Block Wiedemann algorithm on multicore architectures
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Notations

- $\mathbb{K}$ a commutative field.
- $\mathbb{M}_{n \times n}(\mathbb{K})$, ring of matrices of size $n \times m$.
- We denote by $\gamma$ the number of non zeros in a sparse matrix.
- NUMA : Non Uniform Memory Access

Motivations

Solving a linear system with large sparse matrices is a computational kernel used in a wide range of applications. The block version of Wiedemann’s algorithm proposed in [1] take advantage of the sparsity to achieve better performance.

Objectives

An efficient implementation of block Wiedemann algorithm on NUMA multicores architectures.

Contribution

- We efficiently incorporate the sparse block into the first step of BW algorithm.
- We provide an efficient implementation for NUMA multicores using tbb/MPI that provides excellent scaling.

Block Wiedemann algorithm

Let $A \in \mathbb{M}_{n \times n}(\mathbb{K})$, $U, V \in \mathbb{M}_{n \times k}(\mathbb{K})$ random matrices with $k \leq n$.

Block Wiedemann algorithm follows three steps:

1. Compute the first $O(\frac{2n}{k})$ elements of $S = [U^T A V]_{i \in \mathbb{N}}$.
2. Find the minimal matrix polynomial generator of the sequence $S$.
3. Compute the solution using the polynomial found in step 2.

The cost of the first step is dominant, therefore its parallelization is crucial.

We generalize sparse block from [2] in block Wiedemann algorithm. We permute non zeros elements to have a cache efficient version.

\[
U = \begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3 \\
\vdots \\
\delta_{s-1} \\
\delta_{s}
\end{bmatrix}
\]

\[
\begin{array}{c}
\delta_1 \\
\delta_2 \\
\delta_3 \\
\vdots \\
\delta_{s-1} \\
\delta_{s}
\end{array} \quad \begin{array}{c}
\delta_1 \\
\delta_2 \\
\delta_3 \\
\vdots \\
\delta_{s-1} \\
\delta_{s}
\end{array} \quad \begin{array}{c}
\delta_1 \\
\delta_2 \\
\delta_3 \\
\vdots \\
\delta_{s-1} \\
\delta_{s}
\end{array} 
\]

where $s = \lfloor n/k \rfloor$, $\delta_1, \ldots, \delta_{s+1} \in \mathbb{K}$ chosen at random.

Complexities of the first step of block Wiedemann algorithm:

<table>
<thead>
<tr>
<th></th>
<th>Dense blocks</th>
<th>Sparse blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td>$O(n^2 + n^2k^{-2})$</td>
<td>$O(n\gamma + n^2)$</td>
</tr>
<tr>
<td>Parallel $k$ cores</td>
<td>$O(\frac{n^2}{k} + n^2)$</td>
<td>$O(\frac{nk^2}{k} + \frac{n^2}{k})$</td>
</tr>
</tbody>
</table>

We use LinBox [3] for dense blocks code, and tbb for parallelization. We use an NUMA with four Intel XEON E4620 with 8 cores at 2.2Ghz and 384GB of RAM.

We design an Hybrid MPI/tbb implementation that allows good speed-up on NUMA, see table 1.

Sparse blocks perform always better, see figure 1.

Table 1: Timings, in seconds, and speed-up of tbb and MPI/tbb implementations. We use the matrix rand100k.

<table>
<thead>
<tr>
<th>Name</th>
<th>Size</th>
<th>Non zeros</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>rand100k</td>
<td>$100k \times 100k$</td>
<td>1.5M</td>
<td>randomly generated</td>
</tr>
<tr>
<td>EX5</td>
<td>$64k \times 64k$</td>
<td>20.6M</td>
<td>symmetric powers of graphs</td>
</tr>
</tbody>
</table>

Additional Informations

Matrices characteristics:

- Dense blocks (LinBox)
- Sparse blocks
- tbb
- MPI/tbb
- time
- speed-up
- time
- speed-up
- time
- speed-up

References

Solving Homogeneous Linear Equation Over GF(2) via Block Wiedemann Algorithm.

Faster inversion and other block matrix computations using efficient block projections.