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# From Gene Trees to Species Trees through a Supertree Approach

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**Abstract.** Gene trees are leaf-labeled trees inferred from molecular sequences. Due to duplication events arising in genome evolution, gene trees usually have multiple copies of some labels, *i.e.* species. Inferring a species tree from a set of multi-labeled gene trees (MUL trees) is a well-known problem in computational biology. We propose a novel approach to tackle this problem, mainly to transform a collection of MUL trees into a collection of evolutionary trees, each containing single copies of labels. To that aim, we provide several algorithmic building stones and describe how they fit within a general species tree inference process. Most algorithms have a linear-time complexity, except for an FPT algorithm proposed for a problem that we show to be intractable.

## 1 Introduction

An evolutionary tree (or *phylogeny*), is a tree displaying the evolutionary history of a set of sequences or organisms. A *gene tree* is an evolutionary tree built by analyzing a gene family, *i.e.* homologous molecular sequences appearing in the genome of different organisms. Gene trees are primarily used to estimate *species trees*, *i.e.* trees displaying the evolutionary relationships among studied species. Unfortunately, most gene trees can significantly differ from the species tree for methodological or biological reasons, such as long branch attraction, lateral gene transfers, deep gene coalescence and, principally, gene duplications and losses [1]. For this reason, species trees are usually estimated from a large number of gene trees.

Inferring a species tree from gene trees is mostly done in a two-step approach. First, a micro-evolutionary model that takes into account events affecting individual sites is used to infer the gene trees. The species tree is then inferred on the basis of a macro-evolutionary model, *i.e.* minimizing the number of transfer, duplication and loss events [2-6]. To produce more biologically meaningful trees, unified models have been proposed in which the micro and macro-evolutionary

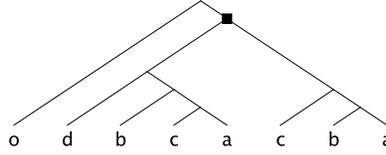
dimensions are entangled [7–9]. However, it is difficult to determine how to incorporate events occurring on different spatial and temporal scales, as well as belonging to neutral and non-neutral processes, in a single model [9]. Lately, a hybrid approach has been proposed, where a first draft of a species tree is inferred with a micro-evolutionary model, the most uncertain parts of which are then corrected according to a macro-evolutionary model [9].

In this paper, we propose instead to take advantage of the very large number of gene trees present in recent phylogenomic projects to avoid entering into the detail of all possible macro-evolutionary scenarios (*e.g.* is a parsimony approach always justified? Should only the most parsimonious scenario be retained?). We propose to extract the non-ambiguous part of the topological information contained in the gene trees, *i.e.* that resulting from speciation events as opposed to duplication events, and then apply a traditional supertree method letting the weight of evidence decide in favor of one candidate species tree [10–12].

This approach is only possible when the number of gene trees is very large, and indeed this is now the case in projects such as the HOMOLENS database (<http://pbil.univ-lyon1.fr/databases/homolens.php>), storing several thousands of gene trees. In the release 04 of this database, 51% of gene families have paralogous sequences, *i.e.* sequences where duplications and losses have actually taken place. Currently, these gene families are discarded when inferring a supertree of the concerned species. Disentangling information derived from speciation events from that resulting from duplication events would thus provide more information for species tree inference.

Supertree methods combine source trees whose leaves are labeled with individual species into a larger species tree. The source trees are *single-labeled*, *i.e.* each species labels at most one leaf. Note that, by definition, the inferred supertree is also single-labeled. In contrast, gene trees are usually *multi-labeled*, *i.e.* a single species can label more than one leaf, since duplication events resulted in the presence of several copies of the genes in the species genomes. The task we therefore have to solve is to extract the largest amount of unambiguous topological information from the multi-labeled gene trees under the form of single-labeled trees. This paper presents a number of results in this direction, that all play a role in the general scheme that is fully described below. The rest of the paper details these results, though in a different order for the sake of dependancies between definitions.

First of all, we propose to separately preprocess MUL trees in order to remove their redundant parts with respect to speciation events. For this purpose, we extend the tree isomorphism algorithm of [13] making it applicable to MUL trees while preserving a linear running time (section 5). This algorithm is then applied to the pairs of subtrees hanging from duplication nodes in MUL trees. This preprocess lowers the number of duplication nodes in gene trees. We also give in passing a linear time algorithm to identify duplication nodes in MUL trees (section 4). For the gene trees that still have duplication nodes, we define a set  $\mathcal{R}$  of triplets (binary rooted trees on three leaves [14, 12]) containing the topological information of a MUL tree that can be thought of as being unambiguously



**Fig. 1.** An example of MUL tree with one odn indicated by a black square

related to speciation events. We show that this set of triplets can be computed in  $O(|\mathcal{R}|)$  time (section 2). When this set is compatible, the MUL tree contributes a coherent topological signal to build the species tree. In such a case, we can replace the MUL tree with a single-labeled tree representing its associated set of triplets by using the BUILD algorithm [14] (section 3). When a MUL tree is not auto-coherent, we propose to extract a maximum subtree that is both auto-coherent and free of duplication events. Surprisingly, this optimization problem can be solved in linear time (section 6). When extracting largest single-labeled subtrees from MUL trees it is possible to obtain an incompatible collection, when a compatible collection could have been obtained by choosing subtrees of MUL trees in a coordinated way. However, solving this problem is computationally harder, as we show by providing an NP-completeness proof (section 7).

## 2 Preliminaries

In this paper we focus on rooted binary multi-labeled (MUL) trees. Let  $M$  be a MUL tree and  $v$  a node of  $M$ . If  $v$  is a leaf node, we denote by  $\mathbf{l}_v$  its label. If  $v$  is an internal node, denote by  $\mathbf{v}_1$  and  $\mathbf{v}_2$  the two sons of  $v$  and by  $\mathbf{sons}(v)$  the set  $\{v_1, v_2\}$ . We denote by  $LM$  the set of leaf nodes of  $M$ . We define by  $M_v$  the subtree with  $v$  as root and by  $\mathbf{L}(v)$  the multiset of labels of  $M_v$ . We denote by  $L(M)$  the multiset  $L(\text{root}(M))$ .

**Definition 1.** A node  $v$  of  $M$  is called an **observed duplication node** (odn) if the intersection of  $L(v_1)$  and  $L(v_2)$  is not empty.

Note that, for an odn  $v$ ,  $L(v)$  will always contain some label more than once. We denote by  $\mathcal{D}(M)$  the set of odn. A label  $l \in L(M)$  is a *repeated label* for  $M$  iff the label  $l$  occurs more than once in  $L(M)$ . We say that  $f$  is a *repeated leaf* for  $M$  iff  $L(f)$  is a repeated label. For every three leaves we can have three different rooted tree binary shapes, called *triplets*. We denote by  $ab|c$  the rooted tree that connects the pair of taxa (a, b) to c via the root.

We denote by  $\mathcal{R}(M)$  the set of triplets of a (single/multi)labeled evolutionary tree  $M$  i.e.  $\mathcal{R}(M) = \{ab|c \text{ s.t. there exist three leaf nodes } x, y, z \in M : l_x = a, l_y = b, l_z = c \text{ and } lca(x, y) \neq (lca(x, z) = lca(y, z))\}^1$ .

<sup>1</sup> The root of the tree is considered to be up and its leaves down.  $lca(x, y)$  denotes the least common ancestor of nodes  $x$  and  $y$ , i.e. the lowest node in the tree that has both  $x$  and  $y$  as descendants.

**Definition 2.** Let  $M$  be a MUL tree. We define by  $\mathcal{R}_{wd}(M)$  ( $\mathcal{R}(M)$  without duplications) the set of triplets  $ab|c$  of  $\mathcal{R}(M)$  s.t. there exist three leaf nodes  $x, y, z \in M : l_x = a, l_y = b, l_z = c$  and  $lca(x, y) \notin \mathcal{D}(M)$ ,  $lca(x, y, z) \notin \mathcal{D}(M)$ ,  $lca(x, y) \neq (lca(x, z) = lca(y, z))$ .

For example, for the MUL tree in Fig. 1,  $\mathcal{R}_{wd}(M) = \{ac|b, ac|d, ab|d, bc|d, ac|o, ab|o, ad|o, bc|o, cd|o, bd|o, ab|c\}$ . Hence, not all the triplets of  $\mathcal{R}(M)$  are kept. This is due to the fact that, once a duplication event occurred in a gene's history, the two copies of the gene evolved independently. The history of each copy is influenced by the species history but, considering them simultaneously may produce information unrelated to the species evolution. Therefore, it is more appropriate to discard the triplets mixing the histories of distinct copies of a gene.

$\mathcal{R}_{wd}(M)$  has size  $O(n^3)$  and can be computed in  $O(n^3)$  time, where  $n$  is the number of leaf nodes of  $M$ . Indeed, once the lca of all pairs of nodes in  $M$  are computed in  $O(n)$  time (see [15, 16]), checking for three leaf nodes  $x, y, z$  of  $M$  if they satisfy Definition 5 can be done in  $O(1)$  time, thus in  $O(n^3)$  for all triplets of leaves in  $M$ .

### 3 Auto-coherency of a MUL Tree

Let  $M$  be a multi-labeled gene tree. Since  $M$  contains several copies of the same gene, we can wonder whether the evolutionary signal of each copy is coherent or not. Let introduce some notations.

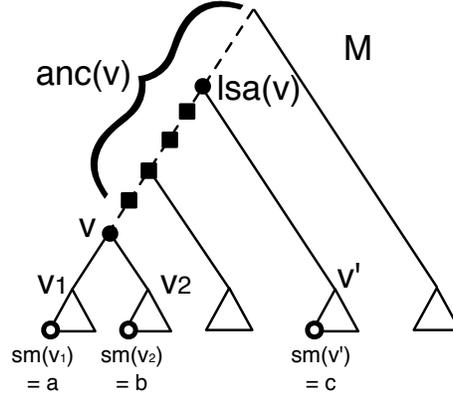
**Definition 3.** A triplet set  $R$  is said to be **compatible** if there exists a single-labeled tree  $T$  such that  $R \subseteq \mathcal{R}(T)$ .

**Definition 4.** A MUL tree  $M$  is said to be **auto-coherent** if the triplet set  $\mathcal{R}_{wd}(M)$  is compatible, i.e. if there exists a single-labeled tree  $T$  such that  $\mathcal{R}_{wd}(M) \subseteq \mathcal{R}(T)$ .

In the case of an auto-coherent MUL tree, we know that there exists at least one tree  $T$  containing all the information in  $\mathcal{R}_{wd}(M)$ , i.e. the information of  $M$  that is considered reliable. To find such a tree, we use the AncestralBuild algorithm of [17]. For a set of triplets  $R$ , this algorithm indicates in  $O(|R| \cdot \log^2(|L(R)|))$  time whether  $R$  is compatible, where  $L(R)$  is the set of leaf labels of the elements of  $R$ . Moreover, in case of a positive answer it returns a tree  $T$  s.t.  $R \subseteq \mathcal{R}(T)$ .

It has been proved in [18] that a binary single-labeled rooted tree  $T$  can be encoded using a triplet set  $\mathcal{R}^l(T)$  whose size is the number of inner nodes of  $T$ . In this section we show that it is possible to check the auto-coherency of a MUL tree  $M$  by using as representation of  $\mathcal{R}_{wd}(M)$  a triplet set  $\mathcal{R}_{wd}^l(M)$  whose size is at most equal to the number of speciation nodes of  $M$ . To univocally define the set  $\mathcal{R}_{wd}^l(M)$ , let  $<$  be a total order on the leaf set  $L(M)$ . For each node  $v$  of  $M$ , we denote by  $\mathbf{sm}(v)$  the smallest element of  $L(M_v)$  according to  $<$  and by  $\mathbf{anc}(v)$  the set of nodes belonging to the path from  $v$  to the root of  $M$ . Let  $\mathbf{lsa}(v)$  be the least speciation ancestor of  $v$  i.e. the speciation node in  $\mathbf{anc}(v)$  closest to  $v$  and  $\mathbf{v}'$  the son of  $\mathbf{lsa}(v)$  such that  $v \notin M_{v'}$ . Note that, if the father of  $v$  is not in  $\mathcal{D}(M)$ , it coincides with  $\mathbf{lsa}(v)$  while  $v'$  is the sibling node of  $v$ .

**Definition 5.** Let  $M$  be a MUL tree and  $<$  a total order on  $L(M)$ . We define by  $\mathcal{R}_{wd}^l(M)$  the set of triplets  $ab|c$  such that  $ab|c \in \mathcal{R}_{wd}(M)$  and there exists a speciation node  $v$  in  $M$  such that  $sm(v_1) = a$ ,  $sm(v_2) = b$  and  $sm(v') = c$ .



**Fig. 2.** The only triplet of  $\mathcal{R}_{wd}^l(M)$  associated to the speciation node  $v$  is  $ab|c$ , while the triplet set associated to  $v$  in  $\mathcal{R}_{wd}(M)$  is composed by the triplets  $l_x l_y | l_z$  of  $\mathcal{R}(M)$ , where  $x \in \mathcal{L}(M_{v_1})$ ,  $y \in \mathcal{L}(M_{v_2})$  and  $z \in \mathcal{L}(M)$  such that  $lca(x, y, z) \notin \mathcal{D}(M)$  and  $lca(x, y) \neq (lca(x, z) = lca(y, z))$ .

Note that, for each speciation node  $v$ , the set  $\mathcal{R}_{wd}^l(M)$  contains at most one triplet while  $\mathcal{R}_{wd}(M)$  typically contains many more triplets (see figure 2).

Once the set of duplication nodes  $\mathcal{D}(M)$  is calculated, Algorithm 1 computes  $\mathcal{R}_{wd}^l(M)$  in linear time (see Theorem 1). We now need to show that checking the auto-coherency of  $\mathcal{R}_{wd}(M)$  and  $\mathcal{R}_{wd}^l(M)$  is equivalent. To do that, we need to introduce some notations. Given a node  $v$  of a MUL tree  $M$ , we define the height of  $v$ , denoted by  $\mathbf{h}(v)$ , as the longest path between  $v$  and its descendants. More formally, the height of a leaf is fixed to zero and that of a internal node  $v$  is  $\max(h(v_1), h(v_2)) + 1$ . We denote by  $\mathcal{G}(R)$  the Aho graph, or clustering graph built from  $R$  (see [14, 19, 18]). The set of vertices of this graph is  $L(R)$ . There is an edge in  $\mathcal{G}(R)$  connecting two vertices  $a$  and  $b$  iff there exists  $ab|c \in R$ . This graph is the traditional way to check the compatibility of a set of rooted trees (see [17] for another approach to this problem).

The proof that the auto-coherency of  $\mathcal{R}_{wd}(M)$  can be tested by checking that of  $\mathcal{R}_{wd}^l(M)$  relies on the following Lemma.

**Lemma 1.** Let  $M$  be a MUL tree and  $v$  a node of  $M$ . Then, if  $anc(v)$  contains at least one speciation node, then  $\mathcal{G}(\mathcal{R}_{wd}^l(M))|L(M_v)$  is connected.

*Proof.* We prove the Lemma by induction on the height of the node  $v$ . Note that we need to consider only those nodes having at least one speciation node as ancestor since Lemma 1 does not apply for nodes not having this property.

**Algorithm 1:** LINEARREPRESENTATION( $M, \mathcal{D}(M), v$ )

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**Data:** A MUL tree  $M$ , the set of duplication nodes  $\mathcal{D}(M)$  of  $M$ , a node  $v$  in  $M$ .  
**Result:** A set of triplets  $\mathcal{R}^l$ .  
 $\mathcal{R}^l \leftarrow \emptyset$ ;  
**if**  $v$  is a leaf **then**  $sm(v) \leftarrow l_v$ ;  
**else**  
    **if** ( $v$  is not the root node) **then**  
         $f \leftarrow$  the father of  $v$ ;  
        **if** ( $f \notin \mathcal{D}(M)$ ) **then**  
            **if** ( $f_1 = v$ ) **then**  $v' \leftarrow f_2$ ;  
            **else**  $v' \leftarrow f_1$ ;  
        **else**  
             $v' \leftarrow f$ ;  
     $\mathcal{R}^l \leftarrow \mathcal{R}^l \cup \text{linearRepresentation}(M, \mathcal{D}(M), v_1)$ ;  
     $\mathcal{R}^l \leftarrow \mathcal{R}^l \cup \text{linearRepresentation}(M, \mathcal{D}(M), v_2)$ ;  
     $sm(v) \leftarrow \min_{<}(sm(v_1), sm(v_2))$ ;  
    **if** ( $v \notin \mathcal{D}(M)$ ) **and** ( $v' \neq \emptyset$ ) **then**  
         $\mathcal{R}^l \leftarrow sm(v_1)sm(v_2) \mid sm(v')$ ;  
**return**  $\mathcal{R}^l$ ;

---

Let start showing that Lemma 1 is valid for all nodes with height 0. In this case  $L(M_v)$  contains a single label, hence  $\mathcal{G}(\mathcal{R}_{wd}^l(M))|L(M_v)$  contains only one vertex *i.e.* it is trivially connected.

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Now suppose that Lemma 1 is valid for all nodes  $v$  such that  $h(v) < \bar{h}$ . We want to prove that this implies that it is true for all nodes  $v$ :  $h(v) \leq \bar{h}$ . Let  $v$  be a node for which  $anc(v)$  contains at least one speciation node and  $h(v) = \bar{h}$ . Since  $h(v_1) = h(v) - 1$  and  $anc(v_1) \supseteq lsa(v)$  we know that  $\mathcal{G}(\mathcal{R}_{wd}^l(M))|L(M_{v_1})$  is connected and the same holds for  $\mathcal{G}(\mathcal{R}_{wd}^l(M))|L(M_{v_2})$ . It remains to prove that there exists an edge connecting the two connected components  $C_1$  and  $C_2$ . Either  $v$  is a speciation node or a duplication node. If  $v$  is a speciation node, then for the definition of  $\mathcal{R}_{wd}^l(M)$  there exists a triplet  $t \in \mathcal{R}_{wd}^l(M)$  such that  $t = sm(v_1)sm(v_2) \mid sm(v')$  and thus  $t$  induces an edge between  $C_1$  and  $C_2$ . If  $v$  is a duplication node, there exists at least a label  $d$  such that  $d \in L(M_{v_1}) \cap L(M_{v_2})$  and this label is represented in  $\mathcal{G}(\mathcal{R}_{wd}^l(M))$  by a single vertex present in both  $C_1$  and  $C_2$ . Then  $\mathcal{G}(\mathcal{R}_{wd}^l(M))|L(M_v)$  is connected in both cases. This concludes the proof.  $\square$

Lemma 1 will be useful while proving Lemma 2. Let introduce the notion of closure of a compatible triplet set. Given a compatible triplet set  $R$ , we say that a triplet  $ab|c$  is the closure of  $R$ , denoted by  $cl(\mathbf{R})$ , iff  $ab|c \in \mathcal{R}(T)$ ,  $\forall T : R \subseteq \mathcal{R}(T)$ . This is equivalent to requiring that both sets  $\{R \cup \{ac|b\}\}$  and  $\{R \cup \{bc|a\}\}$  are incompatible [20]. We introduce a result on the closure of a triplet set that will be useful later on.

**Proposition 1.** *Given a compatible triplet set  $R$ , then  $cl(R)$  is compatible.*

*Proof.* For the definition of compatibility, a triplet set  $R$  is compatible if there exists a tree  $T$  such that  $R \subseteq R(T)$ . From proposition 4(6)<sup>2</sup> of [21] we know that if such a tree exists, this tree has also the property  $cl(R) \subseteq R(T)$ . It follows that  $cl(R)$  is compatible.

We prove the following result.

**Lemma 2.** *Let  $M$  be a MUL tree. If the triplet set  $\mathcal{R}_{wd}^l(M)$  is compatible, then  $\mathcal{R}_{wd}(M) \subseteq cl(\mathcal{R}_{wd}^l(M))$ .*

*Proof.* We prove this statement for all subtrees  $M_v$  of  $M$  by induction on the height of the node  $v$  in  $M$ . As  $M = M_{root(M)}$  this shows the statement.

If  $h(v) = 0$  then  $\mathcal{R}_{wd}(M_v) = cl(\mathcal{R}_{wd}^l(M_v)) = \emptyset$ .

Now suppose that  $\mathcal{R}_{wd}(M_v) \subseteq cl(\mathcal{R}_{wd}^l(M_v))$  for all nodes  $v$  such that  $h(v) < \bar{h}$  and let  $v$  be a node such that  $h(v) = \bar{h}$ . If  $v$  is a duplication node, then  $lca(x, y, z) \in \mathcal{D}(M)$ , for  $x, y \in \mathcal{L}(M_{v_1})$  and  $z \in \mathcal{L}(M_{v_2})$  (and the symmetric case). This implies that  $\mathcal{R}_{wd}(M_v) = \mathcal{R}_{wd}(M_{v_1}) \cup \mathcal{R}_{wd}(M_{v_2})$  and  $\mathcal{R}_{wd}^l(M_v) = \mathcal{R}_{wd}^l(M_{v_1}) \cup \mathcal{R}_{wd}^l(M_{v_2})$ . It follows that  $\mathcal{R}_{wd}(M_v) \subseteq cl(\mathcal{R}_{wd}^l(M_{v_1})) \cup cl(\mathcal{R}_{wd}^l(M_{v_2})) \subseteq cl(\mathcal{R}_{wd}^l(M_{v_1}) \cup \mathcal{R}_{wd}^l(M_{v_2})) = cl(\mathcal{R}_{wd}^l(M_v))$ . Note that, if  $|\mathcal{L}(M_{v_1})| < 3$  (resp  $|\mathcal{L}(M_{v_2})| < 3$ ) then  $\mathcal{R}_{wd}(M_{v_1}) = \emptyset$  (resp  $\mathcal{R}_{wd}(M_{v_2}) = \emptyset$ ) and Lemma 2 still holds.

If  $v$  is a speciation node, then by induction all triplets  $l_x l_y | l_z \in \mathcal{R}_{wd}(M_v)$  with  $x, y, z \in \mathcal{L}(M_{v_1})$  or  $x, y, z \in \mathcal{L}(M_{v_2})$  are in  $cl(\mathcal{R}_{wd}^l(M_v))$ . Let  $t$  be a triplet  $l_x l_y | l_z$  of  $\mathcal{R}_{wd}(M_v)$  with  $x, y \in \mathcal{L}(M_{v_1})$  and  $z \in \mathcal{L}(M_{v_2})$ . We prove that  $t$  is in  $cl(\mathcal{R}_{wd}^l(M_v))$  by proving that  $(\mathcal{R}_{wd}^l(M_v) \cup l_x l_z | l_y)$  and  $(\mathcal{R}_{wd}^l(M_v) \cup l_y l_z | l_x)$  are both incompatible. From Lemma 1 we know that  $\mathcal{G}(\mathcal{R}_{wd}^l(M_v)) | L(M_{v_1})$  and  $\mathcal{G}(\mathcal{R}_{wd}^l(M_v)) | L(M_{v_2})$  are two connected components  $C_1$  and  $C_2$ , since  $v$  is a speciation node above  $v_1$  (resp  $v_2$ ). As  $L(M_v) = L(M_{v_1}) \cup L(M_{v_2})$ , the graph  $\mathcal{G}(\mathcal{R}_{wd}^l(M_v))$  has at most two connected components. Since  $\mathcal{R}_{wd}^l(M)$  is compatible,  $\mathcal{R}_{wd}^l(M_v) \subseteq \mathcal{R}_{wd}^l(M)$  is also compatible then  $\mathcal{G}(\mathcal{R}_{wd}^l(M_v))$  is composed by exactly two connected components (see Theorem 2 in [21]) *i.e.*  $C_1$  and  $C_2$ . Since  $l_x l_y | l_z \in \mathcal{R}_{wd}(M)$  then  $l_x \neq l_y \neq l_z$ : this means that  $l_x, l_y \in C_1$ ,  $l_x, l_y \notin C_2$  and  $l_z \in C_2$ ,  $l_z \notin C_1$ . Then both triplets  $l_x l_z | l_y$  and  $l_y l_z | l_x$  would connect the two connected components. This implies that  $(\mathcal{R}_{wd}^l(M_v) \cup l_x l_z | l_y)$  and  $(\mathcal{R}_{wd}^l(M_v) \cup l_y l_z | l_x)$  are both incompatible and then  $t$  is in  $cl(\mathcal{R}_{wd}^l(M_v))$ . The same result holds for the symmetric case  $x, y \in \mathcal{L}(M_{v_2})$  and  $z \in \mathcal{L}(M_{v_1})$ . Note that Lemma 2 works also if  $|\mathcal{L}(M_{v_1})| = 1$  and/or  $|\mathcal{L}(M_{v_2})| = 1$ . This proves that  $\mathcal{R}_{wd}(M) \subseteq cl(\mathcal{R}_{wd}^l(M))$ .  $\square$

**Lemma 3.** *Let  $M$  be a MUL tree. If the triplet set  $\mathcal{R}_{wd}^l(M)$  is compatible, then  $cl(\mathcal{R}_{wd}^l(M)) = cl(\mathcal{R}_{wd}(M))$ .*

<sup>2</sup> Proposition 4 of [21] is defined for quartets but it remains valid for rooted triplets (see page 441).

*Proof.* Since if  $\mathcal{R}_{wd}^l(M)$  is compatible then it follows from Proposition 1 that  $cl(\mathcal{R}_{wd}^l(M))$  is compatible. Lemma 2 implies that  $\mathcal{R}_{wd}(M)$  is also compatible. Then the closure of  $\mathcal{R}_{wd}(M)$  is well defined and we can then deduce from Lemma 2 that  $cl(\mathcal{R}_{wd}(M)) \subseteq cl(\mathcal{R}_{wd}^l(M))$ , since  $cl(cl(\mathcal{R}_{wd}^l(M))) = cl(\mathcal{R}_{wd}^l(M))$ . By construction  $\mathcal{R}_{wd}(M) \supseteq \mathcal{R}_{wd}^l(M)$ . This implies that  $cl(\mathcal{R}_{wd}(M)) \supseteq cl(\mathcal{R}_{wd}^l(M))$ . This concludes the proof.  $\square$

**Corollary 1.** *The triplet set  $\mathcal{R}_{wd}^l(M)$  is compatible iff the triplet set  $\mathcal{R}_{wd}(M)$  is compatible.*

*Proof.* The fact that  $\mathcal{R}_{wd}(M) \supseteq \mathcal{R}_{wd}^l(M)$  implies that if  $\mathcal{R}_{wd}(M)$  is compatible then  $\mathcal{R}_{wd}^l(M)$  is also compatible while if  $\mathcal{R}_{wd}^l(M)$  is not then  $\mathcal{R}_{wd}(M)$  is not compatible. While proving Lemma 3 we proved that if  $\mathcal{R}_{wd}^l(M)$  is compatible then  $\mathcal{R}_{wd}(M)$  is compatible. This implies that if  $\mathcal{R}_{wd}(M)$  is not compatible then  $\mathcal{R}_{wd}^l(M)$  is also not compatible, otherwise we would have  $\mathcal{R}_{wd}^l(M)$  compatible and  $\mathcal{R}_{wd}(M)$  incompatible and this would violate Lemma 3. This proves the corollary.  $\square$

**Theorem 1.** *Checking the auto-coherency of a MUL tree  $M$  can be done in  $O(n \cdot \log^2 n)$  time.*

*Proof.* From Lemma 3 and Corollary 1 follows that checking the auto-coherency of a MUL tree  $M$  can be done using the triplet set  $\mathcal{R}_{wd}^l(M)$ . This set can be computed in linear time by algorithm 1. Indeed, for each node  $v$  in  $M$ , algorithm 1 computes  $sm(v)$  in a bottom-up approach and  $v'$  in a top-down approach. If  $v \notin \mathcal{D}(M)$  and  $v' \neq \emptyset$ , algorithm 1 inserts in  $\mathcal{R}_{wd}(M)$  the triplet  $sm(v_1)sm(v_2)|sm(v')$ : this is exactly the definition of  $\mathcal{R}_{wd}^l(M)$ . The lca of each pair of nodes and the set of duplication nodes can be computed in linear time (see [15, 16] and section 4).

Algorithm LINEARREPRESENTATION( $M, \mathcal{D}(M), root(M), \mathcal{R}^l$ ) consists in a post-order walk on the MUL tree  $M$  and takes a linear time. Since AncentralBuild checks the compatibility of a triplet set  $R$  on a label set of size  $n$  in  $O(|R| \cdot \log^2 n)$  time, this concludes the proof.  $\square$

*Remark 1.* The trees constructed by AncentralBuild from  $\mathcal{R}_{wd}^l(M)$  and  $\mathcal{R}_{wd}(M)$  are isomorphic.

It follows from Theorem 1 and Remark 1 that, given a MUL tree  $M$ , we can check in  $O(n \cdot \log^2 n)$  time whether there exists a single-label tree  $T$  such that  $\mathcal{R}_{wd}(M) \subseteq \mathcal{R}(T)$  and, in case of positive answer, obtain such a tree.

## 4 Computing $\mathcal{D}(M)$ in Linear Time

The easiest way to compute  $\mathcal{D}(M)$  is checking for each node  $v$  if the sets  $L(v_1)$  and  $L(v_2)$  have at least one label in common; in the case of a positive answer,  $v$  is inserted in  $\mathcal{D}(M)$ . The complexity of this approach is  $O(n^2)$ , since it requires

computing  $O(n)$  intersections of two lists of  $O(n)$  elements. The algorithm 2 uses the lca to find the set of odn  $\mathcal{D}(M)$  and requires only linear time. To demonstrate the correctness of algorithm 2, we need to determine some relationships between the lca and the odn.

**Lemma 4.** *A node is an odn if and only if it is the lca of at least two repeated leaves  $m$  and  $p$ .*

*Proof.* Indeed, from the definition 1,  $v$  is an odn iff  $L(v_1) \cap L(v_2) \neq \emptyset$ . Therefore, there exist two leaf nodes  $m$  and  $p$  with  $m \in M_{v_1}$  and  $p \in M_{v_2}$  such that  $l_m = l_p$ . Thus  $v$  is a common ancestor of the two leaves  $m$  and  $p$  with the same label. Now,  $m$  and  $p$  belong to two different subtrees having  $v$  as father ( $m \in M_{v_1}$  and  $p \in M_{v_2}$ ), hence  $v$  is their lowest common ancestor in  $M$ . Reciprocally, if  $v$  is the lca of two leaves  $m$  and  $p$  with the same label, this means that  $L(v_1) \cap L(v_2) \neq \emptyset$ , then  $v$  is an odn according to definition 1.  $\square$

According to Lemma 4, we can search for the lca of any two leaves  $m$  and  $p$  with the same label. To determine the lca between multiple pairs of nodes, one can use an algorithm in [15] which preprocesses a data structure in  $O(n)$  time, where  $n$  is the number of nodes and returns the lca of any two specific nodes from the data structure in  $O(1)$ . We still have  $O(n^2)$  of these couples, and even constant time for each gives an  $O(n^2)$  total complexity. However, since there are only  $O(n)$  odn, checking the lca of any pair of leaves computes the same lca several times. A smarter approach is used in algorithm 2: first of all, the subtrees of  $M$  are ordered from the left to the right in an arbitrary way. Then, each repeated leaf, starting from the left of the tree and moving to the right, is tagged with the repeated label followed by its occurrence number. Then, for each repeated label  $e$ , the lca of any two successive occurrences of  $e$ ,  $e_i$  and  $e_{i+1}$  is inserted in  $\mathcal{D}(M)$ . This leads to a linear time complexity. Indeed, we have  $O(n)$  of these couples since each leaf of  $M$  is involved in at most two pairs  $(e_i, e_{i+1})$ .

---

**Algorithm 2:** CompDuplicationNodes(r)

---

**Data:** A MUL tree  $M$ .

**Result:** A set of odn  $\mathcal{D}(M)$ .

Order  $M$  in an arbitrary way. In this order, tag each duplicated leaf with the repeated label followed by its occurrence number. Compute the lca for each couple of leaves.

$\mathcal{D}(M) \leftarrow \emptyset$ ;

**foreach** (repeated label  $e$ ) **do**

**foreach** ( $\{e_j, e_{j+1}\}$ ) **do**  $\mathcal{D}(M) \leftarrow lca(e_j, e_{j+1})$ ;

**return**  $\mathcal{D}(M)$ ;

---

The correctness of algorithm 2 is justified by Lemma 5 showing that algorithm 2 retrieves all the odn of  $M$ .

**Lemma 5.** *Let  $M$  be a MUL tree. For each odn  $v$ ,  $\exists$  two successive occurrences of a label  $e$  denoted by  $e_i$  and  $e_{i+1}$  s.t.  $v = lca(e_i, e_{i+1})$ .*

*Proof.* Given an odn  $v$ , there exists at least one label  $e$  present in both subtrees  $v_1$  and  $v_2$ . We denote by  $A$  the set of leaves  $a_i$  s.t.  $a_i \in M_{v_1}$  and  $l_{a_i} = e$  and by

$B$  the set of leaves  $b_j$  s.t.  $b_j \in M_{v_2}$  and  $l_{b_j} = e$ . If we take the last element of  $B$  ( $b_{|B|}$ ) and the first one of  $A$  ( $a_1$ ), we know that  $v$  is their lca. Additionally, due to the way we tagged  $M$ , we know that there is no other occurrence of the label  $e$  between  $b_{|B|}$  and  $a_1$ . Indeed, if there was another leaf  $x$  labeled with  $e$ , it would be either in  $v_1$  (and then  $x = a_1$ ) or in  $v_2$  (and then  $x = b_{|B|}$ ). Then  $b_{|B|}$  and  $a_1$  are two successive occurrences of the same label and their lca is the node  $v$ .  $\square$

## 5 Isomorphic Subtrees

**Definition 6.** *Two rooted trees  $T_1, T_2$  are isomorphic (denoted by  $T_1 = T_2$ ) iff there exists a one-to-one mapping from the nodes of  $T_1$  onto the nodes of  $T_2$  preserving leaf labels and descendance.*

We are interested in testing if, for each odn  $v$ , the two subtrees  $v_1$  and  $v_2$  are isomorphic or not. In the positive, we can prune one of the two isomorphic subtrees without losing any information contained in  $\mathcal{R}_{wd}$  and eliminate the odn  $v$ , as in the example of Fig. 3. For detecting isomorphism of *multi-labelled* trees, we propose Algorithm 3, an extension of the CHECK-ISOMORPHISM-OR-FIND-CONFLICT algorithm [22]. Indeed, the latter does not deal with MUL trees. Alternatively, we could have proposed an appropriate variant of the tree isomorphism algorithm detailed in [23]. However, such an algorithm would likely have been less space efficient than the one we present here due to a number of string sorting steps using a number of queues and lists to ensure linear running time.

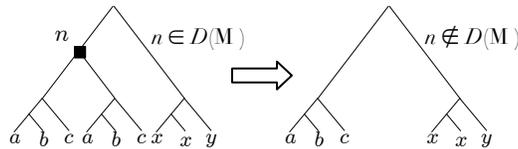
A node that has only two leaves as children is called **cherry**. In the case of single-labeled trees we have the following lemma:

**Lemma 6.** [13] *Let  $T_1, T_2$  be two isomorphic trees and let  $c_1$  be a cherry in  $T_1$ . Then, there is a cherry  $c_2 \in T_2$  s.t.  $L(c_1) = L(c_2)$ .*

In the case of *MUL* trees, we can have several copies of the same cherry. We call a **multiple cherry** the list of cherries on the same two labels. For a multiple cherry  $mc$ , we note  $|mc|$  the number of occurrences of the cherry in its tree.

**Lemma 7.** *Let  $M_1, M_2$  be two isomorphic MUL trees and let  $mc_1$  be a multiple cherry in  $M_1$ . Then, there is a multiple cherry  $mc_2 \in M_2$  s.t.  $L(mc_1) = L(mc_2)$  and  $|mc_1| = |mc_2|$ .*

The proof is inspired from that of Lemma 6 in [13] and left to the reader.



**Fig. 3.** An example of MUL tree where the sons of the duplication node are isomorphic

**Algorithm 3:** CheckIsomorphismMULTree( $M_1, M_2$ )

---

**Data:** Two MUL tree  $M_1$  and  $M_2$ .  
**Result:** TRUE if  $M_1$  and  $M_2$  are isomorphic, FALSE otherwise.  
Let  $L_{mc}$  be the list of multiple cherries in  $M_1$  and  $M_2$ . Let  $H$  be the hashtable where each  $mc \in L_{mc}$  is a key. To each  $H(mc)$ , we associates two lists  $O_2(mc)$  and  $O_1(mc)$ , resp. of the occurrences of  $mc$  in  $M_1$  and  $M_2$ ;  
**while** ( $L_{mc} \neq \emptyset$ ) **do**  
     $mc \leftarrow$  the first multiple cherry in  $L_{mc}$ ; delete  $mc$  form  $L_{mc}$ ;  
    **if** ( $O_2(mc) = O_1(mc)$ ) **then**  
        Turn all cherries in  $O_2(mc)$  and  $O_1(mc)$  into leaves to which a same new label is assigned;  
        add the new multiple cherries in  $L_{mc}$  and  $H$ ;  
    **else** return FALSE;  
**return** TRUE;

---

**5.1 Outline of the Algorithm**

First of all, we find all the multiple cherries for the MUL trees  $M_1$  and  $M_2$ . We store them in the list  $L_{mc}$  using a simple linked list. Additionally, we use a hashtable  $H$  where each  $mc \in L_{mc}$  is a key.  $H$  associates to each multiple cherry  $mc$  two linked lists,  $O_1(mc)$  and  $O_2(mc)$ , storing pointers to nodes of  $M_1$  and  $M_2$  respectively that correspond to the occurrences of  $mc$ . The multiple cherries of a MUL tree are then examined in a bottom-up process. Given a multiple cherry  $mc$  in  $L_{mc}$  we check if the size of  $O_1(mc)$  is the same as that of  $O_2(mc)$ . If this is not the case, we have found a multiple cherry for which we do not have the same number of occurrences in  $M_1$  and  $M_2$ . In this instance,  $M_1$  and  $M_2$  are not isomorphic (Lemma 7) and the algorithm returns FALSE. Otherwise we turn all the cherries in  $O_1(mc)$  and  $O_2(mc)$  into leaves to which a same new label, different from all other labels in  $M_1$  and  $M_2$ , is assigned. This modification of  $M_1$  and  $M_2$  can turn the fathers of some cherries in  $O_1(mc)$  and  $O_2(mc)$  into new cherries. Then  $L_{mc}$  is updated and the processing of cherries in  $M_1$  is iterated until both MUL trees are reduced to a single leaf with the same label if  $M_1$  and  $M_2$  are isomorphic, or a FALSE statement is returned.

**Theorem 2.** *Let  $M_1$  and  $M_2$  be two rooted MUL trees with  $L(M_1) = L(M_2)$  of cardinality  $n$ . In time  $O(n)$ , algorithm 4 returns TRUE if  $M_1$  and  $M_2$  are isomorphic, FALSE otherwise.*

*Proof.* This algorithm is an extension of the CHECK-ISOMORPHISM-OR-FIND-CONFLICT algorithm [22] applicable to MUL trees. We show here that we can keep a linear time execution, using supplementary data structures.

A simple depth-first search of trees  $M_1$  and  $M_2$  initializes  $L_{mc}$  and  $H$  in  $O(n)$  time. At each iteration of the algorithm, choosing a multiple cherry  $mc$  to process is done in  $O(1)$  by removing the first element  $mc$  of  $L_{mc}$ .  $H$  then provides in  $O(1)$  the lists  $O_1(mc)$  and  $O_2(mc)$  of its occurrences in the trees. Checking that these lists have the same number of elements is proportional to the number

of nodes they contain, hence costs  $O(n)$  amortized time, as each node is only once in such a list, and the list is processed once during the whole algorithm. Replacing all occurrences of  $mc$  by a new label is done in  $O(n)$  amortized time, since each replacement is a local operation replacing three nodes by one in a tree and at most  $O(n)$  such replacements can take place in a tree to reduce it down to a single node (the algorithm stops when this situation is reached). Reducing a cherry can create a new occurrence  $o_{mc'}$  of a cherry  $mc'$ . Checking in  $O(1)$  time if  $mc'$  is a key in  $H$  allows to know whether occurrences of  $mc'$  have already been encountered or not. In the positive, we simply add  $o_{mc'}$  to the beginning of the list  $O_1(mc)$  (if  $o_{mc'} \in M_1$ ) or  $O_2(mc)$  (if  $o_{mc'} \in M_2$ ), requiring  $O(1)$  time. In the negative, we add  $mc'$  to the beginning of  $L_{mc}$ , create a new entry in  $H$  for  $mc'$ , and initialize the associated lists  $O_1(mc)$  and  $O_2(mc)$  so that one contains  $o_{mc'}$  and the other is the empty list. Again, this requires only  $O(1)$  time. Thus, performing all operations required by the algorithm globally costs  $O(n)$  time.  $\square$

Apply algorithm 3 to each  $M_{v_1}$  and  $M_{v_2}$  of each odn of a MUL tree  $M$  in a bottom-up approach requires  $O(dn)$  time, where  $d$  is the number of duplication nodes in  $M$ .

## 6 Computing a Largest Duplication-free Subtree of a MUL Tree

If a MUL tree is not auto-coherent, identifying duplication nodes allows for the discrimination of leaves representing orthologous and paralogous sequences. Since only orthologous sequence history reflects the species history, a natural question is to determine the most informative sequence set for a given gene. As long as the gene tree contains odn, it will also contain leaves representing paralogous sequences. Yet, if for each node  $v \in \mathcal{D}(M)$  of  $M$  we choose to keep either  $v_1$  or  $v_2$ , we obtain a pruned single-labeled tree containing only apparent orthologous sequences (observed paralogous have been removed by pruning nodes). Note that the so obtained single-labeled tree is auto-coherent by definition.

**Definition 7.** *Let  $M$  be a MUL tree. We say that  $T$  is obtained by (duplication) pruning  $M$  iff  $T$  is obtained from  $M$  choosing for each odn  $v$  either  $v_1$  or  $v_2$ . We denote this operation by the symbol  $\lesssim$ .*

One can wonder, for a non auto-coherent MUL tree  $M$ , what is the most informative single-labeled tree  $T$  s.t.  $T \lesssim M$ . We define this problem as the *MIPT* (Most Informative Pruned Tree) problem.

To evaluate the informativeness of a tree we can use either the number of triplets of  $T$  (see [24, 25, 11]) that, for binary trees, depends only on the number of leaves, or the CIC criterion (see [26, 12]). The CIC of a not fully resolved and incomplete<sup>3</sup> tree  $T$  with  $|L(T)|$  leaves among the  $n$  possible is a function of both the

<sup>3</sup> A tree is called incomplete when it misses some taxa.

**Algorithm 4:**  $\text{pruning}(v, M, \mathcal{D}(M))$ 


---

**Data:** A node  $v$ , a MUL tree  $M$ , and a set of odn  $\mathcal{D}(M)$ .  
**Result:** The most informative MUL tree  $M'$  s.t.  $M'_v \lesssim M_v$ .  
**foreach** ( $m \in \text{sons}(v)$ ) **do**  $\text{pruning}(m, M, \mathcal{D}(M))$ ;  
**if** ( $v \in \mathcal{D}(M)$ ) **then**  
    **if** ( $|L(v_1)| > |L(v_2)|$ ) **then**  $v \leftarrow v_1$ ;  
    **else**  $v \leftarrow v_2$ ;  
     $\mathcal{D}(M) \leftarrow \mathcal{D}(M) - \{v\}$ ;  
**return**  $M$ ;

---

number  $n_R(T, n)$  of fully resolved trees  $T'$  on  $L(T)$  such that  $\mathcal{R}(T) \subseteq \mathcal{R}(T')$  and the number  $n_R(n)$  of fully resolved trees on  $n$  leaves. More precisely,

$$CIC(T, n) = -\log \left( n_R(T, n) / n_R(n) \right)$$

In the case of binary trees,  $n_R(T, n)$  depends only on the number of source taxa missing in  $T$  since  $T$  does not contain multifurcations. Thus, dealing with binary trees, maximizing the information of a tree (*i.e.* maximizing the number of triplets or minimizing the CIC value) consists in finding the tree with the largest number of leaves. A natural approach for the MIPT problem for binary MUL trees is an algorithm that, after having computed  $\mathcal{D}(M)$ , uses the bottom-up algorithm 4 starting from  $\text{root}(M)$ , to keep the most informative subtree between  $M_{v_1}$  and  $M_{v_2}$ , for each odn  $v$ .

**Theorem 3.** *Let  $M$  a MUL tree on a set of leaves of cardinality  $n$ . In time  $O(n)$ ,  $\text{pruning}(M, \text{root}(M), \mathcal{D}(M))$  returns the most informative tree  $T$  s.t.  $T \lesssim M$ .*

*Proof.* First of all, it's obvious that  $\text{pruning}(M, \text{root}(M), \mathcal{D}(M))$  returns a tree. Indeed, if for each odn  $v$  only one node between  $v_1$  and  $v_2$  is kept, at the end of the bottom-up procedure one copy of each duplicated leaf is present in the modified  $M$ . Now, we have to show that the resulting tree is the most informative tree s.t.  $T \lesssim M$ , *i.e.* the tree with as many leaves as possible. For an odn  $v$  that is the ancestor of other duplication nodes, the choices made for  $v_1$  do not influence the choices for  $v_2$  since for each duplication node we can keep only one of the two subtrees, the most populous one. Thus we can search for the best set of choices left/right for  $v_1$  and  $v_2$  independently and then choose the most populous pruned subtree between  $v_1$  and  $v_2$ . Iterating recursively this reasoning, we demonstrate that the tree obtained by Algorithm 4 is the most informative tree  $T$  s.t.  $T \lesssim M$ . The computation of the set of odn  $\mathcal{D}(M)$  takes linear time. The subroutine  $\text{pruning}(M, \text{root}(M), \mathcal{D}(M))$  requires a tree walk, thus the time complexity of Algorithm 4 is  $O(n)$ .  $\square$

## 7 The Compatibility Issue of Single-labeled Subtrees Obtained from MUL Trees

We can also ask if it is possible, given a collection of MUL tree  $\mathcal{M}$ , to discriminate leaves representing orthologous and paralogous sequences in a gene tree using the information contained in the other gene trees to obtain a compatible forest  $\mathcal{T}$ , *i.e.* a forest for which there exists a tree  $T$  *s.t.*  $\cup_{T_i \in \mathcal{T}} \mathcal{R}(T_i) \subseteq \mathcal{R}(T)$ . We denote this problem by EPCF, Existence of a Pruned and Compatible Forest. Unfortunately, the EPCF problem is NP-complete.

<b>EPCF</b>	<b>Instance</b> : A set of leaves $X$ and a collection $\mathcal{M}=\{M_1, \dots M_k\}$ of MUL trees on $X$ .
	<b>Question</b> : $\exists$ a set $S$ of choices left/right, $S : \mathcal{M} \rightarrow \mathcal{T}$ , with $\mathcal{T}=\{T_1, \dots T_k\}$ <i>s.t.</i> $T_i \lesssim M_i$ and $\mathcal{T}$ is compatible?

**Theorem 4.** *The EPCF problem is NP-complete.*

*Proof.* We start by proving that EPCF is in NP, *i.e.* checking if a set  $S$  of choices left/right is a solution for the instance  $\mathcal{I} = (\mathcal{M}, X)$  can be done in polynomial time. First of all, for each MUL tree  $M_j \in \mathcal{M}$ , we place the choices left/right on  $M_j$ , *i.e.* we discard the subtrees not chosen, obtaining a forest of trees  $\mathcal{T}$ . We check then the compatibility of  $\mathcal{T}$  with the Aho graph[14]. Constructing this graph can be done in polynomial time.

Given that EPCF is in NP, we use a reduction of 3-SAT to EPCF to demonstrate that it is NP-complete.

<b>3-SAT</b>	<b>Instance</b> : A boolean expression $\mathcal{C}=(C_1 \wedge C_2 \wedge \dots \wedge C_n)$ on a finite set $L=\{l_1, l_2, \dots, l_m\}$ of variables with $C_j = (a \vee b \vee c)$ where $\{a, b, c\} \in \{l_1, l_2, \dots, l_m, \bar{l}_1, \bar{l}_2, \dots, \bar{l}_m\}$
	<b>Question</b> : $\exists$ a truth assignment for $L$ that satisfies all $C_j$ in $\mathcal{C}$ ?

We need to show that every instance of 3-SAT can be transformed into an instance of EPCF; then we will show that given an instance  $\mathcal{I} = (\mathcal{C}, L)$  of 3-SAT,  $\mathcal{I}$  is a positive instance, *i.e.* an instance for which a solution exists, iff the corresponding instance for EPCF is positive.

Given an instance  $\mathcal{I} = (\mathcal{C}, L)$  of 3-SAT, we build an instance  $\mathcal{I}' = (\mathcal{M}, X)$  of EPCF associating to each  $l_i$  in  $L$  the binary tree<sup>4</sup>  $T(l_i) = (((x_i, y_i), z_i), d)$  and to  $\bar{l}_i$  the binary tree  $T(\bar{l}_i) = (((z_i, y_i), x_i), d)$  (see Fig. 4 for an example).

The set of subtrees  $\{T(a) \mid a \in \{l_1, l_2, \dots, l_m, \bar{l}_1, \bar{l}_2, \dots, \bar{l}_m\}\}$  is denoted by  $\mathcal{T}_L$ .

<sup>4</sup>  $T(l_i)$  is expressed in the Newick format.

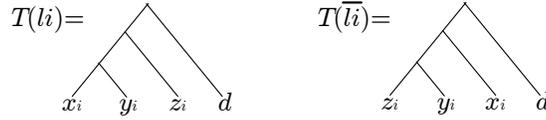


Fig. 4. Binary trees on four leaves associated to  $l_i$  and to  $\bar{l}_i$

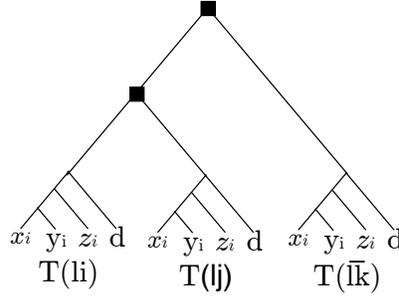


Fig. 5. MUL tree built from the clause  $\{l_i \vee l_j \vee \bar{l}_k\}$ . Odn are indicated by black squares

Then, for each clause  $C_j = (a \vee b \vee c)$  in  $\mathcal{C}$ , a binary MUL tree  $M_j$  is built, formed by three subtrees  $((T(a), T(b)), T(c))$ . Note that  $M_j$  has exactly two duplication nodes due to the presence of  $d$  in  $T(a)$ ,  $T(b)$  and  $T(c)$ , so that any left/right choice of  $M_j$  will reduce it to either  $T(a)$ ,  $T(b)$  or  $T(c)$ . In Fig. 5 an example of a MUL tree built from a clause. In this way we obtain a forest of MUL trees  $\mathcal{M}$  on the leaf set  $X = \left\{ \left\{ \bigcup_{i=1}^m \{x_i, y_i, z_i\} \right\} \cup \{d\} \right\}$ , *i.e.* an instance of the EPCF problem. Clearly  $\mathcal{M}$  can be built in polynomial time.

We now need to show that a positive instance of 3-SAT gives a positive instance of EPCF through the previous transformation. Having a positive instance for 3-SAT implies that for each  $C_j \in \mathcal{C}$  with  $C_j = (a \vee b \vee c)$ , at least one of the three literals is TRUE. Without loss of generality, let us suppose that  $a$  is TRUE. Then in the MUL tree  $M_j$  corresponding to  $C_j$  we set the choice left/right so that only the subtree  $T(a)$  is kept. We then obtain a forest  $\mathcal{T}$  that is a subset of  $\mathcal{T}_L$ . We need to prove that  $\mathcal{T}$  is compatible. Let  $\tilde{T}(a)$  denote the tree  $T(a)|(L(T(a)) - \{d\})$ <sup>5</sup> and  $\tilde{\mathcal{T}}$  the forest composed by all trees  $\{\tilde{T}(a)|T(a) \in \mathcal{T}\}$ . Then, we can build a tree  $T_s = (\tilde{T}_1, \tilde{T}_2, \dots, \tilde{T}_{|\tilde{\mathcal{T}}|}, d)$ . Since  $l_i$  cannot have the value TRUE and FALSE at the same time, we have either  $T(l_i)$  or  $T(\bar{l}_i)$  in  $\mathcal{T}$ . The tree  $T_s$  is therefore a single-labeled tree. Moreover, by construction  $T_s|(L(T(a)))$  is identical to  $T(a)$ , for all  $T(a)$  in  $\mathcal{T}$  ensuring that  $\bigcup_{T_i \in \mathcal{T}} \mathcal{R}(T_i) \subseteq \mathcal{R}(T_s)$ . Thus  $\mathcal{T}$  is compatible.

Now, the only thing left to prove is that a positive instance of EPCF gives a positive instance of 3-SAT.

The repetition of the taxon  $d$  in each subtree makes the two nodes connecting the subtrees in each  $M_j$  be odn. Thus a left/right choice set  $S$  reduces each

<sup>5</sup> Given a tree  $T$  and a label set  $S$ , we denote by  $T|S$  the restriction of  $T$  to the set  $S$ .

$M_j$  in  $\mathcal{M}$  into a tree  $T(a) \in \mathcal{T}_L$ , providing the forest  $\mathcal{T}$ . Setting the value of  $a$  to TRUE ensures that the clause  $C_j$  corresponding to  $M_j$  is TRUE. This can be done simultaneously for all clauses  $\in \mathcal{C}$  since the forest compatibility implies that there is no contradiction among the trees in  $\mathcal{T}$ , all the more so direct contradictions. Then, either  $T(l_i)$  or  $T(\bar{l}_i)$  is in  $\mathcal{T}$ . This ensures us that either  $l_i$  or  $\bar{l}_i$  is assigned to TRUE, but not both.  $\square$

Note that the problem to find the most informative forest  $\mathcal{T} = \{T_1, \dots, T_k\}$  s.t.  $T_i \lesssim M_i$  and  $\mathcal{T}$  is compatible, denoted by MIPCF (Most Informative Pruned and Compatible Forest) is FPT. Indeed, analyzing all possible scenarios of choices left/right is FPT on the total number of duplication nodes in  $\mathcal{M}$ .

## 8 Conclusions

In this paper we presented several algorithms to transform multi-labeled evolutionary trees into single-labeled ones so that they can be used by all existent supertree methods. In the future we plan to go further and extend the algorithm FIND-REFINEMENT-OR-CONFLICT in [22] applicable to MUL trees and to work on a more sophisticated FPT algorithm for the MIPCF problem.

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## References

1. Cotton, J., Page, R.: Rates and patterns of gene duplication and loss in the human genome. In of London. Biological science, R.S., ed.: Proceedings. Volume 272. (2005) 277–283
2. Ma, B., Li, M., Zhang, L.: From gene trees to species trees. *SIAM J. Comput* **30** (2000) 729–752
3. Hallett, M.T., Lagergren, J.: New algorithms for the duplication-loss model. In: RECOMB2000, Fourth Annual International Conference on Computational Molecular Biology. (2000) 138–146
4. Chen, K., Durand, D., Farach-Colton, M.: Notung: a program for dating gene duplications and optimizing gene family trees. *J Comput Biol* **7** (2000) 429–444
5. Vernot, B., Stolzer, M., Goldman, A., Durand, D.: Reconciliation with non-binary species trees. *J Comput Biol* **15**(8) (2008) 981–1006
6. Chauve, C., Doyon, J.P., El-Mabrouk, N.: Gene family evolution by duplication, speciation, and loss. *J Comput Biol* **15**(8) (2008) 1043–1062
7. Goodman, M., Czelusniak, J., Moore, G., A.E., R.H., Matsuda, G.: Fitting the Gene Lineage into its Species Lineage, a Parsimony Strategy Illustrated by Cladograms Constructed from Globin Sequences. *Systematic Zoology* **28**(2) (1979) 132–163
8. Arvestad, L., Berglund, A., Lagergren, J., Sennblad, B.: Bayesian gene/species tree reconciliation and orthology analysis using MCMC. *Bioinformatics* **19 Suppl 1** (2003) 7–15
9. Durand, D., Halldrsson, B., Vernot, B.: A hybrid micro-macroevolutionary approach to gene tree reconstruction. *J. Comput. Biol.* **13** (Mar 2006) 320–335
10. Baum, B.R., Ragan, M.A.: The MRP method. In Bininda-Emonds, O., ed.: *Phylogenetic supertrees: combining information to reveal the Tree of Life*. Kluwer (2004) 17–34

11. Page, R.D.M.: Modified mincut supertrees. In Guigó, R., Gusfield, D., eds.: Proceedings of the 2nd International Workshop on Algorithms in Bioinformatics (WABI'02). (2002) 537–552
12. Scornavacca, C., Berry, V., Lefort, V., Douzery, E.J.P., Ranwez, V.: Physicist: cleaning source trees to infer more informative supertrees. *BMC Bioinformatics* **9**(8) (2008) 413
13. Gusfield, D.: Efficient algorithms for inferring evolutionary trees. *Networks* **21** (1991) 12–28
14. Aho, A.V., Sagiv, Y., Szymanski, T.G., Ullman, J.D.: Inferring a tree from lowest common ancestors with an application to the optimization of relational expressions. *SIAM J. Comp.* **10**(3) (1981) 405–421
15. Harel, D., Tarjan, R.E.: Fast algorithms for finding nearest common ancestors. *SIAM J. Comput.* **13**(2) (1984) 338–355
16. Bender, M.A., Farach-Colton, M.: The lca problem revisited. In Springer-Verlag, ed.: LATIN '00: Proceedings of the 4th Latin American Symposium on Theoretical Informatics. (2000) 88–94
17. Berry, V., Semple, C.: Fast computation of supertrees for compatible phylogenies with nested taxa. *Syst. Biol.* **55** (2006) 270–288
18. Steel, M.: The complexity of reconstructing trees from qualitative characters and subtrees. *Journal of Classification* **9**(1) (January 1992) 91–116
19. Semple, C., Steel, M.A.: Phylogenetics. Volume 24 of Oxford Lecture Series in Mathematics and its Applications. Oxford University Press (2003)
20. Grunewald, S., Steel, M.A., Swenson, M.S.: Closure operations in phylogenetics. *Mathematical Biosciences* **208**(2) (2007) 521–37
21. Bryant, D., Steel, M.: Extension operations on sets of leaf-labelled trees. *Adv. Appl. Math.* **16**(4) (1995) 425–453
22. Berry, V., Nicolas, F.: Improved parameterized complexity of the maximum agreement subtree and maximum compatible tree problems. *IEEE/ACM Trans. Comput. Biol. Bioinformatics* **3**(3) (2006) 289–302
23. Aho, A., Hopcroft, J., Ullman, J.: The Design and Analysis of Computer Algorithms. Addison-Wesley Longman Publishing Co., Inc., Boston, MA, USA (1974)
24. Ranwez, V., Berry, V., Criscuolo, A., Fabre, P., Guillemot, S., Scornavacca, C., Douzery, E.: PhySIC: a veto supertree method with desirable properties. *Syst. Biol.* **56** (Oct 2007) 798–817
25. Semple, C., Steel, M.: A supertree method for rooted trees. *Discrete Appl. Math.* **105** (2000) 147–158
26. Thorley, J., Wilkinson, M., Charleston, M.: The information content of consensus trees. In Rizzi, A., Vichi, M., Bock, H.H., eds.: Advances in Data Science and Classification. Studies in Classification, Data Analysis, and Knowledge Organization. (1998) 91–98