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Block Wiedemann Algorithm on Multicores Architectures

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Notations
- \( \mathbb{K} \) a commutative field.
- \( \mathcal{M}_{n \times m}(\mathbb{K}) \), ring of matrices of size \( n \times m \).
- We denote by \( \gamma \) the number of non zeros in a sparse matrix.
- NUMA : Non Uniform Memory Access

Motivations
Solving a linear system with large sparse matrices is a computational kernel used in a wide range of applications. The block version of Wiedemann’s algorithm proposed in [1] take advantage of the sparsity to achieve better performance.

Objectives
An efficient implementation of block Wiedemann algorithm on NUMA multicores architectures.

Contribution
- We efficiently incorporate the sparse block into the first step of BW algorithm.
- We provide an efficient implementation for NUMA multicores using tbb/MPI that provides excellent scaling.

Block Wiedemann algorithm
Let \( A \in \mathcal{M}_{n \times n}(\mathbb{K}) \), \( U, V \in \mathcal{M}_{n \times k}(\mathbb{K}) \) random matrices with \( k \leq n \).

Block Wiedemann algorithm follows three steps:
1. Compute the first \( O(\frac{2n}{k}) \) elements of \( S = [U^T A V]_{i \in \mathbb{N}} \).
2. Find the minimal matrix polynomial generator of the sequence \( S \).
3. Compute the solution using the polynomial found in step 2.

The cost of the first step is dominant, therefore its parallelization is crucial.

Sparse blocks
We generalize sparse block from [2] in block Wiedemann algorithm. We permute non zeros elements to have a cache efficient version.

\[
U = \begin{pmatrix}
\delta_1 & \delta_2 & \cdots & \delta_s \\
\delta_{s+1} & \delta_1 & \cdots & \delta_s \\
\vdots & \vdots & \ddots & \vdots \\
\delta_{s+k-1} & \delta_{s+k-2} & \cdots & \delta_1 \\
\end{pmatrix}
\]

where \( s = \lfloor n/k \rfloor \), \( \delta_1, \ldots, \delta_{s+k-1} \in \mathbb{K} \) chosen at random.

Complexities of the first step of block Wiedemann algorithm:

<table>
<thead>
<tr>
<th>Dense blocks</th>
<th>Sparse blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td>( O(n \gamma + n k^{\gamma-2}) )</td>
</tr>
<tr>
<td>Parallel ( k ) cores</td>
<td>( O\left(\frac{n^2}{k^2} + n^2\right) )</td>
</tr>
</tbody>
</table>

We use LinBox [3] for dense blocks code, and tbb for parallelization. We use an NUMA with four Intel XEON E4620 with 8 cores at 2.2Ghz and 384GB of RAM.

- The block size does not impact the time with the use of sparse blocks, see figure 1.
- tbb implementation performs badly on NUMA architectures, see table 1.
- We design an Hybrid MPI/tbb implementation that allows good speed-up on NUMA, see table 1.
- Sparse blocks perform always better, see table 1.

Experiments

<table>
<thead>
<tr>
<th>Name</th>
<th>Size</th>
<th>Non zeros</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>rand100k</td>
<td>1000x1000</td>
<td>1.5M</td>
<td>randomly generated</td>
</tr>
<tr>
<td>EX5</td>
<td>64x65</td>
<td>29680</td>
<td>symmetric powers of graphs</td>
</tr>
</tbody>
</table>

References

Figure 1: Sparse blocks size influence in comparison with dense blocks. Computations made with the matrix EX5 with 1 core.

Table 1: Timings, in seconds, and speed-up of tbb and MPI/tbb implementations. We use the matrix rand100k.