Error-free Tables for Trigonometric Function Evaluation
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1. How does a modern processor calculate sine and cosine?

- **Range reduction**: use of trigonometric identities
  - \( \sin(x - \pi) = -\sin(x) \)
  - \( \sin(x + \pi) = \sin(x) \)
  - \( \sin(x) = \sin(x - \pi) \) with \( f_k \in \{\sin, \cos\} \)
  - Range reduction \( f_k \rightarrow f_k \)
  - **Evaluation**: use of other trigonometric properties
    - \( \sin(x + y) = \sin(x) \cos(y) + \cos(x) \sin(y) \)
    - \( \cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y) \)
  - **Tabulated values for sine and cosine** [Tan91]

2. Pythagorean Triples

- **What is a Pythagorean triple?**
  - \( (a, b, c) \in \mathbb{N} \)
  - \( a^2 + b^2 = c^2 \)
  - \( \sin(\theta) = \frac{a}{c} \quad \cos(\theta) = \frac{b}{c} \)

- **Primitive Pythagorean Triple**: a Pythagorean triple \((a, b, c)\) for which \(\gcd(a, b, c) = 1\).

3. Primitive Pythagorean Triple Generation

- **Barning-Hall ternary-tree structure**:
  - \( \begin{array}{c}
  & 1 - 2 2
  \hline
  2 - 1 2 \quad 2 - 2 3
  \end{array} \)
  - \( 5, 12, 13, 6, 20, 21, 4, 5 \)
  - Several equivalent trees, easy to implement
  - Proven to generate all primitive triples by increasing hypotenuse lengths [Bar63]

4. Primitive Pythagorean Triple Selection

- Only one triple per entry needed
- Division by \( c \) removed by incorporating \( c \) in polynomial approximations
- Same hypotenuse \( c \) needed for all entries
- Scale selected PPTs to the least common multiple of their hypotenuses

5. Exhaustive Search for a Small Common Hypotenuse

- **Algorithm**
  1. \( n \leftarrow 4 \)
  2. Repeat
  3. Generate all PPTs \((a, b, c)\) such that \( c \leq n^2 \)
  4. Search for the LCM \( c \) among all generated hypotenuses \( c \)
  5. \( c \leftarrow c + 1 \)
  6. Until such \( a \) is found
  7. Build tabulated values \((A, B, c, \cos)\) for every entry.

- **Results**

6. Heuristic Search

- **Prime factorization of found common multiples**
  - \( c = \prod p_i^{e_i} \) with
  - \( e_i \in \mathbb{N} \)
  - \( p_i \in \mathbb{P} \)

- **Heuristic**: store primitive pythagorean triples satisfying \( c = \prod p_i^{e_i} \) with
  - \( e_i \leq 1 \) if \( p_i \neq 5 \)
  - \( e_i \leq 2 \) if \( p_i = 5 \)
  - Where \( \mathbb{P} \) is the set of Pythagorean primes \( \leq 71 \)

- **Results**

7. Theoretical Gains

- Comparison between three table-based range reductions, for \( p = 10 \).
  - The number of memory accesses \((MA)\) and the number of floating point operations \((FLOP)\) are reported.

8. References


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