



**HAL**  
open science

## Report on GRASTA 2017, 6th Workshop on GRaph Searching, Theory and Applications, Anogia, Crete, Greece, April 10 – April 13, 2017

Spyros Angelopoulos, Pierre Fraigniaud, Fedor V. Fomin, Nicolas Nisse,  
Dimitrios M. Thilikos

### ► To cite this version:

Spyros Angelopoulos, Pierre Fraigniaud, Fedor V. Fomin, Nicolas Nisse, Dimitrios M. Thilikos. Report on GRASTA 2017, 6th Workshop on GRaph Searching, Theory and Applications, Anogia, Crete, Greece, April 10 – April 13, 2017. [Research Report] LIP6, CNRS, Université Pierre et Marie Curie Paris, France; Department of Informatics, University of Bergen, Bergen, Norway; CNRS and Université Paris Diderot, France; Inria & Université Nice Sophia Antipolis, CNRS, I3S, Sophia Antipolis, France; ALGCo project team, CNRS, LIRMM, Montpellier, France. 2017. lirmm-01645614

**HAL Id: lirmm-01645614**

**<https://hal-lirmm.ccsd.cnrs.fr/lirmm-01645614v1>**

Submitted on 23 Nov 2017

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Report on GRASTA 2017

6th Workshop on GRAPh Searching, Theory and Applications,

Anogia, Crete, Greece, April 10 – April 13, 2017

Spyros Angelopoulos\*    Fedor Fomin<sup>†</sup>    Pierre Fraignaud<sup>‡</sup>

Nicolas Nisse<sup>§</sup>    Dimitrios M. Thilikos<sup>¶||</sup>

## 1 Introduction

Graph searching involves a team of mobile agents (called searchers or pursuers or cops) that aims at capturing a set of escaping agents (called evaders or fugitives or robbers) that hide in a network modeled by a graph. There are many variants of graph searching studied in the literature, often referred to as a *pursuit-evasion game* or *cops and robbers game*. These variants are either application driven, i.e. motivated by problems in practice, or are inspired by foundational issues at the intersection of Computer Science, Discrete Mathematics, and Artificial Intelligence. As a result, many researchers from different areas of Mathematics, Computer Science and Operations Research are interested in problems related to graph searching.

**GRASTA 2017** is the main forum on graph searching and was held at the Anogia Academic Village, Anogia, Crete, Greece.

It gathered **39** researchers from diverse areas of Theoretical Computer Science and Discrete Mathematics related to Graph Searching Games and their Applications both from the applied and the theoretical point of view. The scientific program of the workshop consisted of 18 regular talks, and an open problem session.

### 1.1 Objectives

Graph searching was introduced by Breisch (Southwestern Cavers Journal 1967) who proposed a “speleotopological” approach for the problem of finding an explorer who is lost in

---

\*LIP6, CNRS, Université Pierre et Marie Curie Paris, France.

<sup>†</sup>Department of Informatics, University of Bergen, Bergen, Norway.

<sup>‡</sup>CNRS and Université Paris Diderot, France.

<sup>§</sup>Inria & Université Nice Sophia Antipolis, CNRS, I3S, Sophia Antipolis, France.

<sup>¶</sup>AIGCo project team, CNRS, LIRMM, Montpellier, France.

<sup>||</sup>Department of Mathematics, National and Kapodistrian University of Athens, Athens, Greece.

a complicated system of dark caves (see the recent book [Breisch11]). The first mathematical models on Graph Searching were then introduced by Torrence Parsons and Nikolai Petrov in the 70's (e.g., [Parsons78]) while the first variants, along with the corresponding algorithmic and complexity results, appeared during the 80's [MGH+88].

Graph searching revealed the need to express in a formal mathematical way intuitive concepts such as *avoidance*, *surrounding*, *sense of direction*, *hiding*, *persecution*, and *threatening*. Such a project led to the study and introduction of various complicated combinatorial structures. One of the most powerful combinatorial tools used in the study of such structures emerged from the Graph Minors theory, developed by Robertson and Seymour towards proving the long-standing Wagner's Conjecture [RobertsonS85]. The collection of results and methodologies derived from the Graph Minors Theorem are acknowledged as among the most influential results in modern combinatorics. They include deep graph-theoretic results and techniques with direct consequences to problems at the kernel of Graph Searching problems (e.g., [SeymourT93]).

The graph searching games may vary significantly according to the capabilities of the evaders and the pursuers in terms of relative speed, sensor capabilities, visibility, etc. Also, the notion of capture itself admits several interpretations. Therefore, many variants have been studied in the literature [FominT08]. A different, and somehow independent, branch of research on graph searching is the *Cops and Robber games* defined by Winkler and Nowakowski, and independently by Quilliot, in 1983. In this variant, Meyniel conjectured in 1985 that the number of cops needed to capture a robber is  $O(\sqrt{n})$  in any connected  $n$ -node graph. During the last few years, a huge effort of research has been devoted to prove this conjecture which is still open (e.g., see [BKL,BonatoN11,ScottS11]). We do hope that the Workshop will bring us a bit closest to the solution.

Several variants are motivated by problems in practice. For instance, in the seminal variant of Parsons, the problem can be also formulated as the problem of clearing a contaminated network (e.g., by some poisonous gas). The Cleaning with Brushes variant arises from the need to have robots clean networks with conditions that do not allow access to humans (e.g. cleaning the cooling pipes in a nuclear power plant, or cleaning biofilm from small pipes). In what follows, we mention some of the existing applications (practical and fundamental) of Graph Searching.

- *Information Seeking*: Here the searchers represent information sharing models or mobile software agents that are looking for information. Information can be hidden, migrating, moving, and evolving and therefore can be viewed as one or more potential *evaders* from the searchers.
- *Robot motion planning*: Motion planning is one of the central problem in the development of autonomous robots. Can a robot plan its route to achieve a certain goal and to avoid colliding with other robots? Can a team of robots detect a mobile intruder or guard some area from intrusion? To address these type of questions, pursuit-evasion games are the natural setting.
- *Graph Theory*. One of the most powerful combinatorial tools for analyzing cops-and-robbers games emerged from the Graph Minors theory, developed by Robertson and

Seymour towards proving the long-standing Wagner’s Conjecture. The collection of results and methodologies derived by this project are acknowledged as among of the most influential results in modern combinatorics. They include deep graph-theoretic results and techniques with direct consequences to problems at the kernel of the cops-and-robbers games.

- *Database Theory and Cops and Marshals Games:* Among the (practically) most important database query mechanisms are *conjunctive queries*. While general conjunctive query evaluation is NP-complete, Yannakakis proved that it can be done in polynomial time if the queries are acyclic. One of the most convincing concepts for generalising the notion of acyclicity for conjunctive queries has been introduced by Gottlob et al. with the concept of hypergraph decompositions, in particular hypertree-width. Hypertree-width is an adaptation of tree-width to hypergraphs and it has been shown that conjunctive queries of bounded hypertree-width can be evaluated in polynomial time. An elegant and intuitive way to understand hypertree-width is based on Robber and Marshal games, an adaptation of graph searching to hypergraphs. Robber and Marshal games provide valuable insight into hypertree-decompositions and naturally yield a notion of obstructions to small hypertree-width in forms of hyperbrambles.

Following Feder and Vardi’s observation, that conjunctive query evaluation, the graph homomorphism problem and constraint satisfaction problems are essentially the same problem, hypergraph decompositions and hence Robber and Marshal games have found applications in constraint satisfaction also.

- *Game Theory:* From a game-theoretic point of view, graph searching has a very intuitive and useful formulation as a two-player zero-sum game over either pure or mixed strategies in which a mobile *searcher* must locate a *hider* (who may be either mobile or immobile). More precisely, the searcher’s strategy space consists of all possible trajectories it can follow in the environment, whereas the hider’s strategy space consists of all points in which it can hide. Given specific searcher and hider strategies, the cost assigned to the searcher is the time it takes to locate the hider, and can be seen as the game-theoretic payoff to the maximizing hider. The game-theoretic framework offers a simple, yet very inclusive abstraction of search games that has led to an impressive body of work (see, e.g., the textbook [AG03]). The main objective in the study of search games is to determine the value of the game (i.e., the payoff at Nash equilibrium). More recently, algorithmic and computational aspects of search games have also become the topic of study; more precisely, one is interested in polynomial-time strategies that are either optimal, or an approximation of the optimal value of the game. A similar game-theoretic abstraction can be applied in the study of *patrolling* problems: Here, the game involves two players, the *patroller* and the *attacker*, over an environment that can be formulated either as a graph or as a network.
- *Bounded-resource reasoning in AI:* Search problems, even on simple environments such as a set of concurrent rays, offer an abstraction that applies naturally in several

settings in which we seek an intelligent allocation of resources to different tasks. For instance, they capture the decision-making aspect of a management strategy where the objective is to successfully complete at least one task, without knowing in advance the completion time of each task. One such application stems from the area of *bounded-resource reasoning* in artificial intelligence. More specifically, we are interested in algorithms that will return a satisfactory solution (given the constraints of the setting), even if interrupted during their execution; we call these algorithms *interruptible*. It turns out that this problem can be formulated as a *scheduling* problem that has striking similarities with the *star search* or *ray search* problem (which, in turn, is a generalization of the well-known *cow path problem*). For some recent exploration of connections between these problems, see [A15].

- *Logic*: Computational aspects of logical systems are intensively studied in areas such as databases, artificial intelligence and verification. For instance, current approaches to hard- and software verification rely on efficient methods for evaluating logical formulas in process models, i.e. in graphs. Games have always played an important role in logic, for instance in the use of Ehrenfeucht-Fraïssé or pebble games for comparing models of logical formulas, or, more recently, the use of model-checking games as a game based approach to the evaluation problem of logical systems. Among evaluation games, parity games modeling the evaluation problem of the modal  $\mu$ -calculus are perhaps the most prominent and the precise complexity of deciding the winner of a parity game is the most important problem in this area, with significant applications to the theory of verification.

While model-checking games differ in some aspects from graph searching games, they share a core of common methods and problems and it seems likely that there are fruitful connections between the two areas. For instance, Berwanger and Grädel use a graph searching game, called Robber and Detective game, as a tool to analyse the model  $\mu$ -calculus variable hierarchy.

- *Distributed Computing*: currently, Graph Searching is mostly tackled using centralized methods. Nevertheless, recent advances in Mobile Computing enable to envisage tackling graph searching problems in a distributed framework. This framework is in fact the natural one for many applications of graph searching, including network security and decentralized network control.
- *VLSI design*: Circuit design is directly connected to different variants of graph searching. In each such variant, the target is to improve the way a graph (representing a circuit) can be embedded in a specific pattern taking into account different optimization criteria.
- *Models of computation*: The graph represents a computation circuit, and searching the graph is associated with pebble games on the graph that capture various computational complexity measures.
- *Routing in telecommunication networks*: To optimize the usage of resources with the evolution of the traffic in telecommunication networks, it may be necessary to change

the configuration (set of routes of the connections) of the network. It is then required to first determine the new configuration and then to schedule necessary changes to switch from the current configuration to the new one, while limiting possible traffic perturbations to customers (traffic disruption). Coudert *et al.* proposed a modelization of this problem in terms of a graph searching problem in directed graphs. This formulation allowed to provide solutions and tradeoffs for the routing reconfiguration problem.

- *Network security*: Applications of this type concern clearing a network of pipes contaminated by some poisonous gas, capturing intruders resorting in a building or in a road network, disease control, robot motion co-ordination, and virus elimination problems. Franklin, Galil, and Yung used graph searching to model the problem where a set of eavesdroppers is trying to collect information hidden in nodes of a network.

## References

- [A15] S. Angelopoulos. Further connections between contract-scheduling and ray-searching problems. Proceedings of the *24th* International Joint Conference on Artificial Intelligence (IJCAI), pp. 1516–1522, 2015.
- [AG03] S. Alpern and S. Gal. The theory of search games and rendez vous. Kluwer Academic Publishers, 2003.
- [BKL] B. Bollobas, G. Kun, I. Leader, Cops and robbers in a random graph, preprint.
- [BonatoN11] A. Bonato and R. Nowakowski, The Game of Cops and Robbers on Graphs. American Mathematical Society, 2011.
- [Bresich11] R. L. Breisch, Lost in a Cave - Applying Graph Theory to Cave Exploration. 298 pages, National Speological Society, 2011.
- [FominT08] F.V. Fomin and D.M. Thilikos, An annotated bibliography on guaranteed graph searching. Theor. Comput. Sci. 399(3): 236-245, 2008.
- [MHG+88] N. Megiddo, S.L. Hakimi, M.R. Garey, D.S. Johnson, C.H. Papadimitriou, The complexity of searching a graph, J. Assoc. Comput. Mach. 35 pp. 18–44, 1988.
- [Parsons78] T.D. Parsons, Pursuit-evasion in a graph, in: Theory and applications of graphs, in: Lecture Notes in Math., vol. 642, Springer, Berlin, pp. 426–441, 1978.
- [RobertsonS85] N. Robertson, P.D. Seymour, Graph minors – a survey, Surveys in combinatorics, London Math. Soc. Lecture Note Ser., vol. 103, Cambridge Univ. Press, pp. 153–171, 1985.
- [ScottS11] A. Scott and B. Sudakov, A new bound for the cops and robbers problem, SIAM J. Discrete Math. 25(3): 1438-1442, 2011.
- [SeymourT93] P.D. Seymour, R. Thomas, Graph searching and a min-max theorem for tree-width, J. Comb. Theory Ser. B, 58, pp. 22–33, 1993.

## 1.2 Previous editions

This workshop would be the 8th edition of the following fruitful stream of meetings:

- 7th GRASTA 2015: October 19-23rd, 2015, Montréal, Canada
- 6th GRASTA 2014: March 31st–April 4th, 2014, Cargèse, Corsica, France
- 5th GRASTA 2012: October 8–12, 2012, BIRS, Alberta, Canada
- 4th GRASTA 2011: February 13 – 18, 2011, Schloss Dagstuhl, Germany
- 3rd GRASTA 2009: 5-9 October 2009, Valtice, Czech Republic
- 2nd GRASTA 2008, 25-28 February 2008, Praia da Redonda, Brazil
- 1st GRASTA 2006, 9-12 October 2006, Anogia, Crete, Greece

## 1.3 Committees

### Organizing Committee:

*Spyros Angelopoulos*, LIP6, CNRS, Université Pierre et Marie Curie Paris, France  
*Dimitrios M. Thilikos*, AIGCo project-team, CNRS, LIRMM, France

### Scientific Committee:

*Spyros Angelopoulos*, LIP6, CNRS, Université Pierre et Marie Curie Paris France  
*Fedor V. Fomin*, University of Bergen, Norway  
*Pierre Fraigniaud*, CNRS and University Paris Diderot, France  
*Nicolas Nisse*, Inria, Université Nice Sophia Antipolis, CNRS, I3S, UMR 7271, Sophia Antipolis, France  
*Dimitrios M. Thilikos*, AIGCo project-team, CNRS, LIRMM, France

### Local organizers:

*Marina Vasilaki*, *Charalampos Tampakopoulos*, *Dimitrios M. Thilikos*, *Dimitris Zoros*.  
Graduate Program in Logic, Algorithms and Computation, (MPLA)  
Inter-university Postgraduate Program “Algorithms, Logic, and Discrete Mathematics”,  
(ALMA)  
Department of Mathematics, National and Kapodistrian University of Athens  
isoftcloud: <http://www.isoftcloud.gr/>

## 2 Program

<b>Monday, April 10th</b>	
08 : 30	<b>Registration, Welcome</b>
09 : 00	
09 : 00	<ul style="list-style-type: none"> <li>• Cops and Robber in geometric intersection graphs</li> </ul>
10 : 00	<i>Anthony Bonato, Ryerson University, Canada</i>
10 : 00	<i>Coffee Break</i>
10 : 30	
10 : 30	<ul style="list-style-type: none"> <li>• Cops and Robbers with Gangs</li> </ul>
11 : 00	<i>Nancy Clarke, Acadia University, Canada</i>
11 : 00	<ul style="list-style-type: none"> <li>• Cops, Robber and Medianwidth Parameters</li> </ul>
11 : 30	<i>Konstantinos Stavropoulos, RWTH Aachen, Germany</i>
13 : 00	<b>Lunch</b>
14 : 00	
15 : 00	<ul style="list-style-type: none"> <li>• Optimal Strategies for Weighted Search Problems</li> </ul>
16 : 00	<i>Konstantinos Panagiotou, LMU Munich, Germany</i>
16 : 00	<i>Coffee Break</i>
16 : 30	
16 : 30	<ul style="list-style-type: none"> <li>• Connected search against a lazy robber</li> </ul>
17 : 00	<i>Christophe Paul, AIGCo project-team, CNRS, LIRMM, France</i>
17 : 00	<ul style="list-style-type: none"> <li>• Cop number of string graphs</li> </ul>
17 : 30	
<b>Tuesday, April 11th</b>	
09 : 00	<ul style="list-style-type: none"> <li>• Graph Exploration Algorithms</li> </ul>
10 : 00	<i>Dariusz Dereniowski, Gdansk University of Technology, Poland</i>
10 : 00	<i>Coffee Break</i>
10 : 30	
10 : 30	<ul style="list-style-type: none"> <li>• Online graph exploration</li> </ul>
11 : 00	<i>Pascal Schweitzer, RWTH Aachen, Germany</i>
11 : 00	<ul style="list-style-type: none"> <li>• Searching for a Hidden Flaw: Some Results on Minesweeper of 3-Coloring</li> </ul>
11 : 30	<i>Michael Fellows, University of Bergen, Norway</i>
11 : 30	<ul style="list-style-type: none"> <li>• Those Magnificent Blind Cops in Their Flying Machines with Sonars</li> </ul>
12 : 00	<i>Przemyslaw Gordinowicz, Technical University of Łódź, Poland</i>
13 : 00	<b>Lunch</b>
14 : 00	



<b>Wednesday, April 12th</b>	
08 : 30 09 : 00	<b>Registration, Welcome</b>
09 : 00 10 : 00	<ul style="list-style-type: none"> <li>• The sweeping of planar shapes: a geometric analogue of graph searching <i>Minko Markov, Sofia University, Bulgaria</i></li> </ul>
10 : 00 10 : 30	<i>Coffee Break</i>
10 : 30 11 : 00	<ul style="list-style-type: none"> <li>• Minimizing uncertainty in the proximity of moving agents <i>David Kirkpatrick, University of British Columbia, Canada</i></li> </ul>
11 : 00 11 : 30	<ul style="list-style-type: none"> <li>• Using a best response oracle to solve search games on graphs <i>Thomas Lidbetter, London School of Economics, UK</i></li> </ul>
13 : 00 14 : 00	<b>Lunch</b>
15 : 00 16 : 00	<ul style="list-style-type: none"> <li>• From Ants to Query Complexity <i>Amos Korman IRIF, CNRS, Université Paris Diderot, France</i></li> </ul>
16 : 00 16 : 30	<i>Coffee Break</i>
16 : 30 17 : 30	<b>Open Problems session</b>

<b>Thursdy, April 13th</b>	
09 : 00 10 : 00	<ul style="list-style-type: none"> <li>• Open problems on optimal patrolling <i>Akitoshi Kawamura, University of Tokyo, Japan</i></li> </ul>
10 : 00 10 : 30	<i>Coffee Break</i>
10 : 30 11 : 00	<ul style="list-style-type: none"> <li>• Patrolling continuous networks <i>Katerina Papadaki, London School of Economics, UK</i></li> </ul>
11 : 00 11 : 30	<ul style="list-style-type: none"> <li>• How to hunt an invisible rabbit on a graph <i>Petr Golovach, University of Bergen, Norway</i></li> </ul>
11 : 30 12 : 00	<ul style="list-style-type: none"> <li>• Games of selfish cops and robbers <i>Athanasios Kehagias, Aristotle University of Thessaloniki, Greece</i></li> </ul>
13 : 00 14 : 00	<b>Lunch</b>

### 3 Abstracts

**Speaker:** Anthony Bonato

**Title:** Graph Searching Games and Probabilistic Methods

**Abstract:** The intersection of graph searching and probabilistic methods is a new topic within graph theory, with applications to graph searching problems such as the game of Cops and Robbers and its many variants, Firefighting, graph burning, and acquaintance time. Graph searching games may be played on random structures such as binomial random graphs, random regular graphs or random geometric graphs. Probabilistic methods may also be used to understand the properties of games played on deterministic structures. A third and new approach is where randomness figures into the rules of the game, such as in the game of Zombies and Survivors. We give a broad survey of graph searching and probabilistic methods, highlighting the themes and trends in this emerging area.

- The talk is based on my upcoming book (with the same title) co-authored with Pawel Pralat (to be published by CRC Press in late 2017).

**Speaker:** Nancy Clarke

**Title:** Cops and Robbers with Gangs

**Abstract:** A variation of the Cops and Robber game is considered in which the robber side consists of “gangs” of robbers that can win by attacking a cop. We present results for gangs of two robbers on graphs of small girth.

- This is joint work with A. Sanaei.

**Speaker:** Dariusz Dereniowski

**Title:** Graph Exploration Algorithms

**Abstract:** The aim of this talk is to survey some recent results and models of graph exploration. The problem is usually formulated in such a way that a single or multiple mobile agents are placed in a graph and their task is to visit all of its vertices. The goal is to design an algorithm that dictates movements of agents. Typical optimization criteria involve the number of agents or exploration time.

**Speaker:** Michael Fellows

**Title:** Searching for a Hidden Flaw: Some Results on Minesweeper of 3-Coloring

**Abstract:** In the folklore of puzzle-type computer game design there is a fruitful approach based on handles(H) + NP-hard problems(X). A handle is the interaction dynamic or basic interaction script. Examples include the on-line-with-a-wave handle (“Tetris”) and the hidden-solution-with-a-flaw handle (“Minesweeper”). This approach poses many interesting and unexplored algorithms and complexity challenges. The talk will briefly survey the approach and report on some preliminary results concerning the H+X: Minesweeper of 3-COLORING.

- The talk is based on unpublished work with Vladimir Estivill-Castro and Frances Rosamond.

**Speaker:** Petr Golovach

**Title:** How to hunt an invisible rabbit on a graph

**Abstract:** We investigate Hunters & Rabbit game, where a set of hunters tries to catch an invisible rabbit that slides along the edges of a graph. We show that the minimum number of hunters required to win on an  $(n \times m)$ -grid is  $\lfloor \min n, m/2 \rfloor + 1$ . We also show that the extremal value of this number on  $n$ -vertex trees is between  $\Omega(\log n / \log \log n)$  and  $O(\log n)$ .

• The talk is based on the paper: Tatjana V. Abramovskaya, Fedor V. Fomin, Petr A. Golovach, Michal Pilipczuk. How to hunt an invisible rabbit on a graph. *Eur. J. Comb.* 52: 12-26 (2016)

**Speaker:** Przemysław Gordinowicz

**Title:** Those Magnificent Blind Cops in Their Flying Machines with Sonars

**Abstract:** Inspired by the localisation problems in wireless networks we study the variation of Cops and Robber model, in which helicopter cops plays against a slow, but invisible robber. Instead, cops receive the information of a distance from robber's current position to vertices occupied by the cops. The goal for cops is to localise the robber. This model, restricted to one cop, was introduced by Seager (with slightly different rules) and then by Carraher, Choi, Delcourt, Erickson and West. Here we investigate the cop number in this model providing some bounds of it, developed from other graph parameters and giving examples of graph classes on which it is, surprisingly, unbounded (incl. planar graphs).

• The talk is based on still being developed work with B. Bosek, J. Grytczuk, P. Naroski, J. Sokół and M. Śleszyńska-Nowak.

**Speaker:** Akitoshi Kawamura

**Title:** Open problems on optimal patrolling

**Abstract:** In patrolling problems, several mobile agents move on a graph (with edge lengths) and try to cooperate so that every specified point on the graph is (perpetually) visited sufficiently often (that is, no point should be left unattended for a long time). Problems of this kind are studied with various motivations and in various forms: the agents may have the same or different speeds; the underlying graph may be a path, a cycle, a tree, or more general graphs; the points to be visited may be just the vertices or all points on the edges. Finding an optimal patrolling schedule is not straightforward, even in the simplest settings. I will introduce some results and open questions about properties of and algorithms for optimal patrolling.

**Speaker:** Athanasios Kehagias

**Title:** Games of selfish cops and robbers

**Abstract:** I examine variants of the cops and robber game in which is two cops are independently trying to catch a robber. I first consider the variant in which the robber is mobile but does not actively attempt to evade capture. This results in a two-player, zero-sum game between the two cops. I consider two versions of the game: in the first, "sequential" version the players move one after the other and in the second, "concurrent" version they move simultaneously. For both versions I prove that the game possesses a minimax value and optimal strategies which attain the value. I also provide algorithms for the computation of these quantities. I then consider the variant in which the robber

actively attempts to evade capture. This results in a three-player, non-zero-sum game. I only consider the “sequential” version of the game, I prove that the game possesses a Nash equilibrium and optimal strategies and provide an algorithm for the computation of these quantities.

**Speaker:** David Kirkpatrick

**Title:** Minimizing uncertainty in the proximity of moving agents

**Abstract:** Search problems are conventionally understood in terms of minimizing uncertainty in the location of one or more agents (perhaps moving in an adversarial fashion), using some form of spatial queries. We consider, and encourage further consideration of, a natural modification in which the goal is to reduce uncertainty, attributable to unmonitored motion, concerning the proximity, rather than absolute location, of a collection of moving agents. For example, imagine a collection of point agents moving in a graph (or some subset of one-, or higher, dimensional Euclidean space) each with some (known, but possibly different) upper bound on their speed. If we know, by means of an individual location query, the precise location of an individual agent at a particular time, then its location, until the time that it is next queried, lies in a steadily-expanding region of uncertainty. Motivated by the fact that resource demands are often commensurate with congestion (e.g. bandwidth allocation), we consider the problem of minimizing measures of potential congestion—maximum local congestion, over all agent configurations consistent with the current uncertainty of the collection—using individual queries that are restricted to one query per unit of time. Our focus to date has been on the degree of the uncertainty regions (defined as the maximum, over all agents  $a$ , of the number of uncertainty regions that intersect the uncertainty region of  $a$ ), a natural measure of worst-case congestion, for point agents moving continuously in one-, or higher, dimensional Euclidean space. The goal is to minimize this degree continuously. Competitive query strategies are described in terms of a notion of intrinsic degree (the minimum degree achievable by any query strategy, even one that knows the trajectories of all agents). Only partially studied to date are analogous questions for agents moving in a graph.

- Based on joint work with Daniel Busto, and Will Evans.

**Speaker:** Amos Korman

**Title:** From Ants to Query Complexity

**Abstract:** I will talk about my recent adventures with ants. Together with biologists we study *P. longicornis* ants as they collaboratively transport a large food item to their nest. This collective navigation process is guided by pheromones which are laid by individual ants. Using a new methodology to detect scent marks, we identify a new kind of ant trail characterized by very short and dynamic pheromone markings and highly stochastic navigation response to them. We argue that such a trail can be highly beneficial in conditions in which knowledge of individual ants regarding the underlying topological structure is unreliable. This gives rise to a new theoretical search model under unreliable guiding instructions, which is of independent computational interest. To illustrate the model, imagine driving a car in an unknown country that is in the aftermath of a major hurricane which has randomly flipped a certain small fraction of the road-signs. Under

such conditions of unreliability, how can you still reach your destination fast? I will discuss the limits of unreliability that allow for efficient navigation. In trees, for example, there is a phase transition phenomenon that occurs roughly around  $1/\sqrt{D}$ . That is, if noise is above this threshold then any algorithm cannot avoid finding the target in exponential time (in the original distance), while below the threshold we identify an optimal, almost linear, walking algorithm. Finally, I will discuss algorithms that under such a noisy model aim to minimize the number of queries to find a target (rather than the number of moves).

**Speaker:** Tomáš Gavenčíak

**Title:** Cop number of string graphs

**Abstract:** We show that for intersection graphs of curves on a surface, a finite number of cops can always catch the robber (in the Nowakowski-Quilliot Cops and Robber pursuit game).

• The talk is based on Gavenčíak, Gordinowicz, Jelinek, Klavik, Kratochvil: Cops and Robbers on String Graphs, ISAAC 2015.

**Speaker:** Thomas Lidbetter

**Title:** Using a best response oracle to solve search games on graphs

**Abstract:** We consider zero-sum search games between a Searcher and Hider. Such games are often played on a graph, where the Hider's strategy set is the set of vertices of the graph but the Searcher's strategy set is much larger and may correspond to the order in which she visits the vertices. In this case, conventional linear programming methods are not efficient for solving the game (that is, finding optimal mixed strategies and the value of the game). We assume there is an oracle that, for any fixed mixed (randomized) strategy of the Hider, outputs a best response strategy for the Searcher (or an "approximate" best response). Under this assumption, we give efficient algorithms for solving or "approximately" solving the game, and we give many examples of games for which this approach is useful.

• This is joint work with Lisa Hellerstein

**Speaker:** Minko Markov

**Title:** The sweeping of planar shapes: a geometric analogue of graph searching

**Abstract:** The presentation is based on the paper "Decontaminating Planar Regions by Sweeping with Barrier Curves" by Karaivanov, Markov, Snoeyink, and Vassilev. A novel concept of sweeping a planar shape by the coordinated movement of barriers is introduced. The corresponding measure, the sweepwidth, is proven to be NP-hard even for orthogonal polygons. It is still an open question how to prove there always exists a sweep that both avoids recontamination and is optimal. We present a technique for proving lower bounds on the sweepwidth of planar shapes that is a weighted analogue of a technique for proving lower bounds on the vertex separation of graphs.

**Speaker:** Katerina Papadaki

**Title:** Patrolling continuous networks

**Abstract:** We define a zero sum game between a patroller who wants to protect a continuous network and an attacker who wants to disrupt the network. The patroller picks

a walk on a network that she periodically repeats and the attacker picks a point on the network and a time to attack. The attack is successful if the attacker spends time  $r$  at the attack point uninterrupted by the patroller, otherwise the attack is intercepted. We present the value and optimal mixed strategies for various types of networks.

**Speaker:** Konstantinos Panagiotou

**Title:** Optimal Strategies for Weighted Search Problems

**Abstract:** Searching for a hidden target is an important algorithmic problem. We study the general setting in which a number of targets, each with a certain weight, are hidden in a star-like environment that consists of  $m$  infinite, concurrent rays, with a common origin. A mobile searcher, initially located at the origin, explores this environment in order to locate a set of targets whose aggregate weight is at least a given value  $W$ . The cost of the search strategy is defined as the total distance traversed by the searcher, and its performance is evaluated by the worst-case ratio of the cost incurred by the searcher over the cost of an optimal, offline strategy with access to the whole instance. This setting is a broad generalization of well-studied problems in search theory; namely, it generalizes the setting in which either a single target is sought, and the case in which all targets have unit weights. We consider two models depending on the amount of information allowed to the offline algorithm. In the first model, which is the canonical model in search theory, the offline algorithm has complete information. Here, we propose and analyze a strategy that attains optimal performance. In the second model, the offline algorithm has only partial information of the problem instance (i.e., the target locations). Here, we present a strategy of asymptotically optimal performance that is logarithmically related to  $m$ . This is in stark contrast to the full information model in which a linear dependency is unavoidable.

• This is joint work with Spyros Angelopoulos.

**Speaker:** Christophe Paul

**Title:** Connected search against a lazy robber

**Abstract:** The node search game against a lazy/agile (invisible) robber has been introduced as a search-game analogue of the graph parameters of treewidth/pathwidth. In the “connected” variants of the above two games, we additionally demand that, at each moment of the search, the “clean” territories are connected. The connected search game against an agile and invisible robber has been extensively examined. The monotone variant (where we also demand that the clean territories are progressively increasing) of this game, corresponds to the graph parameter of connected pathwidth and has been shown that its value cannot be more than the double (asymptotically) of its non-connected counterpart. This implied that the “price of connectivity” is bounded by 2 for the case of an agile robber. In this paper we initiate the study of the connected variant of this search game where the robber is lazy, in the sense that he/she moves only when the searcher’s strategy threatens the location that he/she currently occupies. We introduce two alternative graph-theoretical formulations of its monotone variant, one in terms of (connected) layouts and one on terms of (connected) tree decompositions, leading to the graph parameter of connected treewidth. For this “lazy-robber” variant we prove that there is no bound in the price of connectivity, which comes in contrast to the case of an agile robber. We

also observe that the corresponding parameter, i.e. connected treewidth, is closed under contractions and we study the contraction-obstruction set class of the class of graphs with connected treewidth at most  $k$ . It follows that this set is infinite for every  $k \geq 2$ . We also provide a complete characterisation for the case where  $k = 2$ .

- This is joint work with Isolde Adler and Dimitrios M. Thilikos.

**Speaker:** Pascal Schweitzer

**Title:** Online graph exploration

**Abstract:** We study the problem of exploring an unknown undirected connected graph. Beginning in some start vertex, a searcher must visit each node of the graph by traversing edges. Upon visiting a vertex for the first time, the searcher learns all incident edges and their respective traversal costs. The goal is to find a tour of minimum total cost. Kalyanasundaram and Pruhs proposed a sophisticated generalization of a Depth First Search that is 16-competitive on planar graphs. The description of the algorithm is not particular to planar graphs. I will explain some results for general graphs from joint work with Nicole Megow and Kurt Mehlhorn and highlight, however, that the main problem is unresolved to-date.

**Speaker:** Konstantinos Stavropoulos

**Title:** Cops, Robber and Medianwidth Parameters

**Abstract:** We present a generalisation of the classical Cops and Robber game where the robber plays not against one, but many teams of cops simultaneously. This is motivated as a game characterisation for medianwidth parameters, which correspond to high-dimensional generalisations of treewidth and pathwidth.

## 4 Open problems

### 4.1 Steve Alpern: A Simple Open Problem in Search Games on an Interval

The Princess and Monster Game on a network is a zero-sum game played between a Searcher (Monster, or Cop) and a mobile Hider (Princess, or Robber). The network is treated as a metric space, where each edge has a given length and distance is the length of the shortest path. We assume both players move with unit speed and no other restrictions, so that they can for instance turn around in the middle of an edge without slowing down. If the Searcher chooses path  $S(t)$ ,  $t \geq 0$ , and the Hider choose a path  $H(t)$ ,  $t \geq 0$  (including their starting points  $S(0)$  and  $H(0)$ ), the payoff to the maximizing Hider is the capture time  $T = T(S, H) = \min \{t : S(t) = H(t)\}$ . A mixed strategy is simply a probability distribution over these pure strategies. Now suppose that the network is simply the unit interval  $[0, 1]$ . What is the solution to this game? That is, what are the optimal mixed strategies for each player, and what is the Value  $V$  of the game (the expected value of  $T$  with best play on both sides)?

An example of a strategy for the Searcher would be an equiprobable mixture of left to right,  $S_+(t) = t$ , and right to left,  $S_-(t) = 1 - t$ . The optimal response to this is for the

Hider to start at  $1/2$  and stay there until just before the Searcher gets there, say till time  $1/2 - \varepsilon$ . At this time the Hider equiprobably goes to 0 or to 1. If the Hider is lucky, the capture time is  $T = 1$ ; if unlucky, the capture time is  $T = 1/2 - \varepsilon/2$ . Thus the expected value of  $T$  is  $3/4 - \varepsilon/4$ . But the Searcher has better strategies, so  $V < 3/4$ .

For references, see [1, 2].

## References

- [1] S. Alpern, R. Fokkink, R. Lindelauf, and G Olsder. The princess and monster game on an interval. *SIAM Journal on Control and Optimization*, 47(3):178–1190, 2008.
- [2] S. Alpern, R. Fokkink, R. Lindelauf, and G Olsder. A numerical approach to the ‘princess and monster’ game on an interval. *Advances in Dynamic Games and Their Applications*, pages 149–157, 2009.

### 4.2 Anthony Bonato: Some topological questions and conjectures on Cops and Robbers

It is known (see [1]) that planar graphs (graphs of genus 0) have cop number at most 3. Nancy Clark, in [2], proved that outerplanar graphs have cop number at most 2.

**Question:** Can we characterize planar (outer-planar) graphs with cop number 1,2, and 3? Is the dodecahedron the unique smallest order planar 3-cop-win graph?

Concerning graphs of higher genus, Schroeder’s conjecture states that if  $G$  has genus  $k$ , then  $c(G) \leq k + 3$ . In regards to this conjecture, the following is known: The conjecture is true for  $k = 0$ ; In [3] it was shown that it is true for  $k = 1$  (toroidal graphs); [4] showed that  $c(G) \leq 2k + 3$ ; and in [3] it was shown that  $c(G) \leq \lfloor \frac{3k}{2} \rfloor + 3$ .

Concerning the capture time of planar graphs, in [6], it was shown that if  $G$  is a connected planar graph and  $k \geq 12\sqrt{n}$  then  $\text{capt}_k(G) \leq 6 \cdot \text{rad}(G) \log n$ . The proof uses the Planar Separator Theorem [5], works also if robber has infinite speed, and generalizes to higher genus (only  $k$  changes, not the bound). Moreover, in [6], it was shown that if  $G$  is a connected planar graph of order  $n$ , then  $\text{capt}_3(G) \leq (\text{diam}(G) + 1)n$ .

**Question:** Can we give bounds on  $\text{capt}_2(G)$  if  $G$  is outerplanar? Are there examples of planar graphs with large 2,3, or even  $\sqrt{n}$ -capture times? Can we give an example of a planar  $G$  with  $\text{capt}_{2,3}(G) = \Theta(n^2)$ ?

## References

- [1] M. Aigner, M. Fromme, A game of cops and robbers, *Discrete Applied Mathematics* 8 (1984) 1–12.
- [2] N.E. Clarke, *Constrained Cops and Robber*, PhD thesis, Dalhousie University, 2002
- [3] B.S.W. Schroeder, The copnumber of a graph is bounded by  $\lfloor \frac{3}{2}\text{genus}(G) \rfloor + 3$ , *Categorical perspectives* (Kent, OH, 1998), Trends in Mathematics, Birkhäuser Boston, Boston, MA, 2001, 243–263.



- [4] A. Quilliot, Jeux et pointes fixes sur les graphes, *Thèse de 3ème cycle*, Université de Paris VI, 1978, 131–145.
- [5] N. Alon, P. Seymour, R. Thomas, Planar separators, *SIAM Journal on Discrete Mathematics*, **7** (1994) 184–193.
- [6] A. Bonato, X. Perez-Gimenez, P. Pralat, B. Reiniger The game of Overprescribed Cops and Robbers played on graphs, Preprint 2017.

### 4.3 Joshua Erde: The Angel and Devil game

**Statement of Problem** The Angel and Devil game is a two player game played on a rooted infinite directed graph  $G$ . In his turn the first player, the *Devil*, chooses a vertex to block, which remains blocked for the rest of the game. The second player, the *Angel*, starts at the root  $r$  and in his turn can move to any unblocked square that is at (graph-)distance at most  $p$  away from his current position, we call  $p$  the *power* of the Angel. The players alternate turns, and the Devil wins if he can trap the Angel, that is if he can force the Angel into a position where he has no available moves. The Angel wins if he has a strategy to keep moving forever.

Originally this game was considered in the undirected case on the  $l_\infty$  grid  $\mathbb{Z}^d$ , which is the strong product of  $d$  double rays. A number of authors [3, 6, 7, 8] independently showed that the angel of power 2 wins in  $\mathbb{Z}^2$ . Blass and Conway [1] showed that if the Angel must increase his  $y$  coordinate in each move then the Devil can beat the Angle of **any** power in  $\mathbb{Z}^2$ . Bollobás and Leader [2] considered an Angel in  $\mathbb{Z}^3$  that must always increase his  $z$  coordinate. They noted that this was equivalent to playing the game in  $\mathbb{Z}^2$  with a weaker devil, who on his turn cannot block a vertex forever, but only at some specific time step in the future. They called this game the *Time-Bomb game* and conjectured

**Conjecture:** [Bollobás and Leader] The Angel of power one wins the Time-Bomb game in  $\mathbb{Z}^2$

Recently the Angel-Devil game has been considered on the directed graphs  $\mathcal{P}^d$  obtained by taking the cartesian product of  $d$  directed rays. The results of [1] shows that the Devil wins against an Angel of any power in  $\mathcal{P}^2$ , and Clarke, Finbow, Fitzpatrick, Messenger and Nowakowski [4] showed, by considering an equivalent game, that the Devil wins against the Angel of power 1 in  $\mathcal{P}^3$ . However, Cranston, Kinnersley, Milans, Puleo and West [5] showed that for  $k \geq 14$  the Angel of power one wins on  $\mathcal{P}^k$ .

**Question:** What is the smallest  $k$  such that the Angel of power one wins on  $\mathcal{P}^k$ ?

Similar methods as in [5] would show that a positive answer to the above Conjecture would imply  $k \leq 8$ .

## References

- [1] E.R. Berlekam, J.J. Conway, and R.K. Guy. *Winning Ways for your Mathematical Plays, volume 2*. Academic Press, 1982.
- [2] B. Bollobás and I. Leader. The angel and the devil in three dimensions. *Journal of Combinatorial Theory, Series A*, **113**(1):176–184, 2006.

- [3] B.H. Bowditch. The angel game in the plane. *Combinatorics Probability and Computing*, **16**:345–362, 2007.
- [4] N.E. Clarke, S. Finbow, S.L. Fitzpatrick, M.E. Messenger, and R.J. Nowakowski. Seepage in directed acyclic graphs. *Australasian Journal of Combinatorics*, 43:91–102, 2009.
- [5] D.W. Cranston, W.B. Kinnersley, K.G. Milans, G.J. Puleo, and D.B. West. Chain-making games in grid-like posets. *Journal of Combinatorics*, **3**(4):633–649, 2012.
- [6] B.H. Gács. The angel wins. *Unpublished*, 2007.
- [7] O. Kloster. A solution to the angel problem. *Theoretical Computer Science*, **389**:152–161, 2007.
- [8] A. Máthé. The angel of power 2 wins. *Combinatorics, Probability and Computing*, **16**:363–374, 2007.

#### 4.4 Fedor V. Fomin: Complexity of connected search when the number of searchers is small

In the connected graph searching problem the task is to identify the minimum number of searchers sufficient to clear the graph such that at every step of searching the cleared area is connected. (For a formal definition, please see [1].) While (not connected) graph searching problem is fixed-parameter tractable with a standard parameterization by the number of searchers, for the connected search problem the situation is much more obscure. Can it be that the problem is NP-hard already for small values of searchers like 3 or 4? Similar questions are open for the monotone version of connected search.

## References

- [1] L. Barrière, P. Flocchini, F. V. Fomin, P. Fraigniaud, N. Nisse, N. Santoro, and D. M. Thilikos. Connected graph searching. *Information and Computation*, 219:1–16, 2012.

#### 4.5 Thanasis Kehagias: Multiplayer pursuit on graphs with a cyclic capture relation

**Problem statement**  $K$  players  $(P_0, P_1, \dots, P_{K-1})$  play a pursuit game on a finite, simple, undirected graph  $G$ . The game has the classic Cops and Robbers rules except for the use of a “Cyclic Capture Relationship”:  $P_0$  chases  $P_1$ ,  $P_1$  chases  $P_2$  etc..  $P_{K-1}$  chases  $P_0$ . More specifically, the rules are the following

- (a) The starting positions (vertices) of the players are given.
- (b) At the  $t$ -th turn the  $i$ -th player, where  $i = (t \bmod K)$ , has the move and the others stay in place.
- (c) The player who has the move, can move to a vertex in the closed neighborhood of his current position.

- (d)  $P_k$  wins (and the game ends) iff at the end of a turn he is in the same vertex as  $P_i$ , where  $i = (k + 1 \bmod K)$ . This is a “capture”. If no capture takes place (at any turn) the game is a draw. .

The basic is to “solve” the game, i.e., to establish the existence of a “reasonable” outcome and “reasonable” strategies for the players.

**Additional remarks** 1. To complete the description of the problem one must define an appropriate payoff function for each player. Many choices are possible. For example: if capture is effected by player  $P_k$ , he receives a payoff of 1 while all other players receive a payoff of -1. Other payoffs functions of a win/lose type can be used; furthermore payoffs may also depend on capture time.

2 Having defined a payoff function, the basic problem can be stated as follows: prove that the game possesses a Nash equilibrium (NE). The existence of a NE may depend on the cop number of the graph.

3. Additional problems include (but are not limited) to the following.

- (a) Prove the existence of Subgame Perfect Equilibrium (SPE).
- (b) Prove that every NE results in capture.
- (c) If (a) and/or (b) are not always true, Characterize graphs for which they hold.
- (d) Find an analog of the cop number.

**Related work:** A similar multiplayer pursuit game has been studied in [1, 2] Useful literature on multiplayer stochastic games includes [4, 3, 5].

## References

- [1] Athanasios Kehagias and Georgios Konstantinidis. Selfish cops and passive robber: Qualitative games. *arXiv:1607.05434*, 2016. Accepted for publication in Theoretical Computer Science.
- [2] Athanasios Kehagias and Georgios Konstantinidis. Selfish cops and adversarial robber: Multi-player pursuit evasion on graphs. *arXiv:1703.07695*, 2017.
- [3] Jean-François Mertens. Stochastic games. In *Handbook of game theory with economic applications* 3, pages 1809–1832. 2002.
- [4] E. Solan. Stochastic games. In *Encyclopedia of Database Systems*. Springer, 2009.
- [5] Nicolas Vieille. Stochastic games: Recent results. In *Handbook of game theory with economic applications* 3, pages 1833–1850. 2002.

## 4.6 Euripides Markou: The Bumblebee Visitation problem

**Statement of the problem** Consider a connected undirected graph  $G = (V, E)$ , where each node  $u \in V$  has been assigned a positive integer  $c(u)$ . A mobile agent  $A$  is able to move along the edges of the graph under the following rules. When agent  $A$  traverses an edge  $(u, v)$ , the numbers  $c(u)$  and  $c(v)$  decrease by 1 (we say that  $A$  collects a value 1 from  $u$  and a value 1 from  $v$ ). An edge  $(u, v)$  cannot be traversed if  $c(u) = 0$  or  $c(v) = 0$ .

Design an algorithm which takes as input a graph  $G$  with positive values at nodes and places and moves an agent  $A$  as above, so that a maximum total value is collected.

**Related work** The problem (decision version) is NP-hard in arbitrary graphs due to a reduction from the Hamilton Circuit problem: assign a value 2 to each node of the given graph  $G$ ; the graph has a Hamilton Circuit iff a total value of at least  $2n$  can be collected, where  $n$  is the number of nodes of  $G$ .

Consider the following variation of the problem: Design an algorithm which takes as input a graph  $G$  with positive values at nodes and places and moves an agent  $A$  as above, so that the total value *remained at visited nodes* is a maximum. Some open questions are as follows:

- Study the above problems in tree or other topologies.
- Design approximation algorithms.
- Study distributed scenarios with more than one mobile agents that might communicate at a cost and have limited information about the topology (e.g., local maps) and/or limited memory, etc.

The above problems can have applications like the following one: Consider a network. The nodes of the network can host an application but each node has to spend some energy in order to execute the application. Initially each node  $u$  has an energy  $c(u)$ . The application can migrate in the network from a node  $u$  to a node  $v$ , if nodes  $u, v$  are adjacent nodes. For the migration procedure, each one of the nodes  $u, v$  spends an energy 1. Where to start the execution of the application and what is the migration tour on the network so that the application can use a maximum energy?

**References** Those problems and their applications were mentioned to me by my colleague assist. prof. Thanasis Loukopoulos. We do not know of any related results.

## 4.7 Sebastian Siebertz: Parameterized complexity of generalized coloring numbers

The *colouring number*  $\text{col}(G)$  of a graph  $G$  is the minimum integer  $k$  such that there is a strict linear order  $<_L$  of the vertices of  $G$  for which each vertex  $v$  has *back-degree* at most  $k - 1$ , i.e. at most  $k - 1$  neighbours  $u$  with  $u <_L v$ . It is well-known that for any graph  $G$ , the chromatic number  $\chi(G)$  satisfies  $\chi(G) \leq \text{col}(G)$ .

Some generalisations of the colouring number of a graph have been studied in the literature. Three natural generalisations of the colouring number are the three series  $\text{adm}_r$ ,  $\text{col}_r$  and  $\text{wcol}_r$  of *generalised colouring numbers* introduced by Kierstead and Yang [7] in the context of colouring games and marking games on graphs. As proved by Zhu [7], these invariants are strongly related to low tree-depth decompositions [7], and can be used to characterise bounded expansion classes of graphs (introduced in [8]) and nowhere dense classes of graphs (introduced in [9]).

An interesting aspect of generalised colouring numbers is that these invariants can also be seen as gradations between the colouring number  $\text{col}(G)$  and two important minor monotone invariants, namely the *tree-width* and the *tree-depth*. More explicitly, for every graph  $G$  we have

$$\begin{aligned} \text{col}(G) &= \text{col}_1(G) \leq \text{col}_2(G) \leq \dots \leq \text{col}_\infty(G) = \text{tw}(G) + 1 \quad \text{and} \\ \text{col}(G) &= \text{wcol}_1(G) \leq \text{wcol}_2(G) \leq \dots \leq \text{wcol}_\infty(G) = \text{td}(G). \end{aligned}$$

See [2] for a structure theorem for graphs with bounded  $\text{adm}_\infty$ . While we can compute  $\text{col}(G)$  of a graph  $G$  in linear time, computing  $\text{col}_2(G)$  is NP-complete in general (this follows from Theorem 16 of [1]) and computing  $\text{wcol}_3(G)$  is NP-complete in general [4]. To the best of my knowledge it is open whether  $\text{wcol}_2(G)$  can be computed in polynomial time. Several approximations, e.g. on bounded tree-width graphs [4], on planar graphs and graphs with excluded minors [10] or on graphs with excluded topological minors [6] are known. It is known how to compute  $\text{adm}_r(G)$  in time  $f(\text{adm}_r(G)) \cdot n$  on an  $n$ -vertex graph  $G$  [3].

**Question:** Is it possible to compute  $\text{col}_r(G)$  and  $\text{wcol}_r(G)$  of an  $n$ -vertex graph in time  $n^{f(\text{col}_r(G))}$  and  $n^{g(\text{wcol}_r(G))}$ , respectively, for computable functions  $f, g$ ?

## References

- [1] Nick D Dendris, Lefteris M Kirov, and Dimitrios M Thilikos. Fugitive-search games on graphs and related parameters. *Theoretical Computer Science*, 172(1-2):233–254, 1997.
- [2] Zdenek Dvorak. A stronger structure theorem for excluded topological minors. *arXiv preprint arXiv:1209.0129*, 2012.
- [3] Zdeněk Dvořák. Constant-factor approximation of the domination number in sparse graphs. *European Journal of Combinatorics*, 34(5):833–840, 2013.
- [4] Martin Grohe, Stephan Kreutzer, Roman Rabinovich, Sebastian Siebertz, and Konstantinos Stavropoulos. Colouring and covering nowhere dense graphs. In *International Workshop on Graph-Theoretic Concepts in Computer Science*, pages 325–338. Springer, 2015.
- [5] H.A. Kierstead and D. Yang. Orderings on graphs and game coloring number. *Order*, 20:255–264, 2003.
- [6] Stephan Kreutzer, Michal Pilipczuk, Roman Rabinovich, and Sebastian Siebertz. The generalised colouring numbers on classes of bounded expansion. In *41st International*

*Symposium on Mathematical Foundations of Computer Science, MFCS 2016, August 22-26, 2016 - Kraków, Poland*, pages 85:1–85:13, 2016.

- [7] J. Nešetřil and P. Ossona de Mendez. Tree-depth, subgraph coloring and homomorphism bounds. *European J. Combin.*, 27:1022–1041, 2006.
- [8] J. Nešetřil and P. Ossona de Mendez. Grad and classes with bounded expansion I. Decompositions. *European J. Combin.*, 29:760–776, 2008. doi:10.1016/j.ejc.2006.07.013.
- [9] J. Nešetřil and P. Ossona de Mendez. On nowhere dense graphs. *European J. Combin.*, 32:600–617, 2011.
- [10] Jan van den Heuvel, Patrice Ossona de Mendez, Roman Rabinovich, and Sebastian Siebertz. On the generalised colouring numbers of graphs that exclude a fixed minor. *Electronic Notes in Discrete Mathematics*, 49:523–530, 2015.
- [11] Xuding Zhu. Colouring graphs with bounded generalized colouring number. *Discrete Mathematics*, 309:5562–5568, 2009.

## 5 Participants

• Steve Alpern, Warwick Business School • Spyros Angelopoulos, LIP6, CNRS, Université Pierre et Marie Curie Paris • Dietmar Berwanger, LSV, CNRS ENS de Cachan Paris • Anthony Bonato, Ryerson University • Brieuc Guinard, IRIF, CNRS Université Paris Diderot • Nancy Clarke, Acadia University • Dariusz Dereniowski, Gdansk University of Technology • Christoph Dürr, Paris 6 • Danny Dyer, Memorial University of Newfoundland • Joshua Erde, Univ Hamburg • Archontia Giannopoulou, Technical University of Berlin • Gavenčiak Tomáš, Charles University • Fedor Fomin, University of Bergen • Petr Golovach, University of Bergen • Przemyslaw Gordinowicz, Technical University of Lodz • Shendan Jin, UPMC Paris • Akitoshi Kawamura, University of Tokyo • Athanasios Kehagias, Aristotle University of Thessaloniki • David Kirkpatrick, University of British Columbia • Amos Korman, IRIF, CNRS Université Paris Diderot • Florian Lehner, University of Hamburg • Thomas Lidbetter, London School of Economics • Euripides Markou, University of Thessaly • Minko Markov, Sofia University • Fionn McInerney, INRIA, Université Nice Sophia Antipolis, CNRS, I3S • Fellows Mike, University of Bergen • Nicolas Nisse, INRIA, Université Nice Sophia Antipolis, CNRS, I3S • Konstantinos Panagiotou, LMU Munich • Katerina Papadaki, London School of Economics • Christophe Paul, AIGCo project-team, CNRS, LIRMM • Marc Renault, Paris 7 • Frances Rosamond, University of Bergen • Pascal Schweitzer, RWTH Aachen • Sebastian Siebertz, Technical University of Berlin • Konstantinos Stavropoulos, Rwth Aachen • Dimitrios Thilikos, AIGCo project-team, CNRS, LIRMM • Peter Widmayer, ETH Zurich • Sandra Zilles, Univ of Regina • Dimitris Zoros, Department of Mathematics, NKUA.

## 6 Workshop photo



Photo of the participants of GRASTA 2017, April 11, 2017

## 7 Other material

In the home page of GRASTA 2017 links to the following sources are available:

- [Photo Repository of GRASTA 2017](#)
- [Talk Repository of GRASTA 2017](#)