A Nonlinear PID Stabilizer With Spherical Projection for Humanoids: From Concept to Real-time Experiments

David Galdeano¹, Ahmed Chemori¹, Sébastien Krut¹ and Philippe Fraisse¹

Abstract—This paper deals with a stabilizer for a hybrid kinematic/dynamic control scheme for humanoid robots. The proposed solution is based on a nonlinear PID regulation of the ZMP, coupled with a spherical projection in the CoM's control space. The result of such a stabilizer is a dynamically stable motion, even with large variations in the inclination of the ground. The effectiveness of this new stabilizer has been demonstrated through real-time experiments on the humanoid robot HOAP-3. The produced motion is smooth and dynamically stable.

I. INTRODUCTION

The goal of developing humanoid robots is to make them working in dull, dirty and dangerous environments where human workers could be in danger. However, up to now, most of humanoid robots are still confined to research laboratories, since they are not reliable enough in real operating conditions. One of the main features that should be improved to produce reliable humanoid robots is the stability of the motions. Their fall can be dangerous for humans standing in a close proximity of the robots. The robot ability to move in its environment, even if the environment is degraded, is needed to bring robots to consumers application.

Even with an optimal joint trajectory design, the stability of the motion could not be guaranteed if a perturbation occurs. In order to produce stable motions, the robot movement should be adapted online to the external environment using the sensors feedback information. This reactive adaptation allows the robot to evolve in a real world environment. The stability indicators are mathematical tools to quantify the degree of stability stability. Mainly two indicators of stability can be used: the Center of Mass (CoM) and the Zero Moment Point (ZMP).

The COM is the mean location of all masses of the robot’s links. This indicator is often used for static stability [1]. The ZMP [2] is the point of junction between the resultant vertical reaction force and the ground. It is the most used indicator for dynamic stability [3], [4]. In the literature, various methods have been proposed to improve the stability of the robot motions based on the ZMP stability indicator.

One of the most used methods is called the Inverted Pendulum Model (IPM) [5], [6] that considers the robot as a single pointwise mass linked through a telescopic leg to a spherical ground/leg joint. Since then, different extensions have been proposed such as the Linear Inverted Pendulum Model (LIPM) [7], [8]. The preview control of the ZMP [9] has improved the LIPM by reducing the error induced by the simplification of the dynamics of the humanoid robot. The previous three methods are very efficient to perform walking motions, but the model equations are based on the assumption that the ground is flat, therefore the range of admissible motions is limited. The ZMP preview control can be extended by using a model predictive control (MPC) to be more robust against unexpected external disturbances [10].

A control law can be design to allows a compliant interaction with external forces. This technique have been implemented on several robots to allows the generation of stable motions by absorbing the external perturbations [11], [12]. However, this technique requires a torque level control [13] to be efficient which is not possible on all humanoids robots.

Some other proposed techniques are based on a sensory feedback system [14], [15]. A stabilizer can be used to control the torque of the ankle joint [16], [17], [18], the angular momentum of the hip [19], [20], the trajectory of the CoM [21], [22] or the decision to take a step [23], [24] in order to ensure the motion stability. The above approaches are very reactive and can adapts to large disturbances. However, they are difficult to be combined with frameworks using the rest of the body of the robot in other useful tasks.

In this paper, a nonlinear ZMP based stabilizer is proposed. This stabilizer is sensory feedback based and has been developed within the framework of an hybrid dynamic/kinematic whole-body control strategy. The whole control architecture with a simpler stabilizer has been proposed in a previous works of the authors [25]. The basic idea of this stabilizer is to use a nonlinear PID controller to regulate the ZMP error and to project this regulation in a CoM tracking objective using a spherical projection. The nonlinear PID controller allows a fast tracking with a favorable damping, improving the stability of the controlled robot. The spherical projection of the ZMP error compensation allows to adapt the robot posture to large variations in the inclination of the ground.

The proposed stabilizer allows to increase the rejection of external disturbances applied to the humanoid robot.

In the literature, most of the proposed stabilizers relies on precomputed trajectories for the desired ZMP trajectory. With the proposed stabilizer, the regulation of the center of pressure (CoP) is used to produce dynamically stable
motions. The stabilizer has been implemented on HOAP-3, a small robot with position control, noisy sensors data and low computational power. The control scheme can be easily deployed on other robots if they are equipped with joint encoders and CoP sensors on both feet.

This paper is organized as follows: in next section, our humanoid robot demonstrator HOAP-3 is introduced. Section II is devoted to the proposed nonlinear ZPD based stabilizer. The ZMP regulation is mapped in the CoM’s control space using a spherical projection. In this section, its basic principle is presented and discussed. Real-time experimental results are introduced in section III, with a presentation and discussion of the obtained results. The paper ends with some concluding remarks and an overview on future work.

II. MAIN CONTRIBUTION: A ZMP ERROR PROJECTION BASED NONLINEAR STABILIZER

A. General overview of the control scheme

In a previous work of the authors, a hybrid kinematic/dynamic whole-body control framework has been proposed [25]. This framework has been experimentally validated for squat-like motions. The framework consist in a kinematic control including four objectives, namely (i) the robot’s feet relative-pose, (ii) the CoM, (iii) the body orientation and (iv) joints’ limits avoidance. The kinematic control is based on the task formalism [26], [27].

A ZMP based dynamic feedback has been considered to produce a dynamically stable motions. This stabilizer uses the ZMP error to modify the CoM trajectory. In this work, we aim at improving this stabilizer through two main contributions:

- A nonlinear feedback control of the ZMP error to improve the rapidity and stability of the stabilizer,
- a sphere projection of this regulation to produce human-like motion even with large variations in the inclination of the ground.

B. Nonlinear ZMP regulation controller

Nonlinear PD (NPD) controllers have been proposed in robotics field as an improvement of the classic linear PD controllers [28], [29], [30], [31]. The nonlinear ZMP regulation control is used to produce a reactive control with a better damping [31]. This allows a faster response without introducing instabilities.

Contrary to classical linear PD controller, the NPD controller uses time-varying gains depending on the tracking errors instead of fixed gains. The general expression of the NPD can be expressed as:

\[ u(t) = k_p(\cdot)e(t) + k_d(\cdot)e(t) \]  

(1)

where \( k_p(\cdot) \) and \( k_d(\cdot) \) are the time-varying proportional and derivative gains, \( e(t) \) is the system error and \( e(t) \) its first time derivative (velocity error).

The time-varying proportional and derivative gains may depend on the system state, its inputs or other variables. In our case, we consider a proportional gain depending on the position error and a derivative gain depending on the velocity error.

The nonlinear proportional gain, illustrated in Fig. 1, is expressed by:

\[ k_p(e) = \begin{cases} k_{p_0}|e|^{\alpha_1-1}, & |e| > \delta_1, \\ k_{p_0}\delta_1^{\alpha_1-1}, & |e| \leq \delta_1. \end{cases} \]  

(2)

where \( k_{p_0} \) is the maximum proportional gain, \( \alpha_1 \) is the non-linearity tuning parameter and \( \delta_1 \) its threshold of activation.

The nonlinear derivative gain, also illustrated in Fig. 1, is given by:

\[ k_d(\dot{e}) = \begin{cases} k_{d_0}|\dot{e}|^{\alpha_2-1}, & |\dot{e}| > \delta_2, \\ k_{d_0}\delta_2^{\alpha_2-1}, & |\dot{e}| \leq \delta_2. \end{cases} \]  

(3)

where \( k_{d_0} \) is the maximum derivative gain, \( \alpha_2 \) is the non-linearity tuning parameter and \( \delta_2 \) its threshold of activation.

![Fig. 1: Typical evolution of the nonlinear proportional \( k_p \) and derivative \( k_d \) gain versus position error \( e \) and velocity error \( \dot{e} \) with \( \alpha_1 = 0.75 \), \( \delta_1 = 1 \), \( \alpha_2 = 1.25 \) and \( \delta_2 = 1 \).](image)

The parameter tuning for \( \alpha_1, \delta_1, \alpha_2, \delta_2 \) is explain in [31]. We consider \( \alpha_1 \in [0.5, 1.0] \) and \( \alpha_2 \in [1.0, 1.5] \) to produce a fast trajectory tracking with improved disturbance rejections.

A NPD controller is not designed to compensate static errors since it does not contain an integral action. Consequently an integral action should be considered to compensate the ZMP static errors. We propose to define the ZMP error compensation as an offset to modify the CoM tracking in order to improve the robot’s dynamic stability. The ZMP error compensation can then be defined as:

\[ o_{ZMP} = k_p(\varepsilon_{ZMP})\varepsilon_{ZMP} + k_d\left(\frac{d\varepsilon_{ZMP}}{dt}\right)\frac{d\varepsilon_{ZMP}}{dt} + k_i\int\varepsilon_{ZMP} \]  

(4)

where \( o_{ZMP} \in \mathbb{R}^{2\times1} \) is the ZMP error compensation, \( k_i \) is the integral gain and \( \varepsilon_{ZMP} \in \mathbb{R}^{2\times1} \) the ZMP tracking error.

The ZMP tracking error is computed from the center of pressure (CoP) measurement as presented in [25]. One of the main advantage of this approach is that the ZMP tracking error, \( \varepsilon_{ZMP} \) is a weighted distribution of ZMP errors on both feet. The ZMP error compensation is not computed using a
ZMP reference trajectory in global coordinates but rather a local coordinates CoP reference trajectory since the desired CoP is at the center of each feet.

C. Spherical projection of the ZMP regulation

In the control scheme proposed in [25], the ZMP error compensation was directly added to the CoM objective as a planar projection. Since the CoM trajectory was modified in the transverse plane, this concept was limited to small ZMP errors and flat ground. However, for the case of large height as illustrated on Fig. 3.

The spherical projection equation of the ZMP error compensation $\varepsilon_{SP} = [\varepsilon_{SPX} \varepsilon_{SPY} \varepsilon_{SPZ}]^T$ from eq. (4), can be expressed as:

$$
\varepsilon_{SPX} = h_{CoM} \sin \left( \frac{\beta \cdot o_{ZMPX}}{h_{CoM}} \right), \\
\varepsilon_{SPY} = h_{CoM} \sin \left( \frac{\gamma \cdot o_{ZMPY}}{h_{CoM}} \right), \\
\varepsilon_{SPZ} = \sqrt{h_{CoM}^2 - o_{ZMPX}^2 - o_{ZMPY}^2},
$$

where $h_{CoM}$ denotes the initial CoM height, $\beta$ and $\gamma$ are spherical projection adjustable parameters that allows to tune the shape of the spherical projection to stay inside the kinematic workspace of the legs.

The projection of the ZMP error compensation is then added as an offset to the CoM tracking as follows:

$$
\varepsilon_{CZ} = \varepsilon_{CoM} + \varepsilon_{SP}
$$

where $\varepsilon_{CZ} \in \mathbb{R}^{2 \times 1}$ is the tracked stability objective containing the CoM tracking and the ZMP regulation; and $\varepsilon_{CoM} \in \mathbb{R}^{2 \times 1}$ is the CoM tracking error [25].

The Fig. 4 illustrate the CoM trajectory in the frontal plane if a disturbance is applied on the ZMP on $y$ axis. This projection space has been verified to be inside the kinematic workspace of the legs if $\beta = 1$ and $\gamma = 1$.

The spherical projection with $u_{ZMPX} = 0$ and $u_{ZMPX}$ varying in $[-h_{CoM} \pi : h_{CoM} \pi ]$ domain. Robot’s structure and footprints are displayed in gray. Spherical projection tuning parameter are set to $\beta = 1$, $\gamma = 1$ for the solid line and $\beta = 0.5$, $\gamma = 0.75$ for the dashed line.

In order to produce an human-like reaction to slope variation of the ground and to avoid auto-collision of the legs of the robot, the torso orientation must be managed. Since the upper-body orientation is not applied on the CoM tracking but on the torso orientation tracking, this does not modify the stability of the robot. The desired torso orientation becomes:

$$
Torso_{ori}(x) = \text{atan2}(o_{ZMPY}, h_{CoM}), \\
Torso_{ori}(y) = \text{atan2}(o_{ZMPX}, h_{CoM}).
$$

Fig. 2: Illustration of the real-time adaptation against ground’s inclination with two different projections.

(a) Planar projection.  
(b) Spherical projection.

Fig. 3: Illustration of the sphere projection space w.r.t. the norm of $o_{ZMP}$ ($\beta = 1$ and $\gamma = 1$). Footprints are displayed in gray.

Fig. 4: Frontal view of the sphere projection with $u_{ZMPX} = 0$ and $u_{ZMPX}$ varying in $[-h_{CoM} \pi : h_{CoM} \pi ]$ domain.
where $\text{Torso}_d(x)$ and $\text{Torso}_d(y)$ are the desired torso orientations. The desired torso orientation is managed in a different regulation objective than the stability objective.

This stabilizer improves the stability locally by modifying the reference position of the CoM. The quick reaction of the ZMP-based stabilizer to CoP errors allows to converge to stable motions if quick and small perturbations occurs. Large and slow disturbances are taken into account using the integral part and the spherical projection to allows a stable posture on inclined ground. It does not guaranties a global stability, which needs the decision of taking a step to keep stability in case of quick and large disturbances.

III. REAL-TIME EXPERIMENTAL VALIDATION

The proposed stabilizer presented in section II has been implemented within the control scheme proposed in [25] on the HOAP-3 humanoid robot presented in the sequel. Two experimental scenarios have been performed to show the efficiency of the proposed stabilizer.

In the first scenario, the slope of the ground has been varied to demonstrate the adaptation of the proposed stabilizer to an inclined ground. The second scenario demonstrates a task of dynamic walking on uneven ground with an unexpected variation in the slope of the ground. In the sequel, the experimental setup and the two scenarios will be detailed.

IV. DESCRIPTION OF THE EXPERIMENTAL SETUP

The proposed control scheme within this work has been be implemented on HOAP-3 humanoid robot, manufactured by Fujitsu Automation. The experimental setup for the following validations consist in a HOAP-3 robot communicating with a host PC as illustrated in Fig. 5.

The robot weights 8.8 kg and is 60 cm tall. Its mechanical structure is composed of 28 degrees of freedom (dof) distributed as follows: 6 dof per leg, one in the waist, 5 per arm, 3 for the neck and one per hand.

The robot is equipped with an incremental encoder per joint and four force sensors per feet which have been used for this work. Beside, it is equipped with a three-axis acceleration sensor, a three-axis gyro sensor and two CDD cameras which have not been used.

A control PC with RT-Linux real-time operating system allows the control of the robot with a sampling time of 1 msec. This robot is a versatile testbed for whole-body motion control. Our stabilizer algorithm is implemented in a C language code for real-time experiments.

A. Scenario 1: Online adaptation toward slope variation of the ground

The objective of this scenario is to demonstrate the adaptation of the proposed stabilizer against slope variation of the ground. The experimental test platform consist of a wooden board (as shown in Fig. 6) which is lifted from one side, therefore creating a rotation around the opposite edge of the board. The spherical projection tuning parameter are $\beta = 1$ and $\gamma = 1$ for this scenario.

The robot adaptation to the ground’s inclination variation can be observed on the illustration of Fig. 6.

![Fig. 5: The experimental setup for the real-time experiments consist in the HOAP-3 humanoid robot linked to a host PC.](image)

![Fig. 6: Illustration of the real-time adaptation against ground’s inclination (scenario 1).](image)

![Fig. 7: Trajectories evolution of the CoM versus time for the first scenario.](image)
no perturbation on this axis has been introduced. Along the $y$ axis, the trajectory of the CoM should be regulated around its desired value (constant). Indeed, the observed variation is due to the adaptation of the stabilizer to the ZMP displacement. The CoM trajectory is modified to keep balance. Along the $z$ axis, the trajectory is also kept constant at the height of the initial CoM position.

Fig. 8: Evolution of the CoM and ZMP trajectories with respect to footprints (scenario 1).

In Fig. 8, the evolution of the measured ZMP and CoM positions are plotted with respect to the footprints of the robot. The trajectory of the desired CoM is constant. The observed variation on the CoM position is due to the adaptation of the stabilizer to the ZMP displacement to keep balance.

The robot’s body adapts to ground’s inclination variation. The combination of the CoM position adjustment and the hip rotation allow a smooth motion. The posture of the robot during large ZMP disturbances on Fig. 6(b) is looking natural, the torso is not inclined like the ground.

B. Scenario 2: Walking on an uneven ground

In Fig. 9, the evolution of the measured ZMP and CoM positions are plotted with respect to the footprints of the humanoid robot. The obtained walking motion is stable and robust. The robot produces the desired walking pattern. It is worth to note that the robot falls down for the same scenario if the stabilizer was not active.

V. CONCLUSION AND FUTURE WORK

This paper deals with a new efficient stabilizer for humanoid robots. The proposed solution is based on a nonlinear PID based regulation, coupled with a spherical projection of the ZMP regulation error in the CoM jacobian control.

The main advantage of this stabilizer lies in the enhancement of the stability of the robot under the control of a previously proposed control framework for whole-body motions control [25]. The obtained results are very promising.
In future work, we aim at extending this work for more complex scenarios including carrying objects or in human-robot interactions.

REFERENCES


