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Predictive control for the stabilization of a fast mechatronic system: from simulation to real-time experiments

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Abstract: In this paper a Generalized Predictive Control (GPC) scheme is proposed for the stabilization of a fast mechatronic system. Namely the inertia wheel inverted pendulum, which has two degrees of freedom and one actuator. The proposed control approach should be able to stabilize this system around its unstable equilibrium point and maintain it in this state. The efficiency and performance of the proposed control scheme are firstly illustrated through simulation results, then its robustness is shown through real-time experiments on the prototype of the system in question.

Keywords: Generalized Predictive Control, stabilization, mechatronic, inertia wheel inverted pendulum.

1. INTRODUCTION

Mechatronics is the synergistic combination of mechanical engineering, electronics, control systems and computers. The key element in mechatronics is the integration of these areas through the design process as in robotics, digitally controlled combustion engines, automated guided vehicles, etc. Mechatronic systems are generally characterized by significant nonlinearities and input and state constraints. Predictive Control can then be a systematic methodology to handle these challenging control problems (Keerthi and Gilbert 1988, Mayne and Michalska 1990).

Predictive controllers are based on the receding horizon methodology that offers a powerful approach to design state feedback controllers for constrained systems (Mayne, Rawlings, Rao, and Scokaert, 2000). The receding horizon control is a form of control in which the current control action is obtained by solving online, at each sample time, a finite horizon open-loop optimal control problem (Richalet et al., 1978), using the current state of the plant as the initial state; the optimization yields to an optimal control sequence and only the first sample in this sequence is applied. The resolution of the optimization problem, often subject to constraints, becomes difficult in the case of nonlinear systems because of the long computational time. Therefore, the optimization algorithm should have speed and convergence properties (Cannon, 2004) through a set of local and global methods (Horst and Pardalos 1995) to ensure a good control performance. In the case of a linear process model, an analytical solution of the optimization problem without constraints is possible and easy to calculate; furthermore the control performance may be better since the global optimum can be reached (Cutler, 1979; Clarke, 1987; Flaus, 1994). Consequently, different model predictive control techniques based on this principle can be applied, especially for fast dynamic mechatronic systems.

Generalized predictive control (GPC) is one of these techniques. It provides an analytical solution of the optimization problem (in the absence of constraints). This control scheme can deal with unstable as well as non-minimum phase systems. It is one of the most popular approaches of Model Predictive Control (MPC) both in industry and academia. In this paper we discuss this control approach for the problem of stabilization of a nonlinear fast mechatronic system, namely the inertia wheel inverted pendulum [16].

The inverted pendulum system is a standard problem in the area of control systems. It is often used to demonstrate concepts in linear and nonlinear control such as stabilization or swinging up of unstable systems. This system belongs to the class of non-minimum systems because of its unstable internal dynamics. One interesting example of inverted pendulum is the inertia wheel inverted pendulum; it is underactuated system since the number of its control inputs is less than the number of its degrees of freedom, which makes it difficult to control. In this paper, the aim of the proposed control scheme (GPC) is to stabilize the inertia wheel inverted pendulum around its unstable equilibrium point and to maintain it in this state, even if it is subject to external disturbances, or in the presence of parametric uncertainties. Numerical simulations as well as real-time experiments are presented to show the effectiveness of the proposed control scheme and its robustness towards external disturbances and changes in system dynamics.

This paper is organized as follows. In section 2, the inertia wheel inverted pendulum is described and its dynamic model is given. The generalized predictive control law is presented in section 3. Section 4 is dedicated to simulation results, while experiments are presented and discussed in section 5. Finally, concluding remarks are drawn in section 6.
2. DESCRIPTION AND DYNAMICS OF THE MECHATRONIC SYSTEM

2.1 Description of the system

The inertia wheel inverted pendulum system (shown in Fig. 1) consists of three main parts: the mechanical part, the computer and the electronic part.

![Inertia wheel inverted pendulum system](image1)

Fig. 1. Inertia wheel inverted pendulum system

**Description of the mechanical part of the system**

The underactuated mechanical system studied in this paper is the inertia wheel inverted pendulum (cf. Fig. 2), which consists of an inverted pendulum equipped with a rotating wheel. The joint between the pendulum body and the frame is unactuated; whereas the joint between the beam and the wheel is actuated by a Maxon EC-powermax 30 DC motor (cf. Fig. 3).

The schematic representation of the inertia wheel inverted pendulum is depicted in Fig. 6. The motor torque produces an angular acceleration of the rotating wheel which generates, thanks to the dynamic coupling between coordinates, a torque acting on the pendulum’s passive joint; therefore this last one can be controlled through the acceleration of the inertia wheel.

![Mechanical part of the system](image2)

Fig. 2. Mechanical part of the system

![Maxon EC-powermax 30 DC motor](image3)

Fig. 3. Maxon EC-powermax 30 DC motor

**Description of the computer and electronic parts of the system**

The actuator of the system is a Maxon EC-powermax 30 DC motor, equipped with an incremental encoder as shown in Fig. 4, allowing the measurement in real-time of the inertia wheel angular position.

In order to measure the angular position of the pendulum with respect to the vertical, the system is equipped with an inclinometer FAS-G of Micro strain (cf. Fig. 5). The system is controlled with a computer equipped with a 2.4 GHz Intel processor. The control approach is implemented using C++ language, and the whole system is running under Ardence RTX real-time OS.

![DC motor equipped with an incremental encoder](image4)

Fig. 4. The DC motor equipped with an incremental encoder

![Inclinometer FAS-G of Micro strain](image5)

Fig. 5. inclinometer FAS-G of Micro strain

2.2 Dynamic modeling of the system

The nonlinear dynamic model of the system (12) is obtained using Lagrange formulation [8], and is given by:

\[
\begin{bmatrix}
I + i_1 + i_2 \\
i_1 \\
i_2
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix} + \begin{bmatrix}
-ml g \sin \theta_1 \\
0
\end{bmatrix} = \begin{bmatrix}
C_1 \\
C_2
\end{bmatrix}
\]

(1)

where:

- \( \theta_1 \) and \( \theta_2 \) are, respectively, the angular positions of the pendulum body and the inertia wheel (cf. Fig. 6);
- \( \dot{\theta}_i \) and \( \dot{\theta}_i (i = 1, 2) \), represent their corresponding velocities and accelerations;
- \( C_1 \) is the external disturbing torque applied on the pendulum (in this study, it is supposed nul);
- \( C_2 \) is the torque generated by the system’s actuator;
- \( i_1, i_2 \) are respectively the moments of inertia of the pendulum body and the wheel;
- \( I = m_1 l_1^2 + m_2 l_2^2 + i_1 \) with \( m_1 \) and \( m_2 \) being the masses of the pendulum and the inertia wheel, \( l_1 \) and \( l_2 \) are the distances from the origin O (cf. Fig. 6) to the gravity centers of the pendulum and the rotating mass (respectively);
- \( ml = m_1 l_1 + m_2 l_2 \).

2.3 State space representation of the model

The state space representation of the inertia wheel inverted pendulum is obtained through linearization of the nonlinear dynamic model presented above around the unstable equilibrium point. Let the state vector \( x_c \) be defined as: \( x_c = [\theta_1 \ \dot{\theta}_1 \ \dot{\theta}_2]^T \) and \( u = C_2 \). The unstable equilibrium point \( (x_c^*, u^*) \) is defined
The state vector $x$ in this case is given as:

$$x = x_e, A = A_c, B = B_c, C = C_c$$

where $y = y_e$.

### 3.2 GPC basic principle

The Generalized Predictive Control (GPC) scheme was initially proposed by Clarke et al [17] and has become one of the most popular Model Predictive Control (MPC) schemas both in industry and academia. The basic idea of GPC is to compute a future control sequence such that it minimizes a multiobjective cost function defined over a prediction horizon. The performance index to be optimized is a quadratic function including a term measuring the distance between the predicted system output and the reference sequence over the horizon plus a term measuring the control effort.

GPC has many ideas in common with the other model predictive controllers since it is based upon the same concepts; nevertheless, it has also some differences. It provides an analytical solution (in the absence of constraints), it can deal with unstable and non-minimum phase systems and incorporates the concept of receding horizon as well as the consideration of weighting of control increments in the cost function.

The proposed GPC formulation should involve all the states of the system, and introduce a penalty condition on the final state. Indeed, the classical formulation of the GPC with a transfer function as input/output model of the system does not guarantee the control of all system states. This is because such a model involves only the outputs of the system and not all the states.

In the case of our system, the objective is to impose: $\theta_{i_f}(t_f) = \hat{\theta}_1(t_f) = \hat{\theta}_2(t_f) = 0$ where $t_f$ characterizes the end of the prediction horizon at each sampling instant. For this formulation, the cost function $J$ to be minimized is as follows:

$$J = \sum_{j=1}^{N_p} (y(k+j) - w(k+j))^T(y(k+j) - w(k+j)) + (x(k+N_p) - w_x(k+N_p))^TQ(x(k+N_p) - w_x(k+N_p)) + \sum_{j=1}^{N_c} \lambda(j)(\Delta u(k+j-1))^T(\Delta u(k+j-1))$$

where: $N_p$ and $N_c$ are the beginning and the end prediction horizons, $N_a$ is the length of the control horizon, $Q$ and $\lambda$ are the weights on the states and the control respectively and $w_x$ is the desired final state of the system.

The calculation of the cost function requires the prediction of future outputs $y(k+j), j = 1 \ldots N_p$ at each sampling instant $k$ based on the information available on the system at previous times. From the state space representation of the system, we have:

$$x(k+1) = A x(k) + B u(k)$$

$$x(k+2) = A x(k+1) + B u(k+1) = A^2 x(k) + Bu(k+1)$$

$$\vdots$$

$$x(k+j) = A^j x(k) + \sum_{i=0}^{j-1} A^{j-i-1} B u(k+i)$$

Therefore, the estimated output of the system at time $k+j$ is written as:

$$y(k+j) = CA^j x(k) + \sum_{i=0}^{j-1} CA^{j-i-1} B u(k+i)$$
Since the control horizon is shorter than the prediction horizon \( N_u \leq N_p \), \( \forall i \geq N_u, u(k+i) = 0 \), and if we consider:

\[
y = \begin{bmatrix} y(k+1) \\ y(k+2) \\ \vdots \\ y(k+N_1) \\ \vdots \\ y(k+N_p) \end{bmatrix}, \quad \Delta u = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \Delta u(k+2) \\ \vdots \\ \Delta u(k+N_u) \end{bmatrix}
\]

(8)

Then the predicted output can be rewritten in matrix form as follows:

\[
y = G\Delta u + f
\]

(9)

where \( G \) and \( f \) are given by:

\[
G = \begin{bmatrix} CB & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ CA^{N_1-1}B & CA^{N_1-2}B & \ldots & CA^{N_1-3}B & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & \ldots & CA^{N_p-3}B & \ldots & CA^{N_p-N_u}B \end{bmatrix}
\]

and \( f = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_1} \\ \vdots \\ CA^{N_p} \end{bmatrix} x(k) \).

The control sequence should be derived from the criterion (5). Since we are interested on the stabilization problem, let’s consider: \( w_s = 0, w(k+j) = 0 \) \( \forall j, N_1 = 1, N_p = N_u = N \). \( \lambda \geq 0 \) et \( \lambda > 0 \).

The criterion (5) can then be rewritten as follows:

\[
J_N = \sum_{j=1}^{N} y(k+j)^T y(k+j) + \lambda \Delta u(k+j-1)^T \Delta u(k+j-1) + x^T (k+N)(Q + C^T C) x(k+N)
\]

(10)

The state of the system at time \( k+N \) is given by:

\[
x(k+N) = A^N x(k) + \tilde{C} \Delta u
\]

(11)

Where: \( \tilde{C} = [A^{N-1}B, A^{N-2}B, \ldots, B] \).

The cost function can then be reformulated as follows:

\[
J_N = \frac{1}{2} [H + 2\tilde{C}^T Q \tilde{C}] \Delta u + \left[ b + 2x^T (k)(A^N)^T Q\tilde{C} \right] \Delta u + f_0 + x^T (k)(A^N)^T Q A^N x(k)
\]

(12)

Where:

- \( H = 2[G^T G + \lambda I] \);
- \( b = 2f^T \tilde{G} \);
- \( f_0 = f^T f \).

The optimal control sequence that minimizes the performance index (12) is obtained from the solution of the equation \( \delta J_N = 0 \).

The obtained optimal solution \( \Delta u^* \) is then written as follows:

\[
\Delta u^* = -Kx(k)
\]

(13)

Where:

\[
K = (G^T G + \lambda I + \tilde{C}^T Q \tilde{C})^{-1}(G^T L + \tilde{C}^T Q A^N)
\]

(14)

It is worth to note, according to the basic principle of predictive control, that only the first sample of the optimal control sequence is applied to the controlled system.

4. SIMULATION RESULTS

Simulation results, obtained using Matlab software, are presented and discussed in this section. As a first validation, they show the feasibility of the proposed control scheme. One simulation scenario is considered to validate the robustness of the proposed control scheme. The first scenario is to consider the system in the nominal case without any external disturbances. While the second one aims to show the robustness of the controller against parameter’s uncertainties.

4.1 Scenario 1: Stabilization in the nominal case

Consider the dynamic model of system (1) with physical parameters summarized in Table 1. These parameters have been identified on the real prototype of the system described in section 2.

Table 1. Description of dynamic parameters of the system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 )</td>
<td>Body mass</td>
<td>3.30810</td>
<td>Kg</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>Wheel mass</td>
<td>3.33081</td>
<td>Kg</td>
</tr>
<tr>
<td>( l_1 )</td>
<td>Body center of mass position</td>
<td>0.06</td>
<td>m</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>Wheel center of mass position</td>
<td>0.044</td>
<td>m</td>
</tr>
<tr>
<td>( I_1 )</td>
<td>Body inertia</td>
<td>0.0314863</td>
<td>Kg m²</td>
</tr>
<tr>
<td>( I_2 )</td>
<td>Wheel inertia</td>
<td>0.0004176</td>
<td>Kg m²</td>
</tr>
</tbody>
</table>

The proposed simulation is started from the initial condition \( x_0 = \begin{bmatrix} \theta_1 = 18^\circ \theta_2 = 0 \end{bmatrix}^T \). Table 2 describes the parameters of the proposed control scheme.

Table 2. Description of the control parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_1 )</td>
<td>Minimum prediction horizon</td>
<td>40</td>
</tr>
<tr>
<td>( N_2 )</td>
<td>Maximum prediction horizon</td>
<td>40</td>
</tr>
<tr>
<td>( N_u )</td>
<td>Length of control horizon</td>
<td>40</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Control weight</td>
<td>40</td>
</tr>
<tr>
<td>( Q )</td>
<td>State weight</td>
<td>( L_2(3,3) )</td>
</tr>
</tbody>
</table>

Fig. 7 displays the obtained simulation results for the nominal case. Fig. 7-(a) and 7-(b) show the evolution of the pendulum body joint position and velocity. The inertia wheel velocity versus time is displayed in Fig. 7-(c). Fig. 7-(d) represents the control input that consists of the motor voltage (proportional to the motor torque).

From the obtained results, it can be observed that the controller is able to stabilize the system around its unstable equilibrium point and keep it around this position.

4.2 Scenario 2: Robustness towards parameter’s uncertainties

The test of the proposed controller robustness allows us to check whether the applied control is capable of compensating the uncertainties on the system parameters. These uncertainties were not considered in the modeling phase of the system.

Let’s now consider the case of an uncertainty on the parameter inertia \( I_1 \) that is:
Three cases are considered: the nominal case corresponding to $\Delta I = 0\%$, and two other cases corresponding to an uncertainty of $\Delta I = 10\%$ and $45\%$, respectively. The objective would then be to see if the proposed controller is able to compensate this uncertainty. Fig. 8 displays the evolution of the system states in these three cases. We notice clearly the ability of the GPC controller to ensure system stabilization despite the misidentification introduced that can go up to $45\%$ uncertainty on the parameter $I$. Note also that the system is becoming slightly slower when increasing the uncertainty amount.

$$\dot{I} = I + \Delta I \times I$$  \hspace{1cm} (14)\

In this scenario, two combined types of disturbances were applied on the inverted pendulum. The first one illustrated in Fig. 10, consists in pushing the pendulum, which generates external punctual torques applied to the pendulum joint at approximately $t = 8s$, $t = 12s$, $t = 17s$ and $t = 26s$. And the second type of disturbances illustrated in Fig. 11 is persistent and applied constantly to the inverted pendulum as an additional mass attached to the pendulum body. The punctual and persistent disturbances can be represented by external forces $F_{1_{ext}}$ and $F_{2_{ext}}$ applied on the inverted pendulum. These forces generate torques $r_{1_{ext}}$ and $r_{2_{ext}}$ around the pendulum pivot. These torques, to be compensated, produce a change in the angular velocity of the rotating wheel.
A. Chemori and M. Alamir. Nonlinear predictive control in inverted pendulum. Our future work will be focused on its robustness towards uncertainties and external disturbances. Namely the inertia wheel inverted pendulum. Numerical simulations as well as real-time experiments show the performance and the effectiveness of the proposed control scheme and its robustness towards uncertainties and external disturbances (both punctual and persistent). Our future work will be focused on the generation of stable limit cycles for the inertia wheel.

6. CONCLUSION AND FUTURE WORK

In this paper, a Generalized Predictive Controller (GPC) is proposed for stabilization of a fast underactuated mechatronic system. Namely the inertia wheel inverted pendulum. Numerical simulations as well as real-time experiments show the performance and the effectiveness of the proposed control scheme and its robustness towards uncertainties and external disturbances (both punctual and persistent). Our future work will be focused on the generation of stable limit cycles for the inertia wheel inverted pendulum.

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