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Synthesis of fixed-point programs based on instruction selection

... the case of polynomial evaluation

Amine Najahi

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Joint work with Christophe Moulleron

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LIRMM, CNRS: UMR 5506 - Univ. Montpellier 2



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Université Perpignan Via Domitia



Laboratoire
d'Informatique
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et de Microélectronique
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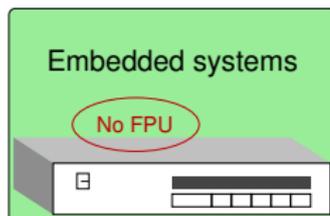


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- **Embedded systems** are ubiquitous
 - ▶ microprocessors and/or DSPs dedicated to one or a few specific tasks
 - ▶ satisfy constraints: area, energy consumption, conception cost

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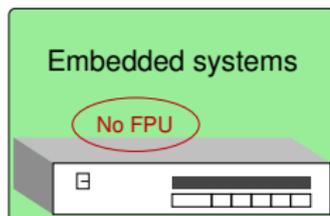
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- Highly used in audio and video applications
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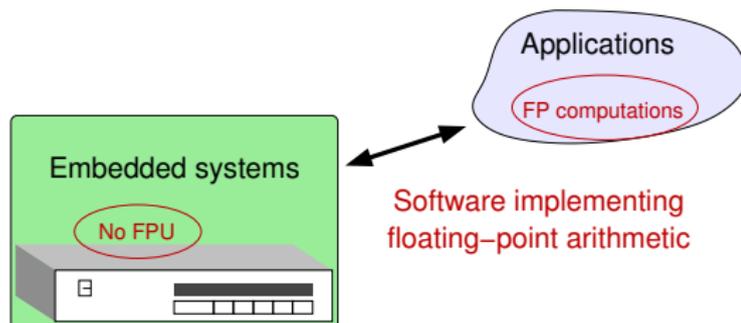
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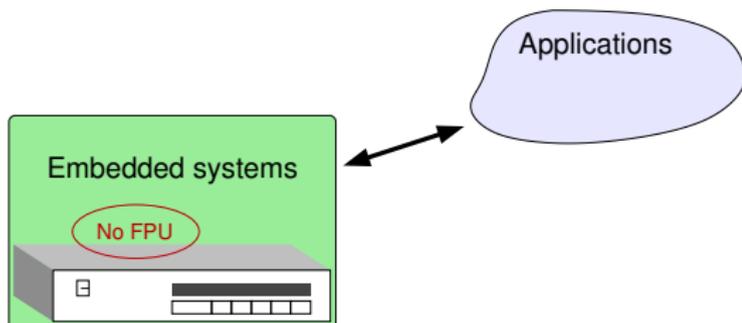
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How to use floating-point programs on embedded systems?

- Two approaches to continue using numerical algorithms on these cores:
 1. convert the entire numerical application from floating to fixed-point arithmetic
 2. write a floating-point emulation library and link the numerical application against it

Fixed-point conversion

- ✓ produces a fast code
- ✓ consumes less energy
- ✗ machine specific: no standard
- ✗ smaller dynamic range than floating-point
- ✗ tedious and time consuming

Floating-point support design

- ✓ tons of code are written using floating-point
- ✓ an algorithm can be synthesized on a PC and then transferred to the device without modifications
- ✗ slower
- ✗ tedious and time consuming

⇒ There is a need for the automation of both processes.

Fixed-point conversion vs. floating-point emulation design

■ Floating to fixed-point conversion tools:

- ▶ addressed by the ANR project DEFIS, with IRISA, LIP6, CEA, THALES, INPIXAL
- ▶ some tools are currently developed: ID.Fix, ...
- ▶ two main approaches:
 1. statistical methods: perform well, but provide no guarantees and may be slow.
 2. analytical methods: usually quite pessimistic, but they are safer to use.

■ Floating-point emulation support:

- ▶ a number of high quality emulation libraries exist: FLIP, SoftFloat, ...
- ▶ more or less compliant with the IEEE-754 standard
- ▶ FLIP: relies on polynomial evaluation to evaluate division and square root
 - a huge number of schemes for evaluating a given polynomial \rightsquigarrow development of CGPE
 - \approx 50 % of FLIP's code was generated by CGPE.

Outline of the talk

1. The CGPE tool
2. Instruction selection: an extension of CGPE
3. Conclusion and perspectives

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Overview of CGPE

- **Goal of CGPE:** automate the design of fast and certified C codes for evaluating univariate/bivariate polynomials
 - ▶ in fixed-point arithmetic
 - ▶ by using the target architecture features (as much as possible)

- **Remarks:**
 - ▶ **fast** \rightsquigarrow that reduce the evaluation latency on a given target
 - ▶ **certified** \rightsquigarrow for which we can bound the error entailed by the evaluation within the given target's arithmetic

Global architecture of CGPE

■ Input of CGPE

1. polynomial coefficients and variables: value intervals, fixed-point format, ...
2. set of criteria: maximum error bound and bound on latency (or the lowest)
3. some architectural constraints: operator cost, parallelism, ...

```
<polynomial>
<coefficient x="0" y="0" inf="0x000000020" sup="0x000000020" sign="0" integer_part="2" fraction_part="30"/>
<coefficient x="0" y="1" inf="0x800000000" sup="0x800000000" sign="0" integer_part="1" fraction_part="31"/>
<coefficient x="1" y="1" inf="0x400000000" sup="0x400000000" sign="0" integer_part="1" fraction_part="31"/>
<coefficient x="2" y="1" inf="0x100000000" sup="0x100000000" sign="1" integer_part="1" fraction_part="31"/>
<coefficient x="3" y="1" inf="0x07fe93e4" sup="0x07fe93e4" sign="0" integer_part="1" fraction_part="31"/>
<coefficient x="4" y="1" inf="0x04eef694" sup="0x04eef694" sign="1" integer_part="1" fraction_part="31"/>
<coefficient x="5" y="1" inf="0x032d6643" sup="0x032d6643" sign="0" integer_part="1" fraction_part="31"/>
<coefficient x="6" y="1" inf="0x01c6cebd" sup="0x01c6cebd" sign="1" integer_part="1" fraction_part="31"/>
<coefficient x="7" y="1" inf="0x00aeb7d" sup="0x00aeb7d" sign="0" integer_part="1" fraction_part="31"/>
<coefficient x="8" y="1" inf="0x00200000" sup="0x00200000" sign="1" integer_part="1" fraction_part="31"/>
<variable x="1" y="0" inf="0x00000000" sup="0xffffe00" sign="0" integer_part="0" fraction_part="32"/>
<variable x="0" y="1" inf="0x800000000" sup="0xb504f334" sign="0" integer_part="1" fraction_part="31"/>
<absolute_evalerror value="250813734883158693012463053528118040380976733198921b-191" strict="false"/>
</polynomial>
```

```
cgpe --degree="[8,1]" --xml-input=cgpe-test1.xml --coefs="[10000000011111111]" \
--latency=lowest --gappa-certificate --output \
--schedule="[4,2]" --max-kept=5 --operators="[1111111111111111:13333333111333331]" ...
```

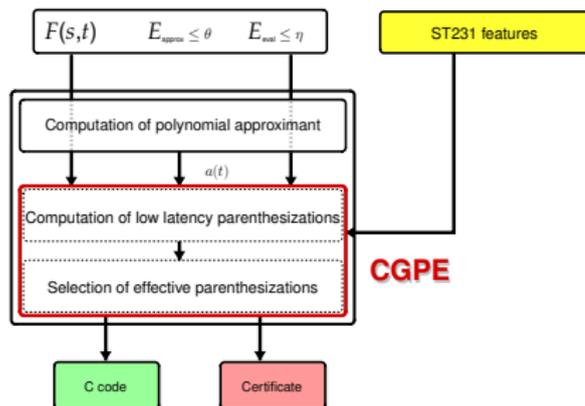
Global architecture of CGPE (cont'd)

Internals of CGPE

CGPE proceeds in two steps:

1. Computation step:

- ▶ computes evaluation schemes while reducing their latency on unbounded parallelism
- ▶ considers only two possible arithmetic operations: addition and multiplication
- ▶ produces DAGs that represent the computed efficient schemes



2. Filtering step:

- ▶ prunes the evaluation schemes that do not satisfy different criteria: latency (\rightsquigarrow scheduling filter), accuracy (\rightsquigarrow numerical filter), ...

Global architecture of CGPE (cont'd)

■ Output of CGPE

```
uint32_t func_d9_0(uint32_t T, uint32_t S)
{
  uint32_t r0 = T >> 2;           // (+) Q[1.31]
  uint32_t r1 = 0x80000000 + r0;  // (+) Q[1.31]
  uint32_t r2 = mul(S, r1);       // (+) Q[2.30]
  uint32_t r3 = 0x00000020 + r2;  // (+) Q[2.30]
  uint32_t r4 = mul(T, r1);       // (+) Q[0.32]
  uint32_t r5 = mul(S, r4);       // (+) Q[1.31]
  uint32_t r6 = mul(T, 0x07fe93e4); // (+) Q[1.31]
  uint32_t r7 = 0x10000000 - r6;   // (-) Q[1.31]
  uint32_t r8 = mul(r5, r7);      // (-) Q[2.30]
  uint32_t r9 = r3 - r8;          // (+) Q[2.30]
  uint32_t r10 = mul(r4, r4);     // (+) Q[0.32]
  uint32_t r11 = mul(S, r10);     // (+) Q[1.31]
  uint32_t r12 = mul(T, 0x032d6643); // (+) Q[1.31]
  uint32_t r13 = 0x04eef694 - r12; // (-) Q[1.31]
  uint32_t r14 = mul(T, 0x00aeb7d); // (+) Q[1.31]
  uint32_t r15 = 0x01c6cebd - r14; // (-) Q[1.31]
  uint32_t r16 = r4 >> 11;       // (-) Q[1.31]
  uint32_t r17 = r15 + r16;      // (-) Q[1.31]
  uint32_t r18 = mul(r4, r17);    // (-) Q[1.31]
  uint32_t r19 = r13 + r18;      // (-) Q[1.31]
  uint32_t r20 = mul(r11, r19);   // (-) Q[2.30]
  uint32_t r21 = r9 - r20;       // (+) Q[2.30]
  return r21;
}
```

Listing 1: C code

```
## Coefficients and variables definition
a0 = fixed<-30,dn>(0x00000020p-30);
a1 = fixed<-31,dn>(0x80000000p-31);
a2 = fixed<-31,dn>(0x40000000p-31);
...
a8 = fixed<-31,dn>(0x00aeb7dp-31);
a9 = fixed<-31,dn>(0x00200000p-31);

T = fixed<-32,dn>(fixed<-23,dn>(var0));
S = fixed<-31,dn>(var1);

CertifiedBound =
25081373483158693012463053528118040380976733198921b-191;

## Evaluation scheme
r0 fixed<-31,dn>= T * a2;      Mr0 = T * a2;
r1 fixed<-31,dn>= a1 + r0;     Mr1 = a1 + Mr0;
...
r21 fixed<-30,dn>= r9 - r20;   Mr21 = Mr9 - Mr20;

## Results
{
  (
    var0 in [0x00000000p-32,0xfffffe00p-32]
    /\ var1 in [0x80000000p-31,0xb504f334p-31]
    ->
    /\ r0 in [0,0xffffffffp-31]
    /\ r0 - Mr0 in ?
    ...
    /\ r21 in [0,0xffffffffp-30]
    /\ |r21 - Mr21| - CertifiedBound <= 0
    /\ CertifiedBound in ?
  )
}
```

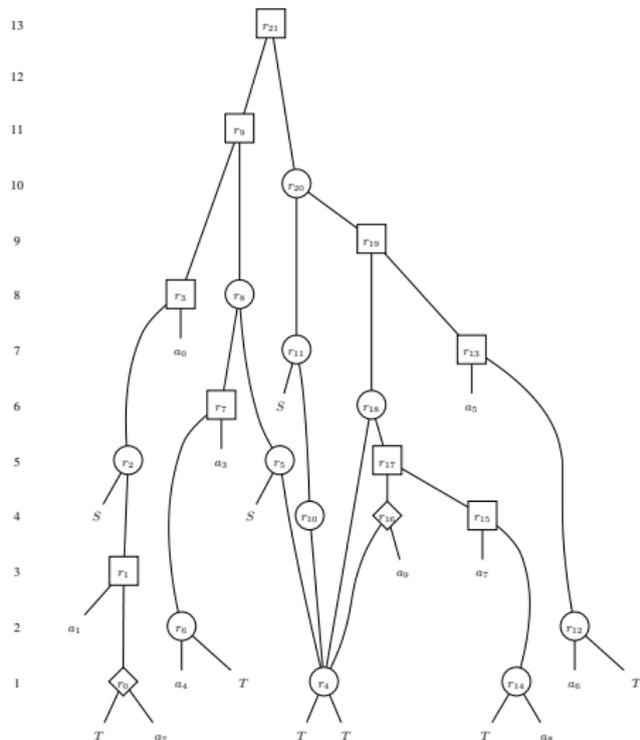
Listing 2: GAPP certificate

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}
```

Listing 3: C code



Achievements and lacking features of CGPE

Features achieved by CGPE

- ✓ validated on the ST200 core
- ✓ so far, no ambushes were encountered for $\sqrt{\quad}$, $\sqrt[3]{\quad}$, $\frac{1}{\sqrt{\quad}}$, $\frac{1}{\sqrt[3]{\quad}}$...
- ✓ produced optimal schemes for some of the above functions such as $\sqrt{\quad}$

Features lacking in CGPE

- ✗ simplistic description of the underlying architecture (ex. no handling of advanced operators such as ST200 `shift_and_add` instruction)
- ✗ the only shifts handled correspond to the multiplication by a power of 2
- ✗ hypotheses are made on the format of the input coefficients

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- ✗ hypotheses are made on the format of the input coefficients

Problem: without hypotheses on the formats of the input coefficients, CGPE fails

Solution: add the handling of multiple shifts to CGPE

Shift handling in CGPE

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 4. **overflow prevention shifts**: used before an arithmetic operation to prevent it from overflowing
 - to prevent the addition of a $Q[1.31]$ and a $Q[1.31]$ from overflowing the $Q[1.31]$ format, both operands are shifted to the $Q[2.30]$ format

- **Remark**: to detect whether one of these shifts is needed, we rely on:
 - ▶ fixed-point arithmetic rules (for case 2)
 - ▶ MPFI computations (for cases 1, 3 and 4).

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Problem: shifts may affect the critical path, potentially increasing the latency of the DAG

Solution: use more advanced instructions to help absorb this increase

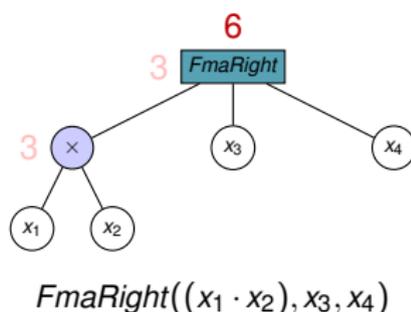
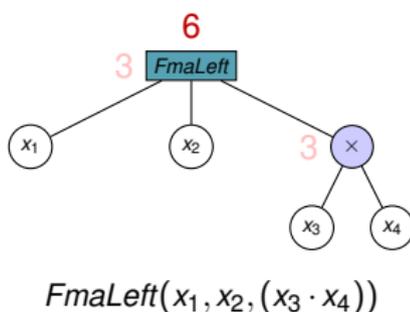
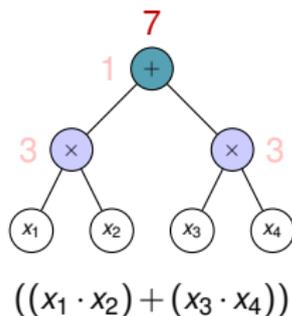
- ▶ ex: shift-and-add instruction available on some fixed-point processors like the ST231

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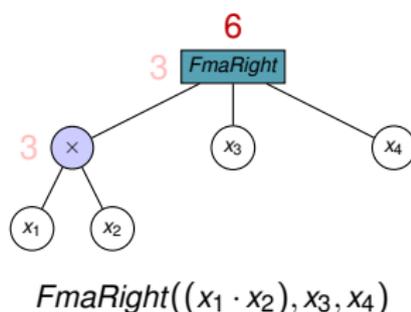
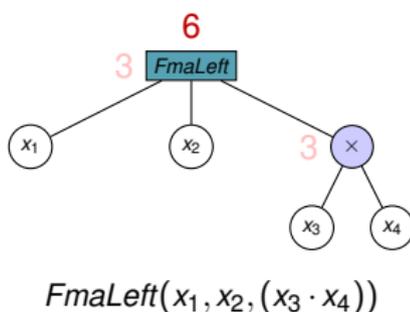
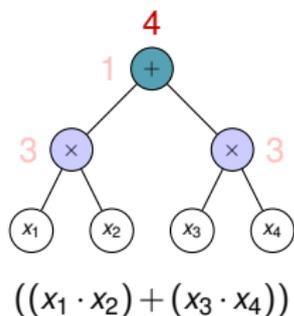
The problem of instruction selection

- A well known problem in compilation that was proven to be NP-complete on DAGs.
- Usually solved using a tiling algorithm:
 - ▶ input:
 - a DAG representing an arithmetic expression.
 - a set of tiles, with a cost for each.
 - a function that associates a cost to a subtree.
 - ▶ output:
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Remark on instruction selection

Some work in the area

Voronenko and Püschel from the Spiral group (2004):

- Automatic Generation of Implementations for DSP Transforms on Fused Multiply-Add Architectures.

✓ They provide a short proof of optimality in the case of trees.

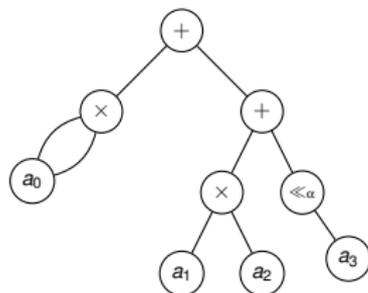
✗ Their method handles FMAs in DAGs but is not generic.

- We wish to integrate numerical verification in the process of instruction selection.

The NOLTIS tiling algorithm

Near-Optimal Instruction Selection algorithm (Koes and Goldstein in CGO-2008)

- 1: BottomUpDP()
 - 2: TopDownSelect()
 - 3: ImproveCSEDecision()
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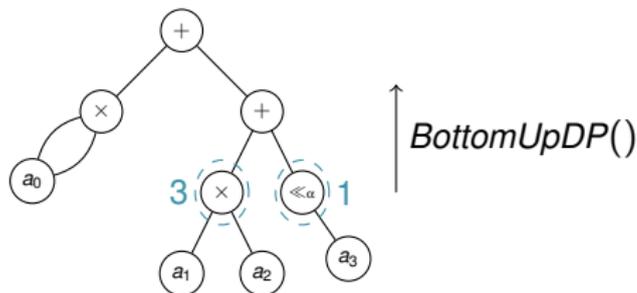


*the progress step by step of the tiling algorithm
on the expression $(a_0^2 + ((a_1 \times a_2) + (a_3 \ll \alpha)))$*

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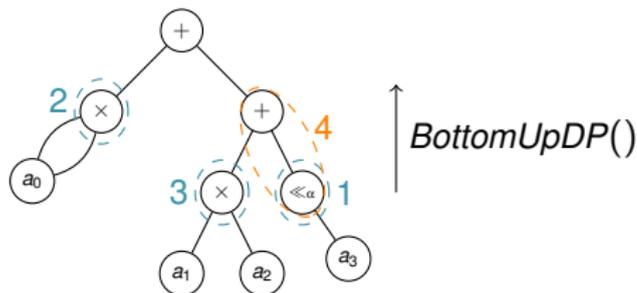


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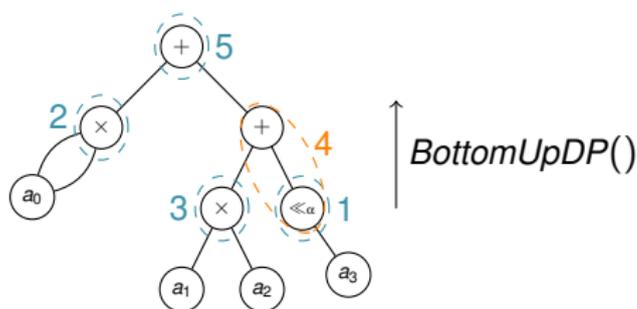


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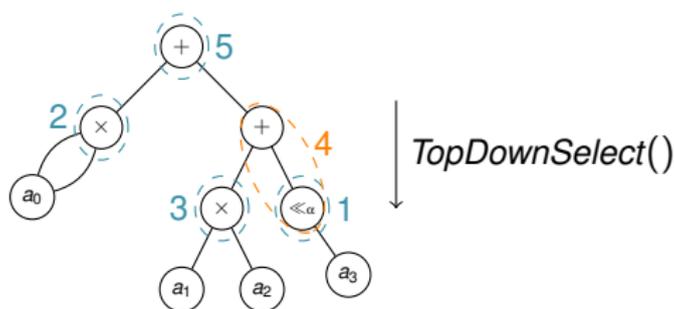


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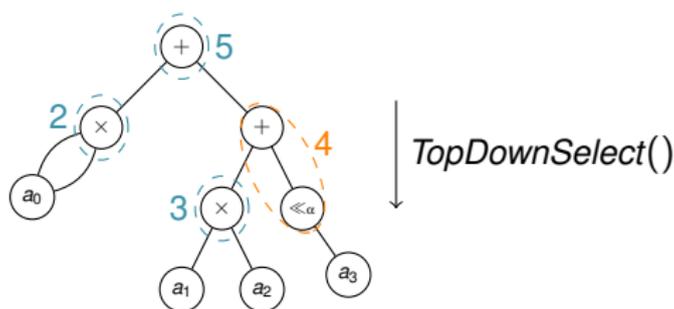


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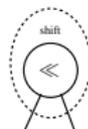
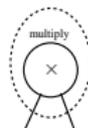
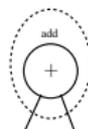


*the progress step by step of the tiling algorithm
on the expression $(a_0^2 + ((a_1 \times a_2) + (a_3 \ll \alpha)))$*

Instruction tiles considered in CGPE

■ Classical tiles

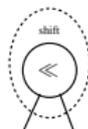
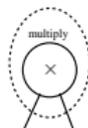
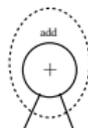
1. addition tile.
2. multiplication tile.
3. shift tile.



Instruction tiles considered in CGPE

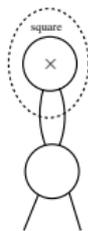
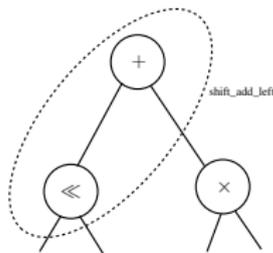
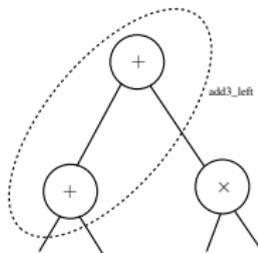
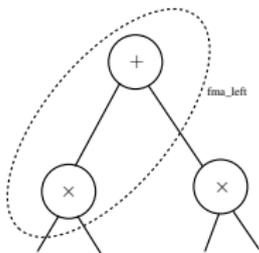
■ Classical tiles

1. addition tile.
2. multiplication tile.
3. shift tile.



■ Advanced tiles

4. fma tiles (left and right).
5. add3 tiles (left and right).
6. shiftAdd tiles (available on the ST200 core).
7. square tile.



Simple example

■ Original code

```
uint32_t func_d9_0(uint32_t T, uint32_t S)
{
    uint32_t r0 = T >> 2;           // (+) Q[1.31]
    uint32_t r1 = 0x80000000 + r0;  // (+) Q[1.31]
    uint32_t r2 = mul(S, r1);       // (+) Q[2.30]
    uint32_t r3 = 0x00000020 + r2;  // (+) Q[2.30]
    uint32_t r4 = mul(T, T);        // (+) Q[0.32]
    uint32_t r5 = mul(S, r4);       // (+) Q[1.31]
    uint32_t r6 = mul(T, 0x07fe93e4); // (+) Q[1.31]
    uint32_t r7 = 0x10000000 - r6;  // (-) Q[1.31]
    uint32_t r8 = mul(r5, r7);      // (-) Q[2.30]
    uint32_t r9 = r3 - r8;          // (+) Q[2.30]
    uint32_t r10 = mul(r4, r4);     // (+) Q[0.32]
    uint32_t r11 = mul(S, r10);     // (+) Q[1.31]
    uint32_t r12 = mul(T, 0x032d6643); // (+) Q[1.31]
    uint32_t r13 = 0x04eef694 - r12; // (-) Q[1.31]
    uint32_t r14 = mul(T, 0x00aabe7d); // (+) Q[1.31]
    uint32_t r15 = 0x01c6cebd - r14; // (-) Q[1.31]
    uint32_t r16 = r4 >> 11;       // (-) Q[1.31]
    uint32_t r17 = r15 + r16;       // (-) Q[1.31]
    uint32_t r18 = mul(r4, r17);    // (-) Q[1.31]
    uint32_t r19 = r13 + r18;       // (-) Q[1.31]
    uint32_t r20 = mul(r11, r19);   // (-) Q[2.30]
    uint32_t r21 = r9 - r20;        // (+) Q[2.30]
    return r21;
}
```

Listing 4: Original C code

■ With the fma in 3 cycles and the shift in 1 cycle

```
uint32_t func_tiled(uint32_t T, uint32_t S)
{
    uint32_t r0 = power(T, -2);
    uint32_t r1 = add(0x80000000, r0);
    uint32_t r2 = fma_right(0x00000020, S, r1);
    uint32_t r3 = square(T);
    uint32_t r4 = mul(S, r3);
    uint32_t r5 = mul(T, 0x07fe93e4);
    uint32_t r6 = sub(0x10000000, r5);
    uint32_t r7 = mul(r4, r6);
    uint32_t r8 = sub(r2, r7);
    uint32_t r9 = square(r3);
    uint32_t r10 = mul(S, r9);
    uint32_t r11 = mul(T, 0x032d6643);
    uint32_t r12 = sub(0x04eef694, r11);
    uint32_t r13 = mul(T, 0x00aabe7d);
    uint32_t r14 = sub(0x01c6cebd, r13);
    uint32_t r15 = power(r3, -11);
    uint32_t r16 = add(r14, r15);
    uint32_t r17 = fma_right(r12, r3, r16);
    uint32_t r18 = mul(r10, r17);
    uint32_t r19 = sub(r8, r18);
    return r19;
}
```

Listing 5: Code after tiling

Simple example

■ Original code

```
uint32_t func_d9_0(uint32_t T, uint32_t S)
{
    uint32_t r0 = T >> 2;           // (+) Q[1.31]
    uint32_t r1 = 0x80000000 + r0;   // (+) Q[1.31]
    uint32_t r2 = mul(S, r1);       // (+) Q[2.30]
    uint32_t r3 = 0x00000020 + r2;   // (+) Q[2.30]
    uint32_t r4 = mul(T, T);        // (+) Q[0.32]
    uint32_t r5 = mul(S, r4);       // (+) Q[1.31]
    uint32_t r6 = mul(T, 0x07fe93e4); // (+) Q[1.31]
    uint32_t r7 = 0x10000000 - r6;   // (-) Q[1.31]
    uint32_t r8 = mul(r5, r7);      // (-) Q[2.30]
    uint32_t r9 = r3 - r8;         // (+) Q[2.30]
    uint32_t r10 = mul(r4, r4);     // (+) Q[0.32]
    uint32_t r11 = mul(S, r10);     // (+) Q[1.31]
    uint32_t r12 = mul(T, 0x032d6643); // (+) Q[1.31]
    uint32_t r13 = 0x04eef694 - r12; // (-) Q[1.31]
    uint32_t r14 = mul(T, 0x00aeb7d); // (+) Q[1.31]
    uint32_t r15 = 0x01c6cebd - r14; // (-) Q[1.31]
    uint32_t r16 = r4 >> 11;       // (-) Q[1.31]
    uint32_t r17 = r15 + r16;      // (-) Q[1.31]
    uint32_t r18 = mul(r4, r17);    // (-) Q[1.31]
    uint32_t r19 = r13 + r18;      // (-) Q[1.31]
    uint32_t r20 = mul(r11, r19);   // (-) Q[2.30]
    uint32_t r21 = r9 - r20;       // (+) Q[2.30]
    return r21;
}
```

Listing 6: Original C code

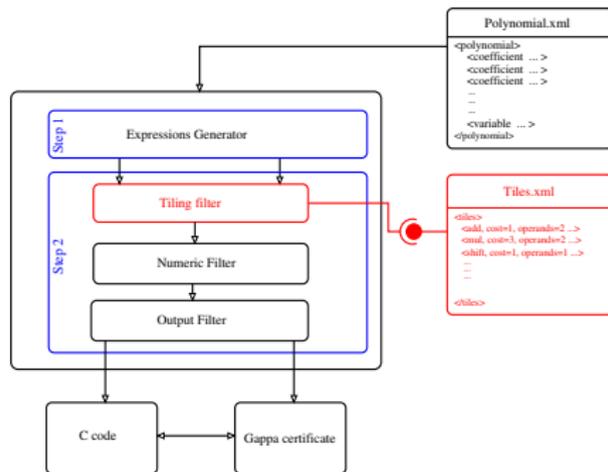
■ With the fma in 3 cycles and the shift in 3 cycle

```
uint32_t func_tiled(uint32_t T, uint32_t S)
{
    uint32_t r0 = fma_right(0x80000000, T, 0x40000000);
    uint32_t r1 = fma_right(0x00000020, S, r0);
    uint32_t r2 = square(T);
    uint32_t r3 = mul(S, r2);
    uint32_t r4 = mul(T, 0x07fe93e4);
    uint32_t r5 = sub(0x10000000, r4);
    uint32_t r6 = mul(r3, r5);
    uint32_t r7 = sub(r1, r6);
    uint32_t r8 = square(r2);
    uint32_t r9 = mul(S, r8);
    uint32_t r10 = mul(T, 0x032d6643);
    uint32_t r11 = sub(0x04eef694, r10);
    uint32_t r12 = mul(T, 0x00aeb7d);
    uint32_t r13 = sub(0x01c6cebd, r12);
    uint32_t r14 = power(r2, -11);
    uint32_t r15 = add(r13, r14);
    uint32_t r16 = fma_right(r11, r2, r15);
    uint32_t r17 = mul(r9, r16);
    uint32_t r18 = sub(r7, r17);
    return r18;
}
```

Listing 7: Code after tiling

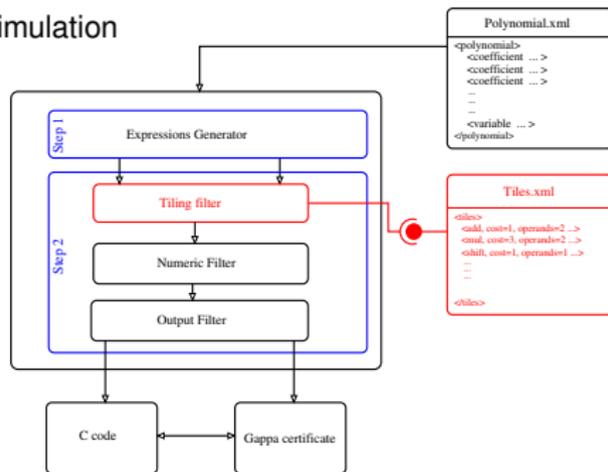
Remarks on instruction selection in CGPE

- A separation is achieved between the computation of DAGs (Intermediate Representation) and the code generation process
 - ▶ the code can be generated according different criteria \rightsquigarrow **cost function**
 - ▶ this general approach allows to tackle other problems (sum, dot-product, ...)



Remarks on instruction selection in CGPE

- A separation is achieved between the computation of DAGs (Intermediate Representation) and the code generation process
 - ▶ the code can be generated according different criteria \rightsquigarrow **cost function**
 - ▶ this general approach allows to tackle other problems (sum, dot-product, ...)
- We are not bound to use these tiles, we can add many others
 - ▶ CGPE can thus serve as a platform of simulation
 - ▶ this general approach allows to give some feedback on the eventual **need or usefulness** of some tiles



Outline of the talk

1. The CGPE tool
2. Instruction selection: an extension of CGPE
3. Conclusion and perspectives

Conclusion

■ Code synthesis for fast and certified polynomial evaluation

- ▶ fast and certified C codes, in fixed point arithmetic
- ▶ tool to automate polynomial evaluation implementation, using at best architectural features
- ▶ implemented in the tool CGPE (Code Generation for Polynomial Evaluation)

`http://cgpe.gforge.inria.fr/`

■ Extension of CGPE based on instruction selection:

- ▶ automatic handling of all input formats.
- ▶ better usage of the **advanced** architectural features (such as fma, add-3, shift-and-add, ...)
- ▶ using a tiling algorithm implies more modularity, as code generation is now an independant process.

Current work and perspectives

■ Current work

- ▶ keep working on instruction selection in CGPE
- ▶ make CGPE more general to tackle other problems, like [matrix inversion](#) and [multiplication](#), ...

Current work and perspectives

■ Current work

- ▶ keep working on instruction selection in CGPE
- ▶ make CGPE more general to tackle other problems, like [matrix inversion](#) and [multiplication](#), ...

■ Further extensions of CGPE

- ▶ handle other arithmetics like floating-point arithmetic, where the fma tile is more and more ubiquitous
- ▶ target other architectures (like FPGAs)

Synthesis of fixed-point programs based on instruction selection

... the case of polynomial evaluation

Amine Najahi

Advisors: Matthieu Martel and Guillaume Revy

Joint work with Christophe Moulleron

Équipe-projet DALI, Univ. Perpignan Via Domitia
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et de Microélectronique
de Montpellier

