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External disturbance rejection in IDA-PBC controller for underactuated mechanical systems: from theory to real time experiments

N. Khraief Haddad¹, A. Chemori² and S. Belghith³

Abstract—Proving the robustness, vis-à-vis external disturbances of IDA-PBC (Interconnexion Damping Assignment, Passive Based Control) controller for underactuated mechanical systems is addressed in this paper. Some sufficient stability conditions on matched and unmatched disturbances are provided. As illustration we propose to revisit the application of IDA-PBC controller to the Inertia Wheel Inverted Pendulum (IWIP) in the presence of external disturbances. Simulations and real-time experimental results are presented as validations of the theoretical results.

I. INTRODUCTION

In this paper we will consider the robustness analysis of IDA-PBC controller when applied to underactuated mechanical systems. The latter, will be, necessarily [1], described by the so-called Port Controlled Hamiltonian models (PCH). PCH systems have been introduced in 1994 by Van der Schaft and Mashke [2]. The central paradigm of complex systems modeling is to have individual open subsystems with well defined port interfaces, hiding an internal model of variable complexity, and a set of rules describing how these subsystems interact through the port variables [3]. To implement this general idea we can use the so called port Hamiltonian systems or port-controlled Hamiltonian systems (PCHS). PCH models represent another alternative to the classical Euler-Lagrange (or standard Hamiltonian) models.

Writing a system in a PCH form has the advantage to cover a large set of physical systems and provide important structural properties. An extended survey of PCHS is presented in [4]. One procedure to control PCH systems is called IDA-PBC [5]. It combines the passivity properties of PCHS with control by interconnection and energy based control. IDA-PBC uses the hamiltonian framework, it consists in solving the PDE associated to the energy balance equation. This technique has been applied to various plants: Mechanical systems [6], [7], magnetic levitation systems [8], [9], mass balance systems [10], electrical machines [11], [12], power converters [13]. For an in-depth review of IDA-PBC the reader is referred to [14].

In this paper we will study the robustness of IDA-PBC, for underactuated mechanical systems. We will focus on the case of external disturbances where little is known. Many studies turn around the improvement of the robustness. Input disturbance suppression for PCHS using an internal model, has been studied in [15]. In [9] an IDA-PBC controller applied to a magnet levitation system was experimentally tested. To solve the robustness problem they add an integral term to the error of the passive output. The same technique was used in [16] to improve the robustness of hamiltonian passive control. Stability and robustness of disturbed port-controlled Hamiltonian systems with dissipation have been addressed in [17]. The authors studied IDA-PBC controller robustness against parameters uncertainties. Recently Jose Guadalupe Romero et al. [18] improve the robustness vis-à-vis external disturbance, of energy shaping controllers for fully actuated mechanical systems. They design a dynamic state feedback controller such that the closed-loop system ensures some stability properties in spite of the presence of external disturbances. Nevertheless two important issues are still open:

1) The proof of the robustness of the controller obtained by IDA-PBC technique.
2) More real-time experimental applications when applying IDA-PBC controller.

Motivated by both issues and as a main contribution of this paper, we prove that, when applying IDA-PBC to underactuated mechanical systems, the stability is preserved in spite of the presence of external disturbances. Some sufficient stability conditions on the external disturbances are given. To strength those results we propose to validate the proposed controller experimentally on the well known inertia wheel inverted pendulum (IWIP). Based on the IDA-PBC controller designed in [6], we did some experiments in presence of external disturbances. The robustness of IDA-PBC controller is proved through these experimental results.

The remaining of the paper is organized as follows. In section II, background on IDA-PBC for underactuated systems is given. Section III introduces the main contribution about external disturbance rejection in IDA-PBC controllers. Simulations and experimental results on an inertia wheel inverted pendulum are given in section IV. Finally, we present some conclusions and future work in section V.

II. BACKGROUND ON IDA-PBC CONTROL

In this section let’s consider previous work proposed in [6], [5]. An underactuated mechanical system with no natural...
damping can be written in PCH form as follows:

\[
\begin{align*}
\left( \begin{array}{c}
\dot{q} \\
\dot{\rho}
\end{array} \right) &= \left( \begin{array}{cc}
0 & I_n \\
-I_n & 0
\end{array} \right) \left( \begin{array}{c}
\nabla_q H \\
\nabla_\rho H
\end{array} \right) + \left( \begin{array}{c}
0 \\
G(q)
\end{array} \right) u \\
y &= G(q)^T \nabla_p H
\end{align*}
\]

with total energy

\[
H(q, p) = \frac{1}{2} p^T M^{-1}(q)p + V(q)
\]

(1)

will be called nominal system, where \( q \in \mathbb{R}^n, p \in \mathbb{R}^n \) are the generalized position and momenta respectively. \( G(q) \in \mathbb{R}^{n \times m} \), is the input matrix. We consider here that the system is underactuated and assume \( \text{rank}(G) = m < n \), \( u \) and \( y \) are the control input vector and the output vector respectively. \( M(q) = M^T(q) > 0 \) is the inertia matrix, and \( V(q) \) is the potential energy. We have that \( \dot{u} = M^{-1}(q)p \).

The energy shaping control to assign the equilibrium \( q^*, 0 \) by the following IDA-PBC controller [6]:

\[
\begin{align*}
\dot{q} &= (J_d(q, p) - R_d(q, p)) \left( \begin{array}{c}
\nabla_q H \\
\nabla_\rho H
\end{array} \right) + \delta_1(x, t) + \delta_2(x, t) \\
y &= G(q)^T \nabla_p H
\end{align*}
\]

(4)

\( u_{es}, u_{di} \) is the energy shaping control to assign the equilibrium and \( u_{di} \) injects damping to achieve asymptotic stability. We obtain finally, the following desired (closed loop) PCH dynamics:

\[
\begin{align*}
\dot{q} &= (J_d(q, p) - R_d(q, p)) \left( \begin{array}{c}
\nabla_q H \\
\nabla_\rho H
\end{array} \right) + \delta_1(x, t) + \delta_2(x, t) \\
y &= G(q)^T \nabla_p H
\end{align*}
\]

(5)

where \( M_d = M_d^T > 0, J_d = -J_d^T = \left( \begin{array}{cc}
0 & M_d^{-1} \\
-M_d^{-1} & J_2
\end{array} \right) \) is the interconnection matrix, and \( R_d = R_d^T = \left( \begin{array}{cc}
0 & 0 \\
0 & G Ke^T
\end{array} \right) \) is the damping matrix [5].

III. IDA-PBC FOR DISTURBED UNDERACTUATED MECHANICAL SYSTEMS

A. Problem formulation

Describe disturbed underactuated mechanical system by the following PCH model:

\[
\begin{align*}
\left( \begin{array}{c}
\dot{q} \\
\dot{\rho}
\end{array} \right) &= \left( \begin{array}{cc}
0 & I_n \\
-I_n & 0
\end{array} \right) \left( \begin{array}{c}
\nabla_q H \\
\nabla_\rho H
\end{array} \right) + \left( \begin{array}{c}
0 \\
G(q)
\end{array} \right) (u + \delta_1(x, t)) + \delta_2(x, t) \\
y &= G(q)^T \nabla_p H
\end{align*}
\]

(6)

where \( t \in \mathbb{R}, x = (q, p)^T \in \mathbb{R}^{2n} \) is the state, \( u(t) \in \mathbb{R}^m \) is the control, \( \delta_1(x, t) \) is the matched uncertainties, and \( \delta_2(x, t) \) is the unmatched uncertainties. Both \( \delta_1(x, t) \) and \( \delta_2(x, t) \) are assumed to be unmeasured and bounded in magnitude, usually their Euclidean norm is denoted by \( \| \| \). In this paper we formulate our IDA-PBC stabilization objective as follows: Given the disturbed PCH system (6) and a desired equilibrium \( (q^*, 0) \), is the IDA-PBC controller (4) capable to reject disturbances and keep \( (q^*, 0) \) always asymptotically stable? In particular we treat two cases:

- Preserving asymptotic stability in spite of the existence of matched disturbances.
- Preserving asymptotic stability in spite of the existence of unmatched disturbances.

B. Main result

Case 1 : Matched uncertainties \( \delta_2(x, t) = 0 \)

Applying the controller (4) to (6) we obtain the following closed loop disturbed system:

\[
\begin{align*}
\left( \begin{array}{c}
\dot{q} \\
\dot{\rho}
\end{array} \right) &= (J_d(q, p) - R_d(q, p)) \left( \begin{array}{c}
\nabla_q H \\
\nabla_\rho H
\end{array} \right) + \delta_1(x, t) \\
y &= G(q)^T \nabla_p H
\end{align*}
\]

(7)

Note that \( \delta_1(x, t) = 0, G(q)\delta_1(x, t)^T \) is also a vector of external disturbances. Let \( \lambda_{\min} \{K_v\} \) the smallest eigenvalue of the matrix \( K_v \), and \( \bar{x} = (q - q^*, p)^T \).

The following proposition set some sufficient conditions on the disturbances boundaries in order to get \( (q^*, 0) \) asymptotically stable.

Proposition 1:

Consider the closed loop dynamics (7) with the desired total energy \( H_d \).

If \( \| \delta_1(x, t) \| \leq \lambda_{\min} \{K_v\} \| (\nabla_p H_d)^T G \|^2 \), then \( \dot{H}_d \leq 0 \) and \( \bar{x} \) of (10) is an asymptotically stable equilibrium point.

Proof:

\[
\begin{align*}
\dot{H}_d &= \nabla_q H_d \dot{q} + \nabla_\rho H_d \dot{\rho} \\
&= (\nabla_q H_d)^T (M_d^{-1} M_d^{-1}) \nabla_q H_d + (\nabla_\rho H_d)^T G(q) \delta_1(x, t) - (\nabla_\rho H_d)^T (M_2^T M_2^{-1}) \nabla_q H_d + (\nabla_\rho H_d)^T G(q) \delta_2(x, t) - (\nabla_\rho H_d)^T (G K_v G^T) \nabla_p H_d
\end{align*}
\]

(8)

If \( \| \delta_1(x, t) \| \leq \lambda_{\min} \{K_v\} \| (\nabla_p H_d)^T G \|^2 \)

\[
\dot{H}_d \leq \| \delta_1(x, t) \| - \lambda_{\min} \{K_v\} \| (\nabla_p H_d)^T G \|^2
\]

(9)

and \( \dot{H}_d \leq 0 \)

Case 2 : Unmatched uncertainties \( \delta_2(x, t) \neq 0 \)

Applying the controller (4) to (6) we obtain the following closed loop disturbed system:

\[
\begin{align*}
\left( \begin{array}{c}
\dot{q} \\
\dot{\rho}
\end{array} \right) &= (J_d(q, p) - R_d(q, p)) \left( \begin{array}{c}
\nabla_q H \\
\nabla_\rho H
\end{array} \right) + \delta_2(x, t) \\
y &= G(q)^T \nabla_p H
\end{align*}
\]

(10)

Note that \( \delta_2(x, t) = (\delta_2_1(x, t), \delta_2_2(x, t))^T \), let \( \lambda_{\min} \{K_v\} \) the smallest eigenvalue of the matrix \( K_v \), and \( \bar{x} = (q - q^*, p)^T \).

The following proposition set some sufficient conditions on the disturbances boundaries in order to get \( (q^*, 0) \) asymptotically stable.

Proposition 2:
Consider the closed loop dynamics (10) with the desired total energy $H_d$. If $|\nabla T H_d \delta \tilde{x}(x, t)| \leq \lambda_{\min} \{K_v\} \|\nabla p H_d\|^2$ then $\tilde{H}_d \leq 0$ and $\tilde{z}$ of (10) is an asymptotically stable equilibrium point. The proof of the proposition 2 is completed proceeding as in the case 1.

IV. APPLICATION : INERTIA WHEEL INVERTED PENDULUM (IWIP)

A. IWIP : Description and modeling

![Diagram of Inertia Wheel Inverted Pendulum]

Fig. 1: Inertia Wheel Inverted Pendulum.

The pendulum device shown in Fig. 1 is called the Inertia Wheel Inverted Pendulum (IWIP). It may be modeled as a two-degrees-of-freedom serial mechanism. The first link is the pendulum and the second one is the rotating disc. The IWIP is an underactuated system, the torque produces an angular acceleration of the end-mass which generates a coupling torque at the pendulum axes.

The parameters of the IWIP are shown in TABLE 1:

<table>
<thead>
<tr>
<th>Description</th>
<th>Notation</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pendulum angle with respect to vertical axis</td>
<td>$\theta_1$</td>
<td>rad</td>
</tr>
<tr>
<td>Wheel angle with respect to pendulum axis</td>
<td>$\theta_2$</td>
<td>rad</td>
</tr>
<tr>
<td>Mass of the pendulum</td>
<td>$m$</td>
<td>kg</td>
</tr>
<tr>
<td>Mass of the Wheel</td>
<td>$M$</td>
<td>kg</td>
</tr>
<tr>
<td>Length from pendulum base (PB) to pendulum center of mass (PC)</td>
<td>$l$</td>
<td>m</td>
</tr>
<tr>
<td>Length from pendulum base (PB) to Wheel center of mass (WC)</td>
<td>$L$</td>
<td>m</td>
</tr>
<tr>
<td>Rotational Inertia of pendulum center of mass (PC)</td>
<td>$I_{PC}$</td>
<td>kg.m$^2$</td>
</tr>
<tr>
<td>Rotational Inertia of pendulum base (PB)</td>
<td>$I_{PB}$</td>
<td>kg.m$^2$</td>
</tr>
<tr>
<td>Rotational Inertia of Wheel about Wheel center of mass (WC)</td>
<td>$I_{WC}$</td>
<td>.m$^2$</td>
</tr>
<tr>
<td>Constant of gravitational acceleration</td>
<td>$g$</td>
<td>m/s$^2$</td>
</tr>
</tbody>
</table>

TABLE 1: Description of dynamical parameters of the IWIP

The Euler-Lagrange equations of motion can be written as

$$\begin{pmatrix} (a + I_{WC}) & I_{WC} \\ I_{WC} & I_{WC} \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} - \begin{pmatrix} bg \sin \theta_1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix} \quad (11)$$

where $\theta = [\theta_1 \ \theta_2]^T$, $u$ is the torque applied by the motor to spin the wheel and:

$$\begin{align*}
a &= ML^2 + I_{PB} \\
b &= ml + ML.
\end{align*} \quad (12)$$

Let’s introduce the following change of coordinates:

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \quad (13)$$

This leads to a simplified model:

$$\begin{pmatrix} q_1 \dot{q}_1 \\ q_2 \dot{q}_2 \\ \dot{p}_1 \\ \dot{p}_2 \end{pmatrix} = \begin{pmatrix} \frac{q_1}{l_{WC}} \\ bg \sin q_1 - u \\ \frac{p_2}{l_{WC}} \\ u \end{pmatrix} \quad (15)$$

Where $q = [q_1 \ q_2]^T$ and $p = [p_1 \ p_2]^T = [aq_1 \ I_{WC}q_2]^T$ are the generalized positions and momenta respectively.

Which leads to write the Hamiltonian function:

$$H(q, p) = \frac{1}{2} p^T M^{-1}(q) p - \frac{1}{2} \frac{p_1^2}{a} + \frac{p_2^2}{I_{WC}} \quad (16)$$

At first we aim at applying IDA-PBC controller in order to globally stabilize the system around its unstable equilibrium (the upward position of the pendulum) with the inertia disk aligned, this equilibrium corresponds to $q_1^a = q_2^a = 0$.

Then the desired Hamiltonian function can be defined as:

$$H_d(q, p) = \frac{1}{2} p^T M_d^{-1}(q) p + V_d(q) \quad (17)$$

where $^3 M_d = \begin{pmatrix} a_1 & a_2 \\ a_2 & a_3 \end{pmatrix}$ with $a_1 > 0$, $a_1 a_3 > a_2^2$ and $a_1 + a_2 < 0$. Secondly, the robustness of such controller will be tested in the presence of external, matched and unmatched, disturbances.

3. Note that the inertia matrix $M$ of the IWIP is independent of $q$, so we can take $J_2 = 0$ and $M_d$ a constant matrix [6]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>pendulum mass</td>
<td>3.228</td>
<td>Kg</td>
</tr>
<tr>
<td>M</td>
<td>mass of the wheel</td>
<td>0.33081</td>
<td>Kg</td>
</tr>
<tr>
<td>l</td>
<td>Pendulum center of mass position</td>
<td>0.06</td>
<td>m</td>
</tr>
<tr>
<td>L</td>
<td>Wheel center of mass position</td>
<td>0.044</td>
<td>m</td>
</tr>
<tr>
<td>I1</td>
<td>Pendulum inertia</td>
<td>0.0314</td>
<td>Kgm²</td>
</tr>
<tr>
<td>I2</td>
<td>Wheel inertia</td>
<td>4.176e-4</td>
<td>Kgm²</td>
</tr>
</tbody>
</table>

**TABLE II: Dynamic Parameters of the IWIP**

**B. IDA-PBC controller**

In [6] the authors proposed the following control law for the IWIP:

\[ u = \gamma_1 \sin(q_1) + k_p(q_2 + \gamma_2 q_1) + k_v(k_2 q_2 + \gamma_2 q_1) \quad (18) \]

where \( \gamma_1 > b, \gamma_2 = -\frac{a_2}{l_W C(\gamma_1 - \delta)} \) and \( k_2 > 0 \).
Together with \( k_p \) and \( k_v \) positive arbitrary constants, define the tuning gains.

In this paper we propose to rewrite the control law (18) in terms of the generalized coordinates \( q \) and momenta \( p \) as:

\[ u = \sum_{i=1}^{4} \left[ \frac{\sin(q_1)}{\lambda_{i1}} + \frac{q_1}{\lambda_{i2}} + \frac{q_2}{\lambda_{i3}} + \frac{k_1 q_1 + k_2 q_2}{\lambda_{i4}} \right] \quad (19) \]

where :\( k_1 = k_p \gamma_2, k_2 = k_p, k_3 = k_v \frac{a_2}{a_1 a_3 - a_2^2} \) and \( k_4 = -k_v \frac{a_1 + a_2}{a_1 a_3 - a_2^2} \).

**C. Simulation results**

We will consider in simulations the dynamics of the PCH platform is designed and developed at LIRMM.

1) Stabilization with external disturbances: To test IDA-PBC controller robustness for the IWIP, we will check in this section, its capability to reject external, matched and unmatched, disturbances.

**Case 1 : Matched disturbances**

In this case a matched disturbance is added to the dynamics (15). It consists of constant torques \( \|\delta_1(x, t)\| = 10^4 \) added to the control (19) at time instants : \( t = 5s, t = 10s \) and \( t = 15s \) during 0.2s at each time. The obtained simulation results are shown in Fig.2 and prove that the IDA-PBC controller is capable to compensate matched disturbances and keep the system around the open loop unstable equilibrium point.

**Case 2 : Unmatched disturbances**

In this case a vector of constant disturbances \( \|\delta_2(x, t)\| = [0.3; -0.315]^T \) is added to the dynamics of the IWIP at time.

4. we can check simply that: \( \|\delta_1(x, t)\| \leq 12 \times \| (\nabla_{p} H_d) \|^2 \), leading the theoretical results.

5. we can check simply that: \( \|\nabla_{p} H_d, \delta_2(x, t) \| \leq 12 \times \| (\nabla_{p} H_d) \|^2 \), leading the theoretical results.

**Fig. 2: Simulation results with matched disturbances**

**t = 10s.** Fig. 3 displays the performance of IDA-PBC controller.
Not that we can check the disturbance boundary by varying the amplitude of the added vector of disturbances and we can observe the lost of stability of the system when exceeding a limit value (determining in section III) of the amplitude disturbance.

**Fig. 3: Simulation results with unmatched disturbances**

**D. Real-time experimental results**

1) Experimental testbed: . To validate the theoretical results, real-time experiments are carried out on the inertia wheel inverted pendulum testbed shown in Fig. 4. This platform is designed and developed at LIRMM². Mechanical stops constrain the movement of the pendulum angle \( \theta_1 \).

6. LIRMM (Montpellier Laboratory of Informatics, Robotics and Microelectronics) : http://www.lirmm.fr
This angle is measured by an encoder linked to the actuator of the system (Maxon EC-powermax 30 DC motor). An inclinometer FAS-G of micro strain served to measure in real time the angle of the pendulum with respect to the vertical. The control approach is implemented on a computer using C++ language. The whole system is running under Ardence RTX real-time OS. Different control schemes have been developed and implemented on this testbed (Fig. 5) [21], [22], [23].

2) Stabilization with external disturbances: The two cases about external, matched and unmatched, disturbances are emphasized experimentally by two scenarios.

Scenario 1: It is illustrated in Fig. 6. The external torques added to (19) are generated by pushing the pendulum at approximately \( t = 6s \), \( t = 8s \) and \( t = 12s \). Experimental results are displayed in Fig. 7. The effect of punctual disturbances at different times can be observed as peaks on the curves. Compensations of such disturbances are observed in the evolution of the angular position and velocity. Matched disturbances are also compensated by the controller. We observe that the IDA-PBC controller compensates the added external torques and maintain the system around the desired equilibrium point.

Scenario 2: The second type of external disturbances is illustrated in Fig. 8. It consists of an additional mass attached to the body of the pendulum. This mass generates a persistent torque applied to the pendulum (passive joint). Fig. 9 shows the obtained experimental results for the second scenario.

Smooth stabilization is always guaranteed in spite of the existence of a persistent disturbance. The permanent rotation of the inertia wheel (\( \dot{\theta}_2 \)) permit to compensate the unmatched disturbances. As a result the IDA-PBC controller reject this type of disturbances and the pendulum is kept around its unstable equilibrium point.
V. CONCLUSION AND FUTURE WORK

This paper characterized the robustness of IDA-PBC control applied to a class of underactuated mechanical systems. The effect of external disturbances is especially studied in this paper. Two sufficient stability conditions are provided to deal with matched and unmatched disturbances. Motivated by the practical matter of IDA-PBC, this paper presented experimental results shown that IDA-PBC is robust with respect to external disturbances. The experimental platform used is an inertia wheel inverted pendulum developed at LIRMM. Based on results presented in this paper we note that in the presence of unmatched disturbances the controller cannot compensate static error. Extension of this work can be done in the sense of improving the robustness of IDA-PBC controller for a class of underactuated systems. An adaptive control can be proposed to compensate errors that are caused by uncertain parameters and unmatched disturbances.

REFERENCES