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Optimal Pattern Generator based on a Three-Mass Linear Inverted Pendulum Model for dynamic walking

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February, 14th, 2011
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2. SHERPA walking robot
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Outline

1. Context and motivation
2. SHERPA walking robot
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Definition

A stability indicator is a mathematical criterion that can characterize the stability margins of a walking robot from the current state of the robot.

Walking mode

Statically stable walk
Indicator: CoM

Dynamically stable walk
Indicators: ZMP, CoP, FRI...
**Context and motivation**

**SHERPA walking robot**

**Basic 3MLIPM pattern generator**

**Limitations and improvements**

**Simulation results**

**Conclusion and Future work**

**Stability indicators**

**State of art**

**Motivations**

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### Center of Mass (CoM)

Projection of the CoM relative to the support polygon [Nunez, 2008]

CoM is the mean location of all masses of the robot links

\[
OG = \sum m_i OG_i
\]

**Static stability criterion**

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### Zero Moment Point (ZMP)

Projection of the ZMP relative to the support polygon [Nunez, 2008]

ZMP is the point where the vertical reaction force intersects the ground

\[
ZMP(t) = f(q(t), \dot{q}(t), \ddot{q}(t), f_e(t))
\]

**Dynamic stability criterion**
State of art on walking pattern generators

- Simplified models
  - LIPM
    - Kajita et al. 2001
    - Hong et al. 2009
    - Tang et al. 2007
    - Ferreira et al. 2009
    - Lee, 2007
  - 2MLIPM
    - Albert et al. 2003
  - 3MLIPM
    - Takenaka et al. 2009

- Biomechanics
  - Biomechanics
  - Bruneau et al. 1998

- Oscillators
  - Van der pol
    - Katoh et al. 1984
  - Sinusoids
    - Harada, 2009
    - Kim et al. 2009
    - Takano et al. 2007
  - FFT
    - Zhao et al. 2008
  - Yamaguchi et al. 1999

- B-Splines
  - Huang et al. 1999

- Others
  - Polynomial Functions
    - Zaier et al. 2007
  - Neural networks
    - Yang et al. 2007
**Motivation**

**Objective**

Design and implementation of a pattern generator for stable dynamic walking

**Assumptions**

- The ground is flat and without obstacles
- The walking cycle is made of single support and impact phases
- The double support phase is not considered
- The solution uses a simplified model of the robot

**Method and application**

- Model: 3 Masses Linear Inverted Pendulum Model (3MLIPM)
- Application: SHERPA biped robot
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SHERPA biped robot

- 7 parts: one waist linked to two legs together articulated with knees and ankles
- 18 degrees of freedom / 12 actuated articulations
- 12 modular transparent actuators (low inertia, low friction and backdrivable)
- Control PC with a real time kernel (RTX)
Prototype

Sensor

- 12 Absolute Shaft Encoders (HENGSTLER AD36) to measure articular positions
- 2 six-axis force sensors (ATI-Mini 85) to measure contact forces with ground

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First movements of the robot

Two motion scenarios are proposed.

Scenario 1

A swing movement Up and down of the hanged leg

Scenario 2

A swing movement Forward-backward movement of the hanged leg

Reference trajectories generation : based on b-splines functions
Objective

Find a trajectory: \( T = x(t), \ t \in [0, t_f] \)

under a set of constraint:

\[
\begin{align*}
  x(0) &= x_0 \\
  \dot{x}(0) &= \dot{x}_0 \\
  x(t_f) &= x_f \\
  \dot{x}(t_f) &= \dot{x}_f \\
  x(t_i) &= x_i
\end{align*}
\]

Proposed solution

CSAPE algorithm from b-splin toolbox of Matlab software
B-splines

Illustration example

Constraints:

\[
\begin{align*}
    x(0) &= 4 \\
    x(1) &= 8 \\
    x(2) &= 6 \\
    \dot{x}(0) &= 0 \\
    \dot{x}(2) &= 3
\end{align*}
\]

B-splines MATLAB function:

```
bspline([0,1,2],[4,8,6],[1,1],[0,3])
```

t₀ = 0, tᵢ = 1, tᵢ = 2

Obtained trajectories
First movements of the robot (Experiments)

Scenario 1

Scenario 2
Context and motivation

SHERPA walking robot

Basic 3MLIPM pattern generator

Limitations and improvements

Simulation results

Conclusion and Future work
Simplified models

LIPM
[Kajita et al., 2009]

2MLIPM
[Albert and Gerth, 2003]

3MLIPM
[Feng and Sun, 2008]

Reduce the dynamic of the robot to the dynamic of a point mass.
The three masses linear inverted pendulum model

"3 Mass Linear Inverted Pendulum Model (3MLIPM)" [Feng and Sun, 2008]

Properties
- Simplified model of the robot
- Three point masses
- Three massless links

Hypothesis
- Walk on flat ground
- No double support phases
The three masses linear inverted pendulum model

"3 Mass Linear Inverted Pendulum Model (3MLIPM)" [Feng and Sun, 2008]

Properties
- Simplified model of the robot
- Three point masses
- Three massless links

Hypothesis
- Walk on flat ground
- No double support phases

Decoupled equations

Motion generated separately

{ Sagittal plane
  Frontal plane

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Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

$x_i$: Cartesian position of mass $m_i$ in $x$ axis
$z_i$: Cartesian position of mass $m_i$ in $z$ axis
Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

\[ \sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i \]

\( x_i \): Cartesian position of mass \( m_i \) in \( x \) axis
\( z_i \): Cartesian position of mass \( m_i \) in \( z \) axis
The three masses linear inverted pendulum model

Motion in sagittal plane

\[ \sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i \]
\[ b \ddot{x}_1 + d \ddot{x}_3 = a x_1 + x_3 \]

- \( x_i \): Cartesian position of mass \( m_i \) in x axis
- \( z_i \): Cartesian position of mass \( m_i \) in z axis

Dynamic of 3MLIPM in the sagittal plane

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Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

\[ \sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i \]

\[ b \ddot{x}_1 + d \ddot{x}_3 = a x_1 + x_3 \]

Choose a trajectory for the swinging foot

\[ x_i : \text{Cartesian position of mass } m_i \text{ in } x \text{ axis} \]

\[ z_i : \text{Cartesian position of mass } m_i \text{ in } z \text{ axis} \]
The three masses linear inverted pendulum model

Motion in sagittal plane

\[ \sum_{i=1}^{3} m_i \ddot{z}_i = \sum_{i=1}^{3} m_i g x_i \]

Choose a trajectory for the swinging foot

Motion of the three masses

\[ b \ddot{x}_1 + d \ddot{x}_3 = a x_1 + x_3 \]

\( x_i \): Cartesian position of mass \( m_i \) in \( x \) axis
\( z_i \): Cartesian position of mass \( m_i \) in \( z \) axis
Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

$x_i$: Cartesian position of mass $m_i$ in $x$ axis
$z_i$: Cartesian position of mass $m_i$ in $z$ axis
Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

\[ \sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i \]

- \( x_i \) : Cartesian position of mass \( m_i \) in \( x \) axis
- \( z_i \) : Cartesian position of mass \( m_i \) in \( z \) axis
Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

\[ \sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i \]

\[ u \ddot{y}_1 - v y_1 = w \]

\( x_i \) : Cartesian position of mass \( m_i \) in \( x \) axis

\( z_i \) : Cartesian position of mass \( m_i \) in \( z \) axis
The three masses linear inverted pendulum model

Motion in sagittal plane

Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

\[ \sum_{i=1}^{3} m_i \dddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i \]

\[ u \dddot{y}_1 - v y_1 = w \]

3D trajectories of hip and ankles

\( x_i \): Cartesian position of mass \( m_i \) in \( x \) axis

\( z_i \): Cartesian position of mass \( m_i \) in \( z \) axis

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Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

$$\sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i$$

$$u \ddot{y}_1 - v y_1 = w$$

3D trajectories of hip and ankles

Inverse kinematics $\rightarrow$ Joints space trajectories

$x_i$ : Cartesian position of mass $m_i$ in $x$ axis
$z_i$ : Cartesian position of mass $m_i$ in $z$ axis
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First contribution

First limitation of the 3MLIPM model:
The dynamic stability is not guaranteed

Proposed improvement: Optimization with respect to ZMP
Principle: optimal value of mass $m_1$ and its position $z_1$:

$$
\begin{bmatrix}
\hat{z}_1 \\
\hat{m}_1
\end{bmatrix} = \arg \min_{z_1, m_1} \max \left( \sqrt{\alpha(x_{zmp} - x_{dzmp})^2 + \beta(y_{zmp} - y_{dzmp})^2} \right)
$$

This optimization aims to find the best fit between the desired and the computed ZMP.
Second contribution

Second limitation of the 3MLIPM model:
Change of walking direction is not allowed

Proposed improvement: Modification of the hip trajectory
Principle: the hip trajectory is modified as follow:

$$\Omega(t) = -\frac{R}{2} \cos\left(\frac{\pi t}{T}\right) \quad t \in [0, T]$$

with $T$: half step period and $R$: amplitude of rotation.
The modification of the hip trajectory allows a change of walking direction.
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Developed simulator

Proposed scenarios:

Scenario 1: Straight walking

Scenario 2: Change of walking direction

Comparison of the original pattern generator with the improved one
First scenario: Straight walking

Joints’ positions

- position: $q_1$ [rad] and $q_7$ [rad]
- position: $q_2$ [rad] and $q_8$ [rad]
- position: $q_3$ [rad] and $q_9$ [rad]
- position: $q_4$ [rad] and $q_{10}$ [rad]
- position: $q_5$ [rad] and $q_{11}$ [rad]
- position: $q_6$ [rad] and $q_{12}$ [rad]

Characteristics:
- Joints’ trajectories are periodic
- Discontinuities in joints’ velocities

Joints’ velocities

- velocity: $\dot{q}_1$ [rad/s] and $\dot{q}_7$ [rad/s]
- velocity: $\dot{q}_2$ [rad/s] and $\dot{q}_8$ [rad/s]
- velocity: $\dot{q}_3$ [rad/s] and $\dot{q}_9$ [rad/s]
- velocity: $\dot{q}_4$ [rad/s] and $\dot{q}_{10}$ [rad/s]
- velocity: $\dot{q}_5$ [rad/s] and $\dot{q}_{11}$ [rad/s]
- velocity: $\dot{q}_6$ [rad/s] and $\dot{q}_{12}$ [rad/s]
First scenario: Straight walking

Stability analysis through ZMP

Without optimization

With optimization

Optimization ⇒ Improvement of the stability margins

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Second scenario: Change of walking direction

Characteristics:
- Joints' trajectories are periodic
- Discontinuities in joints' velocities
Second scenario: Change of walking direction

Stability analysis through ZMP

Without optimization

With optimization

[Graphs showing stability analysis with and without optimization]
Second scenario: Change of walking direction

Stability analysis through ZMP

Without optimization

With optimization

Instable dynamic walking

Stable dynamic walking
Second scenario: Change of walking direction

Stability analysis through ZMP

Without optimization

With optimization

Instable dynamic walking

Stable dynamic walking

Optimization

⇒ Dynamic walking stability is guaranteed
Video
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## Conclusion

**Motivation:**

Design and implementation of a pattern generator for dynamically stable walking

**Deals with:**

- Stability of dynamic walking
- 3D Movements
- Complex nonlinear dynamics
- Low CoM position (no torso)

**Proposed solution:**

- A pattern generator based on a 3 masses simplified model
- Stability margin improvement using optimization
- Change of walking direction is allowed
Future work can include...

- Real-time implementation of the proposed pattern generator on the biped robot SHERPA
- Development of a hybrid Position/Force controller to stabilize dynamic walking (in progress)
- Combine the hybrid Position/Force controller with the developed pattern generator
- Test the effectiveness of controller for walking on uneven ground
- Compare this approach to other pattern generators
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