Robustness enhancement of IDA-PBC controller in stabilising the inertia wheel inverted pendulum: theory and real-time experiments

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Robustness enhancing of IDA-PBC controller for underactuated mechanical systems : Theory and real-time experiments

Nahla Khraief Haddad [a*], Ahmed Chemori[b] and Safya Belghith[a]

[a]*National School of Engineers of Tunis, University of Tunis El Manar, BP.37, le Belvédère, 1002 Tunis, Tunisia ;*
[b]*LIRMM, University of Montpellier, CNRS, 161 rue Ada, 34095 Montpellier, France*

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Improving the robustness, *vis-à-vis* matched input disturbances of IDA-PBC (Interconnection Damping Assignment, Passivity Based Control) for a class of underactuated mechanical systems is addressed in this paper. The characterized class of systems is the one for which IDA–PBC yields a smooth stabilizing controller. Our main contribution consists in combining the so-called IDA-PBC controller with an adaptive control technique. Some sufficient stability conditions on matched input disturbances are given. The comparison of the stability robustness between the classical controller IDA-PBC and the proposed one is then provided. As illustration we propose to revisit the application of IDA-PBC controller to the Inertia Wheel Inverted Pendulum (IWIP) in the presence of matched disturbances. Simulation and real-time experimental results are presented as validations of the theoretical results.

Keywords: IDA-PBC, Hamiltonian systems, matched disturbances, Inertia Wheel Inverted Pendulum, Stability, adaptive control, robustness.

1. Introduction

This paper deals with the robustness improvement of IDA-PBC controller when applied to a class of underactuated mechanical systems (Choukchou-Braham, Cherki, Djemaï, et Busawon (2014), Spong. (1998)). The latter, will be, necessarily [1](F. et der Schaft A.J (2004)) described by the so-called Port-Controlled Hamiltonian models (PCH).
PCH systems have been introduced in 1994 by Van der Shaft and Mashke (Maschke et van der Schaft (1994)). The central paradigm of complex systems modeling is to have individual open subsystems with well defined port interfaces, hiding an internal model of variable complexity, and a set of rules describing how these subsystems interact through the port variables (D'oria-Cerezo (2006)). Port-controlled Hamiltonian systems are used to implement this general idea. Their models represent another alternative to the classical Euler-Lagrange models. Writing a system in a PCH form has the advantage of covering a large set of physical systems and provide important structural properties. An extended survey of PCH systems is presented in (van der Schaft (2006)). The technique used to control PCH systems is called IDA-PBC (Ortega, van der Schaft, Maschke, et Escobar (2002)). It combines the passivity properties of PCH systems with interconnection and energy-based control. IDA-PBC uses the hamiltonian framework, it consists in solving the PDE (Partial Differential Equation) associated to the energy balance equation. This technique has been applied to a large variety of plants : Mechanical systems (Acosta, Ortega, et Astolfi (2004); Ortega,

*Corresponding author. Email : khraiefnahla@gmail.com

1. To stabilize underactuated mechanical systems by energy shaping, it is necessary to modify the total energy function. Which cannot be done with the classical PBC (passivity based control). That is why IDA-PBC is used

Spong, Gomez-Estern, et Blankenstein (2002)), magnetic levitation systems (Rodrıguez, Ortega, et Mareels (2000); Rodrıguez, Siguerdidjane, et Ortega (2000a)), mass balance systems (Ortega, Astolfi, Bastin, et Rodrıguez (2000)), electrical machines (Batlle, Doria-Cerezo, et Ortega (2005); Petrovic, Ortega, et Stankovic (2001)), power converters (Rodrıguez, Ortega, Escobar, et Barabanov (2000)). For an in-depth review of IDA-PBC the reader is referred to (Ortega et Garcıa-Canseco (2004)).

In this paper we will focus on the robusteness improvement of IDA-PBC controller, for a class of underactuated mechanical systems. Especially in the case of matched input disturbances where little is known about the robusteness issue. Many studies turn around the improvement of the robustness of the IDA-PBC controller. In (Gentili et van der Schaft (2003)) an input disturbance suppression for PCH systems is based on internal model. In (Rodrıguez, Siguerdidjane, et Ortega (2000b)) an IDA-PBC controller applied to a magnet levitation system was experimentally tested. To solve the robustness problem an integral term was added to the error of the passive output. The same technique was used in (Batlle et al. (2005)) to improve the robustness of hamiltonian passive control. Stability and robustness of disturbed port-controlled Hamiltonian systems with dissipation have been addressed in (Bechrif et Mendes (2005)). The authors studied IDA-PBC controller robustness against parameter uncertainties. Recently Romero et al. (J. G. Romero (2013)) improved the robustness *vis-à-vis* external disturbances, of energy shaping controllers for fully actuated mechanical systems. They design a dynamic state feedback controller such that the closed-loop system ensures some stability properties in spite of the presence of external disturbances. In (Khraief, Chemori, et Belghith (2014)) the effect of external disturbances is especially studied. Two sufficient stability conditions are provided to deal with matched and unmatched disturbances. Motivated by the practical matter of IDA-PBC, the paper presents experimental results shown that IDA-PBC is robust with respect to external disturbances. To improve the robusteness we propose in this paper the design of a robust Model Reference Adaptive (MRA) control combined with the IDA-PBC. The resulting MRA-IDA-PBC control scheme yields a smooth asymptotically stabilizing controller which increases the disturbance boundary. To strength those results we validate the proposed controllers experimentally on the inertia wheel inverted pendulum (IWIP). Simulation and experimental results on the disturbed IWIP for different scenarios show that the proposed controller gives better performences than the classical IDA-PBC.

The remaining of the paper is organized as follows. In section 2, IDA-PBC for disturbed underactuated mechanical systems is introduced. Section 3 presents the main contribution of the paper resulting in an adaptive control to improve the robusteness of IDA-PBC controller against matched disturbances. Section 4 is devoted to simulation and experimental results. Finally, we present some conclusions and future work in section 5.

2. IDA-PBC for disturbed underactuated mechanical systems

2.1 *Background on IDA-PBC control for underactuated mechanical systems*

This background is based on previous work proposed in (Ortega, Spong, et al. (2002); Ortega, van der Schaft, et al. (2002)). An underactuated mechanical system whith no natural damping can be written in Port Controlled Hamiltonian (PCH) form as follows [2] :

$$\left\{ \begin{aligned} \begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} &= \begin{pmatrix} 0 & I_n \\ -I_n & 0 \end{pmatrix} \begin{pmatrix} \nabla_q H \\ \nabla_p H \end{pmatrix} + \begin{pmatrix} 0 \\ G(q) \end{pmatrix} u \\ y &= G(q)^T \nabla_p H \end{aligned} \right. \tag{1}$$

2. Throughout the whole of the paper we present all vectors, including the gradient($\nabla_x H = \frac{\partial H}{\partial x}$), as column vectors.

with total energy

$$H(q,p) = \frac{1}{2} p^T M^{-1}(q) p + V(q) \tag{2}$$

(1) will be called the nominal system, where $q \in \mathbb{R}^n$, $p \in \mathbb{R}^n$ are the generalized position and momenta respectively. $G(q) \in \mathbb{R}^{n \times m}$, is the input matrix. We consider here that the system is underactuated and assume $rank(G) = m < n$, u and y are the control input vector and the output vector respectively. $M(q) = M^T(q) > 0$ is the inertia matrix, and $V(q)$ is the potential energy. Note that :

$$\dot{q} = M^{-1}(q) p \tag{3}$$

The desired (closed-loop) energy function can be expressed by (Ortega, Spong, et al. (2002)) :

$$H_d(q,p) = \frac{1}{2} p^T M_d^{-1}(q) p + V_d(q) \tag{4}$$

we define $(q^*, 0)$ as the desired equilibrium. V_d is required to have an isolated minimum at q^*. This target can be achieved by the following IDA-PBC controller :

$$u = u_{es} + u_{di} \tag{5}$$

Where :

$$u_{es} = (G^T G)^{-1} G^T (\nabla_q H - M_d M^{-1} \nabla_q H_d + J_2 M_d^{-1} p) \tag{6}$$

$$u_{di} = (-K_v) G^T \nabla_p H_d \tag{7}$$

u_{es} is the energy shaping control to assign the equilibrium and u_{di} injects damping to achieve asymptotic stability. We put the control expression (5) in the nominal system (1) we obtain the following desired (closed-loop) PCH dynamics :

$$\begin{cases} \begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = (J_d(q,p) - R_d(q,p)) \begin{pmatrix} \nabla_q H_d \\ \nabla_p H_d \end{pmatrix} \\ y = G(q)^T \nabla_p H_d \end{cases} \tag{8}$$

where $M_d = M_d^T > 0$, $J_d = -J_d^T = \begin{pmatrix} 0 & M^{-1} M_d \\ -M_d M^{-1} & J_2 \end{pmatrix}$ is the interconnection matrix, and $R_d = R_d^T = \begin{pmatrix} 0 & 0 \\ 0 & G K_v G^T \end{pmatrix}$ is the damping matrix (Ortega, Spong, et al. (2002)).

2.2 *Matched dsiturbances rejection in IDA-PBC controller*

2.2.1 Problem formulation

Let us describe the underactuated mechanical system in presence of matched input disturbances by the following PCH model :

$$\begin{cases} \begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & I_n \\ -I_n & 0 \end{pmatrix} \begin{pmatrix} \nabla_q H \\ \nabla_p H \end{pmatrix} + \begin{pmatrix} 0 \\ G(q) \end{pmatrix} (u + \delta(x,t)) \\ \quad y = G(q)^T \nabla_p H \end{cases} \tag{9}$$

where $t \in \mathbb{R}^+$, $x = (q \quad p)^T \in \mathbb{R}^{2n}$ is the system's state, $u(t) \in \mathbb{R}^m$ is the control input, $\delta(x,t)$ is the matched input distubances. $\delta(x,t)$ is assumed to be unmeasured and bounded in magnitude, usually its Euclidean norm is denoted by $\|\delta(x,t)\|$. In this paper we formulate the IDA-PBC stabilization objective as follows : Given the disturbed PCH system (9) and a desired equilibrium $(q^*, 0)$, is the IDA-PBC controller (5) capable to reject disturbances and keep $(q^*, 0)$ as a stable equilibrium in spite of the existence of matched input disturbances ?
Applying the controller (5) to the system (9) we obtain the following closed-loop disturbed system :

$$\begin{cases} \begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = (J_d(q,p) - R_d(q,p)) \begin{pmatrix} \nabla_q H_d \\ \nabla_p H_d \end{pmatrix} + \delta_1(x,t) \\ \quad y = G(q)^T \nabla_p H_d \end{cases} \tag{10}$$

Note that $\delta_1(x,t) = (0 \quad G(q)\delta(x,t))^T$ is also a vector of external disturbances. Let $\lambda_{min}\{K_v\}$ be the smallest eigenvalue of the matrix K_v, and $\widetilde{x} = (q - q^* \quad p)^T$.
The following proposition sets some sufficient conditions on the disturbance boundaries in order to have $(q^*, 0)$ as a stable equilibrium.

Proposition 1: *(Khraief et al. (2014)) Consider the closed-loop dynamics (10) with the desired total energy H_d.*
If $\|\delta(x,t)\| \leq \lambda_{min}\{K_v\} \left|(\nabla_p H_d)^T G\right|^2$, then $\dot{H}_d \leq 0$ and $\widetilde{x}$ of (10) is a stable equilibrium point.

Proof. The derivative of H_d defined in (4) gives :

$$\dot{H}_d = (\nabla_q H_d)^T \dot{q} + (\nabla_p H_d)^T \dot{p} \tag{11}$$

We remplace $dotq$ and $dot(p)$ by their expression in (10) we obtain :

$$\begin{aligned} \dot{H}_d = &(\nabla_q H_d)^T (M^{-1} M_d) \nabla_p H_d + (\nabla_p H_d)^T G(q)\delta_1(x,t) - (\nabla_p H_d)^T (M_d M^{-1}) \nabla_q H_d + \\ &(\nabla_p H_d)^T (J_2 - G K_v G^T) \nabla_p H_d \\ &= (\nabla_p H_d)^T G(q)\delta_1(x,t) - (\nabla_p H_d)^T (G K_v G^T) \nabla_p H_d \end{aligned} \tag{12}$$

If $\|\delta_1(x,t)\| \leq \lambda_{min}\{K_v\}\left|(\nabla_p H_d)^T G\right|^2$ then

$$\dot{H}_d \leq \|\delta_1(x,t)\| - \lambda_{min}\{K_v\}\left|(\nabla_p H_d)^T G\right|^2 \quad (13)$$

and $\dot{H}_d \leq 0$. □

3. Main result : Robusteness improvement of the IDA-PBC controller

This section deals with the robusteness improvement of the IDA-PBC controller. Our main contribution consists in combining IDA-PBC controller with an adaptation law (Khraief-Haddad, Chemori, Pena, et S.Belghith (2015)) in order to reduce the tracking error rapidly and to increase the admissible disturbance boundries. The proposed approach consists on a direct adaptive control in which the controller parameters (feedback gains) are estimated online.

3.1 *Basic principle of MRA-IDA-PBC*

In this work the idea is to introduce an adaptive control scheme combined with the IDA-PBC controller. The latter, yields smooth stabilization for disturbed underactuated systems (Khraief et al. (2014)). Nevertheless the controller gains depend strongly on the matching conditions. They have to be online adapted in order to get better performances.
Motivated by this issue an adaptive control is proposed to improve the convergence of the IDA-PBC controller in presence of matched input disturbances. Before going further, the whole block diagram of the proposed controller is illustrated in figure (1).

3.2 *Proposed adaptive control solution*

3.2.1 Controller design

In the previous section the standard (IDA-PBC) was introduced to stabilize a class of disturbed underactuated mechanical systems. However, this controller needs the well known of the control gains which required full information about the interconnection and damping matrices. Consequently, ill-known adequate gains can result in a control input which doesn't lead to the desired closed-loop behavior of the closed-loop system. To resolve this issue we propose to describe the control input signal u in terms of a nominal part and an unknown part (based on unknown errors in the control gains).

Proposition 2: *Let's consider the control law u^* :*

$$u^* = u(x,t) + \Delta(x,t)\hat{z} \quad (14)$$

Where $u(x,t)$ is the nominal controller defined in (5), $\Delta(x,t)$ is a matrix of known functions,

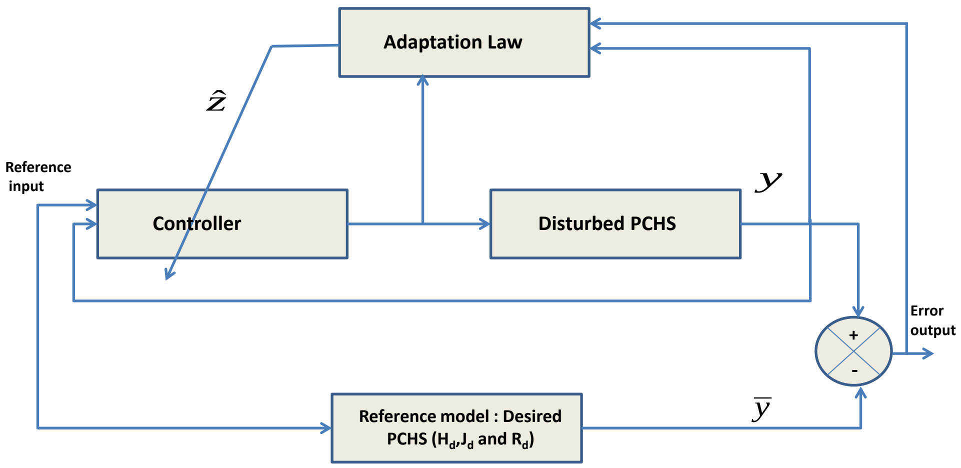

Figure 1.: Block diagram of the proposed MRA-IDA-PBC approach.

$z = [z_1, z_2, ..., z_p]$ is the vector of unknown parameters and $\hat{z}$ is the estimate of z with the following adaptation law :

$$\dot{\hat{z}} = -K_a \Delta^T(x,t) y \tag{15}$$

Where K_a is a diagonal positive definite matrix.
u^* renders the equilibrium point stable.
if $\|\delta_1(x,t)\| \leq (\alpha + \lambda_{min}\{K_v\}) \left|(\nabla_p H_d)^T G\right|^2$.

Proof. With the control law (14) the resulting closed-loop of the disturbed system can be written as follows :

$$\begin{cases} \begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = (J_d(q,p) - R_d(q,p)) \begin{pmatrix} \nabla_q H_d \\ \nabla_p H_d \end{pmatrix} + \begin{pmatrix} 0 \\ G(q)(\Delta(x,t)\hat{z} + \delta(x,t)) \end{pmatrix} \\ y = G(q)^T \nabla_p H_d \end{cases} \tag{16}$$

Define the estimation error by $\tilde{z} = \hat{z} - z$ and a hamiltonian $\overline{H}$ as :

$$\overline{H}(q,p) = H_d(q,p) + \frac{1}{2}\tilde{z}^T K_a^{-1} \tilde{z} \tag{17}$$

Then the closed-loop system (16) with the error $\tilde{z}$ can be rewritten as :

$$\begin{cases} \begin{pmatrix} \dot{q} \\ \dot{p} \\ \dot{\tilde{z}} \end{pmatrix} = \begin{pmatrix} 0 & M^{-1}M_d & 0 \\ -M_dM^{-1} & J_2 - Gk_vG^T & G\Delta K_a \\ 0 & -K_a\Delta^T G^T & 0 \end{pmatrix} \begin{pmatrix} \nabla_q\overline{H} \\ \nabla_p\overline{H} \\ \nabla_{\tilde{z}}\overline{H} \end{pmatrix} + \delta_1(x,t) \\ \quad = (J_{eq}(q,p) - R_{eq}(q,p)) \begin{pmatrix} \nabla_q\overline{H} \\ \nabla_p\overline{H} \\ \nabla_{\tilde{z}}\overline{H} \end{pmatrix} + \delta_1(x,t) \end{cases} \tag{18}$$

Note that $\delta_1(x,t) = (0 \quad G(q)\delta_1(x,t) \quad 0)^T$ is the new vector of external disturbances. It can be easily proved that (18) is in a PCH form ($J_{eq} = -J_{eq}^T$ and $R_{eq} = R_{eq}^T \geq 0$)[3].
Let's Choose $\overline{H}$ as a Lyapunov candidate function, then its first derivative can be expressed by :

$$\begin{aligned} \dot{\overline{H}} &= (\nabla_q\overline{H})^T\dot{q} + (\nabla_p\overline{H})^T\dot{p} + (\nabla_{\tilde{z}}\overline{H})^T\dot{\tilde{z}} \\ &= (\nabla_q\overline{H})^T(M^{-1}M_d)\nabla_p\overline{H} + (\nabla_p\overline{H})^T G(q)\delta_1(x,t) - (\nabla_p\overline{H})^T(M_dM^{-1})\nabla_q\overline{H} + (\nabla_p\overline{H})^T(J_2 - GK_vG^T)\nabla_p\overline{H} + \\ &\nabla_p\overline{H}^T G(q)\Delta(x,t)K_a\nabla_{\tilde{z}}\overline{H} - \nabla_{\tilde{z}}\overline{H}K_a\Delta(x,t)^T G^T\nabla_p\overline{H} \\ &= \nabla_q H_d\,\dot{q} + \nabla_p H_d\,\dot{p} + K_a^{-1}\tilde{z}\Delta^T(x,t)G^T\nabla_p H_d \end{aligned} \tag{19}$$

If $\|\delta_1(x,t)\| \leq (\alpha + \lambda_{min}\{K_v\}) \left|(\nabla_p H_d)^T G\right|^2$ then

$\dot{\overline{H}} \leq \delta_1(x,t) - (\alpha + \lambda_{min}\{K_v\}) \left|(\nabla_p H_d)^T G\right|^2$ where $\alpha > 0$.
Hence $\dot{\overline{H}} \leq 0$, and $(q^*;\ 0;\ \tilde{z})$ is a stable equilibrium. □

4. Application : Inertia Wheel Inverted Pendulum

4.1 *Description and modelling*

The system illustrated in figure (2) is called the Inertia Wheel Inverted Pendulum (IWIP). It is an underactuated mechanical system. The IWIP can be modeled as a two-degrees-of-freedom serial mechanism. The first link is the pendulum (passive joint) and the second one is the rotating disc (active joint). The generated torque produces an angular acceleration of the end-mass which induces a coupling torque at the pendulum axis. The dynamic parameters of the system are summarized in table (1).

The Euler-Lagrange equations of motion can be written as (Spong et Vidyasagar (1989)) :

$$\begin{pmatrix} (a + I_{WC}) & I_{WC} \\ I_{WC} & I_{WC} \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} - \begin{pmatrix} bg\sin\theta_1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix} \tag{20}$$

3. See (D. Dirksz et Scherpen (2010),Dirksz et Scherpen (2012),Fujimoto et Sugie (1998),K. Fujimoto et Sugie (2000),K. Fujimoto, Sakurama, et Sugie (2003)) for more theoretical details.

(a) Shematic view of the IWIP. (b) Mechanical structure of the IWIP.

Figure 2.: The Inertia Wheel Inverted Pendulum (IWIP)

Table 1.: Description of dynamical parameters of the IWIP

Description	**Notation**	**Value**	**Unit**
Pendulum angle with respect to vertical axis	θ_1		rad
Wheel angle with respect to pendulum axis	θ_2		rad
Mass of pendulum	m	3.228	kg
Mass of the Wheel	M	0.33081	kg
Length from pendulum base (PB) to pendulum center of mass (PC)	l	0.06	m
Length from pendulum base (PB) to Wheel center of mass (WC)	L	0.044	m
Rotational Inertia of pendulum about pendulum center of mass (PC)	I_{PC}	0.0314	$kg.m^2$
Rotational Inertia of pendulum about its base (PB)	I_{PB}		$kg.m^2$
Rotational Inertia of Wheel about its center of mass (WC)	I_{WC}	$4.176e-4$	$.m^2$
Constant of gravitational acceleration	g	9.8	m/s^2

where :
$\theta = [\theta_1 \quad \theta_2]^T$ is the vector of generalized positions, u is the torque generated and applied by the actuator on the inertia wheel and :

$$\begin{aligned} a &= ML^2 + I_{PB} \\ b &= ml + ML. \end{aligned} \tag{21}$$

Let us now introduce the following change of coordinates :

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{22}$$

This leads to the following simplified model :

$$\begin{bmatrix} a & 0 \\ 0 & I_{WC} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} - \begin{bmatrix} bg\sin q_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} u. \tag{23}$$

The model (23) can be rewritten through Hamilton's equations of motion as (Santibanez, Kelly, et Sandoval (2005)) :

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{p}_1 \\ \dot{p}_2 \end{bmatrix} = \begin{bmatrix} \frac{p_1}{a} \\ \frac{p_2}{I_{WC}} \\ bg\sin q_1 - u \\ u \end{bmatrix} \tag{24}$$

Where $q = [q_1 \quad q_2]^T$ and $p = [p_1 \quad p_2]^T = [a\dot{q}_1 \quad I_W C\dot{q}_2]^T$ are the generalized positions and momenta respectively. Which leads to write the Hamiltonian function as follows :

$$H(q,p) = \frac{1}{2}p^T M^{-1}(q)p = \frac{1}{2}[\frac{p_1^2}{a} + \frac{p_2^2}{I_{WC}}] \tag{25}$$

The equilibrium point to be stabilized is the upward position whith the inertia disk aligned ($q_1 = q_2 = 0$)

4.2 *Simulation results*

In (Ortega, Spong, et al. (2002))the authors proposed the following control law for the IWIP :

$$u = \gamma_1 sin(q_1) + k_p(q_2 + \gamma_2 q_1) + k_v k_2(\dot{q}_2 + \gamma_2 \dot{q}_1) \tag{26}$$

where $\gamma_1 > b, \gamma_2 = -\frac{a\gamma_1}{I_W C(\gamma_1 - b)}$ and $k_2 > 0$.
Together with k_p and k_v positive arbitrary constants, define the tuning gains.
In (Khraief et al. (2014), Khraief-Haddad et al. (2015)) we propose to rewrite the control law (26) in terms of the generalized coordinates q and momenta p as :

$$u = \underbrace{\gamma_1 sin(q_1) + k_1 q_1 + k_2 q_2}_{u_{es}} + \underbrace{k_3 p_1 + k_4 p_2}_{u_{di}} \tag{27}$$

where : $k_1 = k_p\gamma_2, k_2 = k_p, k_3 = k_v(\frac{a_2+a_3}{a_1 a_3 - a_2^2})$ and $k_4 = -k_v(\frac{a_1+a_2}{a_1 a_3 - a_2^2})$.
In the following paragraphs simulation results are obtained using Matlab software. Two simulation scenarios are considered in order to compare IDA-PBC controller with the proposed adaptive one.

The first scenario is to consider the system in the nominal case without any external disturbances. While the second one aims to show the robustness of both controllers against matched input disturbances.

4.2.1 Stabilization in the nominal case by both controllers

In the forthcoming simulations the dynamics of the PCH model (24) is considered. The parameters of the IWIP are described in table (1). They have been experimentally identified on the real prototype of the system. In this section simulation results obtained on the IWIP (in the nominal case) when applying both controllers are presented and discussed. The first controller is the IDA-PBC given in (27). We choose the initial configuration $(q_1 \quad q_2 \quad p_1 \quad p_2)^T{=}(0.2 \quad 0 \quad 0 \quad 0)^T$. The remaining control parameters were selected as $\gamma_1 = 6.1284$, $k_1 = 1.0367$ and $k_2 = 0.0011$. $k_3 = 16.62$ and $k_4 = 3.4640$.
The second controller is the proposed MRA-IDA-PBC when the control input is described by the following expression :

$$\begin{aligned} u^* &= \gamma_1 sin(q_1) + k_1 q_1 + k_2 q_2 + k_3 p_1 + k_4 p_2 + \Delta(x,t)\hat{z} \\ \Delta(x,t) &= [p_2, p_1]^T \\ \hat{z} &= [\hat{k_3}, \hat{k_4}] \\ \dot{\hat{z}} &= -K_a [p_2; p_1]\,(\dot{q_2} + \gamma_2\,\dot{q_1}) \end{aligned} \tag{28}$$

We consider the initial conditions :
$(q_1 \quad q_2 \quad p_1 \quad p_2 \quad k_3 \quad k_4)^T{=}(0.2 \quad 0 \quad 0 \quad 0 \quad 2 \quad 10)^T$. The remaining control parameters were selected as : $\gamma_1 = 6.1284$, $\gamma_2 = 942.447$, $k_1 = 1.0367$, $k_2 = 0.0011$ and $K_a = \begin{pmatrix} 2.08e-4 & 0 \\ 0 & 2.08e-4 \end{pmatrix}$.
Figure (3) displays the evolution of joint positions versus time for both controllers. A simple observation shows a better performance of the proposed adaptive IDA-PBC controller which assure the convergence towards the equilibrium point faster than the standard IDA-PBC controller. Figure (4) shows the efficiency of the MRA-IDA-PBC through the evolution of the angular velocities of the pendulum and the inertia wheel. When applying the proposed adaptive controller the evolution of the control input converges more rapidly than the one of the IDA-PBC controller and presents less oscillations. These observations can also be seen in figure(5). The effectiveness of the proposed controller can also be observed through the phase portrait in figure(6) and figure (7) which displays the evolution of estimated parameters versus time.

4.2.2 Disturbances rejection by both controllers

To test and compare the robustness of both controllers we propose in this section to check their capability to reject external matched disturbances.
A matched input disturbance is added to the dynamics (24). It consists of constant torques [4] $\|\delta_1(x,t)\| = 10$ added to the control (27) at time instants : $t = 5s$, $t = 10s$ and $t = 15s$ during $0.2s$ at each time. The obtained simulation results are shown in figure (8) and (9). According to the obtained result it can be noticed that the MRA-IDA-PBC controller is better than IDA-PBC

4. we can check simply that$\|\delta_1(x,t)\| \leq 12 * \left\|\left|(\nabla_p H_d)^T G\right|^2\right.$, leading to the theoretical results.

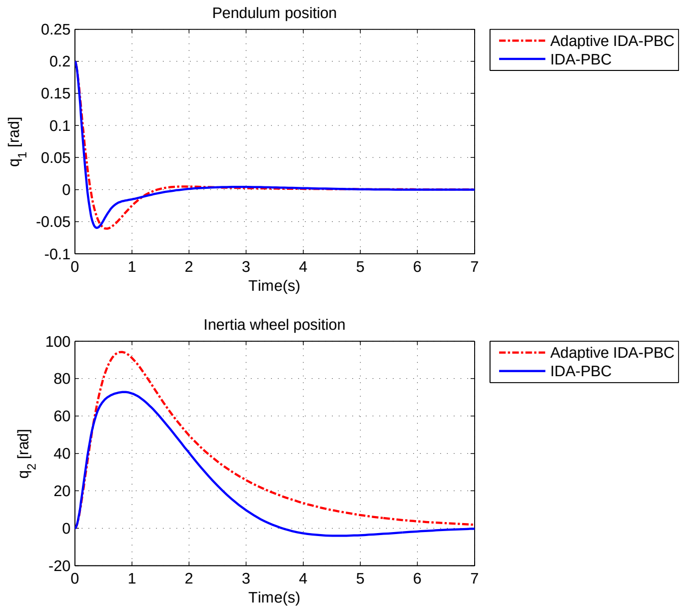

Figure 3.: Evolution of angular positions versus time

controller to compensate matched disturbances and keep the system around its open-loop unstable equilibrium point.

4.3 *Real-time experimental results*

4.3.1 Description of the experimental testbed

To validate the theoretical results, real-time experiments have been carried out on the inertia wheel inverted pendulum testbed shown in figure (10). This platform is designed and developed at LIRMM [5]. Mechanical stops constrain the movement of the pendulum angle θ_1. This angle is measured by an encoder fixed to the actuator of the system (Maxon EC-powermax 30 DC motor). An inclinometer FAS-G of micro strain served to measure in real-time the angle of the pendulum with respect to the vertical. The control approach is implemented on a computer using C++ language. The whole system is running under Ardence RTX real-time OS. Different control schemes have been already implemented on this testbed (Andary, Chemori, et Krut (2009), Touati et Chemori (2013), S. Andary, Chemori, et Krut (2009)). In the remaining of this section we

will compare the performance of both controllers experimentally.Two scenarios are represented

5. LIRMM (Montpellier Laboratory of Informatics, Robotics and Microelectronics) : http ://www.lirmm.fr

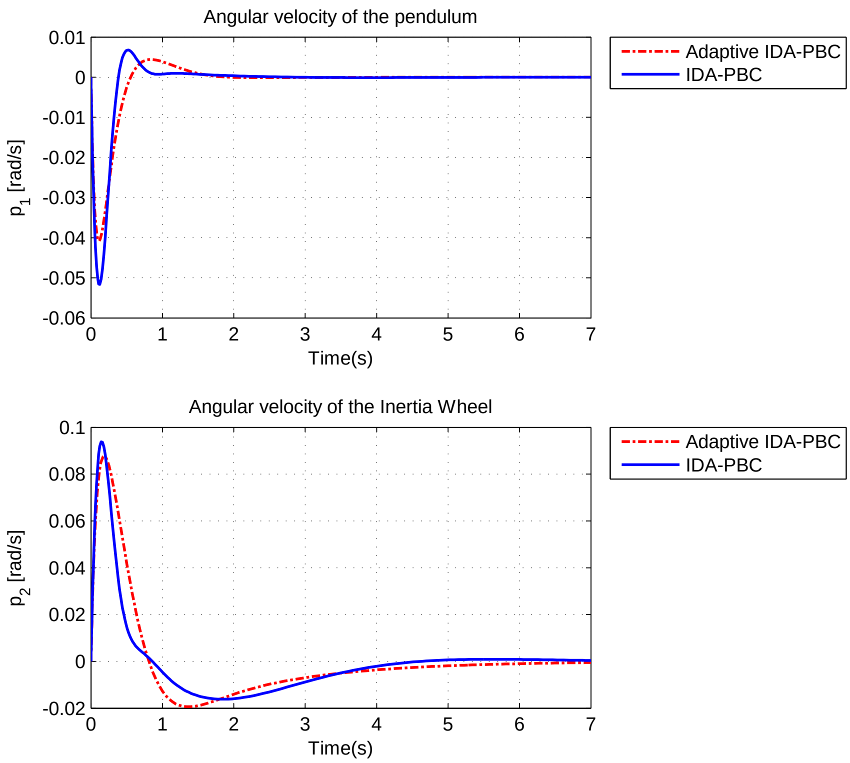

Figure 4.: Simulation results of the angular velocities

whith implementation issues. The first one concerns the control of the nominal system without perturbations ; however in the second one the system is subject to an external perturbation.

4.3.2 Stabilization in the nominal case

Real-time experiments have been carried out thanks to the experimental testbed described above. We apply at first the IDA-PBC controller (27) with the following parameters : $\gamma_1 = 6.1284$, $k_1 = 0.0471$, $k_2 = 0.00005$, $k_3 = 15.9840$ and $k_4 = 3.9960$. The proposed experiments are started from the initial condition $(q_1 \quad q_2 \quad p_1 \quad p_2)^T = (0.17 \quad 0 \quad 0 \quad 0)^T$. Secondly we apply the proposed MRA-IDA-PBC controller (28) starting with the configuration : $(q_1 \quad q_2 \quad p_1 \quad p_2 \quad K_3 \quad K_4)^T = (0.17 \quad 0 \quad 0 \quad 0 \quad 2 \quad 10)^T$. The remaining parameters are chosen as follows : $\gamma_1 = 6.1284$, $\gamma_2 = 942.447$, $k_1 = 0.0471$, $k_2 = 0.00005$, and $K_a = \begin{pmatrix} 0.02547 & 0 \\ 0 & 0.02547 \end{pmatrix}$. Figure (11) displays the obtained results in terms of the angular positions and velocities as well as the control input versus time. It can be clearly observed that the convergence in the case of MRA-IDA-PBC is better than the case with the standard IDA-PBC controller. The phase portrait in figure(12) illustrates experimentally the performance of the adaptation law added to the IDA-PBC controller.

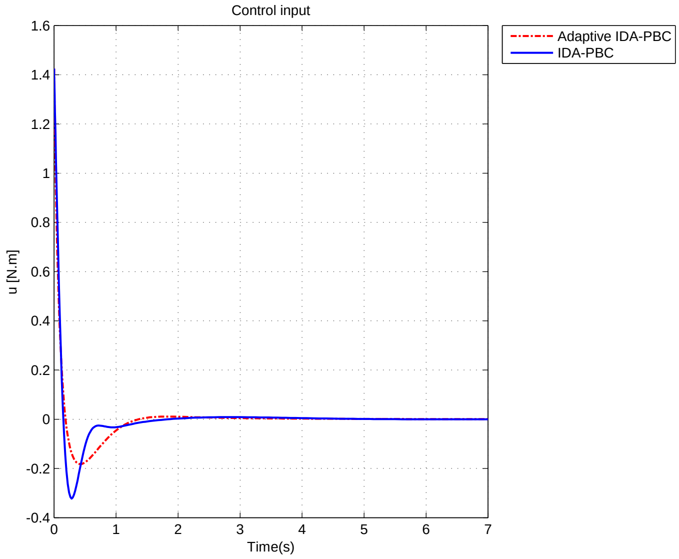

Figure 5.: Simulation results of the input control

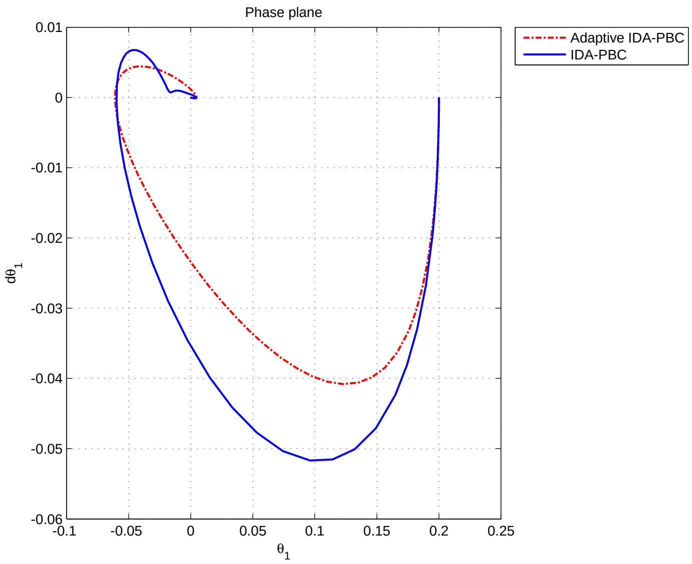

Figure 6.: Plot of the phase portrait $(\theta_1, \dot{\theta}_1)$

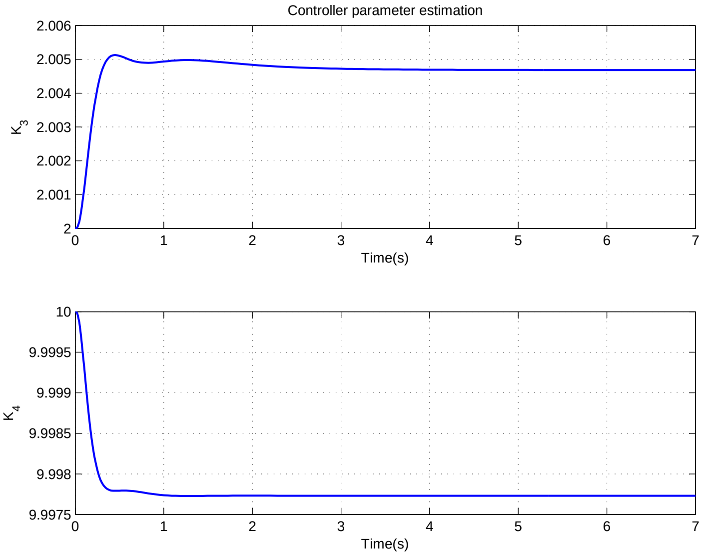

Figure 7.: Evolution versus time of the estimated parameters

4.3.3 Stabilization with matched input disturbances

In this scenario matched input disturbances rejection is emphasized experimentally by adding external torques to (27) which are generated by pushing the pendulum at approximately $t = 6s$, $t = 8s$ and $t = 12s$. Experimental results are displayed in figure (13). The effect of punctual disturbances at different times can be observed as peaks on the curves. Compensations of such disturbances are observed in the evolution of the angular position and velocity. These matched disturbances are better rejected by the MRA-IDA-PBC controller. We observe that the MRA-IDA-PBC controller compensates better the added external torques and maintain the system around the desired equilibrium point. These observations are also proved by displaying the phase portrait in figure (14).

5. Conclusion and future work

In this work we have studied the effect of matched input disturbances in the IDA-PBC method. We have considered the robustness improvement of IDA-PBC control applied to a class of underactuated mechanical systems. We propose an adaptive control scheme combined with the IDA-PBC design methodology. As an illustration we have presented the proposed adaptation law for the IDA-PBC controller of the Inertia Wheel Inverted Pendulum. The objective was to stabilize the system at its upward position (unstable equilibrium). Simulation as well as experimental results was presented to show the efficiency of the proposed controller compared to a classical IDA-PBC. The results illustrate how the adaptive control estimates and compensates for the errors on the

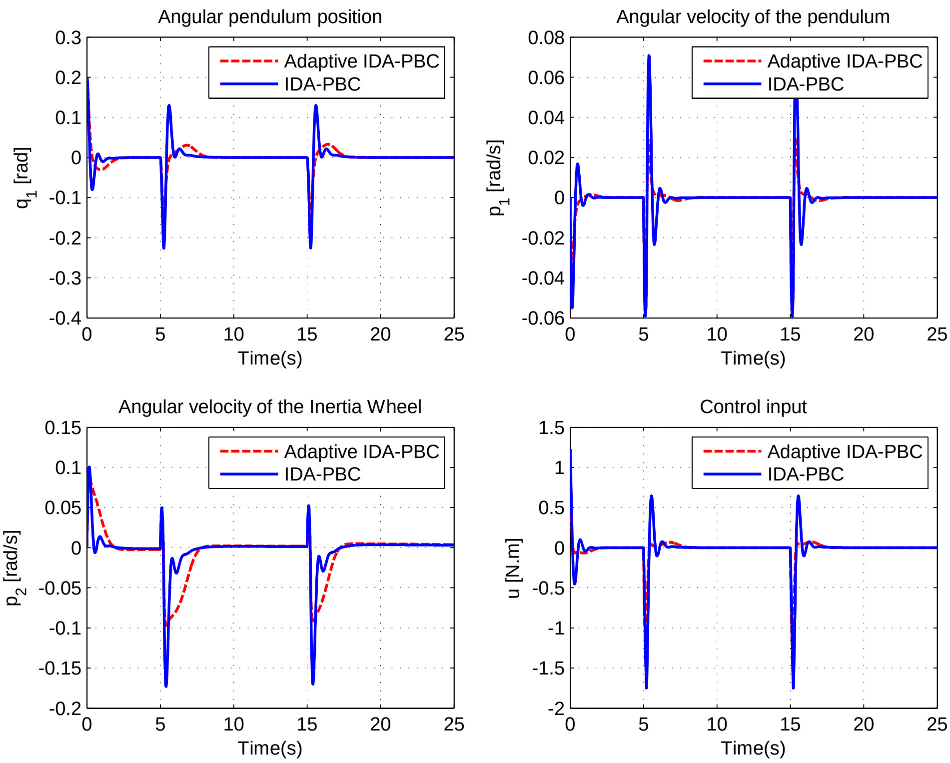

Figure 8.: Simulation results with matched disturbances

gain parameters, which yields smooth stabilization better than the classical IDA-PBC controller. Various possible extensions of this work can be investigated. At first, we can improve the adaptation law in order to increase the robustness of the IWIP against unmatched uncertainties. Secondly, discussions can be investigated about the generalization of the proposed adaptive control scheme to the case of other classes of underactuated mechanical systems. Besides, such control scheme can also be tested to track some reference trajectories for stable limit cycle generation.

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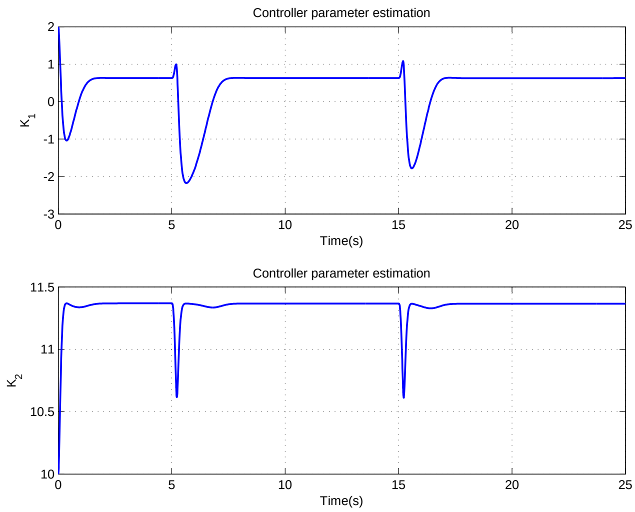

Figure 9.: Simulation results with matched disturbances

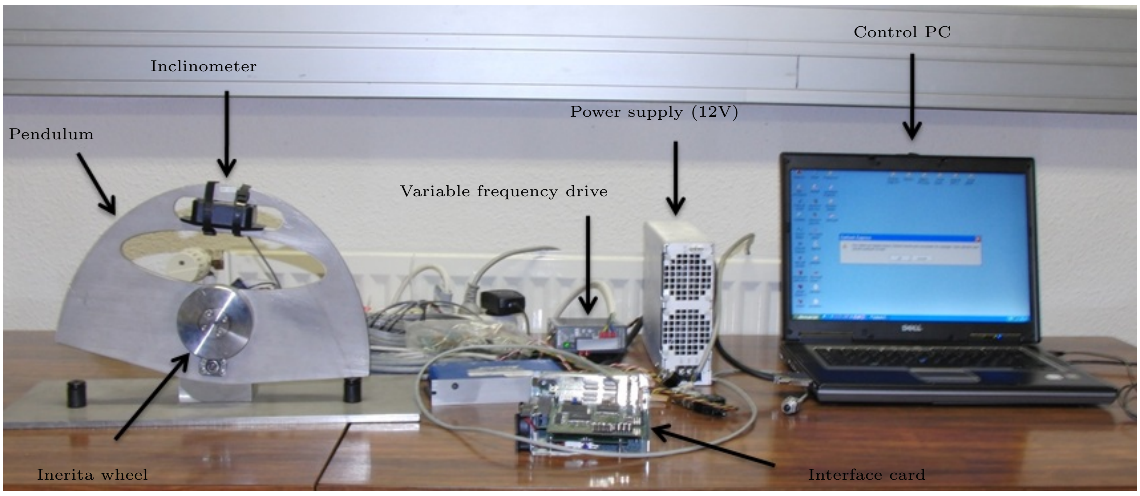

Figure 10.: IWIP experimental testbed

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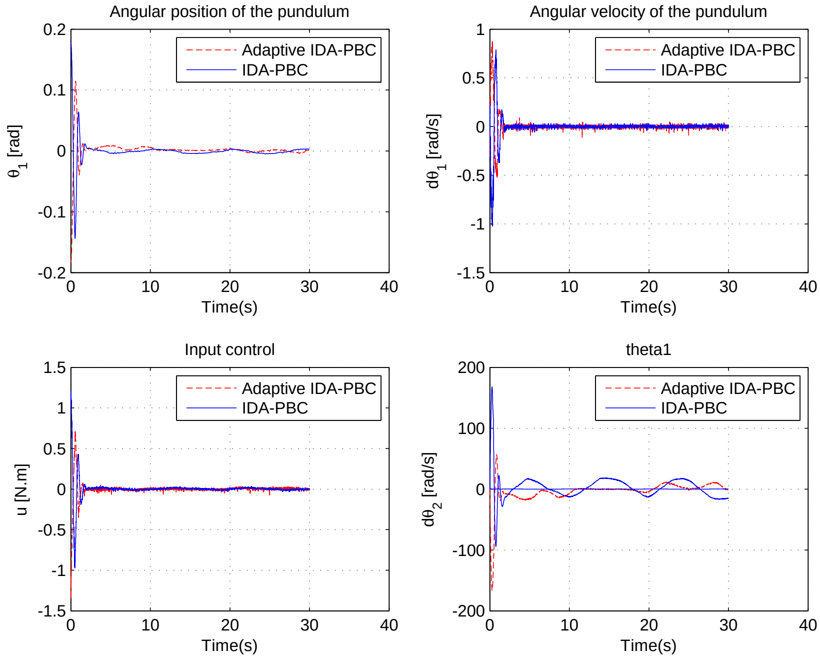

Figure 11.: Experimental results for both controllers

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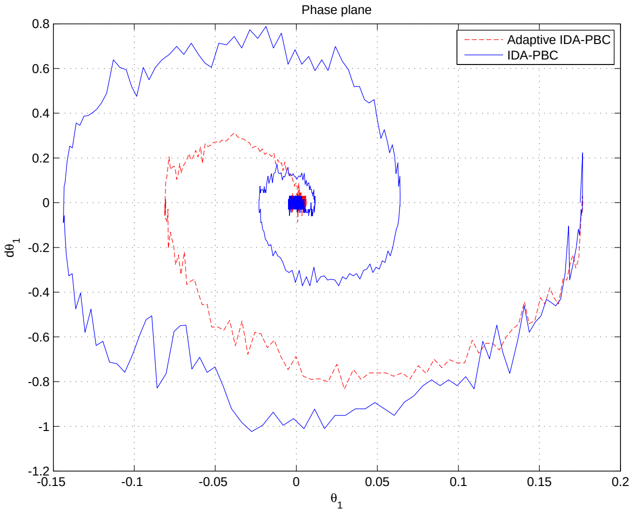

Figure 12.: Phase portrait $(\theta_1, \dot{\theta}_1)$ *obtained experimentally for both controllers*

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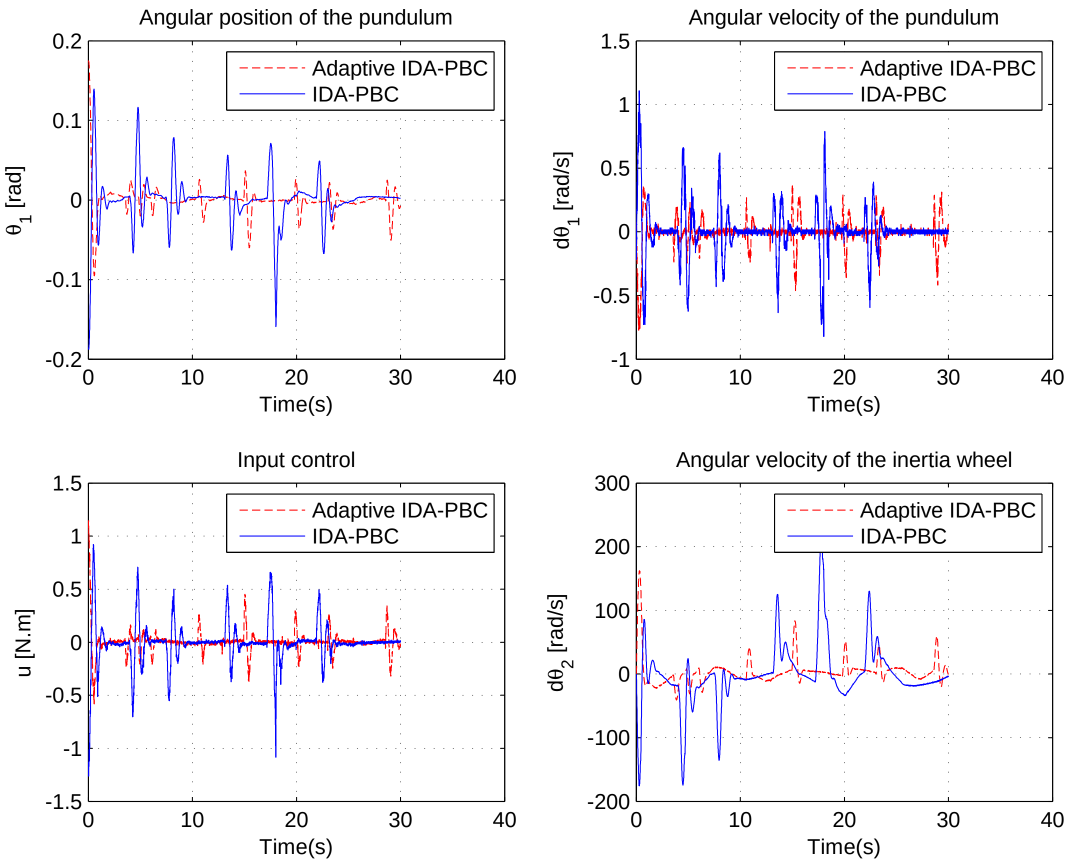

Figure 13.: Experimental results for both controllers

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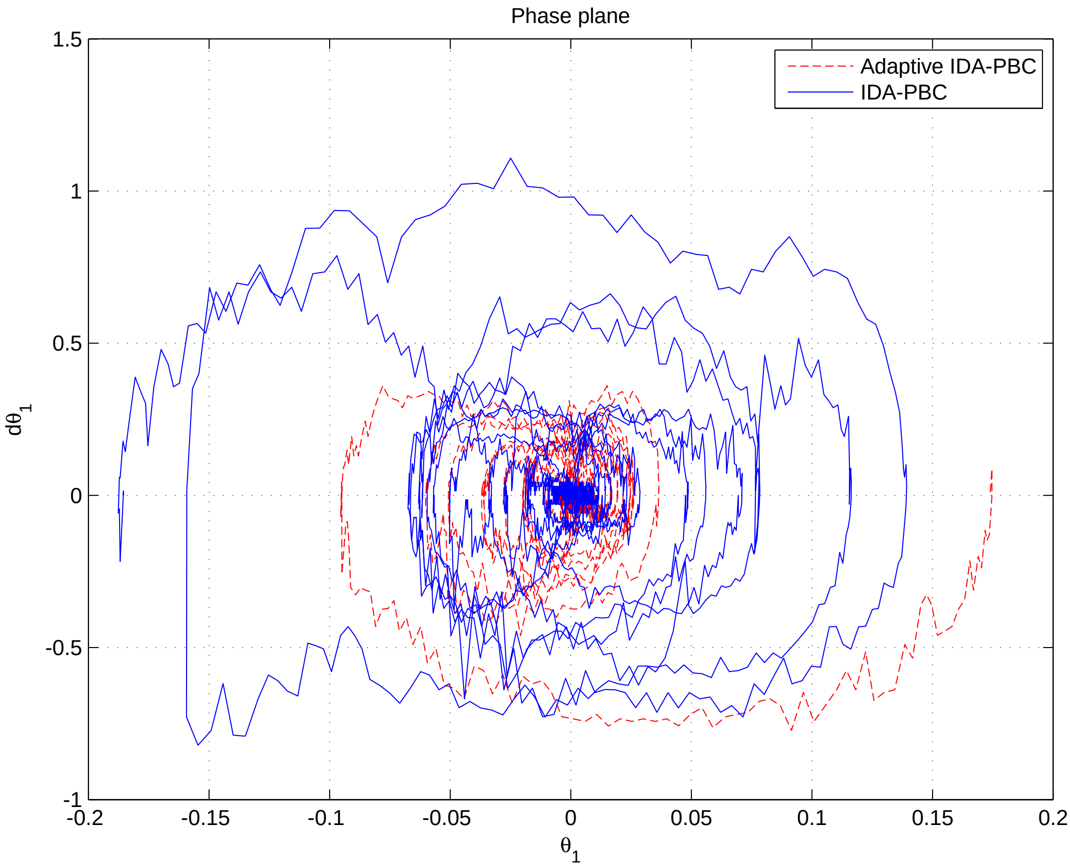

Figure 14.: Phase portrait obtained experimentally for both controllers

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