# EKF-based state estimation for train localization

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# EKF-based State Estimation for Train Localization

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***Abstract*—Determination of longitudinal acceleration of a land-vehicle regardless its inclination is a common problem for systems of localization. This paper addresses a solution for railway applications by combining a low-cost MEMS IMU (Inertial Measurement Unit) equipped with a 3-axis accelerometer and a 3-axis gyrometer and an algorithm for data fusion. In particular, the impact of adding attitude and velocity observations into a Kalman filter is studied. Compared to conventional methods that use regular Kalman filter with external aiding sensors such as GPS or tachometers, the proposed approach uses an Extended Kalman Filter which exploits an augmented state vector. A velocity estimation obtained by a method observing the spectral analysis of the vertical accelerometer and the attitude estimation obtained by a complementary filter compose the observation vector with the accelerometer and the gyrometer data. At last, experimental results performed on an urban train are presented.**

**Keywords - Kalman filter; Compelmentary filter; MEMS; Spectral analysis; Data fusion; Train localization**

## I. Introduction

The European Train Control System (ETCS) aims to provide an accurate train localization [1]. This information is vital for the traffic and the safety. To achieve this localization, several onboard sensors and a clever fusion guarantees good performance and reliability of the velocity and position estimations. GPS reciever and tag reader - tags are positioned along the track - are usually used to detect the train position. During the time laps where no information are available, inertial sensors, tachometers and Doppler radar are used to estimate the position and velocity of the train. The goal of this plurality of sensors is to provide an accurate reliable data in any environmental conditions (weather, wheels/rails adhesion conditions, GPS signal, tunnels or buildings presence, ...). Our contribution is to enhance the current inertial system by providing the longitudinal acceleration (gravity compensation) and the velocity of the train based only on inertial sensors, 3-axis gyrometer and 3-axis accelerometer. The purpose of this paper is to enhance the method proposed in [2] thanks to the use of an Extended Kalman Filter (EKF) integrating, in its observation vector, velocity and attitude estimations obtained respectively from a spectral analysis and a complementary filter. Fig. 1 shows an overview of the proposed state estimator.

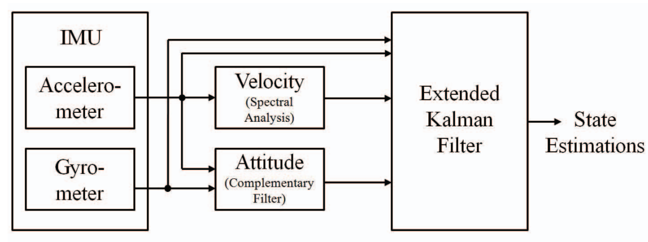


Fig. 1: State estimator overview

## II. Estimation Method

### A. Velocity Observation

The accelerometer measures the specific force applied on its three axis. This specific force includes the gravity, the train accelerations (longitudinal and centripetal) and the Coriolis acceleration. The centrifugal force due to the earth rotations is neglected in this work. However, accelerometer measures the vibrations too. According to [3], vibrations are directly impacted by the sensors placement, the undercarriage, the motion and the velocity of the train. Thus, the analysis of vibratory phenomena wheels/rails has contributed of the estimation of the velocity [3], [4] and geometrical characteristic of railways [5]. Differents sources of vibrations are shown in Fig. 2.

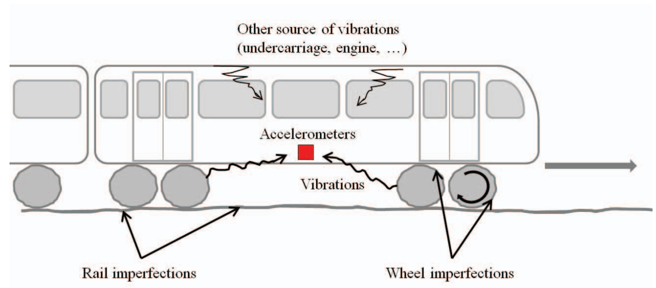


Fig. 2: Sources of vibrations measured by the accelerometers

The method proposed in [2] allows an observation of the velocity $v_{vel}$ by using over the time a linear fitting (1) of the spectral analysis of data from the vertical accelerometer.

$$v_{vel} = \rho\sqrt{S} + \beta \tag{1}$$

Where $\rho$ is a scale factor which has been determined by means of experimental results. $\beta$ is a bias which is calculated during the initialization phase of the inertial system. $S$ is the sum of euclidean norms of the Fast Fourier Transformation (FFT) components of the vertical accelerometer output. $S$ is defined by the equation (2).

$$S = \sum_{k=0}^{N/2} \| H_k \|_2 \tag{2}$$

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Where $H_k$ are the components of the $N$-points FFT vector ($N = 1024$) calculated with a Hamming window.

### *B. Attitude Observation*

Complementary filter is well known for attitude estimation [6] because it uses a good compromise between different sensors by choosing the best frequency response of each sensor. In the case of accelerometers and gyrometers, the attitude estimation is given by the low-frequencies response of the accelerometer and the high-frequencies response of the gyrometer. As the ranges of the roll and pitch angles are $\pm 8°$ in the railway applications, the values are too large for not considering the cross-coupling of the gyrometer and too low for considering the "*gimbal lock*" singularities. The complementary filter can be expressed as:

$$\begin{aligned} \phi_{inc} &= H_1(s)\phi_m + H_2(s)\frac{\dot{\phi}_m}{s} \\ \theta_{inc} &= H_1(s)\theta_m + H_2(s)\frac{\dot{\theta}_m}{s} \end{aligned} \quad (3)$$

Where $H_1$ and $H_2$ are a low-pass filter and a high-pass filter, $\phi_{inc}$ and $\theta_{inc}$ are the estimates of the roll and pitch angles, $\phi_m$ and $\theta_m$ are the roll and pitch angles obtained by the measures of the accelerometer $(f_x, f_y, f_z)^T$ with,

$$\begin{aligned} \phi_m &= atan(-f_y/9.81) \\ \theta_m &= asin(-f_x/9.81) \end{aligned} \quad (4)$$

and $\dot{\phi}_m$ and $\dot{\theta}_m$ are the rate of the roll and pitch angles obtained by the gyrometer measurements $(\omega_x, \omega_y, \omega_z)^T$ after cross-coupling compensation [7].

### *C. Extended Kalman Filter*

The Extended Kalman Filter (EKF) is a commun estimator for data fusion [7]. The main interest of this filter is its ability to deal with non linear equations. Fig. 3 shows an overview of the EKF algorithm.

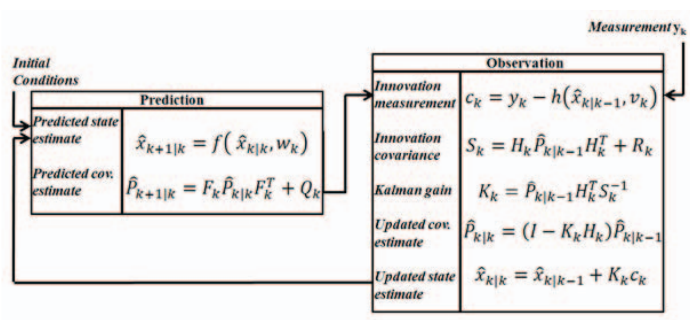


Fig. 3: Extended Kalman Filter overview

In our approach, the state vector $x$ is composed of 11 elements and the observation vector $y$ is composed of 8 measurements.

$$\begin{aligned} x &= [\dot{\phi}, \dot{\theta}, \dot{\psi}, \phi, \theta, \ddot{x}_n^b, \ddot{y}_n^b, \ddot{z}_n^b, \dot{x}_n^b, b_{gx}, b_{gy}]^T \\ y &= [\omega_x, \omega_y, \omega_z, f_x, f_y, f_z, \phi_{inc}, \theta_{inc}, v_{vel}]^T \end{aligned} \quad (5)$$

Where $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ are the rates of the roll, pitch and yaw angles, $(\phi, \theta)$ are the roll and pitch angles, $(\ddot{x}_n^b, \ddot{y}_n^b, \ddot{z}_n^b)$ are the linear accelerations of the train defined in the body frame, $\dot{x}_n^b$ is the velocity of the train defined in the body frame and $(b_{gx}, b_{gy})$ are the $X$-axis and $Y$-axis biases of the gyrometer. These vectors are integrated in the functions $f$ and $h$ of the Kalman algorithm. The prediction model, i.e. the function $f$, is defined by the system of equations (6):

$$\begin{aligned} \dot{\phi}_{k+1} &= \dot{\phi}_k + w_{\dot{\phi}_k} \\ \dot{\theta}_{k+1} &= \dot{\theta}_k + w_{\dot{\theta}_k} \\ \dot{\psi}_{k+1} &= \dot{\psi}_k + w_{\dot{\psi}_k} \\ \phi_{k+1} &= \Delta t * \dot{\phi}_k + \phi_k + w_{\phi_k} \\ \theta_{k+1} &= \Delta t * \dot{\theta}_k + \theta_k + w_{\theta_k} \\ \ddot{x}_{n_{k+1}}^b &= \ddot{x}_{n_k}^b + w_{\ddot{x}_k} \\ \ddot{y}_{n_{k+1}}^b &= \dot{x}_{n_k}^b \dot{\psi}_k + w_{\ddot{y}_k} \\ \ddot{z}_{n_{k+1}}^b &= \dot{x}_{n_k}^b \dot{\theta}_k + w_{\ddot{z}_k} \\ \dot{x}_{n_{k+1}}^b &= \Delta t * \ddot{x}_{n_k}^b + \dot{x}_{n_k}^b + w_{\dot{x}_k} \\ b_{gx_{k+1}} &= b_{gx_k} + w_{b_{gx_k}} \\ b_{gy_{k+1}} &= b_{gy_k} + w_{b_{gy_k}} \end{aligned} \quad (6)$$

Where $\Delta t$ is the sampling period of the Kalman filter and $w_k$ represents the process noise defined by the covariance matrix $Q$. By the same way, the observation model, i.e. the function $h$, is defined by the system of equations (7):

$$\begin{aligned} \omega_{x_k} &= \dot{\phi}_k - sin\theta_k \dot{\psi}_k + b_{gx_k} + \nu_{\omega_{x_k}} \\ \omega_{y_k} &= cos\phi_k \dot{\theta}_k + sin\phi_k cos\theta_k \dot{\psi}_k + b_{gy_k} + \nu_{\omega_{y_k}} \\ \omega_{z_k} &= -sin\phi_k \dot{\theta}_k + cos\phi_k cos\theta_k \dot{\psi}_k + \nu_{\omega_{z_k}} \\ f_{x_k} &= \ddot{x}_{n_k}^b - g_D sin\theta_k + \nu_{f_{x_k}} \\ f_{y_k} &= \ddot{y}_{n_k}^b + g_D sin\phi_k cos\theta_k + \nu_{f_{y_k}} \\ f_{z_k} &= \ddot{z}_{n_k}^b + g_D cos\phi_k cos\theta_k + \nu_{f_{z_k}} \\ \phi_{inc_k} &= \phi_k + \nu_{\phi_k} \\ \theta_{inc_k} &= \theta_k + \nu_{\theta_k} \\ v_{vel_k} &= \dot{x}_{n_k}^b + \nu_{v_{vel_k}} \end{aligned} \quad (7)$$

Where $g_D$ is the gravity and $\nu_k$ represents the measurement noise defined by the covariance matrix $R$. In order to conserve the covariance matrix $P$ symmetric and positive definite, a factorized form of the algorithm is used [7].

## III. EXPERIMENTAL RESULTS

### *A. Experimental Setup*

These experimentations have been carried out on an urban train from the company ZELC running on the metro network in Ankara (Turkish). The track route corresponds to the M1 line which is 14.7km long route and crosses 12 stations. All data have been recorded by the IMU SX43030 from MEGGITT Sensorex [8]. In the following figures, the results of our approach is compared to the data recorded by a high performances inertial navigation system, *HPINS*, (cost$\geq$ 100k$) and by the tachometers from the ETCS. Both systems can be considered as references. The presented results have been obtained after the train has run several times the line M1 for 1h30 corresponding to a covered distance of 36.768km.

## B. Results

Fig. 4 presents the relative errors on the longitudinal and centripetal accelerations (*HPINS* as reference). The RMS error is equal to 4mg (Max error 25mg) for a $\pm 140$mg acceleration range. The acceleration errors depend directly of the accuracy of the attitude estimations. Fig. 5 presents the relative errors on the estimation of the roll and pitch angles (*HPINS* as reference). We observe along the experimentation a RMS error equals to $0.2°$ (Max error $1.0°$) for a $\pm 6°$ angle range.

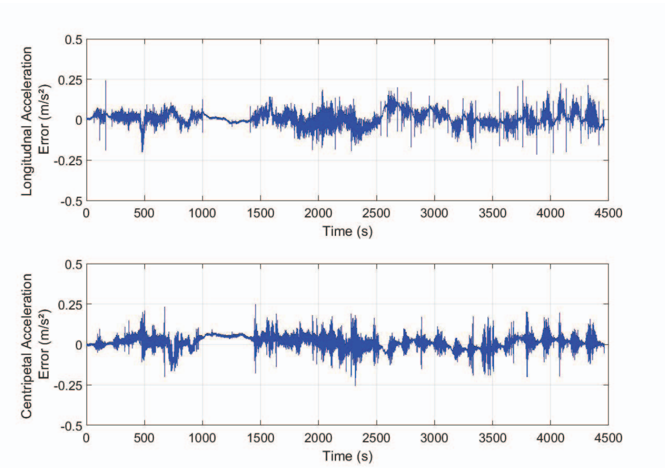


Fig. 4: Train acceleration errors during the experimentation

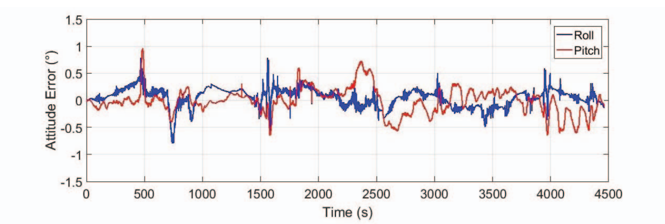


Fig. 5: Train attitude errors during the experimentation

Fig. 6 presents the velocity error with *Tachometers* as reference. During this experimentation, the RMS error is equal to 2.5m/s and it presents the interest to be centered on 0 even if an high maximum error of 8m/s is observed. Fig. 7 presents the relative errors (*Tachometers* as reference) on the covered distance on this experimentation. The covered distance is calculated by a double integration of the longitudinal acceleration estimate (*EKF(ACC)*) and a simple integration of the velocity estimate (*EKF(VEL)*). A comparison is also possible with the covered distance calculated from the *HPINS*. Thanks to the centered error of the velocity estimate, we can observe an error equals to 6.4km (17.4%) on the covered distance after 1h30 running while the double integrations of data from *HPINS* and longitudinal acceleration estimation tend to diverge.

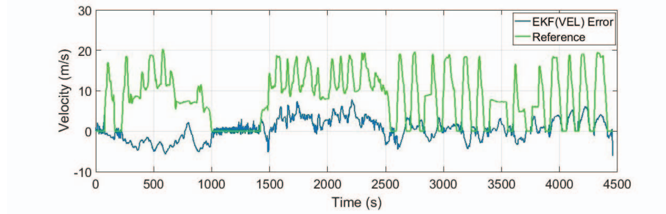


Fig. 6: Train velocity error observed during the experimentation

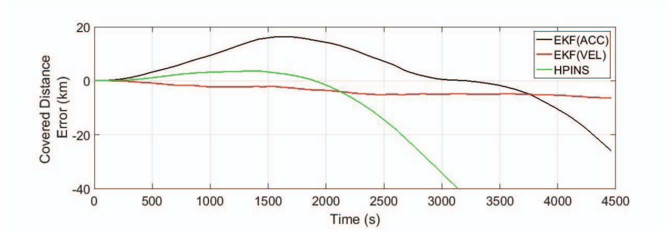


Fig. 7: Train covered distance error observed during the experimentation

## IV. Conclusion

In this paper, a state estimator method combining an Extended Kalman Filter with velocity observation based on a spectral analysis and attitude observations based on a complementary filter is presented in order to estimate the longitudinal acceleration of a train. The interest of this method is to propose a state estimation for train localisation without any external aiding source. The performances of the method have been tested on an urban train and results are compared to an estimation given by an high performances inertial navigation system (navigation grade) and the train tachometers. Finally, the performances during 1h30-experiment lead to a RMS error of 4mg on the longitudinal acceleration and an error of 6.4km on the covered distance.

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