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Optimal Pattern Generator based on a Three-Mass Linear Inverted Pendulum Model for dynamic walking

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February, 14th, 2011
Outline

1. Context and motivation
2. SHERPA walking robot
3. Basic 3MLIPM pattern generator
4. Limitations and improvements
5. Simulation results
6. Conclusion and Future work
A stability indicator is a mathematical criterion that can characterize the stability margins of a walking robot from the current state of the robot.

Walking mode

Statically stable walk
Indicator: CoM

Dynamically stable walk
Indicators: ZMP, CoP, FRI...
Center of Mass (CoM)

Projection of the CoM relative to the support polygon [Nunez, 2008]

CoM is the mean location of all masses of the robot links

\[ OG = \sum m_i OG_i \]

Static stability criterion

Zero Moment Point (ZMP)

Projection of the ZMP relative to the support polygon [Nunez, 2008]

ZMP is the point where the vertical reaction force intersects the ground

\[ ZMP(t) = f(q(t), \dot{q}(t), \ddot{q}(t), f_e(t)) \]

Dynamic stability criterion
State of art on walking pattern generators

Pattern generators

Simplified models
- LIPM
  - [Kajita et al. 2001]
  - [Hong et al. 2009]
  - [Tang et al. 2007]
  - [Ferreira et al. 2009]
  - [Lee, 2007]
- 2MLIPM
  - [Albert et al. 2003]
- 3MLIPM
  - [Takenaka et al. 2009]

Biomechanics
- Biomechanics
  - [Bruneau et al. 1998]
  - Motion capture
    - [Harada, 2009]
    - [Kim et al. 2009]
    - [Takano et al. 2007]
- Van der pol
  - [Katoh et al. 1984]
  - Sinusoids
    - [Zhao et al. 2008]
    - FFT
    - [Yamaguchi et al. 1999]

B-Splines
- [Huang et al. 1999]

Others
- Polynomial Functions
  - [Zaier et al. 2007]
- Neural networks
  - [Yang et al. 2007]
Motivation

Objective
Design and implementation of a pattern generator for stable dynamic walking

Assumptions
- The ground is flat and without obstacles
- The walking cycle is made of single support and impact phases
- The double support phase is not considered
- The solution uses a simplified model of the robot

Method and application
- Model: 3 Masses Linear Inverted Pendulum Model (3MLIPM)
- Application: SHERPA biped robot
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Prototype

SHERPA biped robot

- 7 parts: one waist linked to two legs together articulated with knees and ankles
- 18 degrees of freedom / 12 actuated articulations
- 12 modular transparent actuators (low inertia, low friction and backdrivable)
- Control PC with a real time kernel (RTX)
Prototype

Sensor

- 12 Absolute Shaft Encoders (HENGSTLER AD36) to measure articular positions
- 2 six-axis force sensors (ATI-Mini 85) to measure contact forces with ground

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Two motion scenarios are proposed.

**Scenario 1**

A swing movement Up and down of the hanged leg

**Scenario 2**

A swing movement Forward-backward movement of the hanged leg

Reference trajectories generation: based on b-splines functions
**Objective**

Find a trajectory: \( T = x(t), \ t \in [0, t_f] \)

under a set of constraint:

\[
\begin{align*}
    x(0) & = x_0 \\
    \dot{x}(0) & = \dot{x}_0 \\
    x(t_f) & = x_f \\
    \dot{x}(t_f) & = \dot{x}_f \\
    x(t_i) & = x_i
\end{align*}
\]

**Proposed solution**

CSAPE algorithm from b-splin toolbox of Matlab software
B-splines

Illustration example

Constraints: \[
\begin{align*}
    x(0) &= 4 \\
    x(1) &= 8 \\
    x(2) &= 6 \\
    \dot{x}(0) &= 0 \\
    \dot{x}(2) &= 3
\end{align*}
\]

B-splines MATLAB function: \[
\text{csape( [0,1,2], [4,8,6],[1,1],[0,3])}
\]

Obtained trajectories

Position

Velocity

Acceleration

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First movements of the robot (Experiments)

**Scenario 1**

**Scenario 2**
Outline

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Simplified models

LIPM
[Kajita et al., 2009]

2MLIPM
[Albert and Gerth, 2003]

3MLIPM
[Feng and Sun, 2008]

Reduce the dynamic of the robot to the dynamic of a point mass.
The three masses linear inverted pendulum model

"3 Mass Linear Inverted Pendulum Model (3MLIPM)" [Feng and Sun, 2008]

Properties
- Simplified model of the robot
- Three point masses
- Three massless links

Hypothesis
- Walk on flat ground
- No double support phases
The three masses linear inverted pendulum model

"3 Mass Linear Inverted Pendulum Model (3MLIPM)" [Feng and Sun, 2008]

Properties
- Simplified model of the robot
- Three point masses
- Three massless links

Hypothesis
- Walk on flat ground
- No double support phases

Decoupled equations
Motion generated separately
The three masses linear inverted pendulum model

Motion in sagittal plane

Motion in frontal plane

Context and motivation
SHERPA walking robot
Basic 3MLIPM pattern generator
Limitations and improvements
Simulation results
Conclusion and Future work

Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

\[ x_i : \text{Cartesian position of mass } m_i \text{ in } x \text{ axis} \]

\[ z_i : \text{Cartesian position of mass } m_i \text{ in } z \text{ axis} \]
Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

\[ \sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i \]

\( x_i \) : Cartesian position of mass \( m_i \) in \( x \) axis
\( z_i \) : Cartesian position of mass \( m_i \) in \( z \) axis
Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

\[ \sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i \]

\[ b \ddot{x}_1 + d \ddot{x}_3 = a x_1 + x_3 \]

- \( x_i \): Cartesian position of mass \( m_i \) in \( x \) axis
- \( z_i \): Cartesian position of mass \( m_i \) in \( z \) axis

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Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

\[ \sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i \]

\[ b \ddot{x}_1 + d \ddot{x}_3 = a x_1 + x_3 \]

Choose a trajectory for the swinging foot

\( x_i \) : Cartesian position of mass \( m_i \) in \( x \) axis

\( z_i \) : Cartesian position of mass \( m_i \) in \( z \) axis
Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

\[ \sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i \]

\[ b \ddot{x}_1 + d \ddot{x}_3 = a x_1 + x_3 \]

Choose a trajectory for the swinging foot

Motion of the three masses

\( x_i \): Cartesian position of mass \( m_i \) in \( x \) axis

\( z_i \): Cartesian position of mass \( m_i \) in \( z \) axis
Context and motivation
SHERPA walking robot
Basic 3MLIPM pattern generator
Limitations and improvements
Simulation results
Conclusion and Future work

Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

$x_i$ : Cartesian position of mass $m_i$ in $x$ axis
$z_i$ : Cartesian position of mass $m_i$ in $z$ axis

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Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

\[ \sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i \]

- \( x_i \): Cartesian position of mass \( m_i \) in \( x \) axis
- \( z_i \): Cartesian position of mass \( m_i \) in \( z \) axis

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Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

\[ \sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i \]

\[ u \ddot{y}_1 - v y_1 = w \]

\( x_i \) : Cartesian position of mass \( m_i \) in \( x \) axis

\( z_i \) : Cartesian position of mass \( m_i \) in \( z \) axis
Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

\[ \sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i \]

\[ u \ddot{y}_1 - v y_1 = w \]

3D trajectories of hip and ankles

- \( x_i \): Cartesian position of mass \( m_i \) in x axis
- \( z_i \): Cartesian position of mass \( m_i \) in z axis
### Motion in frontal plane

**Dynamic of 3MLIPM in the frontal plane**

\[
\sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i
\]

\[
u \ddot{y}_1 - v y_1 = w
\]

**3D trajectories of hip and ankles**

**Inverse kinematics** → **Joints space trajectories**

\(x_i\): Cartesian position of mass \(m_i\) in \(x\) axis

\(z_i\): Cartesian position of mass \(m_i\) in \(z\) axis
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First contribution

First limitation of the 3MLIPM model:
The dynamic stability is not guaranteed

Proposed improvement: Optimization with respect to ZMP
Principle: optimal value of mass $m_1$ and its position $z_1$:

$$
\begin{bmatrix}
\hat{z}_1 \\
\hat{m}_1
\end{bmatrix}
= \text{Arg Min}_{z_1, m_1} \text{Max} \left( \sqrt{\alpha(x_{zmp} - x_{dzmp})^2 + \beta(y_{zmp} - y_{dzmp})^2} \right)
$$

This optimization aims to find the best fit between the desired and the computed ZMP.
Second contribution

Second limitation of the 3MLIPM model:
Change of walking direction is not allowed

Proposed improvement: Modification of the hip trajectory
Principle: the hip trajectory is modified as follow:

$$\Omega(t) = -\frac{R}{2} \cos\left(\frac{\pi t}{T}\right) \quad t \in [0, T]$$

with $T$: half step period and $R$: amplitude of rotation.
The modification of the hip trajectory allows a change of walking direction.
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Developed simulator

Proposed scenarios:

Scenario 1: Straight walking

Scenario 2: Change of walking direction

Comparison of the original pattern generator with the improved one
First scenario: Straight walking

**Joints’ positions**

- position: $q_1$ [rad] and $q_7$ [rad]
- position: $q_2$ [rad] and $q_8$ [rad]
- position: $q_3$ [rad] and $q_9$ [rad]
- position: $q_4$ [rad] and $q_{10}$ [rad]
- position: $q_5$ [rad] and $q_{11}$ [rad]
- position: $q_6$ [rad] and $q_{12}$ [rad]

**Joints’ velocities**

- velocity: $\dot{q}_1$ [rad/s] and $\dot{q}_7$ [rad/s]
- velocity: $\dot{q}_2$ [rad/s] and $\dot{q}_8$ [rad/s]
- velocity: $\dot{q}_3$ [rad/s] and $\dot{q}_9$ [rad/s]
- velocity: $\dot{q}_4$ [rad/s] and $\dot{q}_{10}$ [rad/s]
- velocity: $\dot{q}_5$ [rad/s] and $\dot{q}_{11}$ [rad/s]
- velocity: $\dot{q}_6$ [rad/s] and $\dot{q}_{12}$ [rad/s]

**Characteristics:**

- Joints’ trajectories are periodic
- Discontinuities in joints’ velocities
First scenario: Straight walking

Stability analysis through ZMP

Without optimization

With optimization

Optimization ⇒ Improvement of the stability margins
Second scenario: Change of walking direction

Characteristics:
- Joints’ trajectories are periodic
- Discontinuities in joints’ velocities
Second scenario: Change of walking direction

Stability analysis through ZMP

Without optimization

With optimization

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Second scenario: Change of walking direction

Stability analysis through ZMP

Without optimization

With optimization

Instable dynamic walking

Stable dynamic walking

Developed simulator
First scenario: Straight walking
Second scenario: Change of walking direction
Video

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Second scenario : Change of walking direction

Stability analysis through ZMP

Without optimization

With optimization

Instable dynamic walking  Stable dynamic walking

Optimization  ⇒  Dynamic walking stability is guaranteed
Context and motivation
SHERPA walking robot
Basic 3MLIPM pattern generator
Limitations and improvements
Simulation results
Conclusion and Future work

Developed simulator
First scenario: Straight walking
Second scenario: Change of walking direction
Video

Video

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Conclusion

Motivation:
Design and implementation of a pattern generator for dynamically stable walking

Deals with:
- Stability of dynamic walking
- 3D Movements
- Complex nonlinear dynamics
- Low CoM position (no torso)

Proposed solution:
- A pattern generator based on a 3 masses simplified model
- Stability margin improvement using optimization
- Change of walking direction is allowed
Future work can include...

- Real-time implementation of the proposed pattern generator on the biped robot SHERPA
- Development of a hybrid Position/Force controller to stabilize dynamic walking (in progress)
- Combine the hybrid Position/Force controller with the developed pattern generator
- Test the effectiveness of controller for walking on uneven ground
- Compare this approach to other pattern generators
Analytic path planning algorithms for bipedal robots without a trunk.

Dynamic transition simulation of a walking anthropomorphic robot.

Biped robot walking using three-mass linear inverted pendulum model.

ZMP trajectory reference for the sagittal plane control of a biped robot based on a human CoP and gait.
In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS’09)*, pages 1588–1593, St. Louis, USA.

Toward human-like walking pattern generator.

A walking pattern generation method with feedback and feedforward control for humanoid robots.
In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'09)*, pages 1078–1083, St. Louis, USA.

A high stability, smooth walking pattern for a biped robot.
In *IEEE International Conference on Robotics and Automation (ICRA'99)*, pages 65–71, Detroit, Michigan, USA.

*Introduction à la commande des robots humanoïdes. Translated in French by Sakka, S.* Springer.

The 3d linear inverted pendulum mode: A simple modeling for a biped walking pattern generation.

Control method of biped locomotion giving asymptotic stability of trajectory.

Stable whole-body motion generation for humanoid robots to imitate human motions.
In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'09)*, pages 2518–2524, St. Louis, USA.

Modifiable walking pattern generation using real-time zmp manipulation for humanoid robots.

Étude de la commande des mouvements dynamiques d’un robot humanoïde.
*ITL - LISV*.

Takano, W., Yamane, K., and Nakamura, Y. (2007).
Capture database through symbolization, recognition and generation of motion patterns.
In *IEEE International Conference on Robotics and Automation (ICRA’07)*, pages 3092–3097, Roma, Italy.

Real time motion generation and control for biped robot-1st report: Walking gait pattern generation.
In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'09)*, pages 1084–1091, St. Louis, USA.

Humanoid 3d gait generation based on inverted pendulum model.
Development of a bipedal humanoid robot - control method of whole body cooperative
dynamic biped walking.
pages 368–374, Detroit, Michigan.

Self-adapting humanoid locomotion using a neural oscillator network.
In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS’07)*,
pages 309–316, San Diego, USA.

Piecewise-linear pattern generator and reflex system for humanoid robots.
In *IEEE International Conference on Robotics and Automation (ICRA’07)*, pages 2188–2194,
Roma, Italy.

Humanoid robot gait generation based on limit cycle stability.