Block Wiedemann algorithm on multicore architectures
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Notations

- $\mathbb{K}$ a commutative field.
- $\mathcal{M}_{n\times n}(\mathbb{K})$, ring of matrices of size $n \times n$.
- We denote by $\gamma$ the number of non zeros in a sparse matrix.
- $\text{NUMA : Non Uniform Memory Access}$

Motivations

Solving a linear system with large sparse matrices is a computational kernel used in a wide range of applications. The block version of Wiedemann’s algorithm proposed in [1] take advantage of the sparsity to achieve better performance.

Objectives

An efficient implementation of block Wiedemann algorithm on NUMA multicores architectures.

Contribution

- We efficiently incorporate the sparse block into the first step of BW algorithm.
- We provide an efficient implementation for NUMA multicores using tbb/mpi that provides excellent scaling.

Block Wiedemann algorithm

Let $A \epsilon \mathcal{M}_{n\times n}(\mathbb{K})$, $U, V \epsilon \mathcal{M}_{n\times k}(\mathbb{K})$ random matrices with $k \leq n$.

Block Wiedemann algorithm follows three steps:

1. Compute the first $O\left(\frac{n\gamma}{k}\right)$ elements of $S = [U^T A V]_{\in \mathbb{N}}$.
2. Find the minimal matrix polynomial generator of the sequence $S$.
3. Compute the solution using the polynomial found in step 2.

The cost of the first step is dominant, therefore its parallelization is crucial.

Sparse blocks

We generalize sparse block from [2] in block Wiedemann algorithm. We permute non-zeros elements to have a cache efficient computation. We generalize sparse block from [2] in block Wiedemann algorithm. We permute non-zeros elements to have a cache efficient computation. We efficiently incorporate the sparse block into the first step of BW algorithm.

Complexities of the first step of block Wiedemann algorithm:

![Block Wiedemann Algorithm](image.png)

where $s = \lfloor n/k \rfloor$, $\delta_1, \ldots, \delta_{k+1} \epsilon \mathbb{K}$ chosen at random.

Table 1: Timings, in seconds, and speed-up of tbb and MPI/tbb implementations. We use the matrix rand100k.

<table>
<thead>
<tr>
<th>Name</th>
<th>Dense blocks (LinBox)</th>
<th>Sparse blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>Speed-up</td>
<td>Time</td>
</tr>
<tr>
<td>1 cpu : 1 core</td>
<td>2205</td>
<td>1</td>
</tr>
<tr>
<td>1 cpu : 8 cores</td>
<td>540</td>
<td>4</td>
</tr>
<tr>
<td>2 cpus : 16 cores</td>
<td>643</td>
<td>3.5</td>
</tr>
<tr>
<td>3 cpus : 24 cores</td>
<td>798</td>
<td>2.7</td>
</tr>
<tr>
<td>4 cpus : 32 cores</td>
<td>960</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Table 1: Timings, in seconds, and speed-up of tbb and MPI/tbb implementations. We use the matrix EX5 with 1 core.

We use LinBox [3] for dense blocks code, and tbb for parallelization. We use an NUMA with four Intel XEON E4620 with 8 cores at 2.2Ghz and 384GB of RAM.

We design an Hybrid MPI/tbb implementation that allows good speed-up on NUMA, see table 1.

Additional Informations

Matrices characteristics:

<table>
<thead>
<tr>
<th>Name</th>
<th>Size</th>
<th>Non zeros</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>rand100k</td>
<td>100k</td>
<td>1.5M</td>
<td>randomly generated</td>
</tr>
<tr>
<td>EX5</td>
<td>5645</td>
<td>209680</td>
<td>symmetric powers of graphs</td>
</tr>
</tbody>
</table>

References