

Block Wiedemann algorithm on multicore architectures

Bastien Vialla

► **To cite this version:**

Bastien Vialla. Block Wiedemann algorithm on multicore architectures. ACM Communications in Computer Algebra, Association for Computing Machinery (ACM), 2014, 47 (3/4), pp.102 - 103. <10.1145/2576802.2576814>. <lirmm-01372535>

HAL Id: lirmm-01372535

<https://hal-lirmm.ccsd.cnrs.fr/lirmm-01372535>

Submitted on 27 Sep 2016

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Notations

- \mathbb{K} a commutative field.
- $\mathcal{M}_{n \times m}(\mathbb{K})$, ring of matrices of size $n \times m$.
- We denote by γ the number of non zeros in a sparse matrix.
- NUMA : Non Uniform Memory Access

Motivations

Solving a linear system with large sparse matrices is a computational kernel used in a wide range of applications. The block version of Wiedemann's algorithm proposed in [1] take advantage of the sparsity to achieve better performance.

Objectives

An efficient implementation of block Wiedemann algorithm on NUMA multicores architectures.

Contribution

- We efficiently incorporate the sparse block into the first step of BW algorithm.
- We provide an efficient implementation for NUMA multicores using tbb/MPI that provides excellent scaling.

Block Wiedemann algorithm

Let $A \in \mathcal{M}_{n \times n}(\mathbb{K})$, $U, V \in \mathcal{M}_{n \times k}(\mathbb{K})$ random matrices with $k \leq n$.

Block Wiedemann algorithm follows three steps:

- ① Compute the first $O(\frac{2n}{k})$ elements of $S = [U^T A^i V]_{i \in \mathbb{N}}$.
- ② Find the minimal matrix polynomial generator of the sequence S .
- ③ Compute the solution using the polynomial found in step 2.

The cost of the first step is dominant, therefore its parallelization is crucial.

Sparse blocks

We generalize sparse block from [2] in block Wiedemann algorithm. We permute non zeros elements to have a cache efficient version.

$$U = \begin{pmatrix} \delta_1 & & & & & & & & \\ \vdots & & & & & & & & \\ \delta_s & & & & & & & & \\ \delta_{s+1} & & & & & & & & \\ & \delta_1 & & & & & & & \\ & \vdots & & & & & & & \\ & \delta_s & & & & & & & \\ & \delta_{s+1} & & & & & & & \\ & \dots & & & & & & & \\ & & \delta_1 & & & & & & \\ & & \vdots & & & & & & \\ & & \delta_s & & & & & & \\ & & \delta_{s+1} & & & & & & \\ & & & \dots & & & & & \\ & & & & \delta_1 & & & & \\ & & & & \vdots & & & & \\ & & & & \delta_s & & & & \\ & & & & & \dots & & & \\ & & & & & & \delta_1 & & \\ & & & & & & \vdots & & \\ & & & & & & \delta_s & & \end{pmatrix} \begin{matrix} \\ \\ \\ \\ [n/k] \text{ times} \\ \\ \\ \\ \\ \\ \\ \\ [n \bmod k] \text{ times} \end{matrix}$$

where $s = \lfloor n/k \rfloor$, $\delta_1, \dots, \delta_{s+1} \in \mathbb{K}$ chosen at random.
Complexities of the first step of block Wiedemann algorithm:

	Dense blocks	Sparse blocks
Sequential	$O(n\gamma + n^2 k^{\omega-2})$	$O(n\gamma + n^2)$
Parallel k cores	$O(\frac{n\gamma}{k} + n^2)$	$O(\frac{n\gamma + n^2}{k})$

Experimentations

- We use LinBox [3] for dense blocks code, and tbb for parallelization. We use an NUMA with four Intel XEON E4620 with 8 cores at 2.2Ghz and 384GB of RAM.
- The block size does not impact the time with the use of sparse blocks, see figure 1.
- tbb implementation performs badly on NUMA architectures, see table 1.
- We design an Hybrid MPI/tbb implementation that allows good speed-up on NUMA, see table 1.
- Sparse blocks perform always better, see table 1.

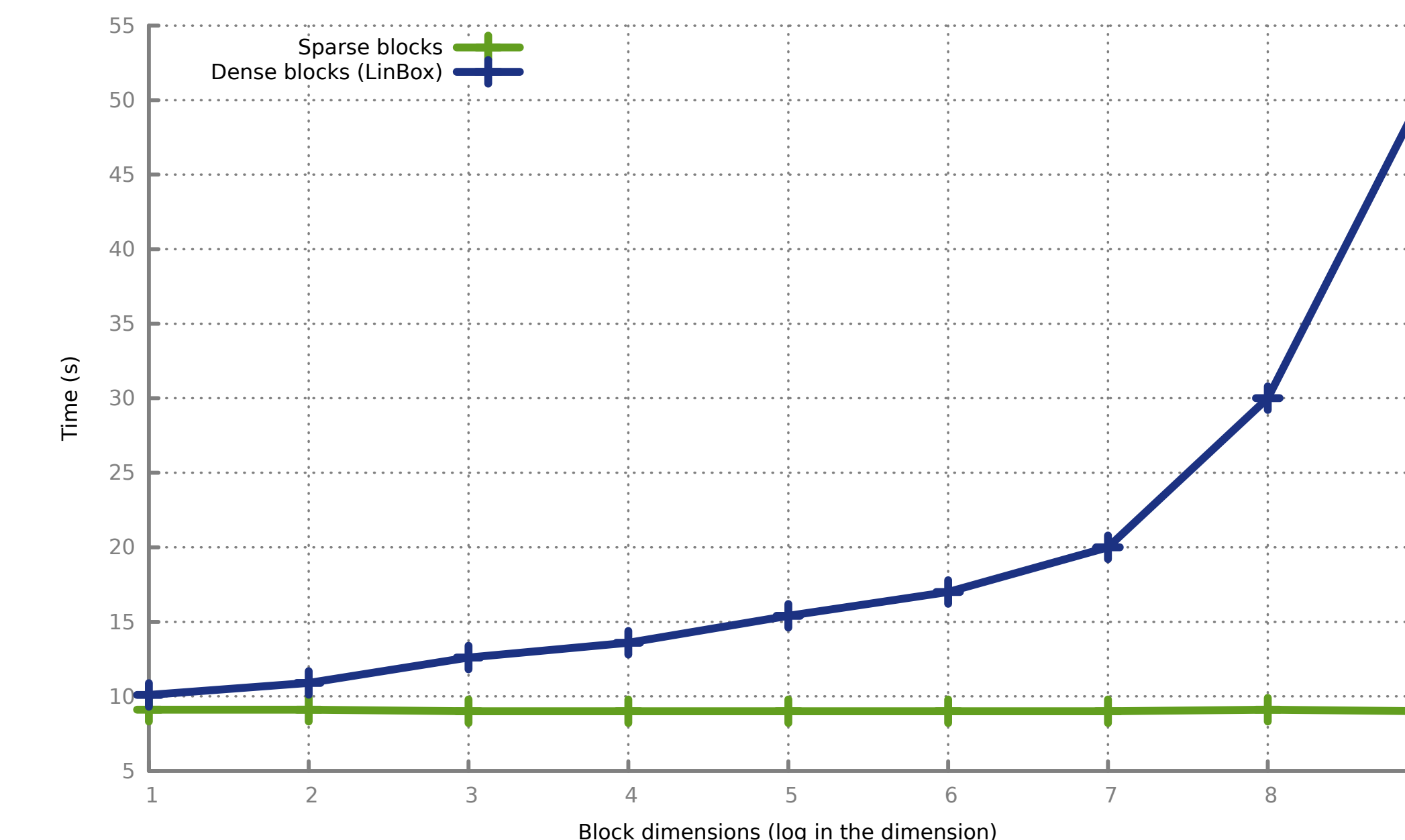


Figure 1: Sparse blocks size influence in comparison with dense blocks. Computations made with the matrix EX5 with 1 core.

	Dense blocks (LinBox)				Sparse blocks			
	tbb		MPI/tbb		tbb		MPI/tbb	
	time	speed-up	time	speed-up	time	speed-up	time	speed-up
1 cpu : 1 core	2205	1	2207	1	2160	1	2165	1
1 cpu : 8 cores	540	4	540	4	308	7	308	7
2 cpus : 16 cores	623	3.5	279	7.9	310	6.8	154	14
3 cpus : 24 cores	798	2.7	183	12	242	8.8	102	21.2
4 cpus : 32 cores	960	2.2	135	16.3	177	12	77	28.1

Table 1: Timings, in seconds, and speed-up of tbb and MPI/tbb implementations. We use the matrix rand100k.

Additional Informations

Matrices characteristics :

Name	Size	Non zeros	Problem
rand100k	$100k \times 100k$	1.5M	randomly generated
EX5	6545×6545	295680	symmetric powers of graphs

References

- [1] Don Coppersmith. Solving Homogeneous Linear Equation Over GF(2) via Block Wiedemann Algorithm. *Mathematics of Computation*, 62(205):333–350, 1994.
- [2] Wayne Eberly, Mark Giesbrecht, Pascal Giorgi, Arne Storjohann, and Gilles Villard. Faster inversion and other black box matrix computations using efficient block projections. *Proceedings of the 2007 international symposium on Symbolic and algebraic computation - ISSAC '07*, 3(1):143, 2007.
- [3] <http://www.linalg.org>.