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Block Wiedemann Algorithm on Multicores Architectures

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Notations

- \( K \) a commutative field.
- \( M_{n \times n}(K) \), ring of matrices of size \( n \times m \).
- We denote \( \gamma \) the number of non zeros in a sparse matrix.
- NUMA : Non Uniform Memory Access

Motivations

Solving a linear system with large sparse matrices is a computational kernel used in a wide range of applications. The block version of Wiedemann’s algorithm proposed in [1] take advantage of the sparsity to achieve better performance.

Objectives

An efficient implementation of block Wiedemann algorithm on NUMA multicores architectures.

Contribution

- We efficiently incorporate the sparse block into the first step of BW algorithm.
- We provide an efficient implementation for NUMA multicores using tbb/MPI that provides excellent scaling.

Block Wiedemann algorithm

Let \( A \in M_{n \times n}(K) \), \( U, V \in M_{n \times k}(K) \) random matrices with \( k \leq n \).

Block Wiedemann algorithm follows three steps:
- Compute the first \( O(\frac{2n^3}{k}) \) elements of \( S = [U^T A V]_{i \in \mathbb{N}} \).
- Find the minimal matrix polynomial generator of the sequence \( S \).
- Compute the solution using the polynomial found in step 2.

The cost of the first step is dominant, therefore its parallelization is crucial.

Sparse blocks

We generalize sparse block from [2] in block Wiedemann algorithm. We permute non zeros elements to have a cache efficient version.

\[
U = \begin{bmatrix}
\delta_i & \delta_i & \cdots & \delta_i \\
\delta_i & \cdots & \delta_i \\
\cdots & \cdots & \cdots \\
\delta_i & \cdots & \delta_i \\
\end{bmatrix} 
\]

\[
V = \begin{bmatrix}
\delta_{k+1} & \cdots & \delta_{k+1} \\
\delta_{k+1} & \cdots & \delta_{k+1} \\
\cdots & \cdots & \cdots \\
\delta_{k+1} & \cdots & \delta_{k+1} \\
\end{bmatrix} 
\]

\[
\delta_i \in K \text{ chosen at random.}
\]

\[
U V = \begin{bmatrix}
\delta_i & \cdots & \delta_i \\
\delta_i & \cdots & \delta_i \\
\cdots & \cdots & \cdots \\
\delta_i & \cdots & \delta_i \\
\end{bmatrix}
\]

where \( s = \lfloor n/k \rfloor \), \( \delta_1, \ldots, \delta_{s+1} \in K \) chosen at random.

Complexities of the first step of block Wiedemann algorithm:

- Sequential: \( O(\gamma n^{\gamma + n^2 k - 2}) \)
- \( O(\frac{n^\gamma}{k^2} + n^2) \)
- Parallel \( k \) cores: \( O(\frac{n^\gamma}{k^2} + n^2) \)

Experimentations

- We use LinBox [3] for dense blocks code, and tbb for parallelization. We use an NUMA with four Intel XEON E4620 with 8 cores at 2.2Ghz and 384GB of RAM.
- The block size does not impact the time with the use of sparse blocks, see figure 1.
- tbb implementation performs badly on NUMA architectures, see table 1.
- We design an Hybrid MPI/tbb implementation that allows good speed-up on NUMA, see table 1.
- Sparse blocks perform always better, see table 1.

Additional Informations

Matrices characteristics:

<table>
<thead>
<tr>
<th>Name</th>
<th>Size</th>
<th>Non zeros</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>rand100k</td>
<td>100k x 100k</td>
<td>1.5M</td>
<td>randomly generated</td>
</tr>
<tr>
<td>EX5</td>
<td>654 x 654</td>
<td>20680</td>
<td>symmetric powers of graphs</td>
</tr>
</tbody>
</table>

References