Optimal Pattern Generator Based on a Three-Mass Linear Inverted Pendulum Model for Dynamic Walking

David Galdeano, Ahmed Chemori, Sébastien Krut

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Laboratoire d'Informatique de Robotique et de Microélectronique de Montpellier
LIRMM - UMR 5506
161, Rue Ada 34392 Montpellier, France

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Table of contents

Outline

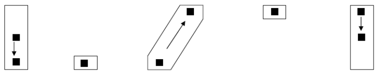
Stability indicators

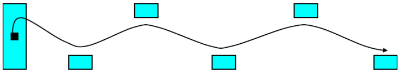
Definition

A stability indicator is a mathematical criterion that can characterize the stability margins of a walking robot from the current state of the robot.

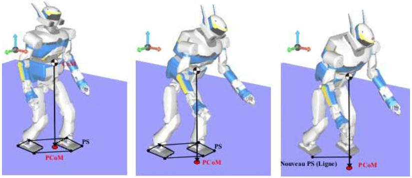
Walking mode

Statically stable walk	Dynamically stable walk
Indicator : CoM	Indicators : ZMP, CoP, FRI...





Center of Mass (CoM)



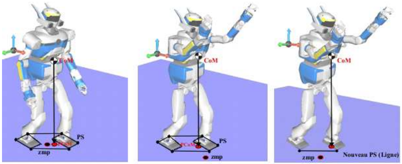
Projection of the CoM relative to the support polygon [Nunez, 2008]

CoM is the mean location of all masses of the robot links

$$OG = \sum m_i OG_i$$

Static stability criterion

Zero Moment Point (ZMP)



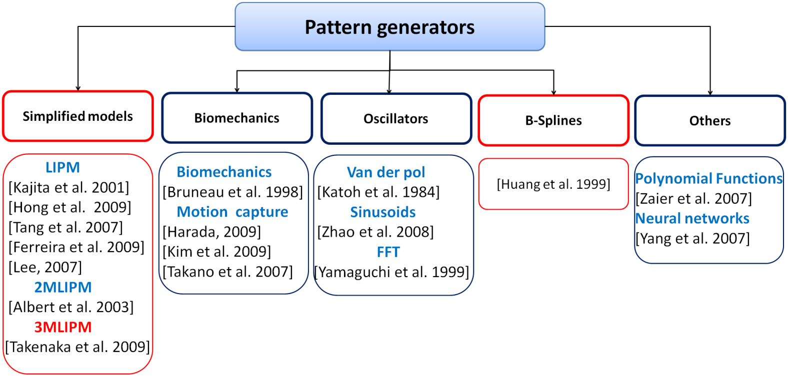
Projection of the ZMP relative to the support polygon [Nunez, 2008]

ZMP is the point where the vertical reaction force intersects the ground

$$ZMP(t) = f(q(t), \dot{q}(t), \ddot{q}(t), f_e(t))$$

Dynamic stability criterion

State of art on walking pattern generators



Motivation

Objective

Design and implementation of a pattern generator for stable dynamic walking

Assumptions

- The ground is flat and without obstacles
- The walking cycle is made of single support and impact phases
- The double support phase is not considered
- The solution use a simplified model of the robot

Method and application

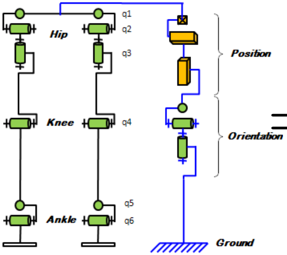
- Model : 3 Masses Linear Inverted Pendulum Model (3MLIPM)
- Application : SHERPA biped robot

Outline

Prototype

SHERPA biped robot

- 7 parts : one waist linked to two legs together articulated with knees and ankles
- 18 degrees of freedom / 12 actuated articulations
- 12 modular transparent actuators (low inertia, low friction and backdrivable)
- Control PC with a real time kernel (RTX)



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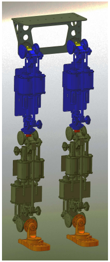

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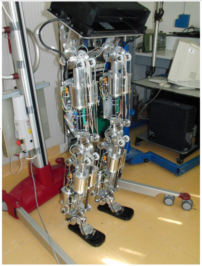


Prototype

Sensor

- 12 Absolute Shaft Encoders (HENGSTLER AD36) to measure articular positions
- 2 six-axis force sensors (ATI-Mini 85) to measure contact forces with ground

HENGSTLER AD36

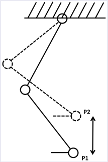
ATI-Mini 85

First movements of the robot

Two motion scenarios are proposed.

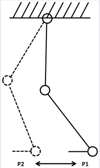
Scenario 1

A swing movement Up and down of the hanged leg



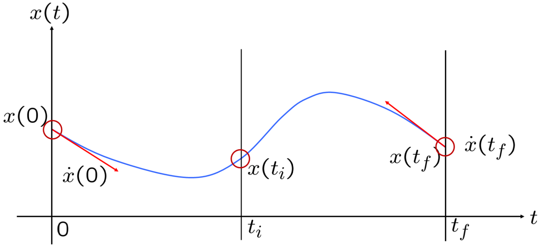
Scenario 2

A swing movement Forward-backward movement of the hanged leg



Reference trajectories generation : based on b-splines functions

B-splines



Objective

Find a trajectory : $T = x(t),\ t \in [0, t_f]$

under a set of constraint : $\begin{cases} x(0) & = & x_0 \\ \dot{x}(0) & = & \dot{x}_0 \\ x(t_f) & = & x_f \\ \dot{x}(t_f) & = & \dot{x}_f \\ x(t_i) & = & x_i \end{cases}$

Proposed solution

CSAPE algorithm from b-splin toolbox of Matlab software

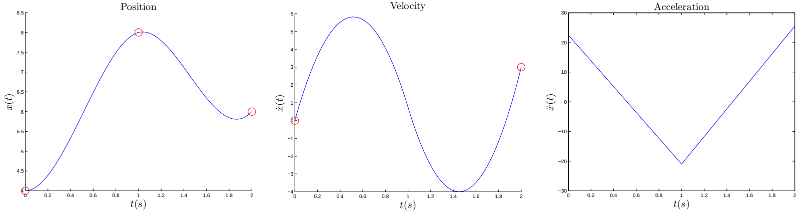
B-splines

Illustration example

Constraints : $\begin{cases} x(0) & = & 4 \\ x(1) & = & 8 \\ x(2) & = & 6 \\ \dot{x}(0) & = & 0 \\ \dot{x}(2) & = & 3 \end{cases}$

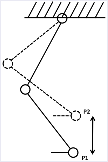
B-splines MATLAB function :
csape([0,1,2], [4,8,6],[1,1],[0,3])
$t_0 = 0, \ t_i = 1, \ t_f = 2$

Obtained trajectories

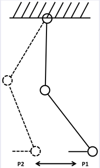


First movements of the robot (Experiments)

Scenario 1

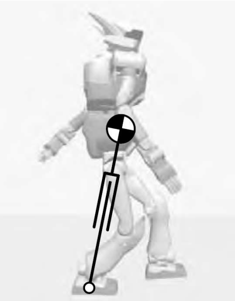


Scenario 2

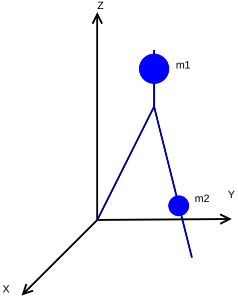


Outline

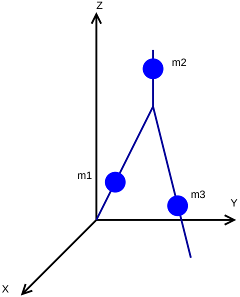
Simplified models



LIPM
[Kajita et al., 2009]



2MLIPM
[Albert and Gerth, 2003]

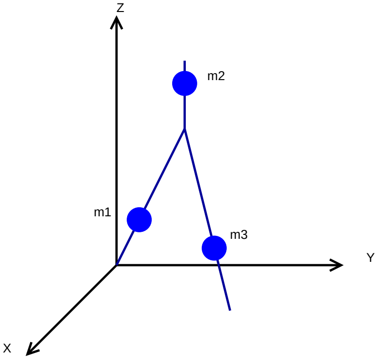


3MLIPM
[Feng and Sun, 2008]

Reduce the dynamic of the robot to the dynamic of a point mass.

The three masses linear inverted pendulum model

"3 Mass Linear Inverted Pendulum Model (3MLIPM)" [Feng and Sun, 2008]



Properties

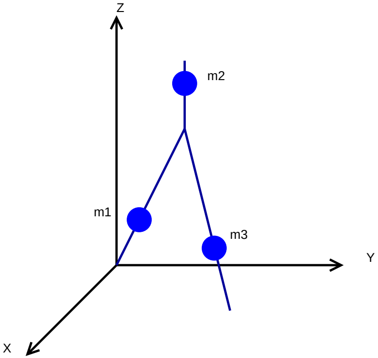
- Simplified model of the robot
- Three point masses
- Three massless links

Hypothesis

- Walk on flat ground
- No double support phases

The three masses linear inverted pendulum model

"3 Mass Linear Inverted Pendulum Model (3MLIPM)" [Feng and Sun, 2008]



Properties

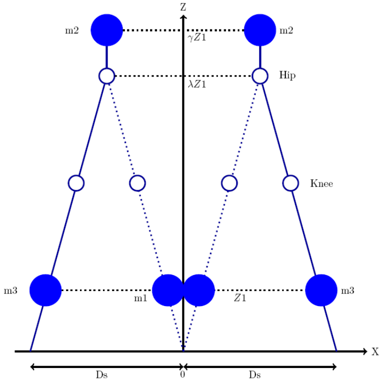
- Simplified model of the robot
- Three point masses
- Three massless links

Hypothesis

- Walk on flat ground
- No double support phases

Decoupled equations   Motion generated separately { Sagittal plane / Frontal plane

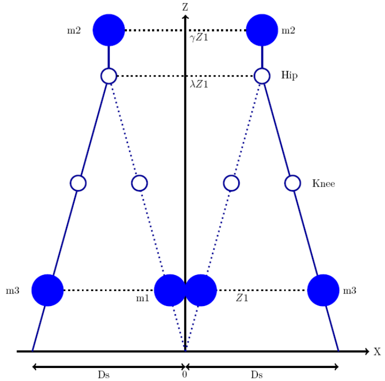
Motion in sagittal plane



Dynamic of 3MLIPM in the sagittal plane

x_i :Cartesian position of mass m_i in x axis
z_i :Cartesian position of mass m_i in z axis

Motion in sagittal plane



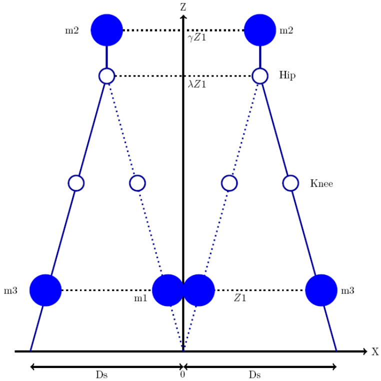
Dynamic of 3MLIPM in the sagittal plane

$$\sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i$$

x_i :Cartesian position of mass m_i in x axis

z_i :Cartesian position of mass m_i in z axis

Motion in sagittal plane



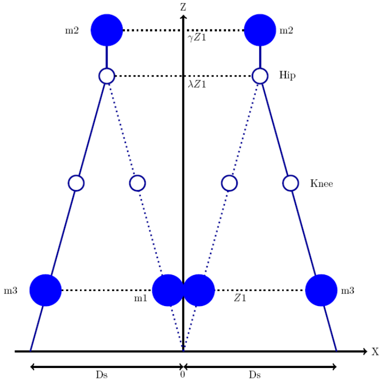
Dynamic of 3MLIPM in the sagittal plane

$$\sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i$$

$$b\ddot{x}_1 + d\ddot{x}_3 = ax_1 + x_3$$

x_i :Cartesian position of mass m_i in x axis
z_i :Cartesian position of mass m_i in z axis

Motion in sagittal plane



Dynamic of 3MLIPM in the sagittal plane

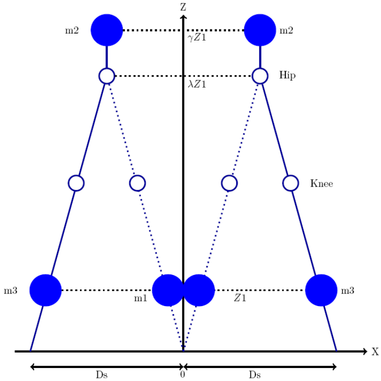
$$\sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i$$

$$b\ddot{x}_1 + d\ddot{x}_3 = a x_1 + x_3$$

Choose a trajectory for the swinging foot

x_i :Cartesian position of mass m_i in x axis
z_i :Cartesian position of mass m_i in z axis

Motion in sagittal plane



Dynamic of 3MLIPM in the sagittal plane

$$\sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i$$

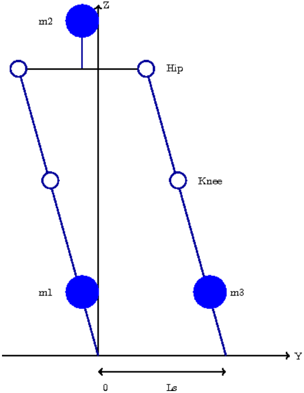
$$b\ddot{x}_1 + d\ddot{x}_3 = a x_1 + x_3$$

Choose a trajectory for the swinging foot

Motion of the three masses

x_i :Cartesian position of mass m_i in x axis
z_i :Cartesian position of mass m_i in z axis

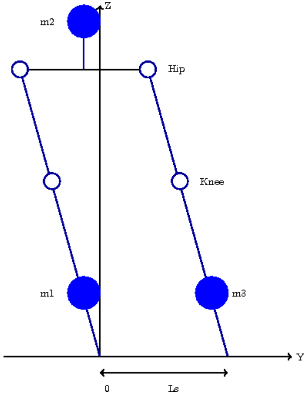
Motion in frontal plane



Dynamic of 3MLIPM in the frontal plane

x_i :Cartesian position of mass m_i in x axis
z_i :Cartesian position of mass m_i in z axis

Motion in frontal plane

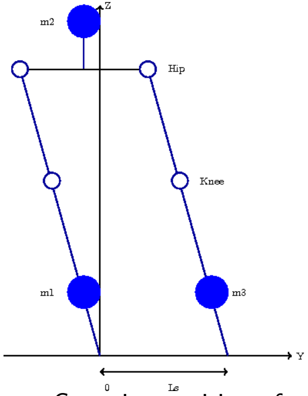


Dynamic of 3MLIPM in the frontal plane

$$\sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i$$

x_i :Cartesian position of mass m_i in x axis
z_i :Cartesian position of mass m_i in z axis

Motion in frontal plane



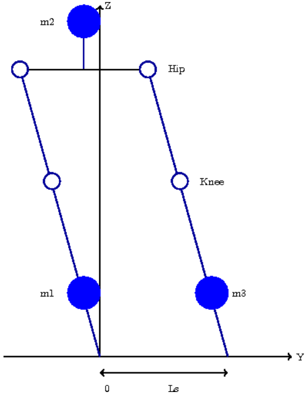
Dynamic of 3MLIPM in the frontal plane

$$\sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i$$

$$u\ddot{y}_1 - v y_1 = w$$

x_i :Cartesian position of mass m_i in x axis
z_i :Cartesian position of mass m_i in z axis

Motion in frontal plane



Dynamic of 3MLIPM in the frontal plane

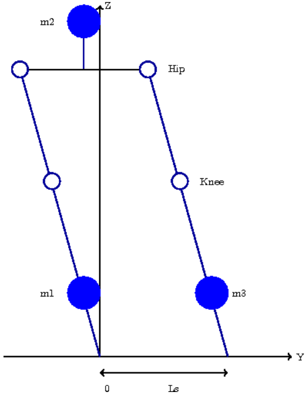
$$\sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i$$

$$u\ddot{y}_1 - v y_1 = w$$

3D trajectories of hip and ankles

x_i :Cartesian position of mass m_i in x axis
z_i :Cartesian position of mass m_i in z axis

Motion in frontal plane



Dynamic of 3MLIPM in the frontal plane

$$\sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i$$

$$u\ddot{y}_1 - v y_1 = w$$

3D trajectories of hip and ankles

Inverse kinematics → Joints space trajectories

x_i :Cartesian position of mass m_i in x axis
z_i :Cartesian position of mass m_i in z axis

Outline

First contribution

First limitation of the 3MLIPM model :

The dynamic stability is not guaranteed

Proposed improvement : Optimization with respect to ZMP

Principle : optimal value of mass m_1 and its position z_1 :

$$\begin{bmatrix} \hat{z}_1 \\ \hat{m}_1 \end{bmatrix} = Arg \underset{\begin{bmatrix} z_1 \\ m_1 \end{bmatrix}}{Min} \; Max \left(\sqrt{\alpha(x_{zmp} - x_{dzmp})^2 + \beta(y_{zmp} - y_{dzmp})^2} \right)$$

This optimization aims to find the best fit between the desired and the computed ZMP.

Second contribution

Second limitation of the 3MLIPM model :

Change of walking direction is not allowed

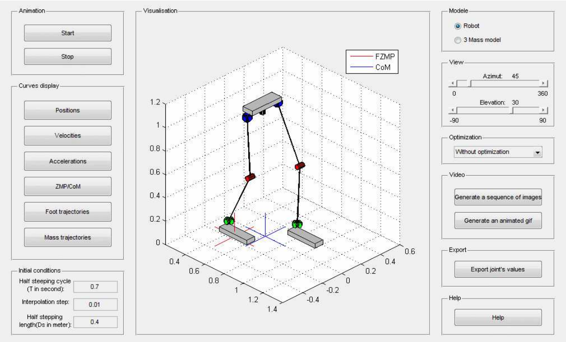
Proposed improvement : Modification of the hip trajectory
Principle : the hip trajectory is modified as follow :

$$\Omega(t) = -\frac{R}{2}\cos(\frac{\pi t}{T}) \quad t \in [0, T]$$

with T : half step period and R : amplitude of rotation.
The modification of the hip trajectory allows a change of walking direction.

Outline

Developed simulator



Proposed scenarios :

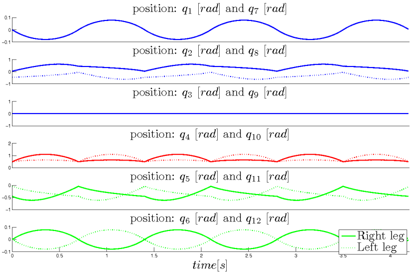
Scenario 1 :
Straight walking

Scenario 2 :
Change of walking direction

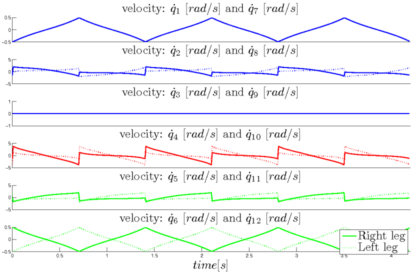
Comparison of the original pattern generator with the improved one

First scenario : Straight walking

Joints' positions



Joints' velocities



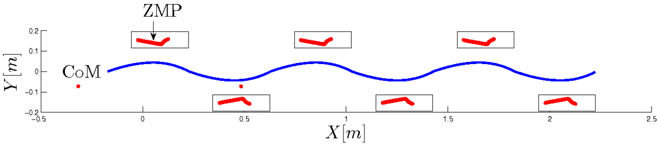
Characteristics :

- Joints' trajectories are periodic
- Discontinuities in joints' velocities

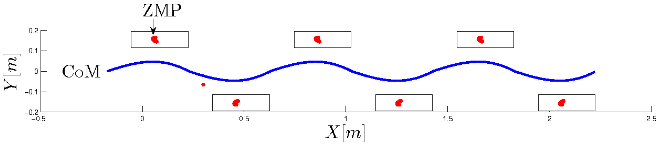
First scenario : Straight walking

Stability analysis through ZMP

Without optimization



With optimization



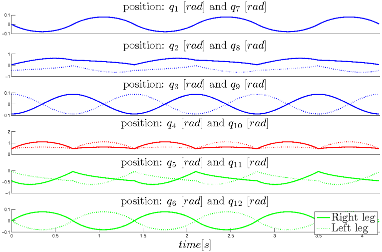
Optimization $\Rightarrow$ Improvement of the stability margins

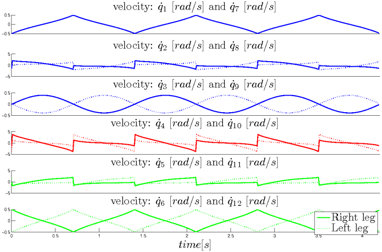
Second scenario : Change of walking direction

Joints' positions

Joints' velocities

Change of walking direction





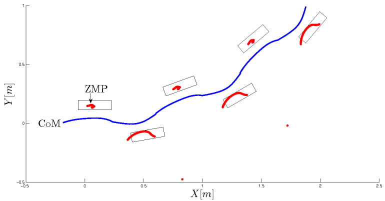
Characteristics :

- Joints' trajectories are periodic
- Discontinuities in joints' velocities

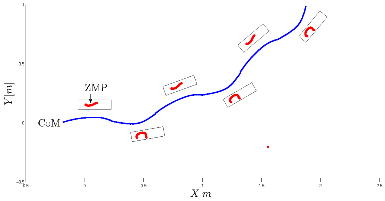
Second scenario : Change of walking direction

Stability analysis through ZMP

Without optimization



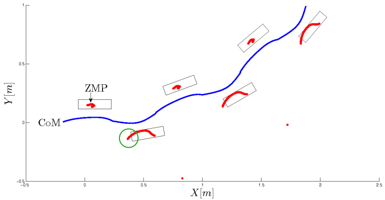
With optimization



Second scenario : Change of walking direction

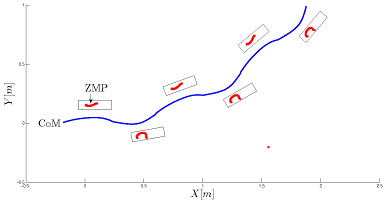
Stability analysis through ZMP

Without optimization



Instable dynamic walking

With optimization

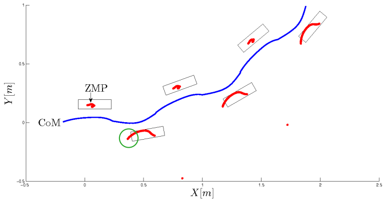


Stable dynamic walking

Second scenario : Change of walking direction

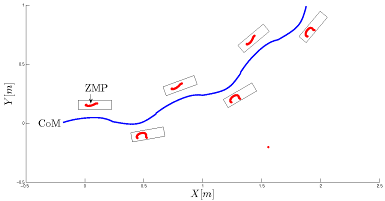
Stability analysis through ZMP

Without optimization



Instable dynamic walking

With optimization



Stable dynamic walking

Optimization $\Rightarrow$ Dynamic walking stability is guaranteed

Video

Outline

Conclusion

Motivation :

Design and implementation of a pattern generator for dynamically stable walking

Deals with :

- Stability of dynamic walking
- 3D Movements
- Complex nonlinear dynamics
- Low CoM position (no torso)

Proposed solution :

- A pattern generator based on a 3 masses simplified model
- Stability margin improvement using optimization
- Change of walking direction is allowed

Future work

Future work can include...

- Real-time implementation of the proposed pattern generator on the biped robot SHERPA
- Development of a hybrid Position/Force controller to stabilize dynamic walking (in progress)
- Combine the hybrid Position/Force controller with the developed pattern generator
- Test the effectiveness of controller for walking on uneven ground
- Compare this approach to other pattern generators

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