Computing Time for Summation Algorithm: Less Hazard and More Scientific Research

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Less Hazard and More Scientific Research

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How to manage accuracy and speed?

A new "better" algorithm every year since 1999

- 1965 Møller, Ross
- 1969 Babuska, Knuth
- 1970 Nickel
- 1971 Dekker, Malcolm
- 1972 Kahan, Pichat
- 1974 Neumaier
- 1975 Kulisch/Bohlender
- 1977 Bohlender, Mosteller/Tukey
- 1981 Linnaimaa
- 1982 Leuprecht/Oberaigner
- 1983 Jankowski/Semoktunowicz/-Wozniakowski
- 1985 Jankowski/Wozniakowski
- 1987 Kahan
- 1991 Priest
- 1992 Clarkson, Priest
- 1993 Higham
- 1997 Shewchuk
- 1999 Anderson
- 2001 Hlavacs/Uberhuber
- 2002 Li et al. (XBLAS)
- 2003 Demmel/Hida, Nievergelt, Zielke/Drygalla
- 2005 Ogita/Rump/Oishi, Zhu/Yong/Zeng
- 2006 Zhu/Hayes
- 2008 Rump/Ogita/Oishi
- 2009 Rump, Zhu/Hayes
- 2010 Zhu/Hayes

Accuracy of the floating point summation

Precision

- **u**= arithmetic precision
- $\mathbf{u} = 2^{-53} \approx 10^{-16}$ for b64 in IEEE-754 (2008)

Accuracy for backward stable algorithms

- Accuracy of the computed sum $\leq (n-1) \times \mathit{cond} \times \mathbf{u}$
- $\mathit{cond}(\sum x_i) = \frac{\sum |x_i|}{|\sum x_i|}$
- No more significant digit in IEEE-b64 for large cond, *i.e.* $> 10^{16}$

More accuracy ...

- More precision: double-double, quad-double, ...
- Compensated algorithms: Kahan(72), ..., Sum2(05), SumK(05)
- Accuracy of the computed sum $\lesssim \mathbf{u} + \mathit{cond} \times \mathbf{u}^K$

... but still depending on the conditioning

Skip over the conditioning

Distillation: iterate until faithful or exact rounding

- Error free transformation of $[x] \rightarrow [x^{(1)}] \rightarrow \cdots \rightarrow [x^*]$ such that $\sum x_i = \sum x_i^*$ and $[x^*]$ provides the expected rounded value.
- Kahan (87), ..., Zhu-Hayes: iFastSum (SISC-09)

More space to keep everything

- Long accumulator, hardware oriented: Malcolm (71), Kulish (80)
- Cut the summands: AccSum (SISC-08), FastAccSum (SISC-09)
- Sum by fixed exponent: HybridSum (SISC-09), OnLineExact (TOMS-10)

From faithful to exact rounding

- costly choice of the right side when closed to breakpoints
- *e.g.* $1 + 2^{-53} \pm 2^{-106}$

Skip over the conditioning

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From faithful to exact rounding

$\longrightarrow$ **Run-time and memory efficiencies are now the discriminant factors**

Reliable and significant measure of the time complexity?

The classic way: count the number of flop

- A usual problem: double the accuracy of a computed result
- A usual answer for polynomial evaluation (degree n)

Metric	Horner	CompHorner	DDHorner
Flop count	2n	$22n + 5$	$28n + 4$
Flop count ratio	1	≈ 11	≈ 14
Measured #cycles ratio	1	2.8 – 3.2	8.7 – 9.7

Flop count vs. run-time measures

- Flop counts and measured run-times are not proportional
- Run-time measure is a very difficult experimental process
- Which one trust?

How to trust non-reproducible experiment results?

Measures are mostly non-reproducible

- The execution time of a binary program varies, even using the same data input and the same execution environment.

Why? Experimental uncertainty of the hardware performance counters

- Spoiling events: background tasks, concurrent jobs, OS interrupts
- Non deterministic issues: instruction scheduler, branch predictor
- External conditions: temperature of the room
- Timing accuracy: no constant cycle period on modern processors (i7...)

Uncertainty increases as computer system complexity does

- Architecture and micro-architecture issues: multicore, hybrid, speculation
- Compiler options and its effects

How to read the current literature?

Numerical results in S.M. Rump contributions (for summation)

- 26% for Sum2-SumK (SISC-05) : 9 pages over 34
- 20% for AccSum (SISC-08) : 7 pages over 35
- 20% for AccSumK-NearSum (SISC-08b) : 6 pages over 30
- **less that 3%** for FastAccSum (SISC-09) : 1 page over 37

Lack of proof, or at least of reproducibility

> *Measuring the computing time of summation algorithms in a high-level language on today's architectures is more of a hazard than scientific research.* *S.M. Rump (SISC, 2009)*

. . . in the paper entitled *Ultimately Fast Accurate Summation*

Software and System Performance experts' point of view

The limited *Accuracy of Performance Counter Measurements*

We caution performance analysts to be suspicious of cycle counts . . . gathered with performance counters.

D. Zaparanuks, M. Jovic, M. Hauswirth (2009)

Can Hardware Performance Counters Produces Expected, Deterministic Results?

In practice counters that should be deterministic show variation from run to run on the x86_64 architecture. . . . it is difficult to determine known "good" reference counts for comparison.

V.M. Weaver, J. Dongarra (2010)

The picture is blurred: the computing chain is wobbling around

If we combine all the published speedups (accelerations) on the well known public benchmarks since four decades, why don't we observe execution times approaching to zero? *S. Touati (2009)*

Outline

ILP and the performance potential of an algorithm

Instruction Level Parallelism (ILP) describes the potential of the instructions of a program that can be executed simultaneously

Hennessy-Patterson's ideal machine (H-P IM)

- every instruction is executed one cycle after the execution one of the producers it depends
- no other constraint than the true instruction dependency (RAW)

Measure the #cycles and the #IPC running the code with the H-P IM

- maximal exploitation of the program ILP
- processor and ILP in practice: superscalar and out-of-order execution
- ILP measures the potential of the algorithm performance

What is ILP?

A synthetic sample: $e = (a+b) + (c+d)$

x86 binary

	...
i1	mov eax,DWP[ebp-16]
i2	mov edx,DWP[ebp-20]
i3	add edx,eax
i4	mov ebx,DWP[ebp-8]
i5	add ebx,DWP[ebp-12]
i6	add edx,ebx
	...

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Instruction and cycle counting

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Instruction and cycle counting

Cycle 0: i1 i2 i4

What is ILP?

A synthetic sample: e = (a+b) + (c+d)

x86 binary

```
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i1  mov   eax,DWP[ebp-16]
i2  mov   edx,DWP[ebp-20]
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i4  mov   ebx,DWP[ebp-8]
i5  add   ebx,DWP[ebp-12]
i6  add   edx,ebx
    ...
```
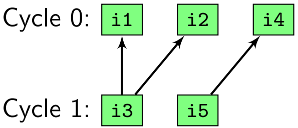

Instruction and cycle counting



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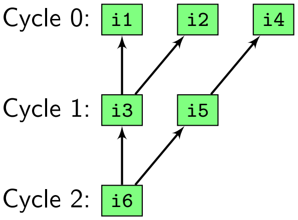
```

Instruction and cycle counting



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A synthetic sample: e = (a+b) + (c+d)

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i6  add    edx,ebx
    ...
```
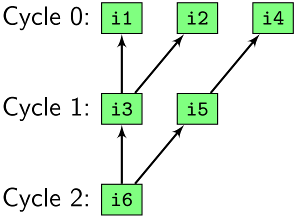

Instruction and cycle counting



\# of instructions = 6, # of cycles = 3

ILP = # of instructions/# of cycles = 2

ILP explains why compensated algorithms run fast

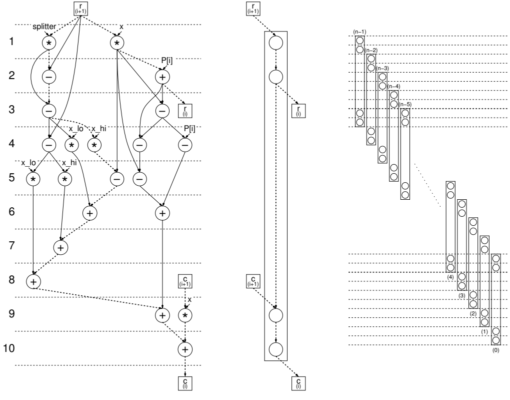
N. Louvet, PhD (07)

CompHorner

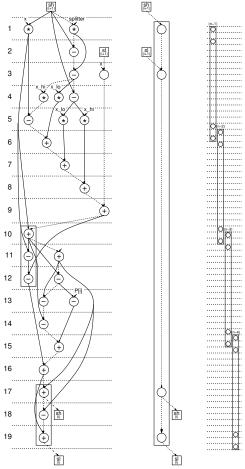
#C=2n+8, ILP≈ 11

DDHorner

#C=17n+2, ILP≈ 1.65



(a) (b) (c)



(a) (b) (c)

The PerPI Tool automatizes this ILP analysis

PerPI: a pintool to analyse and visualise the ILP of x86-coded algorithms

- Pin (Intel) tool (http://www.pintool.org)
- Outputs: ILP measure (#C, #I), IPC histogram, data-dependency graph
- Input: x86_64 binary file
- Developed and maintained by B. Goossens and D. Parello (DALI)

Some recent accurate and fast summation algorithms

Twice more precision

- Sum2: Compensated with a VectSum that uses TwoSum
- DDSum: Recursive sum + double-double arithmetic

Faithful or exact rounding

- iFastSum: SumK with dynamic error control
- AccSum and FastAccSum: Adaptative computational effort wrt cond.
  Split the summands by chunk that sums exactly (width depends on n), careful sum of the chunks.
  Chunk cutting-line fixed in AccSum while more dynamic in FastAccSum
- HybridSum and OnLineExactSum: Exponent extraction of the summands, careful accumulation in one (HS) or two vectors (OLE) of fixed and short length (2048 in IEEE-b64), and distillate the (very for OLE) short vector with iFastSum.

How to chose the data test?

Time complexity parameters of the summation algorithms

- Only n for Sum2, SumK: constant accuracy improvement
- n and *cond* for AccSum, iFastSum: adaptive accuracy improvement
- Exponent range of the summands: Z-H exhibit no influence for HybridSum and OnlineExactSum for large n
- Rump's generator of arbitrary ill-conditioned dot product (SISC-05), modified for summation and to cover an arbitrary exponent range.
- Length: $n \in [10^3, 10^7]$ and $cond \in [10^8, 10^{40}] = [\sqrt{1/\mathbf{u}}, 1/\mathbf{u}^{2.5}]$

Our fuzzy PAPI picture

Parameters : sum length: 10^3 to 10^6, cond: 10^8 to 10^{40}

cond	Sum	Sum2	FastAccSum	iFastSum	HybridSum	OnLineExact
10^8	1	2–3	4–5	7–8	5 ($n > 10^5$)	4 ($n > 10^5$)
10^{16}	1	2–3	5–6	7–8	5 ($n > 10^5$)	4 ($n > 10^5$)
10^{24}	1	–	7	13	5 ($n > 10^5$)	4 ($n > 10^5$)
10^{32}	1	–	8	18	5 ($n > 10^5$)	4 ($n > 10^5$)
10^{40}	1	–	9	18+	5 ($n > 10^5$)	4 ($n > 10^5$)

Experimental process: PAPI, counter delay, hot caches, average over 50 samples for each *n* and cond. . . .

Intel(R) Core(TM) i7 CPU870 2.93GHz, x86_64, GNU/Linux noyau 2.6.38-8-generic
– gcc (4.6) -std=c99 -march=corei7 -mfpmath=sse -O3 -funroll-all-loops
– icc ($12.0.4_{20110427}$) -std=c99 -O3 -mtune=corei7 -xSSE -axsse4.2 -funroll-all-loops

Focus inside OnLineExact

Low level choices are crucial

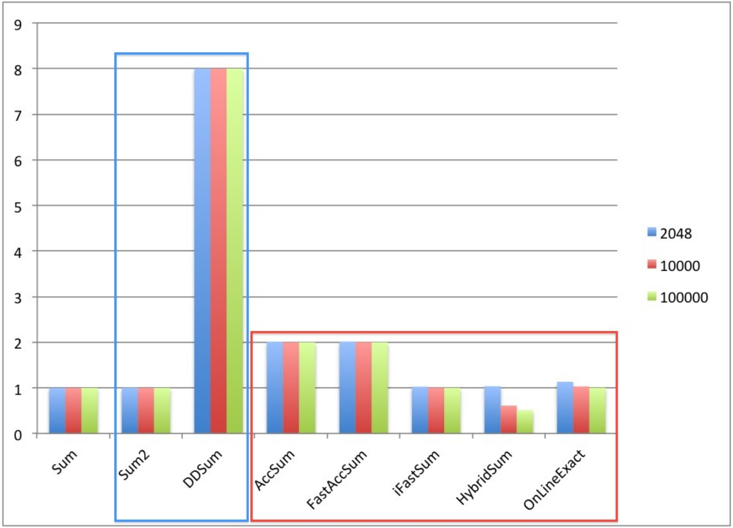
cond $= 10^{16}$	gcc		icc	
n	2sum	Fast2sum	2sum	Fast2sum
10^3	13	11	13.7	9
10^4	6.5	6.5	5.8	4
10^5	5	5.5	3.8	3.3
10^6	4.7	5.6	3.8	3.3

Cycle ratios (vs. Sum) vary for different EFT and compilers

PerPI and reproducibility: one run is enough

```
start : <main>   (depth: 2, lcid: 104)
  stop  : <Sum>   (depth: 3, lcid: 10201)(cid: 10201)    I[13781]::C[10000]::ILP[1.3781]
  stop  : <Sum>   (depth: 3, lcid: 10203)(cid: 10203)    I[13781]::C[10000]::ILP[1.3781]
  stop  : <Sum>   (depth: 3, lcid: 10205)(cid: 10205)    I[13781]::C[10000]::ILP[1.3781]
  stop  : <iFastSumIn>   (depth: 3, lcid: 10207)(cid: 10207)    I[696088]::C[18043]::ILP[3
  stop  : <iFastSumIn>   (depth: 3, lcid: 10241)(cid: 10241)    I[696076]::C[18043]::ILP[3
  stop  : <iFastSumIn>   (depth: 3, lcid: 10275)(cid: 10275)    I[696076]::C[18043]::ILP[3
  start : <OnlineExactSum>   (depth: 3, lcid: 10309)
    stop  : <iFastSumIn>   (depth: 4, lcid: 10320)(cid: 10320)    I[29704]::C[611]::ILP[48
  stop  : <OnlineExactSum>   (depth: 3, lcid: 10309)(cid: 10309)    I[301467]::C[10607]::I
 stop  : <main>   (depth: 2, lcid: 104)(cid: 104)    I[2884900]::C[49320]::ILP[58.4935]
Global ILP (cid: 0)    I[2895541]::C[49572]::ILP[58.4108]
```

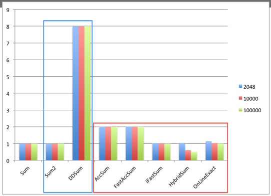
PerPI: # cycle ratios for summation algorithms



Number of cycles: ratios vs. Sum

for *cond* $= 10^{32}$ and $n = 2048, 10^4, 10^5$

PerPI: # cycle ratios for summation algorithms



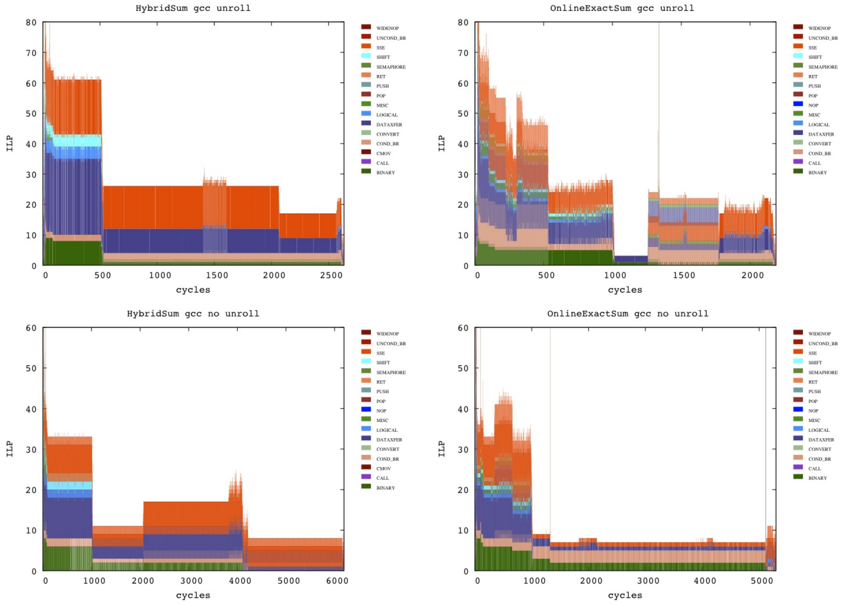
Twice more accurate computed sum

- No overhead: compensation is the right choice

Faithfully or exactly rounded computed sum

- The newest, the potentially fastest but be cautious: sensitive in practice
- FastAccSum (3n) not faster than AccSum (4n) [GLPP-Para10]
- OnLineExact for large n, else iFastSum
- PerPI highlights the control, *e.g.* iteration counters
- Less #C in HybridSum than in Sum?
  - Sum is unrolled 8 times by gcc but C forbids to change the evaluation order of the arithmetic expression
  - Every cycle of HybridSum has enough parallel work with different summands: 2 here
  - OnLineExact introduces dependency between iterations: $x[i]$ and $x[i+1]$ may have the same exponent

HS (left) and OLE (right), unrolled (up) or not (down)



Conclusion

Conclusion

- Highly accurate algorithm $\longrightarrow$ reliable performance evaluation
- Flop count: not significant
- Hardware counter based measure: uncertainty and no reproducibility
- PerPI: a software platform to analyze and visualise ILP
  - Reliable: reproducibility both in time and location
  - Realistic: correlation with measured ones
  - Useful: a detailed picture of the intrinsic behavior of the algorithm
  - Optimisation tool: analyse the effect of some hardware constraints [GLPP-Para10]
  - Exploratory tool: gives us the taste of the behavior of our algorithms running on "tomorrow" processors

Conclusion

Computing time: More science? Less hazard?

- No definitive answer
- PerPI result is far from perfect
  - Not abstract enough: instruction set dependence, compiler choice
  - Good abstraction level? Assembler program or high level programming language?

Next step for f.p. summation: reproducibility to improve productivity

- Web site with common and shared resources: tested + test + make file sources, data files and generators, real and abstract associated measures
- Open and dynamic interaction: load your new algorithm, your new data, run them and let's contribute
- architectures? compilers?
- suggestions and partners are welcome!

References I

D. H. Bailey.
Twelve ways to fool the masses when giving performance results on parallel computers.
Supercomputing Review, pages 54–55, Aug. 1991.

B. Goossens, P. Langlois, D. Parello, and E. Petit.
PerPI: A tool to measure instruction level parallelism.
In K. Jónasson, editor, *Applied Parallel and Scientific Computing - 10th International Conference, PARA 2010, Reykjavík, Iceland, June 6-9, 2010, Revised Selected Papers, Part I*, volume 7133 of *Lecture Notes in Computer Science*, pages 270–281. Springer, 2012.

N. J. Higham.
Accuracy and Stability of Numerical Algorithms.
SIAM, 2nd edition, 2002.

J.-M. Muller, N. Brisebarre, F. de Dinechin, C.-P. Jeannerod, V. Lefèvre, G. Melquiond, N. Revol, D. Stehlé, and S. Torres.
Handbook of Floating-Point Arithmetic.
Birkhäuser Boston, 2010.

References II

T. Ogita, S. M. Rump, and S. Oishi.
Accurate sum and dot product.
SIAM J. Sci. Comput., 26(6):1955–1988, 2005.

S. M. Rump.
Ultimately fast accurate summation.
SIAM J. Sci. Comput., 31(5):3466–3502, 2009.

S. M. Rump, T. Ogita, and S. Oishi.
Accurate floating-point summation – part I: Faithful rounding.
SIAM J. Sci. Comput., 31(1):189–224, 2008.

V. Weaver and J. Dongarra.
Can hardware performance counters produce expected, deterministic results?
In *3rd Workshop on Functionality of Hardware Performance Monitoring*, 2010.

D. Zaparanuks, M. Jovic, and M. Hauswirth.
Accuracy of performance counter measurements.
In *ISPASS*, pages 23–32. IEEE, 2009.

References III

Y.-K. Zhu and W. B. Hayes.
Correct rounding and hybrid approach to exact floating-point summation.
SIAM J. Sci. Comput., 31(4):2981–3001, 2009.

Y.-K. Zhu and W. B. Hayes.
Algorithm 908: Online exact summation of floating-point streams.
ACM Transactions on Mathematical Software, 37(3):37:1–37:13, Sept. 2010.