

# Combining logical and distributional methods in type-logical grammars

Richard Moot

► **To cite this version:**

Richard Moot. Combining logical and distributional methods in type-logical grammars. Journal of Language Modelling, Institute of Computer Science, Polish Academy of Sciences, Poland, In press. <lirmm-01651508>

**HAL Id: lirmm-01651508**

**<https://hal-lirmm.ccsd.cnrs.fr/lirmm-01651508>**

Submitted on 29 Nov 2017

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Combining logical and distributional methods in type-logical grammars

*Richard Moot*

CNRS (LIRMM), Montpellier University

## ABSTRACT

In this paper, we will look at a low-level way of combining distributional and logical ideas into a single formal system. This will be an instantiation of a more general system, adding weights to proof rules. These weights will not measure some sort of “confidence the proof is valid”, but rather act as a way to *prefer* some proofs over others, where preference can mean “easier to process (for humans)” or “more coherent (combining words that make sense together)”. The resulting system of weighted theorem proving can be implemented either as a best-first proof search strategy or as a polynomial-time approximation of proof search for NP-complete parsing problems.

*Keywords:*  
*Type-logical*  
*grammar, Lambek*  
*calculus,*  
*processing,*  
*theorem proving*

1

## INTRODUCTION

Type-logical grammar (and formal semantics in general) are agnostic about the meaning of atomic terms, such as those corresponding to nouns and verbs (though not about the meaning corresponding to words with logical content such as “not”, “and”, “all”, “which”). Another way to see this is that in standard formal semantics, entailment only holds under strict identity of predicates. As a consequence, practical use of the output of a system computing such formal semantics depends to a large extent on the available world knowledge (Bos and Markert 2005), possibly stated in the form of additional axioms or meaning postulates, stating that “pub” and “bar” (in one meaning of the word) are synonyms, and that “good and “bad” are antonyms, ie. “bad” entails “not good” and inversely.

In contrast to formal semantics in the tradition of Montague, distributional or vector-based semantics take semantic similarity, as measured by word cooccurrences, as their basic notion. Systems using only semantic similarity are agnostic about argument structure and agnostic about the meaning of words with logical content. This is generally referred to as the problem of compositionality in vector space semantics.

Whereas compositional formal semantics, unless augmented by specific lexical meanings or meaning postulates, concludes that “good” and “bad” are unrelated unary predicates, vector semantics concludes that “good” and “bad” are very similar. Other semantically similar words are “animal” and “veterinarian”, and “sweater” and “warm”. The absence of argument structure (who does what to whom) from the vectors makes “animal” and “veterinarian” similar: even though many sentences contain both words, these tend to be sentences where the veterinarian treats or examines the animal, or where the owner takes an animal to the veterinarian.

There appears to be some complementarity to these two approaches: formal semantics takes compositionality as its basic principle but has little to say about the meaning of individual predicates, vector semantics takes the similarity of predicates as its basic concept but then has little to say about compositionality and the words with logical content.

In this paper, we will look at a low-level way of combining distributional and logical ideas into a single formal system. This will be an instantiation of a more general system, adding weights to proof rules. These weights will not measure some sort of “confidence the proof is valid”, but rather act as a way to *prefer* some proofs over others, where preference can mean “easier to process (for humans)” or “more coherent (combining words that make sense together)”. The resulting system of weighted theorem proving can be implemented either as a best-first proof search strategy or as a polynomial-time approximation of proof search for NP-complete parsing problems.

## 2 TYPE-LOGICAL GRAMMAR AND FORMAL SEMANTICS

**Definition 2.1 (Type-logical grammar)** Given a logic  $\mathcal{L}$  with formulas  $\mathcal{F}$ , a type-logical grammar over  $\mathcal{L}$  is a tuple  $\langle \Sigma, Lex, goal \rangle$  where

1.  $\Sigma$  is the set of words in the language,
2. the lexicon  $Lex$ , is a function from  $w \in \Sigma$  to a (non-empty) subset of  $\mathcal{F}$ ,
3.  $goal$ , the set of goal formulas is a (non-empty) subset of  $\mathcal{F}$ ,
4.  $yield$  is a function from antecedents of  $\mathcal{L}$  to sequences of formulas,
5.  $h$  is a homomorphism from proofs in  $\mathcal{L}$  to proofs in multiplicative intuitionistic linear logic representing their “deep structure”.

We say a sentence  $w_1, \dots, w_n$  is *grammatical* if for all  $i$ ,  $w_i \in \Sigma$ ,  $A_i \in Lex(w_i)$ ,  $C \in goal$ , and  $\Gamma \vdash C$ , with  $yield(\Gamma)$  is  $A_1, \dots, A_n \vdash C$ , is a theorem of the logic  $\mathcal{L}$ . A sentence is *ungrammatical* otherwise.

Many authors choose the set  $\{s\}$  for  $goal$ . However, for more elaborate grammars, we may be interested not only in declarative sentences, but also in yes-no questions, *wh* questions, imperatives, etc., and it seems reasonable to allow such sentences to have a different type of meaning from declarative sentences.

For the Lambek calculus (Lambek 1958), the logic is  $L$ , the yield function is the identity function (since antecedents  $\Gamma$  of  $L$  are already sequences of formulas), and  $h$  translates the Lambek calculus slashes “/” and “\” to the multiplicative linear logic implication “ $\multimap$ ” (and the product “ $\bullet$ ” to multiplicative conjunction “ $\otimes$ ”).

For multimodal type-logical grammars (Moortgat 1997), sequents are of the form  $\Gamma \vdash C$  where the antecedent  $\Gamma$  is a labelled tree with unary and binary branches and with formulas as its leaves. The yield function is simply the left-to-right sequence of formulas occurring as its leaves (ie. we use the standard definition of the yield of a tree).

### 2.1 *The Lambek calculus*

To make this more concrete, we’ll instantiate the general type-logical grammar framework to Lambek’s Syntactic Calculus,  $L$  (Lambek 1958). Formulas of the Lambek calculus are inductively defined from a set of atomic formulas, including  $np$  (noun phrase),  $n$  (common

Figure 1:  
Proof rules and  
corresponding  
lambda term  
operations

$$\begin{array}{c}
 \frac{A/B : M^{U \rightarrow T} \quad B : N^U}{A : (MN)^T} /E \qquad \frac{B : N^U \quad B \setminus A : M^{U \rightarrow T}}{A : (MN)^T} \setminus E \\
 \\
 \dots [B : x^U]_i \qquad \qquad [B : x^U]_i \dots \\
 \vdots \qquad \qquad \qquad \vdots \\
 \frac{A : M^T}{A/B : \lambda x^U . M} /I_i \qquad \frac{A : M^T}{B \setminus A : \lambda x^U . M} \setminus I_i
 \end{array}$$

noun),  $s$  (sentence) and  $pp$  (prepositional phrase). A formula in the Lambek calculus is

- an atomic formula
- if  $A$  and  $B$  are formulas, then  $A/B$  (pronounced “A over B”, it looks for a  $B$  formula to its right to produce an  $A$ ),  $B \setminus A$  (pronounced “B under A”, it looks for a  $B$  formula to its left to produce an  $A$ ) are formulas<sup>1</sup>.

Figure 1 shows the natural deduction proof rules for the Lambek calculus (and the associated lambda term assignments).

The elimination rule for “/”, labeled “/E” states that if we have a proof with conclusion  $A/B$  which is assigned term  $M$  (of type  $U \rightarrow T$ ) and a proof with conclusion  $B$  which is assigned term  $N$  (of type  $U$ ), then we can combine these two proofs to form a proof of  $A$  which is assigned lambda-term  $(MN)$ . The order of the premisses is important:  $B$  must occur adjacent to and to the right of  $A/B$ . The elimination rule for  $\setminus$  is left-right symmetric, with  $B$  occurring to the immediate left of  $B \setminus A$ .

The introduction rule, labeled “/I”, states that if we have a proof of  $A$  with lambda-term  $M$  of some type  $T$ , which we have derived while using a hypothesis  $B$ , which is assigned a variable  $x$  of type  $U$  and which is the rightmost undischarged hypotheses of this proof, then we can discharge this  $B$  hypothesis to derive  $A/B$  of type  $U \rightarrow T$  with term  $\lambda x . M$ . The discharged hypothesis is co-indexed with the rule, using an index  $i$  unique to the proof (for the Lambek calculus without

<sup>1</sup>To keep the discussion simple, we do not present the natural deduction proof rules for the product  $A \bullet B$ , representing the concatenation of  $A$  and  $B$ .

product, this index is strictly speaking superfluous, since the leftmost and rightmost undischarged hypotheses are uniquely determined for each subproof). The introduction rule for  $\setminus I$  is again left-right symmetric, requiring  $B$  to be the leftmost undischarged hypothesis.

We will write  $A_1, \dots, A_n \vdash C$  for a proof with undischarged hypotheses  $A_1, \dots, A_n$  (in the given order) and conclusion  $C$ .

As an example, the following Lambek calculus proof shows that “moons which Galileo discovered” is a noun  $n$ . To make the proof more readable, the lexical entries have been indicated as the conclusions of a rule *Lex* with the word occurring above it and the corresponding formula assigned by the lexicon below it (we will add the lambda terms later).

$$\frac{\frac{\frac{\text{moons}}{n} \text{ Lex} \quad \frac{\frac{\text{which}}{(n \setminus n)/(s/np)} \text{ Lex} \quad \frac{\frac{\text{Galileo}}{np} \text{ Lex} \quad \frac{\frac{\text{discovered}}{(np \setminus s)/np} \text{ Lex} \quad [np]_1 /E}{np \setminus s} \setminus E}}{n \setminus n} \setminus E} \quad \frac{s}{s/np} /I_1 /E}{n} \setminus E}$$

We can read of the lexical assignments from the undischarged leaves of the proof above. So “discovered” is a transitive verb, looking for a noun phrase ( $np$ , its object) to its right, then for a noun phrase (its subject) to its left to form a sentence  $s$ . The relativiser “which” looks for a complex formula  $s/np$  (that is a sentence missing a noun phrase in its rightmost position) to its right and for a noun to its left. The hypothetical  $np$  corresponds to a trace in mainstream syntactic theory. A weakness of the Lambek calculus is that this analysis does not extend to only slightly more complicated examples such as “moons which Galileo discovered in 1610”, where the hypothetical noun phrase no longer occurs in a peripheral position. Many variants and extensions of the Lambek calculus have been developed with the goal of solving this and other problems (see, for example Moortgat 1997; Morrill 2011).

Given the lexicon, the phrase “moons which Galileo discovered” is a noun  $n$  iff the following holds.

$$n, (n \setminus n)/(s/np), np, (np \setminus s)/np \vdash n$$

The proof above shows this statement holds. Even though it is easy to verify this proof is correct by inspecting each rule application, it may not be immediately obvious how to find natural deduction proofs. In the next section, we will present a proof search procedure for the implicational fragment of Lambek calculus natural deduction.

## 2.2 Proof search in natural deduction

For our proof search procedure, the notion of *result* is useful.

**Definition 2.2** *Given a formula  $F$ , its result is the atomic subformula of  $F$  defined as follows.*

$$\begin{aligned} \text{result}(A) &= A && \text{if } A \text{ atomic} \\ \text{result}(A/B) &= \text{result}(A) \\ \text{result}(B \setminus A) &= \text{result}(A) \end{aligned}$$

Essentially, the result is the atomic formula we obtain once we have combined a formula with all its arguments. So the result of  $(np \setminus s)/np$  is  $s$  and the result of  $(n \setminus n)/(s/np)$  is  $n$ . Proof search for a sequent  $A_1, \dots, A_n \vdash C$  in natural deduction works as follows (a more precise description can be found in Moot and Retoré 2012).

1. If  $C$  is a complex formula, apply the appropriate introduction rules until we obtain an atomic formula  $p$  (this may add formulas to the left of  $A_1$  and to the right of  $A_n$ ).
2. Select an active hypothesis  $H$  of the proof such that  $\text{result}(H) = p$  (that is, select a formula which eventually produces the current atomic goal formula).
3. Our current sequent is of the form  $A_1, \dots, A_{i-1}, H, A_{i+1}, \dots, A_n \vdash p$  and we need to subdivide the formulas to the left of  $H$  ( $A_1, \dots, A_{i-1}$ ) into  $m$  subsequences  $\Gamma_1, \dots, \Gamma_m$ , where  $m$  is the number of arguments  $H$  takes to its left, and we need to subdivide the formulas to the right of  $H$  ( $A_{i+1}, \dots, A_n$ ) into  $k$  subsequences  $\Delta_1, \dots, \Delta_k$  where  $k$  is the number of arguments  $H$  takes to its right (this fails if  $H$  selects no arguments to its left and  $A_1, \dots, A_{i-1}$  is not empty, and similarly if  $H$  has no arguments to its right and  $A_{i+1}, \dots, A_n$  is not empty). We apply all elimination rules to  $H$  until we arrive at atomic formula  $p$ , then recursively find the proofs from step 1 for each of the arguments: proofs  $\Gamma_q \vdash B_q$  for arguments to the left

and proofs  $\Delta_r \vdash D_r$  for arguments to the right. In the simplest case with a single argument on the right and a single argument on the left,  $H = (B \setminus p)/D$ ; there is no need for splitting the  $A_i$  further and we simply try to find proofs for  $A_1, \dots, A_{i-1} \vdash B$  and for  $A_{i+1}, \dots, A_n \vdash D$ . Succeed if all recursive steps succeed. If not try other subdivisions of the hypotheses. Fail when there is no way to divide the hypotheses such that all subproofs succeed.

The algorithm above has non-determinism in two places. The first step is deterministic, but in the second step there may be several choices for the atomic goal formula and in the third step there may be several ways to split up the sequence of formulas (we need multiple arguments either to the right or to the left for this).

As an example, the sequent  $n, (n \setminus n)/(s/np), np, (np \setminus s)/np \vdash n$  has an atomic conclusion, so nothing needs to be done for the first step. For the second step, there is the choice of two formulas: either  $n$  (corresponding to “moons”) or  $(n \setminus n)/(s/np)$  (corresponding to “which”). The first choice fails immediately since there are still formulas to the right which are not arguments of the formula producing the result (since it is atomic). The second choice provides a formula looking for an  $s/np$  to its right and an  $n$  to its left. Simply writing out the required elimination rules and separating the hypotheses produces the following.

$$\begin{array}{c}
 \frac{\text{moons}}{n} \text{Lex} \quad \frac{\text{Galileo}}{np} \text{Lex} \quad \frac{\text{discovered}}{(np \setminus s)/np} \text{Lex} \\
 \vdots \delta_1 \quad \frac{\text{which}}{(n \setminus n)/(s/np)} \text{Lex} \quad \vdots \delta_2 \\
 \frac{n}{n} \quad \frac{n \setminus n}{n \setminus n} \setminus E \quad \frac{s/np}{s/np} /E \\
 \hline
 n
 \end{array}$$

We now need to complete the procedure recursively to find the proofs  $\delta_1$  and  $\delta_2$ . The first subproof is trivial: we are looking for a noun and there is one, so  $\delta_1$  is empty and the  $n$  premiss and the  $n$  conclusion of  $\delta_1$  become the same formula occurrence. The second subproof has a complex goal, so according to step 1 we apply the introduction rule for “/” which produces the following.



$$\frac{\frac{\frac{\text{moons}}{n} \text{ Lex} \quad \frac{\text{which}}{(n \setminus n) / (s / np)} \text{ Lex} \quad \frac{\frac{\text{Galileo}}{np} \text{ Lex} \quad \frac{\text{discovered}}{(np \setminus s) / np} \text{ Lex} \quad [np]_1}{\vdots \delta_3} \quad \frac{s}{s / np} / I_1}}{n \setminus n} \setminus E}{n} / E$$

Our subproof  $\delta_3$  requires us to prove  $np, (np \setminus s) / np, np \vdash s$ . Since  $s$  is atomic and only the transitive verb has  $s$  as its goal, this produces the following.

$$\frac{\frac{\frac{\text{moons}}{n} \text{ Lex} \quad \frac{\text{which}}{(n \setminus n) / (s / np)} \text{ Lex} \quad \frac{\frac{\frac{\text{Galileo}}{np} \text{ Lex} \quad \frac{\text{discovered}}{(np \setminus s) / np} \text{ Lex} \quad [np]_1}{\vdots \delta_4} \quad \frac{s}{np \setminus s} \setminus E}}{np} / E}{n \setminus n} \setminus E}{n} / E$$

We can complete the proof by identifying the atomic noun phrases in  $\delta_4$  and in  $\delta_5$ . The given proof procedure is top-down and enumerates long normal form proofs. In addition, different proofs correspond to different meanings, that is, different proofs will have different lambda terms assigned to them using the term assignment of Figure 1. This correspondence between natural deduction proofs and lambda-terms is the well-know Curry-Howard correspondence. It is not an isomorphism for the Lambek calculus, since not all intuitionistic proofs have a corresponding Lambek calculus proof. Even stronger, not all multiplicative intuitionistic linear logic proofs have a corresponding Lambek calculus proof: the Lambek calculus is a logic without contraction and weakening, like linear logic, but also without the exchange rule.

Adding the term assignment of Figure 1 produces the following

proof.

$$\frac{\frac{\text{moons}}{n : m} \text{Lex} \quad \frac{\text{which}}{(n \setminus n) / (s / np) : w} \text{Lex} \quad \frac{\text{Galileo}}{np : g} \text{Lex} \quad \frac{\text{discovered}}{(np \setminus s) / np : d} \text{Lex} \quad [np : x]_1 / E}{\frac{n \setminus n : (w (\lambda x. ((d x) g)))}{n : ((w (\lambda x. ((d x) g))) m)} \setminus E} \quad \frac{s : ((d x) g)}{s / np : \lambda x. ((d x) g)} / I_1 \quad / E$$

Each lexical assumption of the proof is assigned a unique variable (in the example above, this variable is the first letter of the corresponding word for the convenience of the reader) and these have exactly one free occurrence in the final term (these are linear lambda terms, since each abstraction binds exactly one variable as well). The lexicon assigns both a formula and a corresponding lambda term to each lexical entry. Computing the formal semantics corresponds to replacing the lexical semantics for word. In the current case, ignoring complications such as tense and the plural, many words have a trivial meaning assignment, so we replace  $m$  by the constant  $moon^{e \rightarrow t}$  (or, if we prefer, by its eta expansion  $\lambda x^e. moon(x)$ ),  $d$  by the constant  $discover^{e \rightarrow (e \rightarrow t)}$ ,  $g$  by the constant  $Galileo^e$ . The crucial case is the lexical assignment for “which”, for which we replace  $w$  by  $\lambda Q^{e \rightarrow t} \lambda P^{e \rightarrow t} \lambda y^e. (P y) \wedge (Q y)$ . Making all lexical substitutions in our original term  $((w (\lambda x. ((d x) g))) m)$  produces the following term.

$$((\lambda Q \lambda P \lambda y. ((P y) \wedge (Q y)) (\lambda x. ((discover x) Galileo))) moon)$$

We apply beta-reduction by substituting  $\lambda x. ((discover x) Galileo)$  for  $Q$ , which produces the following.

$$(\lambda P \lambda y. ((P y) \wedge (\lambda x. ((discover x) Galileo)) y)) moon)$$

A second beta-reduction replace  $x$  by  $y$  as follows.

$$(\lambda P \lambda y. ((P y) \wedge ((discover y) Galileo)) moon)$$

The final beta-reduction replaces  $P$  by  $moon$  to produce the following normal form.

$$\lambda y. ((moon y) \wedge ((discover y) Galileo))$$

According to the standard notational conventions of Montague semantics (Gamut 1991), this corresponds to the following more natural term.

$$\lambda y.(moon(y) \wedge discover(Galileo, y))$$

That is, the  $y$  such that they are moons and were discovered by Galileo.

### 2.3

#### *Proof nets*

Type-logical grammars generally have multiple proof systems which are provably equivalent (in the sense that they derive the same theorems). Having multiple proof systems available is a great benefit, because meta-theoretical properties are often easier to prove in one system than in another.

Even though natural deduction is a nice proof system producing proofs which are fairly easy to read and which have a direct connection to the semantics, we will introduce a second proof system, *proof nets*, which makes some aspects of proof combinatorics easier to see<sup>2</sup>.

Proof nets are a proof system introduced for linear logic by Girard (1987). Proof nets represent proofs as (hyper)graphs, where the vertices are (polarized) formulas and the hyperlinks represent a connection between the main formula of a rule and its immediate subformulas. The links for the Lambek calculus are shown in Table 1. The formulas above a link are its premisses whereas the formulas below it are its conclusions.

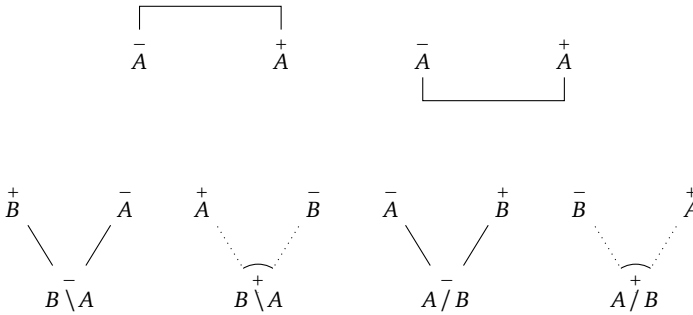
The links for the negative implications correspond to the natural deduction elimination rules, though the complex formula is the conclusion of the link and the main premiss of the elimination rule. The links for the positive implications correspond to the introduction rules, with the withdrawn hypothesis as the negative premiss of the rule.

Positive and negative formulas correspond essentially to un-negated and negated formulas (as in the classical equivalences between  $B \rightarrow A \equiv \neg B \vee A$  and  $\neg(B \rightarrow A) \equiv B \wedge \neg A$ ). In the context of linear logic, polarities function as a restriction on classical formulas to ensure the intuitionistic restriction to a single conclusion. The distinction between solid and dotted links corresponds to the distinction between a logical conjunction (solid) and a logical disjunction (dot-

---

<sup>2</sup>Another advantage of proof nets is that, unlike natural deduction, adding the product rules to the proof net calculus presents no complications.

Table 1:  
Links for Lambek  
calculus proof  
structures



ted). This distinction plays a key role in deciding the correctness of proof structures below.

Given a statement  $A_1, \dots, A_n \vdash C$ , we obtain a proof frame by unfolding the formulas according to the logical links on the bottom row of Table 1, using the negative unfolding for the  $A_i$  and the positive unfolding for  $C$ , until we reach the atomic formulas. We then connect the atomic formulas by means of the axiom link shown on the top left of Table 1. We need to respect the linear order of the premisses for the logical links (and the linear order of the formulas in the sequent), but the axiom link can connect a positive and a negative atom in either order.

Figure 2 shows the formula unfolding corresponding to “moons which Galileo discovered”. We saw the natural deduction proof for this noun in Section 2.1. When all axioms of a formula unfolding have been linked we call the resulting structure a *proof structure*. Not all proof structures correspond to proofs. The proof structures which do are *proof nets*. As we will see, we can distinguish proof nets from other proof structures just by looking at properties of the graph.

For the formula unfolding, it is immediately clear what the search space for potential proofs is: we need to find a 1-1 matching between positive and negative occurrences of the same atomic formula. So for Figure 2, we need to match the positive  $s$  to the negative  $s$  (there is only one possible solution here), the two positive  $n$ 's to the two negative ones, and the two positive  $np$ 's to the two negative ones.

For Lambek calculus proof nets, the matching of atomic formulas must be planar. Planarity corresponds to non-commutativity of the logic and it therefore holds for the Lambek calculus but not for



to reduce this cycle to a self-loop, but we can never eliminate it. Similarly, the contractions only shorten paths, but do not create new ones. Therefore, if a proof structure is disconnected, it will never contract to a single point. The leftmost reduction corresponds to the dotted links for the positive implications and it essentially requires us to “join” the two premisses of the link. As the previous discussion suggests, the contraction criterion is quite close to the more well-known acyclicity and connectedness condition (Danos and Regnier 1989). However, the contraction condition has the advantage of allowing a compact representation of intermediate structures in proof search, and is therefore more suitable for proof search (Moot 2017).

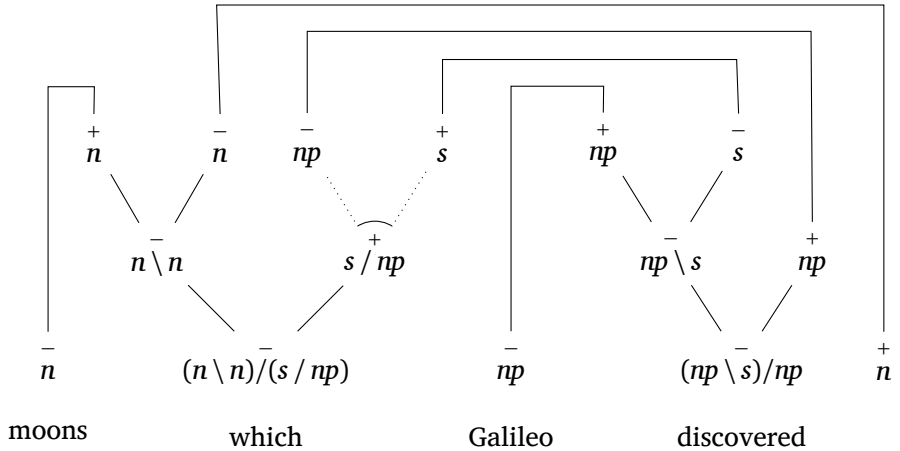
With this in mind, it is clear that of the two possible connections between the  $n$  formulas, connecting the two  $n$  formulas of “which” together produces a cycle of solid links, and therefore a structure which doesn’t contract to a point. In addition, connecting the noun “moons” to the goal formula produces an axiom link disconnected from the rest of the structure. Therefore, the structure shown in Figure 4 is the only proof net given this sequence of formulas. It is easy to verify it contracts to a point given the contractions of Figure 3. One way of proceeding is eliminating all “dangling” links (that is, links connecting a single vertex to the rest of the structure). When such links have been recursively removed, we end up with the pair of dotted links and a solid path between the positive  $s$  and the negative  $np$  of this dotted link. We can contract this path to a single vertex, producing the correct configuration for contracting the pair of dotted links, apply the final contraction to the resulting solid link to produce a single vertex.

### 3 COMBINATORICS AND COMPLEXITY

Using type-logical grammars for computing the meaning of sentences and using the resulting meaning for different tasks (entailment, question answering, etc.) has the following bottlenecks.

1. Lexical lookup. For wide-coverage grammars, the number of formulas that the lexicon assigns to many common words is rather large.
2. Proof combinatorics. Finding a proof (or the best proof for some numerical definition of best) is NP-complete for most type-logical

Figure 4:  
Proof net for  
“moons which  
Galileo  
discovered”



grammars.

3. Meaning computation. Computing the meaning of a sentence is done by substituting lexical lambda terms, then normalising the resulting simply typed lambda term. Normalising simply typed lambda terms is known to be of non-elementary complexity.
4. Meaning use. Questions of logical entailment between sentences are undecidable, even in the first-order case.

Item 1, lexical ambiguity, has a well-known solution which we will briefly mention in Section 3.1. Item 3, lambda term normalisation, is more a problem in theory than it is in practice and will be briefly discussed in Section 3.2. Item 4 will be discussed briefly in Section 3.3. The focus of the rest of this article will be on proof combinatorics.

### 3.1 *A probabilistic lexicon*

When writing a type-logical grammar for a linguistic phenomenon, we usually have a limited number of words in the lexicon, each with only a few (typically only one) formula assigned to it. However, when we are interested in wide-coverage parsing, we have two complementary problems.

1. Regardless of the size of the corpus from which our lexicon has been extracted, there are still many words absent.
2. Many of the frequent words, such as “and”, the forms of “to be” etc. have many different lexical formulas.

When we fix a set of possible formulas, we can define a probability model over words, together with a limited amount of context (typically the two preceding and succeeding words). This general approach is called supertagging (Bangalore and Joshi 2011) and it has been supplied successfully to many formalisms including type-logical grammars (Moot 2010, 2014b, presents supertaggers for multimodal type-logical grammars for Dutch and for French).

Since the topic of supertagging has been discussed at length elsewhere and provides good, practical solutions to the problem of lexical ambiguity we will not elaborate on it here.

### 3.2 *Meaning computation*

Schwichtenberg (1982) shows that normalising simply typed lambda terms has non-elementary worst-case complexity. This complexity result essentially exploits recursive copying. However, there are many implementations of the simply typed lambda calculus for computational linguistics which perform rather efficiently, and this in spite of the fact that, in general little effort is spent on optimising the implementation of the normalisation component. We claim that the meaning recipes necessary for the lexicon are all terms of soft linear logic, and hence can be reduced in polynomial time (Lafont 2004; Baillot and Mogbil 2004). This accounts for the observed fact that lambda term normalisation is not a real bottleneck in practice (Moot and Retoré 2016).

### 3.3 *Logical entailment*

Given that logical entailment is undecidable in general, there are two basic strategies we can use:

1. we can use an off-the-shelf theorem prover (generally using some time limit) and simply see whether it finds a proof. Bos and Markert (2005) use this approach (as well as some more approximative measures),
2. we can use an incomplete but decidable logical fragment for compute entailment. Abzianidze (2017) uses this approach.

In both cases, the result is a high-precision but low-recall system (that is, when the system produces an answer, it is usually right, but there are many correct answers for which no proofs are found). The



main bottleneck to improving recall is adding a logical formalisation of a sufficient amount of world knowledge (without reducing prover performance), a classic problem in artificial intelligence.

4

## WEIGHTED PROOF SYSTEMS

Given a sequent  $\Gamma \vdash C$ , the formula unfolding into a proof frame gives a compact representation of the proof combinatorics for the given sequent. We can combine positive and negative occurrences of the same atomic formulas until we obtain a proof structure. An atomic formula  $a$  with  $n$  positive and  $n$  negative occurrences (the number of positive and negative occurrences for each atomic formula must be equal if the sequent is derivable, this is called the *count check*) corresponds to an  $n \times n$  matrix, and a potential reading for this sequent, that is a proof structure, is a perfect matching between positive and negative occurrences.

When we fill the matrix with weights (we will discuss some different ways of computing these weights below), it becomes possible to compute the best proof structure according to these weights. We can either use best-first search, connecting the “best” axiom links first for each local choice, or use a  $k$ -best proof structure computation, computing the  $k$  total links which are the best globally. Given that computing the  $k$ -best proof structures this way can be done in polynomial time (Kuhn 1955), there is no guarantee that the best proof structure is actually a proof net (unless  $P=NP$ ). However, we can use a polynomial  $k$ -best system as an incomplete approximation of proof search.

Given a two atomic formulas  $a_1$  and  $a_2$ , which are atomic subformulas of lexical formulas  $F_1$  and  $F_2$  corresponding to words  $w_1$  and  $w_2$  ( $F_1$  and  $F_2$  might be the same formula occurrence, similarly  $w_1$  and  $w_2$  may be identical), there are different ways of assigning weights to the different pairings of atomic formulas. The trivial weight assignment assigns all pairs identical weight. More interesting weight assignments are the following.

1. We can use the distance between words as weight, using distance 0 when the two atomic formulas are subformulas of the same formula occurrence, distance 1 between adjacent words, etc.

2. We can use a probability-like measure, estimated from proof nets in a large corpus.
3. We can use the word similarity measure between  $w_1$  and  $w_2$

We will discuss each of these alternative measures in turn in the next sections.

#### 4.1 *Word distance*

One simple metric to use is word distance, preferring axiom connections between closer words. This gives a strong preference to linking atomic subformulas of the same formula, followed by linking atomic subformulas of adjacent formulas.

Given a proof net with  $k$  axiom links, when processing this proof net left-to-right performing axiom links as soon as possible, this will mean each link of distance of  $n$  will cause the leftmost corresponding atom to be open/unlinked for  $n$  steps. Measuring the number of open axioms after each word has been proposed as a simple model of human sentence processing which, in spite of its simplicity, makes a number of correct predictions about processing (Johnson 1998; Morrill 1998, 2011).

We illustrate this by examining the contrast between Dutch and German verb clusters. As is well known, verb clusters in both Dutch and German can have a sequence of verb arguments followed by a sequence of verbs selecting these arguments. The difference between German and Dutch is the German verbs select these arguments right-to-left (that is, the leftmost verb selects the rightmost argument) and the Dutch verb select these argument left-to-right (that is, the leftmost verb selects the leftmost argument). This is illustrated by the following contrast.

- (1) Wolfgang hat die Lehrerin die Murmeln aufräumen helfen  
Wolfgang has the teacher the marbles collect  
*Wolfgang helped the teacher collect the marbles*
- (2) Jantje heeft de lerares de knikkers helpen opruimen  
Jantje has the teacher the marbles help collect  
*Jantje helped the teacher collect the marbles*

Bach et al. (1986) find that, in experimental setting, German sentences



failure and it is possible that this effects processing complexity (at least it does so for a computer implementation). The *size* of the partial proof net constructed so far may also play a role (the size of the contracted partial proof net according to the contractions of Figure 3 seems a good candidate).

Morrill and Johnson use the simplest solution here, measuring complexity by the successful proof nets when processed left to right, counting the number of unlinked axioms at each step.

Figure 5 shows a dotted column after each word, together with a count of the number of axioms it crosses. In the example, after the first word, “Wolfgang”, there is a single unlinked *np*, therefore the count is 1. After “hat”, the *np* of “Wolfgang” becomes linked but an unlinked *s* and an unlinked *inf* are added, leaving a total of 2 unlinked atoms. In general, the complexity profile of verb clusters in German (and in Dutch) rises when the arguments of the verbs in the cluster are encountered (the noun phrases “die Lehrerin” and “die Murmeln”) then descends when the verbs start selecting their arguments. This provides an explanation for why these sentences quickly become unacceptable with multiple levels of embedding.

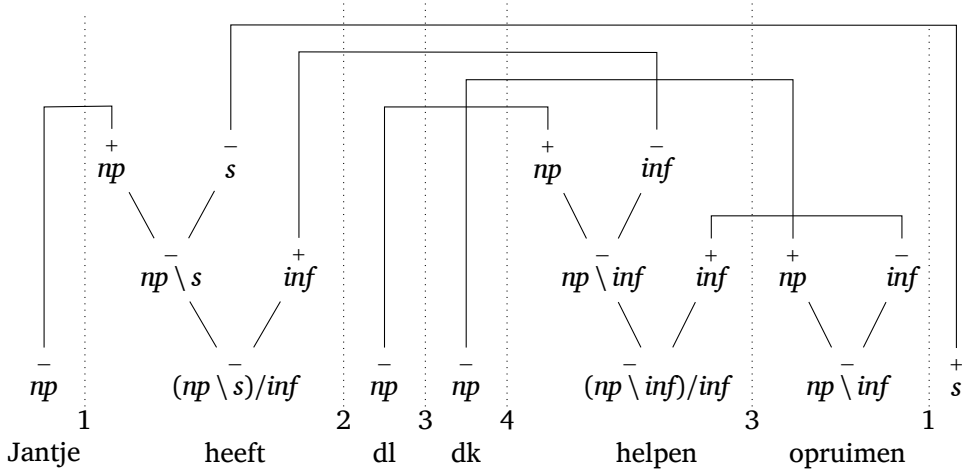
The complexity profile of Figure 5 has a maximum complexity of 4 and a total complexity of 15.

To provide a proof net for the Dutch example (2), we need somewhat more complex proof net machinery, since the crossed dependencies of Dutch cannot be handled by planar structures. Since for these more complicated proof nets do not affect our chosen complexity measure, we will simply look at the complexity profile for the non-planar proof net in Figure 6: this is not a Lambek calculus proof net, and we assume a proof net calculus which allows only this structure for the example sentence.

The complexity profiles of the two sentences are the same until the first infinitive, but then the Dutch sentence has a slight advantage: its maximum complexity, like the German example is 4, but its total complexity is 14, compared to 15 for the German example. This advantage becomes somewhat more pronounced when we add a third and a fourth verb.

These examples are interesting, because with rather minimal assumptions about a model of processing, many known psycholinguistic facts fall out, even some fairly subtle ones like shown here. Morrill

Figure 6:  
Proof net for  
“Jantje heeft de  
lerares de  
knikkers helpen  
opruimen”



(2011) presents many other examples.

#### 4.2

#### *Corpus estimation*

Given a sequent, it is not hard to define a probability distribution over its proof structures: to obtain such a probability distribution we simply need to fill the  $n \times n$  matrix for each atomic formula in such a way that all rows and all columns sum to 1.

Mathematically, it is much harder to define a probability distribution over proof *nets*. When we define a probability distribution over proof structures, we assign a non-zero probability to structures which do not correspond to proofs. Even though it makes sense a priori to want non-proofs to have zero probability, this entails that an undervivable sequent should fail to be assigned a probability distribution at all, since no axiom link can contribute to a proof. However, this means assigning probabilities becomes an NP-hard problem, since we would be able to decide derivability of a sequent from the success or failure to compute a probability distribution.

Accepting the assignment of non-zero probability to axiom links which are not part of any proof is comparable to probabilistic context-free grammar parsers assigning non-zero probability to constituents which cannot be part of a derivation of the complete string.

The question for probability assignment is how likely is the  $n$ th atomic formula of word  $w_1$  to combine with the  $k$ th atomic formula of

$w_2$  (with some form of backoff, for example to the two part-of-speech tags).

In terms of assigning weights to atomic formulas, this is not fundamentally different from the other weighted approaches discussed here. It has the advantage over the other methods that it can distinguish between different arguments of the same formula (neither the distance measure nor the vector similarity measure does this). However, it has the disadvantage that it requires a large amount of annotated data to estimate the probabilities.

### 4.3 *Vector similarity*

An advantage of using vector similarity rather than a large corpus of parsed text is that it is much easier to obtain the former than it is to obtain the latter: a high-quality parsed corpus of sufficiently large size requires an enormous effort in times of man-hours; on the other hand, computing word vectors can be done automatically and, using the enormous amount of text available on the internet, on a scale unrealistic for any manual method. Computing word vectors from the web still requires an important effort in crawling, cleaning, duplicate detection, etc. but nowhere near the man-hours needed to manually annotate a similar size corpus, something especially relevant for under-resourced languages: as discussed by Kilgarriff and Grefenstette (2003), even ‘smaller’ languages such as Icelandic, Basque, Latin and Esperanto have over 50 million words of text available according to conservative estimates. For many of the most-used languages on the internet, cleaned-up and (automatically) annotated versions of this content are freely available and can be used to extract word vectors (Baroni et al. 2009).

Weighing axiom links according to the similarity of the words according to distributional semantics means preferring connections between words with related meanings (this appears to be close to the notion of *discourse coherence* as used by Asher and Lascarides 2003, only in a more shallow, syntactic context).

Even though this basic idea is easily stated, implementing it requires making some choices. While distributional semantic closeness is easily defined for two words, defining it for two complex expressions is essentially the compositionality problem for vector space semantics.

A simple solution would be to choose the vector sum for composi-

tion, and this already performs surprisingly well on several similarity tasks. However, the vector sum approach is ill-adapted to preferences in type-logical proofs: given that we need to match the atomic subformulas of all words in a sentence in any case, and given that the vector sum operation is associative and commutative, this would not allow us to distinguish between different word groupings (or even between different word orders).

We therefore need a slightly more sophisticated method for combining vector similarity with proof rules. We adapt the basic idea of lexicalised parsing with context-free grammars and use a *head word* for each expression — the verb for a verb phrase, the noun for a noun phrase, etc. In the context of type-logical grammar, we therefore specify the following general principles, as sort of type-logical equivalent to the head percolation principles of Magerman (1994):

1. the head of a lexical hypothesis is the word itself,
2. the head of the combination of  $A/A$  and  $A\backslash A$  is the head of  $X$
3. the head of  $np/n$  (resp.  $pp/np$ ) is the head of the noun (resp. the head of the noun phrase)
4. the head of other formulas  $A/B$  and  $B\backslash A$  is the head of  $A$ .

Moreover, for the semantic assignments in a type-logical lexicon, we can distinguish between the lexical entries whose semantic content is purely logical (using only the logical constants like “ $\neg$ ”, “ $\wedge$ ”, “ $\forall$ ”, and a few other predicates whose meaning is invariant across models, like “ $=$ ” and “ $<$ ”) and those whose semantic content is not (these typically contain predicates like “*love*” and “*book*” corresponding to the lexical entry itself<sup>4</sup>).

The basic elements in our formal setup are now triples containing a *formula*, a *head word*, and *vector distance weight*, with each lexical entry starting with the word lemma as its head and distance zero. For each elimination rule, the new head word is defined by the propagation rules above (it is the head of the argument when the functor is a modifier, a determiner or a preposition, and the head of the functor

---

<sup>4</sup>This contrast is unfortunately not as sharp as we would like it to be: while it is simple to see “all” and “some” as purely logical, it is much less easy to see how other words such as “few” and “many” can be interpreted in terms of purely logical operators.

otherwise) and the weight is updated by adding the weights of the two premisses and additionally adding the distance of the two head words according to the vector model. The rule below presents a general elimination rule operating on triples (with “ $\circ$ ” generalising over both “/” and “\”) according to this description.

$$\frac{\langle w_1, h_1, A \circ B \rangle \quad \langle w_2, h_2, A \rangle}{\langle w_1 + w_2 + d(h_1, h_2), h, A \rangle} \circ E$$

From a purely logical point of view (that is, looking only at the third element of the triple), this rule operates just like a normal elimination rule. The head  $h$  of the conclusion will be either  $h_1$  or  $h_2$  depending on the head propagation rules described above. The weight computation uses a distance measure  $d$  computing the distance between the two head words and simple addition. Nothing in particular hinges on the use of “+” here, any function monotone in both its arguments can be used here. Many choices are also possible for the distance measure  $d$ , but in the examples below we use the simple cosine measure which is the most commonly used for distributional similarity.

For the introduction rules, it is somewhat more difficult to define the proper elements for the discharged hypothesis of the rule. We can simply choose zero for its weight, but it is unclear what the proper head word for a hypothesised constituent is. One simple solution is to assign the empty word  $\epsilon$  to such hypotheses and stipulate that the empty word has distance zero to all other words (ie. for all  $w$ ,  $d(\epsilon, w) = d(w, \epsilon) = 0$ ). This would give the following introduction rule.

$$\frac{\langle 0, \epsilon, B \rangle \quad \vdots \quad \langle w, h, A \rangle}{\langle w, h, B \circ A \rangle} \circ I$$

We can also use a somewhat more sophisticated rule for subproofs of the following form.

$$\frac{\langle w_1, h_1, (B \circ A) \circ C \rangle \quad \vdots \quad \frac{[\langle 0, h_1, B \rangle]^k \quad \vdots \quad \langle w_2, h_2, A \rangle}{\langle w_2, h_2, B \circ A \rangle} \circ I_k}{\langle w_1 + w_2, h_1, C \rangle} \circ E$$



Where for relative pronouns  $A = s$ ,  $B = np$  and  $C = n \setminus n$ , and for generalised quantifiers  $A = s$ ,  $B = np$  and  $C = s$ . Essentially,  $h_1$  is propagated from the left premiss of the elimination rule to the hypothesis of the introduction rule. This works well since the head word of a determiner phrase (of type  $(np \multimap s) \multimap s$ ) is its noun, and similarly, the noun argument of the relative pronoun is semantically identical to the extracted noun phrase  $B = np$ .

As a concrete example, a French sentence like “concert de piano gratuit”, like its English translation “free piano concert”, has two possible readings, one where there is a piano concert which is free and one where a free piano is used to give a concert. This type of ambiguity, although somewhat reduced by noun/adjective agreement, is quite common in the French Treebank (Abeillé et al. 2003) and apparently a difficult construction both for journalists and annotators. Treating this example according to the method described above provides the following (to reduce horizontal space, we have used  $w_1$ ,  $w_2$  and  $w_3$  for the weight terms to be discussed later)<sup>5</sup>.

$$\frac{\frac{\text{concert}}{\langle 0, \text{concert}n \rangle} \text{Lex} \quad \frac{\frac{\text{de}}{\langle 0, \text{de}, (n \setminus n)/n \rangle} \text{Lex} \quad \frac{\text{piano}}{\langle 0, \text{piano}, n \rangle} \text{Lex}}{\langle w_1, \text{piano}, n \setminus n \rangle} \text{/E}}{\langle w_2, \text{concert}, n \rangle} \text{\E} \quad \frac{\text{gratuit}}{\langle 0, \text{gratuit}n \setminus n \rangle} \text{Lex}}{\langle w_3, \text{concert}, n \rangle} \text{\E}$$

The weight  $w_1$  of the proof showing “de piano” is of type  $n \setminus n$  is equal to the weight of its two premisses (both zero) plus the distance between the heads of the two premisses of the rule, “de” and “piano” in our case, which have a distance of 0.0740, so we conclude  $w_1$  is 0.0740. We can now compute the weight of the proof showing “concert de piano” to be of type  $n$  by combining the weight 0 of “concert” with the weight 0.0740 of “de piano” and adding the distance between the two head words “concert” and “piano”, which is 0.4398 (that is, these words are fairly close) to arrive at  $w_2 = 0.5138$ . Finally, we combine this  $n$  with the adjective “gratuit” to compute the final weight  $w_3$  by adding the previously computed  $w_2$  to 0 (the weight of “gratuit”) and adding the distance between “concert” and “gratuit”, which is 0.1921.

<sup>5</sup> Here and elsewhere, all weights are computed using the models provided by Fauconnier (2016) at <http://fauconnier.github.io/#software>

This gives us a total weight  $w_3$  of 0.7059 for this reading (note that this number is not a probability and meaningful only in comparison to similarly computed numbers).

The second proof looks as follows.

$$\frac{\frac{\text{concert}}{\langle w_3, \text{concert}, n \rangle} \text{Lex} \frac{\text{de}}{\langle 0, \text{de}, (n \setminus n) / n \rangle} \text{Lex} \frac{\text{piano}}{\langle 0, \text{piano}, n \rangle} \text{Lex} \frac{\text{gratuit}}{\langle 0, \text{gratuit}, n \setminus n \rangle} \text{Lex}}{\langle w_2, \text{piano}, n \setminus n \rangle} / E \text{Lex}}{n} \setminus E$$

Even though the second reading uses the exact same words, it combines them in a different way, and this affects the weight calculations. We now combine “piano” and “gratuit” first, to obtain  $w_1 = 0 + 0 + d(\text{piano}, \text{gratuit}) = 0.0695$ . We then combine this result with “de” and calculate  $w_2 = 0 + w_1 + d(\text{de}, \text{piano}) = 0.0695 + 0.0740 = 0.1435$ . Finally, we combine the previous result with “concert” and calculate  $w_3 = 0 + w_2 + d(\text{concert}, \text{piano}) = 0.1435 + 0.4398 = 0.5833$ .

These calculations show a preference for the first reading, where the concert is free rather than the piano. The key difference between the two readings is that  $d(\text{concert}, \text{gratuit}) > d(\text{piano}, \text{gratuit})$ , whereas the other computed terms are equal.

We can use the same method to compute “voir la fille avec les lunettes” (to see the girl with the glasses), since  $d(\text{voir}, \text{lunettes}) > d(\text{fille}, \text{lunettes})$ , which gives a preference for “fille avec les lunettes” to be a constituent over “avec les lunettes” modifying the verb phrase. However, in this case we have two proofs using different lexical formulas, which may get assigned different weights when we use a supertagger. This means that when combining the current weight assignment with a supertagger, the result depends on the way in which we combine the supertagger weights with the proof weights.

Using semantic relatedness like this is, of course, not without its defects. For example, the verb phrase “saw the star with the telescope” is structurally identical to the example above, but has only one plausible reading, where “with the telescope” is a verb phrase modifier. However, “star” and “telescope” are closer semantically than “girl” and “telescope” are. The problem here is essentially that semantic relatedness doesn’t give us much information about argument structure.

Table 2:  
Lexical  
assignments with  
head word and  
weight  
computation  
information

$$\begin{aligned} \text{lex}(\text{book}) &= n(\text{book}, 0) \\ \text{lex}(\text{the}) &= np(X, w) / np(X, w) \\ \text{lex}(\text{interesting}) &= n(X, w + d(\text{interesting}, X)) / n(X, w) \\ \text{lex}(\text{read}) &= (np(X, w_1) \setminus s(\text{read}, w_1 + w_2 + d(w_1, \text{read}) + d(w_2, \text{read})) / np(Y, w_2) \\ \text{lex}(\text{which}) &= (n(X, w_1) \setminus n(X, w_1 + w_2)) / (s(Y, w_2) / np(X, 0) \\ \text{lex}(\text{every}) &= (s(Y, w_1 + w_2) / (np(X, 0) \setminus s(Y, w_2))) / n(X, w_1) \end{aligned}$$

#### 4.4 Vector similarity and proof nets

We can adapt the above strategy with minor modifications to a proof net parser. This is done by replacing atomic formulas by binary predicates, where the first argument represents the head and the second argument the weight. For weights, we use the real numbers with the usual function “+” and the  $d(w_1, w_2)$  function computing the distance between two word  $w_1$  and  $w_2$ . Instead of having extra-logical head percolation and weight computation principles, these now form a part of the lexical entries (although these extra-logical principles would explain many common patterns occurring in the lexical entries). Using first-order arguments has many applications, including as a solution for the Dutch verb clusters we’ve seen in Section 4.1 (Moot 2014a). As we use them here, these arguments are extra-grammatical and serve only as a way to compute preferences among different proofs using the same formulas.

As before, atomic content words, like the noun “book” are assigned the entry  $n(\text{book}, 0)$ . That is, the head constituent of the noun is “book” itself and the weight assigned to this expression is zero. Table 2 lists some other lexical entries in the current context.

As shown in the table, the determiner “the” is assigned an entry simply copying both the head word and the weight, following the principles of a purely logical word in the previous section.

An adjective such as “interesting” on the other hand copies the head word, but adds the distance between the head word to the previous weight<sup>6</sup>.

---

<sup>6</sup>Modifiers of modifiers (eg. adverbs like “very”) cannot be handled in the same way as in the natural deduction based account on the previous section. Since they modify the adjective or adverb they select, and since these no longer

Similarly, the transitive verb, “read” adds both the distance between the verb and its subject and the distance between the verb and its object to the weight of its two arguments.

The weighted entry for “which” requires some explanation. First of all, the head word  $X$  of the resulting noun is the same as the head of the argument noun (this behaviour is consistent with a noun modifier, and it makes, for example, “book which ...” behave the same as “book” in this respect). Second, the extracted  $np$ , being a hypothetical element, starts at weight 0 and shares the head with the noun argument. This approach therefore expects the long-distance dependency between a noun and a relative clause to occur most likely between the head noun and the verb which is semantically closest to it.

Compared to the standard proof net matching algorithm using minimisation (or maximisation) of weight, we have now added computation of weights to the matching process. This presents somewhat of a complication. However, since we need to do only simple computations (addition and vector cosine) in each cell of the matrices representing the search space, this doesn’t make a big difference computationally.

#### 4.5

#### *Limitations*

The current method doesn’t distinguish between object and subject arguments and this is an important weakness<sup>7</sup>. This limitation is essentially a consequence of the division of labour between the distributional and type-logical approaches: the type-logical component of the system is solely responsible for word order while the distributional component only tests for similarity between a verb and its argument, taking neither grammatical nor structural considerations into account. We therefore need either a more subtle similarity measure or another

---

contain this adjective or adverb as their head word, we can no longer compute the distance between eg. “very” and “interesting”, and between “very” and “quickly” since both “interesting” and “quickly” are not arguments of any of their atomic subformulas. The natural deduction approach of the previous section assigns heads to formulas with no requirement these are atomic, and this provides a potential benefit for these cases.

<sup>7</sup>As discussed in Section 4.2, we can incorporate this when estimating probabilities from a corpus, but not for the other methods discussed here, ie. vector similarity and word distance.

way of distinguishing the likely relations a verb and its different arguments.

## 5 CONCLUSIONS AND FUTURE WORK

We have given an overview of several methods of adding weights to proof search in type-logical grammars. With the exception of Bonfante and de Groot (2001), this possibility has been surprisingly little discussed in the type-logical grammar literature. We have given applications of weighted proof search to modelling human processing, to finding parses most similar to those found in a given corpus, and to finding parses which prefer grouping similar words together. These methods still need to be thoroughly evaluated beyond the manually calculated examples shown here. Fortunately, the groundwork for incorporating weighted proof search has already been done for some of the currently available type-logical grammars. Given the many potential applications of weighted proof search, we look forward to testing these methods against available data for parsing and human processing.

## REFERENCES

- Abeillé, A., Clément, L. and Toussanel, F. (2003), Building a treebank for french, in A. Abeillé, ed., ‘Treebanks’, Vol. 20 of *Text, Speech and Language Technology*, Springer, pp. 165–187.
- Abzianidze, L. (2017), A natural proof system for natural language, PhD thesis, Tilburg University.
- Asher, N. and Lascarides, A. (2003), *Logics of Conversation*, Cambridge University Press.
- Bach, E., Brown, C. and Marslen-Wilson, W. (1986), ‘Crossed and nested dependencies in German and Dutch: A psycholinguistic study’, *Language and Cognitive Processes* 1(4), 249–262.
- Baillet, P. and Mogbil, V. (2004), Soft lambda-calculus: a language for polynomial time computation, in ‘Foundations of software science and computation structures’, Springer, pp. 27–41.
- Bangalore, S. and Joshi, A. (2011), *Supertagging: Using Complex Lexical Descriptions in Natural Language Processing*, MIT Press, Cambridge, Massachusetts.

- Baroni, M., Bernardini, S., Ferraresi, A. and Zanchetta, E. (2009), 'The WaCky Wide Web: A collection of very large linguistically processed web-crawled corpora', *Language Resources and Evaluation* 43(3), 209–226.
- Bonfante, G. and de Groote, P. (2001), Stochastic Lambek categorial grammars, in G.-J. Kruijff, L. Moss and R. T. Oehrle, eds, 'Proceedings of FGMOL 2001', Vol. 53 of *Electronic Notes in Theoretical Computer Science*, Elsevier.
- Bos, J. and Markert, K. (2005), Recognising textual entailment with logical inference, in 'Proceedings of the 2005 Conference on Empirical Methods in Natural Language Processing (EMNLP 2005)', pp. 628–635.
- Danos, V. (1990), La Logique Linéaire Appliquée à l'étude de Divers Processus de Normalisation (Principalement du  $\lambda$ -Calcul), PhD thesis, University of Paris VII.
- Danos, V. and Regnier, L. (1989), 'The structure of multiplicatives', *Archive for Mathematical Logic* 28, 181–203.
- Fauconnier, J.-P. (2016), Acquisition de liens sémantiques à partir d'éléments de mise en forme des textes : exploitation des structures énumératives, PhD thesis, Université de Toulouse.
- Gamut, L. T. F. (1991), *Logic, Language and Meaning*, Vol. 2, The University of Chicago Press.
- Girard, J.-Y. (1987), 'Linear logic', *Theoretical Computer Science* 50, 1–102.
- Johnson, M. (1998), 'Proof nets and the complexity of processing center-embedded constructions', *Journal of Logic, Language and Information* 7(4), 443–447.
- Kilgarriff, A. and Grefenstette, G. (2003), 'Introduction to the special issue on the web as corpus', *Computational Linguistics* 29, 333–347.
- Kuhn, H. W. (1955), 'The Hungarian method for the assignment problem', *Naval Research Logistics Quarterly* 2, 83–97.
- Lafont, Y. (2004), 'Soft linear logic and polynomial time', *Theoretical Computer Science* 318(1), 163–180.
- Lambek, J. (1958), 'The mathematics of sentence structure', *American Mathematical Monthly* 65, 154–170.
- Magerman, D. M. (1994), Natural Language Parsing as Statistical Pattern Recognition, PhD thesis, University of Pennsylvania.
- Moortgat, M. (1997), Categorial type logics, in J. van Benthem and A. ter Meulen, eds, 'Handbook of Logic and Language', Elsevier/MIT Press, chapter 2, pp. 93–177.
- Moot, R. (2010), Automated extraction of type-logical supertags from the spoken dutch corpus, in S. Bangalore and A. Joshi, eds, 'Complexity of Lexical Descriptions and its Relevance to Natural Language Processing: A Supertagging Approach', MIT Press, Cambridge, Massachusetts, chapter 12, pp. 291–312.

- Moot, R. (2014a), Extended Lambek calculi and first-order linear logic, in C. Casadio, B. Coecke, M. Moortgat and P. Scott, eds, 'Categories and Types in Logic, Language, and Physics: Essays dedicated to Jim Lambek on the Occasion of this 90th Birthday', number 8222 in 'Lecture Notes in Artificial Intelligence', Springer, Heidelberg, pp. 297–330.
- Moot, R. (2014b), 'A type-logical treebank for french', *Journal of Language Modelling* 2(2).
- Moot, R. (2017), The Grail theorem prover: Type theory for syntax and semantics, in Z. Luo and S. Chatzirykiakidis, eds, 'Modern Perspectives in Type Theoretical Semantics', Springer.
- Moot, R. and Retoré, C. (2012), *The Logic of Categorical Grammars: A Deductive Account of Natural Language Syntax and Semantics*, number 6850 in 'Lecture Notes in Artificial Intelligence', Springer, Heidelberg.
- Moot, R. and Retoré, C. (2016), Natural language semantics and computability, Technical report, LIRMM.
- Morrill, G. (1998), Incremental processing and acceptability, Technical Report LSI-98-46-R, Departament de Llenguatges i Sistemes Informàtics, Universitat Politècnica de Catalunya.
- Morrill, G. (2011), *Categorical Grammar: Logical Syntax, Semantics, and Processing*, Oxford University Press, Oxford.
- Schwichtenberg, H. (1982), Complexity of normalization in the pure typed lambda-calculus, in 'The L. E. J. Brouwer Centenary Symposium', North-Holland, pp. 453–457.

*This work is licensed under the Creative Commons Attribution 3.0 Unported License.*

<http://creativecommons.org/licenses/by/3.0/>

