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Optimal Pattern Generator based on a Three-Mass Linear Inverted Pendulum Model for dynamic walking

David GALDEANO, Ahmed CHEMORI and Sébastien KRUT

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February, 14th, 2011
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1. Context and motivation
2. SHERPA walking robot
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6. Conclusion and Future work
A stability indicator is a mathematical criterion that can characterize the stability margins of a walking robot from the current state of the robot.

**Walking mode**

- **Statically stable walk**
  - Indicator: CoM

- **Dynamically stable walk**
  - Indicators: ZMP, CoP, FRI...
Center of Mass (CoM)

Projection of the CoM relative to the support polygon [Nunez, 2008]

CoM is the mean location of all masses of the robot links

\[ OG = \sum m_i OG_i \]

Static stability criterion

Zero Moment Point (ZMP)

Projection of the ZMP relative to the support polygon [Nunez, 2008]

ZMP is the point where the vertical reaction force intersects the ground

\[ ZMP(t) = f(q(t), \dot{q}(t), \ddot{q}(t), f_e(t)) \]

Dynamic stability criterion
State of art on walking pattern generators

Pattern generators

- Simplified models
  - LIPM
    - [Kajita et al. 2001]
    - [Hong et al. 2009]
    - [Tang et al. 2007]
    - [Ferreira et al. 2009]
    - [Lee, 2007]
    - 2MLIPM
      - [Albert et al. 2003]
    - 3MLIPM
      - [Takenaka et al. 2009]
  - Biomechanics
    - [Bruneau et al. 1998]
    - Motion capture
      - [Harada, 2009]
      - [Kim et al. 2009]
      - [Takano et al. 2007]
  - Oscillators
    - Van der pol
      - [Katoh et al. 1984]
    - Sinusoids
      - [Zhao et al. 2008]
    - FFT
      - [Yamaguchi et al. 1999]
  - B-Splines
    - [Huang et al. 1999]
  - Others
    - Polynomial Functions
      - [Zaier et al. 2007]
    - Neural networks
      - [Yang et al. 2007]
## Motivation

### Objective

Design and implementation of a pattern generator for stable dynamic walking

### Assumptions

- The ground is flat and without obstacles
- The walking cycle is made of single support and impact phases
- The double support phase is not considered
- The solution uses a simplified model of the robot

### Method and application

- Model: 3 Masses Linear Inverted Pendulum Model (3MLIPM)
- Application: SHERPA biped robot

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February, 14th, 2011  
HLR 2011 WORKSHOP
Outline

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Prototype

SHERPA biped robot

- 7 parts: one waist linked to two legs together articulated with knees and ankles
- 18 degrees of freedom / 12 actuated articulations
- 12 modular transparent actuators (low inertia, low friction and backdrivable)
- Control PC with a real time kernel (RTX)
Prototype

Sensor

- 12 Absolute Shaft Encoders (HENGSTLER AD36) to measure articular positions
- 2 six-axis force sensors (ATI-Mini 85) to measure contact forces with ground
First movements of the robot

Two motion scenarios are proposed.

**Scenario 1**

A swing movement Up and down of the hanged leg

**Scenario 2**

A swing movement Forward-backward movement of the hanged leg

Reference trajectories generation: based on b-splines functions
Objective

Find a trajectory: \( T = x(t), \ t \in [0, t_f] \)

under a set of constraint:

\[
\begin{align*}
    x(0) &= x_0 \\
    \dot{x}(0) &= \dot{x}_0 \\
    x(t_f) &= x_f \\
    \dot{x}(t_f) &= \dot{x}_f \\
    x(t_i) &= x_i
\end{align*}
\]

Proposed solution

CSAPE algorithm from b-splin toolbox of Matlab software
Illustration example

Constraints:
\[
\begin{align*}
    &x(0) = 4 \\
    &x(1) = 8 \\
    &x(2) = 6 \\
    &\dot{x}(0) = 0 \\
    &\dot{x}(2) = 3
\end{align*}
\]

B-splines MATLAB function:
\[
\text{csape}( [0,1,2], [4,8,6],[1,1],[0,3])
\]

Obtained trajectories
First movements of the robot (Experiments)

Scenario 1

Scenario 2
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Simplified models

LIPM
[Kajita et al., 2009]

2MLIPM
[Albert and Gerth, 2003]

3MLIPM
[Feng and Sun, 2008]

Reduce the dynamic of the robot to the dynamic of a point mass.
The three masses linear inverted pendulum model

"3 Mass Linear Inverted Pendulum Model (3MLIPM)" [Feng and Sun, 2008]

Properties
- Simplified model of the robot
- Three point masses
- Three massless links

Hypothesis
- Walk on flat ground
- No double support phases
The three masses linear inverted pendulum model

"3 Mass Linear Inverted Pendulum Model (3MLIPM)" [Feng and Sun, 2008]

Properties
- Simplified model of the robot
- Three point masses
- Three massless links

Hypothesis
- Walk on flat ground
- No double support phases

Decoupled equations
Motion generated separately

{ Sagittal plane
Frontal plane }
Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

$x_i$: Cartesian position of mass $m_i$ in $x$ axis
$z_i$: Cartesian position of mass $m_i$ in $z$ axis
Motion in sagittal plane

\[ \sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i \]

- \( x_i \): Cartesian position of mass \( m_i \) in \( x \) axis
- \( z_i \): Cartesian position of mass \( m_i \) in \( z \) axis
Motivation and context

SHERPA walking robot

Basic 3MLIPM pattern generator

Limitations and improvements

Simulation results

Conclusion and Future work

The three masses linear inverted pendulum model

Motion in sagittal plane

Motion in frontal plane

Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

\[
\sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i
\]

\[
b \ddot{x}_1 + d \ddot{x}_3 = a x_1 + x_3
\]

\(x_i\) : Cartesian position of mass \(m_i\) in \(x\) axis

\(z_i\) : Cartesian position of mass \(m_i\) in \(z\) axis

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Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

\[ \sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i \]

\[ b \ddot{x}_1 + d \ddot{x}_3 = a x_1 + x_3 \]

Choose a trajectory for the swinging foot

\[ x_i : \text{Cartesian position of mass } m_i \text{ in } x \text{ axis} \]

\[ z_i : \text{Cartesian position of mass } m_i \text{ in } z \text{ axis} \]
Motion in sagittal plane

Dynamic of 3MLIPM in the sagittal plane

\[ \sum_{i=1}^{3} m_i \ddot{x}_i z_i = \sum_{i=1}^{3} m_i g x_i \]
\[ b \ddot{x}_1 + d \ddot{x}_3 = ax_1 + x_3 \]

Choose a trajectory for the swinging foot

Motion of the three masses

\( x_i \): Cartesian position of mass \( m_i \) in \( x \) axis

\( z_i \): Cartesian position of mass \( m_i \) in \( z \) axis
Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

\[ x_i : \text{Cartesian position of mass } m_i \text{ in } x \text{ axis} \]
\[ z_i : \text{Cartesian position of mass } m_i \text{ in } z \text{ axis} \]
Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

\[
\sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i
\]

\(x_i\): Cartesian position of mass \(m_i\) in \(x\) axis

\(z_i\): Cartesian position of mass \(m_i\) in \(z\) axis
Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

\[ \sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i \]

\[ u \ddot{y}_1 - v y_1 = w \]

\( x_i \): Cartesian position of mass \( m_i \) in x axis

\( z_i \): Cartesian position of mass \( m_i \) in z axis
Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

\[ \sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i \]

\[ u\ddot{y}_1 - vy_1 = w \]

3D trajectories of hip and ankles

\( x_i \): Cartesian position of mass \( m_i \) in \( x \) axis

\( z_i \): Cartesian position of mass \( m_i \) in \( z \) axis
Motion in frontal plane

The three masses linear inverted pendulum model

Motion in sagittal plane

Motion in frontal plane

Context and motivation
SHERPA walking robot
Basic 3MLIPM pattern generator
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The three masses linear inverted pendulum model

Motion in sagittal plane

Motion in frontal plane

The three masses linear inverted pendulum model

Motion in sagittal plane

Motion in frontal plane

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The three masses linear inverted pendulum model

Motion in sagittal plane

Motion in frontal plane

Dynamic of 3MLIPM in the frontal plane

\[
\sum_{i=1}^{3} m_i \ddot{y}_i z_i = \sum_{i=1}^{3} m_i g y_i
\]

\[
u \ddot{y}_1 - v y_1 = w
\]

3D trajectories of hip and ankles

Inverse kinematics \(\rightarrow\) Joints space trajectories

\(x_i\) : Cartesian position of mass \(m_i\) in \(x\) axis

\(z_i\) : Cartesian position of mass \(m_i\) in \(z\) axis
Outline

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First contribution

First limitation of the 3MLIPM model:
The dynamic stability is not guaranteed

Proposed improvement: Optimization with respect to ZMP
Principle: optimal value of mass $m_1$ and its position $z_1$:

$$
\begin{bmatrix}
\hat{z}_1 \\
\hat{m}_1 
\end{bmatrix} = \text{Arg Min}_{z_1} \text{Max}_{m_1} \left( \sqrt{\alpha(x_{zmp} - x_{dzmp})^2 + \beta(y_{zmp} - y_{dzmp})^2} \right)
$$

This optimization aims to find the best fit between the desired and the computed ZMP.
Second contribution

Second limitation of the 3MLIPM model:
Change of walking direction is not allowed

Proposed improvement: Modification of the hip trajectory
Principle: the hip trajectory is modified as follow:

$$\Omega(t) = -\frac{R}{2} \cos\left(\frac{\pi t}{T}\right) \quad t \in [0, T]$$

with $T$: half step period and $R$: amplitude of rotation. The modification of the hip trajectory allows a change of walking direction.
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Developed simulator

Proposed scenarios:

Scenario 1: Straight walking
Scenario 2: Change of walking direction

Comparison of the original pattern generator with the improved one
First scenario: Straight walking

Joints’ positions

- position: \( q_1 \) [rad] and \( q_7 \) [rad]
- position: \( q_2 \) [rad] and \( q_8 \) [rad]
- position: \( q_3 \) [rad] and \( q_9 \) [rad]
- position: \( q_4 \) [rad] and \( q_{10} \) [rad]
- position: \( q_5 \) [rad] and \( q_{11} \) [rad]
- position: \( q_6 \) [rad] and \( q_{12} \) [rad]

Joints’ velocities

- velocity: \( \dot{q}_1 \) [rad/s] and \( \dot{q}_7 \) [rad/s]
- velocity: \( \dot{q}_2 \) [rad/s] and \( \dot{q}_8 \) [rad/s]
- velocity: \( \dot{q}_3 \) [rad/s] and \( \dot{q}_9 \) [rad/s]
- velocity: \( \dot{q}_4 \) [rad/s] and \( \dot{q}_{10} \) [rad/s]
- velocity: \( \dot{q}_5 \) [rad/s] and \( \dot{q}_{11} \) [rad/s]
- velocity: \( \dot{q}_6 \) [rad/s] and \( \dot{q}_{12} \) [rad/s]

Characteristics:
- Joints’ trajectories are periodic
- Discontinuities in joints’ velocities

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First scenario: Straight walking

Stability analysis through ZMP

Without optimization

With optimization

Optimization \Rightarrow Improvement of the stability margins

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Second scenario: Change of walking direction

Joints’ positions

- $q_1$ [rad] and $q_7$ [rad]
- $q_2$ [rad] and $q_8$ [rad]
- $q_3$ [rad] and $q_9$ [rad]
- $q_4$ [rad] and $q_{10}$ [rad]
- $q_5$ [rad] and $q_{11}$ [rad]
- $q_6$ [rad] and $q_{12}$ [rad]

Joints’ velocities

- $\dot{q}_1$ [rad/s] and $\dot{q}_7$ [rad/s]
- $\dot{q}_2$ [rad/s] and $\dot{q}_8$ [rad/s]
- $\dot{q}_3$ [rad/s] and $\dot{q}_9$ [rad/s]
- $\dot{q}_4$ [rad/s] and $\dot{q}_{10}$ [rad/s]
- $\dot{q}_5$ [rad/s] and $\dot{q}_{11}$ [rad/s]
- $\dot{q}_6$ [rad/s] and $\dot{q}_{12}$ [rad/s]

Change of walking direction

Characteristics:

- Joints’ trajectories are periodic
- Discontinuities in joints’ velocities
Second scenario: Change of walking direction

Stability analysis through ZMP

Without optimization

With optimization
Second scenario: Change of walking direction

Stability analysis through ZMP

Without optimization

With optimization

Instable dynamic walking

Stable dynamic walking
Second scenario: Change of walking direction

Stability analysis through ZMP

Without optimization

With optimization

Instable dynamic walking

Stable dynamic walking

Optimization ⇒ Dynamic walking stability is guaranteed
Context and motivation
SHERPA walking robot
Basic 3MLIPM pattern generator
Limitations and improvements

Simulation results
Conclusion and Future work

Developed simulator
First scenario: Straight walking
Second scenario: Change of walking direction

Video
Outline

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Conclusion

Motivation:
Design and implementation of a pattern generator for dynamically stable walking

Deals with:
- Stability of dynamic walking
- Complex nonlinear dynamics
- 3D Movements
- Low CoM position (no torso)

Proposed solution:
- A pattern generator based on a 3 masses simplified model
- Stability margin improvement using optimization
- Change of walking direction is allowed
Future work can include...

- Real-time implementation of the proposed pattern generator on the biped robot SHERPA
- Development of a hybrid Position/Force controller to stabilize dynamic walking (in progress)
- Combine the hybrid Position/Force controller with the developed pattern generator
- Test the effectiveness of controller for walking on uneven ground
- Compare this approach to other pattern generators


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