-vegetable	meat	fish	dairy
tomato-sauce	×			
cream				×
onion	×			
bacon		×		
salmon			×	
soy-cream	×			
mozza				×
goat-cheese				×
emmental				×
fourme-ambert				×
eggplant	×			
mushroom	×			

Concept lattice family



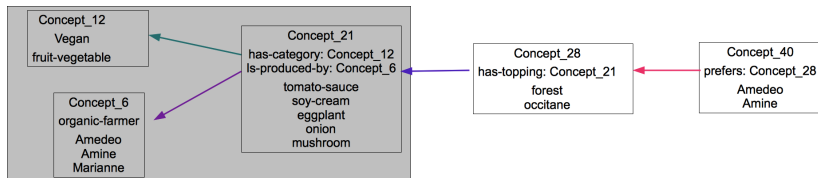
\exists prefers $\forall \exists$ has-topping $\forall \exists$ has-category $\forall \exists$ is-produced-by

Concept lattice family



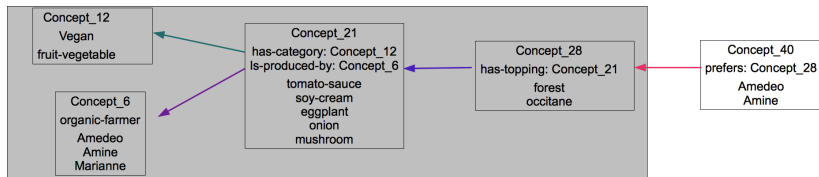
Step 0

Concept lattice family



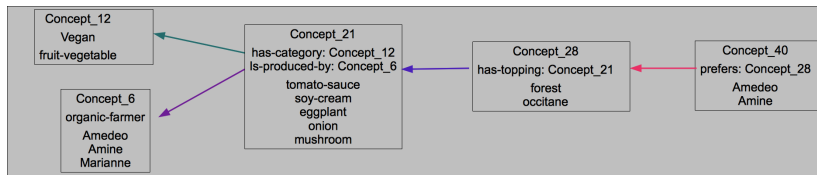
Step 1

Concept lattice family



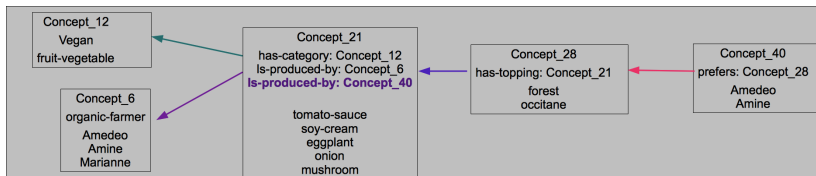
Step 2

Concept lattice family



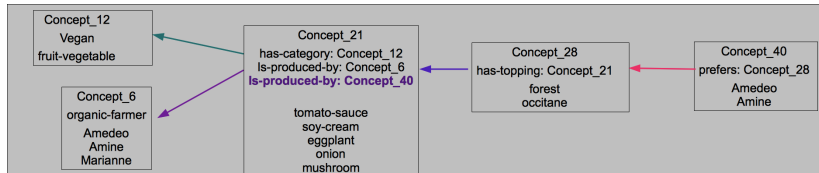
Step 3

Concept lattice family



Step 4

Concept lattice family



- ▶ People: $\exists \text{prefers. Concept_28} \Rightarrow \text{organic farmer}$
- ▶ Ingredient: $\forall \exists \text{has-category. Concept_12} \Leftrightarrow \forall \exists \text{is-produced-by. Concept_6}$ (organic farmers)
- ▶ Amedeo/Amine prefer at least one pizza with only vegan topping ingredients and produced only by organic farmers

A synthesis on RCA

- ▶ an iterative method to produce interconnected classifications
- ▶ converges after a number of iterations that depends on the structure
- ▶ variations on scaling can be done

Tools

- ▶ Galicia: <http://galicia.sourceforge.net/>
- ▶ eRCA: <http://code.google.com/p/erca/>

RCA - Current tracks

- ▶ Querying the concept lattice family
- ▶ Exploratory RCA: select, divide, step-by-step
- ▶ Metrics for guiding the process and filtering concepts
- ▶ Build Galois sub-hierarchy (AOC-poset)