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Error-free Tables for Trigonometric Function Evaluation

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1. How does a modern processor calculate sine and cosine?

- **Range reduction:** use of trigonometric identities
  - \[ \sin(x - k \pi) = \sin(x) \]
  - \[ \sin(x) = \pm \sin(x \pm k \pi) \]
  \[ f_k \in \{\sin, \cos\} \]

- **Evaluation:** use of other trigonometric properties
  - \[ \sin(x) = \sin(x - \pi) \]
  - \[ \cos(x) = \cos(x - \pi) \]
  - \[ \sin(x) = \sin(x - 2\pi) \]

- **Tabulated values** for sine and cosine [Tan91]

- **Reconstruction:** scheme reducing the error on tabulated values [GB91]

2. Pythagorean Triples

- **What is a Pythagorean triple?**

- \[ (a, b, c) \in \mathbb{N}^3 \]
  - \[ a^2 + b^2 = c^2 \]
  - \[ \sin(\theta) = \frac{a}{c}, \cos(\theta) = \frac{b}{c} \]

- **Primitive Pythagorean Triplet:** a Pythagorean triplet \((a, b, c)\) for which \(\gcd(a, b, c) = 1\).

- **Barning-Hall ternary-tree structure:**
  
  \[
  \begin{array}{ccccccc}
  & 1 & -2 & 2 & -1 & 2 & 2 \\
  & 2 & -1 & 2 & 1 & 2 & 2 \\
  & -2 & 2 & -2 & 2 & 2 & 2 \\
  \end{array}
  \]

- **Several equivalent trees, easy to implement**

- **Proven to generate all primitive triples by increasing hypotenuse lengths** [Bar63]

3. Primitive Pythagorean Triple Generation

- **Algorithm**
  
  1. \(n \leftarrow 4\)
  2. repeat
  3. Generate all PPTs \((a, b, c)\) such that \(c \leq 2n\)
  4. \(\text{Search for the LCM } k \text{ among all generated hypotenuses } c\)
  5. \(n \leftarrow n + 1\)
  6. until such a \(k\) is found
  7. Build tabulated values \((A, B, c, corr)\) for every entry.

- **Results**

4. Primitive Pythagorean Triple Selection

- **Only one triple per entry needed**

- **Division by \(c\) removed by incorporating it in polynomial approximations**

- **Same hypotenuse:** \(c\) needed for all entries

- **Scale** selected PPTs to the least common multiple of their hypotenuses

5. Exhaustive Search for a Small Common Hypotenuse

- **Algorithm**
  
  1. \(n \leftarrow 4\)
  2. repeat
  3. Generate all PPTs \((a, b, c)\) such that \(c \leq 2n\)
  4. \(\text{Search for the LCM } k \text{ among all generated hypotenuses } c\)
  5. \(n \leftarrow n + 1\)
  6. until such a \(k\) is found
  7. Build tabulated values \((A, B, c, corr)\) for every entry.

- **Results**

6. Heuristic Search

- **Prime factorization of found common multiples**

- **Prime factorization**

- **Black-box**

- **Heuristic store primitive pythagorean triples**

- **Satisfying** \(c = \prod P_i\)

- **with**

- \[ r_i \in \{0, 1\} \]

- **P** is the set of Pythagorean primes \(\leq 72\)

- **Results**

7. Theoretical Gains

- **Comparison between three table-based range reductions, for \(p = 10\). The number of memory accesses (MA) and the number of floating point operations (FLOP) are reported.**

- **References**


  [Tan91] Ping Tak Peter Tang. Table-lookup algorithms for elementary functions and their error analysis.


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