Block Wiedemann algorithm on multicore architectures
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To cite this version:

HAL Id: lirmm-01372535
https://hal-lirmm.ccsd.cnrs.fr/lirmm-01372535
Submitted on 27 Sep 2016

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Notations
- $\mathbb{K}$ a commutative field.
- $\mathcal{M}_{n \times n}(\mathbb{K})$, ring of matrices of size $n \times m$.
- We denote by $\gamma$ the number of non zeros in a sparse matrix.
- NUMA : Non Uniform Memory Access

Motivations
Solving a linear system with large sparse matrices is a computational kernel used in a wide range of applications. The block version of Wiedemann’s algorithm proposed in [1] take advantage of the sparsity to achieve better performance.

Objectives
An efficient implementation of block Wiedemann algorithm on NUMA multicores architectures.

Contribution
- We efficiently incorporate the sparse block into the first step of BW algorithm.
- We provide an efficient implementation for NUMA multicores using tbb/MPI that provides excellent scaling.

Block Wiedemann algorithm
Let $A \in \mathcal{M}_{n \times n}(\mathbb{K})$, $U, V \in \mathcal{M}_{n \times k}(\mathbb{K})$ random matrices with $k \leq n$. Block Wiedemann algorithm follows three steps:
- Compute the first $O(\frac{\gamma n}{k})$ elements of $S = [U^T A V]_{i \in \mathbb{N}}$.
- Find the minimal matrix polynomial generator of the sequence $S$.
- Compute the solution using the polynomial found in step 2.

The cost of the first step is dominant, therefore its parallelization is crucial.

Sparsity blocks
We generalize sparse block from [2] in block Wiedemann algorithm. We permute non zeros elements to have a cache efficient version.

\[
U = \begin{bmatrix}
\delta_i & \delta_i & \cdots & \delta_i & \delta_i & \cdots & \delta_i \\
\delta_i & \delta_i & \cdots & \delta_i & \delta_i & \cdots & \delta_i \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\delta_i & \delta_i & \cdots & \delta_i & \delta_i & \cdots & \delta_i \\
\end{bmatrix} \mod k \times n
\]

where $s = \lfloor n/k \rfloor$, $\delta_1, \ldots, \delta_{s+1} \in \mathbb{K}$ chosen at random.
Complexities of the first step of block Wiedemann algorithm:

- Sequential $O(n\gamma + n^2 k^{-2})$
- Parallel $O(n\gamma + n^2)$
- Sparse blocks $O(\frac{n^2}{k}) + n^2$

Experiments
- We use LinBox [3] for dense blocks code, and tbb for parallelization. We use an NUMA with four Intel XEON E4620 with 8 cores at 2.2Ghz and 384GB of RAM.
- The block size does not impact the time with the use of sparse blocks, see figure 1.
- tbb implementation performs badly on NUMA architectures, see table 1.
- We design an Hybrid MPI/tbb implementation that allows good speed-up on NUMA, see table 1.
- Sparse blocks perform always better, see table 1.

Additional Informations
Matrices characteristics:

<table>
<thead>
<tr>
<th>Name</th>
<th>Size</th>
<th>Non zeros</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>rand100k</td>
<td>100k</td>
<td>1.5M</td>
<td>randomly generated</td>
</tr>
<tr>
<td>EX5</td>
<td>6545</td>
<td>20680</td>
<td>symmetric powers of graphs</td>
</tr>
</tbody>
</table>

References