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## Fractional order model reference adaptive control for SCARA robot trajectory tracking

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**Abstract:** This paper proposes a new control strategy to the problem of trajectory tracking of SCARA robotic manipulator system by using a fractional order model reference adaptive controller (FMRAC) for sufficiently smooth trajectories. Over the few last years the idea of introducing fractional calculus and systems in adaptive control has found a great interest, for the benefits one can win in the performances given by such systems. The main contribution of this paper, is to show that the proposed fractional adaptive controller is able to reduce the delay time and the overshoot existing in classical control approach. Simulation results illustrate the effectiveness of the proposed control approach comparatively to the classical integer order one.

**Keywords:** fractional adaptive control; model reference adaptive control; MRAC; SCARA robot; trajectory tracking; approximation methods.

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## 1 Introduction

Modern industry depends increasingly on robotic manipulators for their ability of improving the product quality and increasing the production efficiency. Industrial tasks needing more and more accuracy in trajectory tracking involved a great research effort on motion control systems design (Borenstein and Koren, 1987). Adaptive control techniques have been intensively used for robotic systems as the problem of controlling the motion of rigid-link manipulators which require precise knowledge of the complete manipulator dynamic model for implementation suffer from the lack of accuracy of joint velocity measurements (Colbaugh et al., 1996; Pagilla and Tomizuka, 2001; Alonge et al., 2003; Yildirim, 2005; Yanling, 2015).

Model reference adaptive control (MRAC) remains one of the most popular approaches of adaptive control, regarding its simplicity of implementation and its high efficiency (Tung et al., 2000; Cortesão, 2009). In this adaptive system, the desired performance is specified by a chosen reference model. The difference between the plant output and the reference model output is used to adjust the controller parameters. Its ability to deal with unknown or slowly varying plants attracted many researchers, who tried to increase the MRAC control scheme robustness in order to better deal with industrial application problems in presence of noises and disturbances. Various modified approaches were proposed for this aim (Chien et al., 1996; Makoudi and Radouane, 1996).

In this paper we propose a fractional order MRAC approach (Ladaci and Charef, 2006; Bensafia and Ladaci, 2011) for trajectory tracking of a robot manipulator. The proposed controller introduces in classical MRAC scheme a fractional order systems as referential dynamics in order to improve the global control system behaviour. Since a decade and the pioneering works on FMRAC design (Hadjili and Abida, 1994), many variants of FMRAC control scheme have been developed (Vinagre et al., 2002; Ladaci et al., 2009; Shi et al., 2014; Wei et al., 2015).

This control approach has been applied to different types of processes such as a robot arm (Ladaci and Charef, 2002), a DC motor (Ladaci and Charef, 2006), a hydraulic driven flight motion simulator (Ma et al., 2009), an isotope separation plant (Dumitrache et al., 2012), an automatic voltage regulator (Aguila-Camacho and Duarte-Mermoud, 2013).

In this paper the trajectory tracking control of a SCARA type robot is considered. SCARA manipulators are widely used in industrial factories and academic laboratories, gathering various research works for different control strategies (Garg, 1991; Whitcomb et al., 1993; Visioli and Legnani, 2002; Roudbari, 2010).

Some fractional order control schemes have been already proposed for the problem of robotic manipulators control for trajectory tracking: Delavari et al. (2010) proposed a fractional adaptive PID controller for a robot manipulator showing that the trajectory of the robot converges to the desired one asymptotically. Marcos et al. (2008) studied the fractional chaotic dynamics of robots with kinematical redundancy. Whereas, Dumlu and Erenturk (2002) introduced a fractional-order  $PI^\lambda D^\mu$  controller to determine the control action for 3-degrees-of-freedom parallel manipulator and were able to improve the results obtained with the classical PID controller.

The main contribution of this work is the use the fractional model reference adaptive control (FMRAC) to determine the control action for trajectory tracking of a SCARA type manipulator. This control application is a major challenge as the trajectory tracking

control of industrial parallel mechanisms involves high speed, high acceleration, and high accuracy.

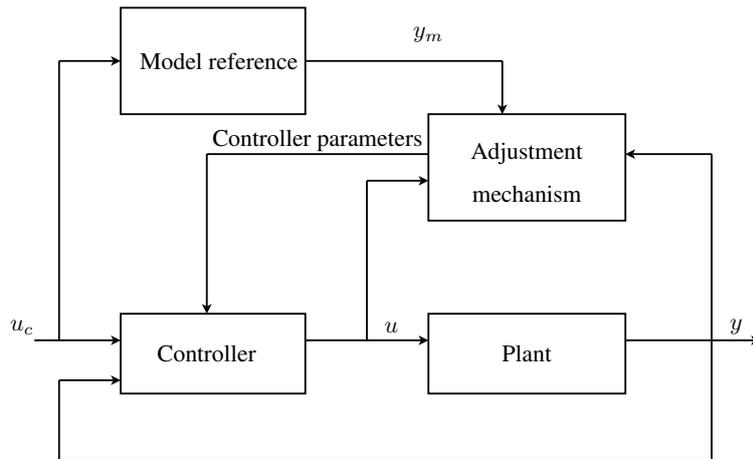
This paper is structured as follows: Section 2 is an background on MRAC and MIT rule approach. Section 3 presents fractional order systems and their approximation methods with rational transfer functions. The FMRAC with application time varying reference trajectory tracking in robotics are given in Section 4. Section 5 presents the simulation results of the proposed fractional adaptive controller for SCARA robot. Finally, some concluding remarks with future work are presented in Section 6.

## 2 Background on MRAC: description and limitations

### 2.1 MRAC description

The parallel structure (Figure 1) is the most famous MRAC structure, called the method of the output error; this approach involves plant model identification (Johnson et al., 1984; Landau, 1979).

**Figure 1** Parallel structure of MRAC



Consider an single input, single output system, which can be a model of continuous time or discrete-time model:

$$y(t) = \frac{B}{A} u(t) \tag{1}$$

where  $u$  is the control signal and  $y$  is the output signal. The symbols  $A$  and  $B$  denote polynomials in the differential operator  $p$ . It is assumed that the degree level  $deg(A) \geq deg(B)$ , the system is causal.

We assume that the performance specifications are given in terms of a reference model of the form,

$$y_m(t) = \frac{B_m}{A_m} u_c(t) \tag{2}$$

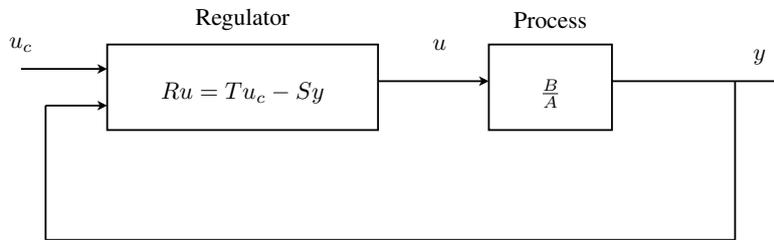
where  $u_c$  is the reference signal and  $y_m$  is the desired model output signal.  $A_m$  and  $B_m$  are polynomials in the differential operator  $p$ .

The general linear control law represented in Figure 2, is described as:

$$Ru = Tu_c - Sy \quad (3)$$

where  $R$ ,  $S$ , and  $T$  are polynomials. This control law represents a negative feedback with the transfer operator  $-S/R$  and a feedforward with the transfer operator  $T/R$ . It thus has two degrees of freedom.

**Figure 2** Closed-loop system with a linear regulator



The gradient approach of MRAC is based on the assumption that the parameters change more slowly than the other variables in the system. This assumption, which admits a quasi-stationary treatment, is essential for the computation of the sensitivity derivatives that are needed in the adaptation (Makoudi and Radouane, 1996; Åström and Wittenmark, 1995).

Let  $e$  denote the error between the system output  $y$  and the reference output  $y_m$ , and let  $\theta$  denote the parameters' vector to be updated. By using the criterion:

$$J(\theta) = \frac{1}{2}e^2 \quad (4)$$

the adjustment rule for changing the parameters in the direction of the negative gradient of  $J$  is,

$$\frac{d\theta}{dt} = -\gamma \frac{\delta J}{\delta \theta} = -\gamma e \frac{\delta e}{\delta \theta} \quad (5)$$

The derivative  $\frac{\delta e}{\delta \theta}$ , i.e., the sensitivity derivative of the system, can be evaluated under the assumption that  $\theta$  is constant (Landau, 1979).

The control law is given by the following equation,

$$u = \theta^T \varphi \quad (6)$$

where  $\varphi$  is the regression vector or measurement vector such that,

$$\varphi = -\frac{\delta e}{\delta \theta} \quad (7)$$

## 2.2 Limitations

Let us study a simulation example to illustrate the limitations of this control scheme in case of a time varying reference.

Consider the system described by the following equation,

$$G(s) = \frac{y(s)}{u(s)} = \frac{2s + 8}{s^2 - 3s - 2} \quad (8)$$

The referential model is defined by:

$$G_m(s) = \frac{y_m(s)}{u_c(s)} = \frac{1}{0.2s + 1} \quad (9)$$

The control objective is to minimise the error:

$$e = y - y_m \quad (10)$$

The recurrence equation of the system described above which is obtained after the discretisation (with the sampling period  $T = 0.04$  sec.) is given by:

$$y(k + 2) = 2.131y(k + 1) - 1.127y(k) + 0.09171u(k + 1) - 0.07811u(k) \quad (11)$$

The recurrence equation of the reference model (9) which is obtained after the discretisation ( $T = 0.04$ ) is given by:

$$y_m(k + 1) = 0.8187y_m(k) + 0.1813u_c(k) \quad (12)$$

Let  $k$ ,  $l$  and  $m$  be respectively the degree of the polynomials  $R$ ,  $S$  and  $T$ , such that Åström and Wittenmark (1995):

$$\begin{aligned} k &= \text{deg}R = \text{deg}A_m + \text{deg}B - \text{deg}A = 1 + 1 - 2 = 0 \\ l &= \text{deg}S = \text{deg}R = 0 \\ m &= \text{deg}T = \text{deg}B_m = 0 \end{aligned} \quad (13)$$

Therefore, the vector of regulation parameters is:

$$\theta = (s_0 \ t_0) \quad (14)$$

Let us define the regression vector  $\varphi$  as follows,

$$\begin{aligned} \varphi^T &= - \left[ \frac{\delta e}{\delta r_1} \ \frac{\delta e}{\delta r_2} \ \dots \ \frac{\delta e}{\delta r_k} \ \frac{\delta e}{\delta s_0} \ \frac{\delta e}{\delta s_1} \ \dots \ \frac{\delta e}{\delta s_l} \ \frac{\delta e}{\delta t_0} \ \frac{\delta e}{\delta t_1} \ \dots \ \frac{\delta e}{\delta t_m} \right] \\ &= \frac{b_0}{A_o A_m} [s^{k-1}u \dots u \ s^l y \dots y \ - s^m u_c \dots - u_c] \\ &= \frac{b_0}{A_o A_m} [y - u_c] \end{aligned} \quad (15)$$

such that  $b_0 = 1$  and the  $A_o$  the observer polynomial is chosen as  $A_o = s + 1$ ; thus,

$$\varphi^T = \frac{1}{(s+1)(0.2s+1)} [y - u_c] \quad (16)$$

which is equivalent in the discrete time using the process sampling period  $T = 0.04$  sec. to,

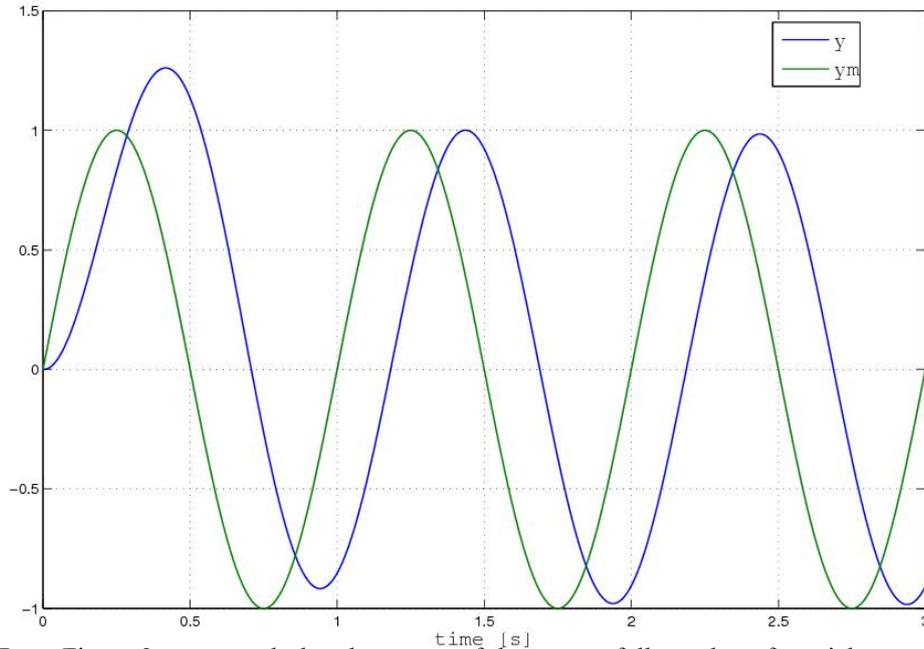
$$\varphi^T(k) = \frac{0.003696z + 0.003412}{z^2 - 1.78z + 0.7866} [y(k) - u_c(k)]$$

giving the following regression,

$$\begin{aligned} \varphi^T(k+2) &= 1.78 \varphi^T(k+1) - 0.7866 \varphi^T(k) \\ &+ [0.003696 y(k+1) + 0.003412 y(k) \\ &- 0.003696 u_c(k+1) - 0.003412 u_c(k)] \end{aligned} \quad (17)$$

Using the command law of equation (6), we obtain the system output response shown in Figure 3.

**Figure 3** Reference trajectory  $y_m$  and output trajectory  $y$  using the classical MRAC approach (see online version for colours)



From Figure 3, we remark that the output of the system follows the referential one with the overshoot  $D = 1.25$  and the delay time  $t_d = 0.21$  s, this characteristic presents the major limitation of the MRAC approach in the tracking control strategy.

Besides, as robot systems are strongly nonlinear, the only way to apply this control strategy is to consider sufficiently smooth or slowly varying referential trajectories.

### 3 Fractional order systems

#### 3.1 Fractional calculus

Fractional calculus is a subdivision of calculus theory which generalises the derivative or integral of a function to non-integer order (Oustaloup, 1995). The number of applications where fractional calculus has been used grows rapidly mainly for the reason that these mathematical phenomena allow to describe a real object more accurately than the classical methods. Approximation methods of fractional derivative and integral to rational functions permitted to use very easily fractional order systems in wide areas of applications such as control theory (Sabatier et al., 2003; Ladaci et al., 2008, 2010), economical systems (Dadras and Momeni, 2010), renewable energy (Melício et al., 2010; Neçaibia et al., 2015), vehicle motion systems (Neçaibia et al., 2015b), Spacecraft attitude control (Manabe, 2002), etc.

The generalised fundamental operator which includes the differentiation and integration is given as:

$${}_a D_t^\mu = \begin{cases} \frac{d^\mu}{dt^\mu} & \Re(\mu) > 0 \\ 1 & \Re(\mu) = 0 \\ \int_a^t (d\tau)^{-\mu} & \Re(\mu) < 0 \end{cases} \quad (18)$$

where

- $a$ : lower limit of integration.
- $t$ : upper limit of integration.
- $\mu$ : order of fractional differentiation or integration.  $\mu$  with negative value indicates integration while  $\mu$  with positive value indicates differentiation.

The theory of fractional-order derivative was developed mainly in the 19th century. There are several definitions of fractional order derivative. Two important and widely applied definitions are Riemann-Liouville definition and Grünwald-Letnikov definition which is perhaps the best known due to its most suitability for the realisation of discrete control algorithms (Podlubny, 1999).

The Grünwald-Letnikov definition is expresses as:

$${}_a D_t^\mu f(t) = \lim_{h \rightarrow 0} h^{-\mu} \sum_{r=0}^{\frac{t-a}{h}} (-1)^r \frac{\Gamma(\mu+1)}{r! \Gamma(\mu-r+1)} f(t-rh) \quad (19)$$

where the binomial coefficients ( $r > 0$ ) are given by:

$$\binom{\mu}{0} = 1, \quad \binom{\mu}{r} = \frac{\mu(\mu-1)\dots(\mu-r+1)}{r!} \quad (20)$$

The Riemann-Liouville definition is expresses as:

$${}_a D_t^\mu f(t) = \frac{1}{\Gamma(\eta-\mu)} \frac{d^\eta}{dt^\eta} \left[ \int_a^t \frac{f(\tau)}{(t-\tau)^{\mu-\eta+1}} d\tau \right] \quad (21)$$

where:

$$\Gamma(x) = \int_t^a y^{\mu-1} e^{-y} dy \quad (22)$$

$\Gamma(\cdot)$  is the Euler's Gamma function.

For a wide class of functions which appear in real physical and engineering applications, the Riemann-Liouville and the Grünwald-Letnikov definitions are equivalent.

### 3.2 Approximation methods of a fractional order transfer function

Many approximations methods ave been developed in the fractional calculus literature. The most popular are those of Charef Charef et al. (1992) called the singularity function method and Oustaloup's method (Oustaloup et al., 2000). This second approach is a generalised differential action which covers the frequency space, based on a recursive distribution of an infinite number of zeros and negative real poles (to ensure phase behaviour minimum). As part of a realist synthesis (practice) based on a finite number of zeros and poles, it should reduce the differential behaviour of a generalised bounded frequency range, chosen according to the needs of the application.

The method is based on the function approximation from:

$$H(s) = s^\alpha, \quad \alpha \in \mathfrak{R} \quad (23)$$

By a rational function:

$$G_f(s) = K \prod_{k=1}^N \frac{s + \omega'_k}{s + \omega_k} \quad (24)$$

where the poles, zeros, and gain are evaluated from:

$$\omega'_k = \omega_b \cdot \omega_u^{(2k-1-\gamma)/N}, \quad \omega_k = \omega_b \cdot \omega_u^{(2k-1+\gamma)/N}, \quad K = \omega_h^\gamma \quad (25)$$

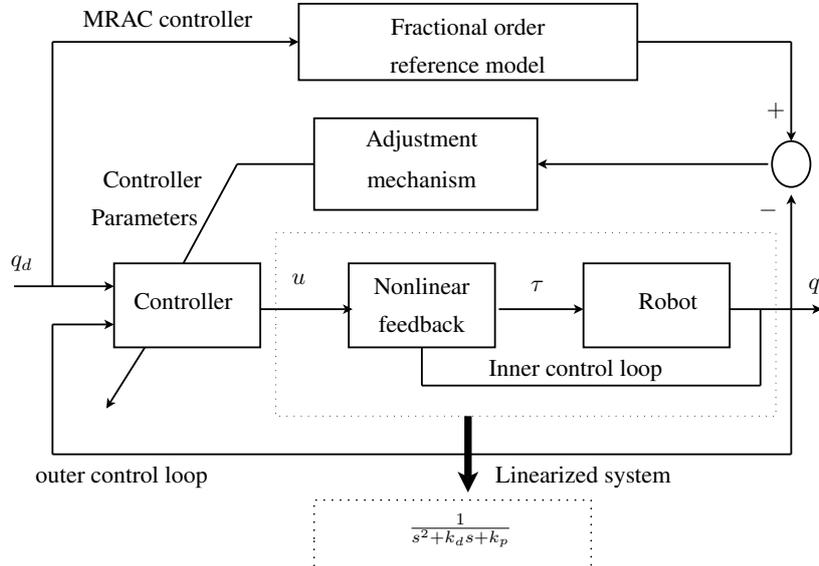
where  $\omega_u$  is the unity frequencies' gain and the central frequency of a band of frequencies distributed geometrically. We have  $\omega_u = \sqrt{\omega_h \cdot \omega_b}$ , where  $\omega_h$  and  $\omega_b$  are respectively the upper and lower frequencies,  $\gamma$  is the order of derivative, and  $N$  is the order of the filter.

## 4 Fractional model reference adaptive control

The proposed global control scheme for trajectory tracking control of robot manipulator is based on a position-feedback as illustrated in Figure 4.

It is composed of three parts:

- the robot
- the linearised dynamics expressed in articular space
- the fractional order model reference controller.

**Figure 4** Global control scheme for robot manipulator: FMRAC approach


#### 4.1 Dynamic model of the SCARA robot

This dynamic model can be written in the following matrix form Amiri et al. (2009):

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \quad (26)$$

where

- $B(q) \in \mathfrak{R}^3 \times \mathfrak{R}^3$  is the inertia matrix
- $C(q, \dot{q}) \in \mathfrak{R}^3$  is the matrix of the centrifugal and Coriolis terms
- $G(q) \in \mathfrak{R}^3$  is the vector of gravitational forces
- $\tau \in \mathfrak{R}^3$  is the vector of control inputs (torques generated by the actuators)
- $q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \in \mathfrak{R}^3$  is the vector of articular positions
- $\dot{q} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} \in \mathfrak{R}^3$  is the vector of articular velocities
- $\ddot{q} = \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{pmatrix} \in \mathfrak{R}^3$  is the vector of articular accelerations.

#### 4.2 Linearised model of the SCARA robot

The linearisation model of the robot with a nonlinear feedback is given by the following equivalent transfer function:

$$G(s) = \frac{1}{s^2 + k_d s + k_p} \quad (27)$$

where  $s$  is the complex variable of the transfer function,  $k_p$  is the position gain and  $k_d$  the velocity gain.

### 4.3 Fractional order reference model

The reference model is used to specify the desired performance and to monitor the state of the robot. It is chosen and adjusted in practice according to the operation conditions and the environment (Feng et al., 1994). For the present study we will propose a fractional model reference given by:

$$G_{mf}(s) = \frac{\omega^2}{s^\alpha(s^2 + 2\omega\xi s + \omega^2)} \quad (28)$$

with  $\alpha \in [0 \ 1]$ .

## 5 Application: trajectory tracking in SCARA robot

### 5.1 Experimental configuration

This dynamic model can be written in the matrix form (26):

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau$$

The Dynamical parameters of the SCARA robot are summarised in Table 1.

**Table 1** Parameters of SCARA robot

Segment	Segment 0	Segment 1	Segment 2	Segment 3
Mass (kg)	$m_0 = 19 : 5$	$m_1 = 8$	$m_2 = 6$	$m_3 = 0 : 5$
Length (m)	$d_0 = 0 : 65$	$d_1 = 0 : 4$	$d_2 = 0 : 3$	$d_3 = 0 : 3$
Inertia (kg m <sup>2</sup> )	$I_0 = 1 : 0.298$	$I_1 = 0 : 16$	$I_2 = 0 : 0.675$	$I_3 = 0 : 0.056$

The reference trajectory is set as follows:

$$q_1 = q_2 = q_3 = \frac{10\pi}{180} \sin(2\pi t) \quad (29)$$

The integer order model reference ( $\alpha = 1$ ) is given by:

$$G_m(s) = \frac{\omega^2}{s(s^2 + 2\omega\xi s + \omega^2)}$$

with  $\omega = 20$ ,  $\xi = 1$ .

The fractional model reference is given by (28):

$$G_{mf}(s) = \frac{\omega^2}{s^\alpha(s^2 + 2\omega\xi s + \omega^2)}$$

with  $\omega = 20$ ,  $\xi = 1$  and  $\alpha = 0.3$ .

The parameters of the linearised model of the robot are:  $k_p = 2,500$  and  $k_d = 500$ .

The reference models are discretised using a sampling period  $T = 0.04$  sec.

Now if we reconsider the plant and reference models (1) and (2) in discrete time we have,

$$A(z)y(k) = B(z)u(k) \quad (30)$$

and

$$A_m(z)y_m(k) = B_m(z)u_c(k) \quad (31)$$

We will denote by  $\theta(k)$  a vector containing the unknown controller parameters, and by  $\varphi(k)$  the regression vector (7) discretised with the sampling constant  $T$ .

Using a simple approximation of the adaptation rule (5) in its normalised form, we get

$$\theta(k+1) = \theta(k) - \frac{T\gamma(y(k) - y_m(k))\varphi(k)}{\epsilon + \varphi(k)^T\varphi(k)} \quad (32)$$

where  $\epsilon$  is a positive real.

The adaptive control law (6) is given by,

$$u(k+1) = \theta^T(k+1)\varphi(k) \quad (33)$$

## 5.2 Numerical simulations

This section presents simulation results of the proposed architecture based on the linearising feedback and the FMRAC controller.

The parameter vector  $\theta$  is initialised as:

$$\theta_0 = [0.0 \quad 235.2 \quad 1771.4 \quad 1771.4]^T.$$

The output response and the corresponding torques of the SCARA robot using the classical MRAC controller with the above model and control parameters are shown in Figure 5 and Figure 6 respectively.

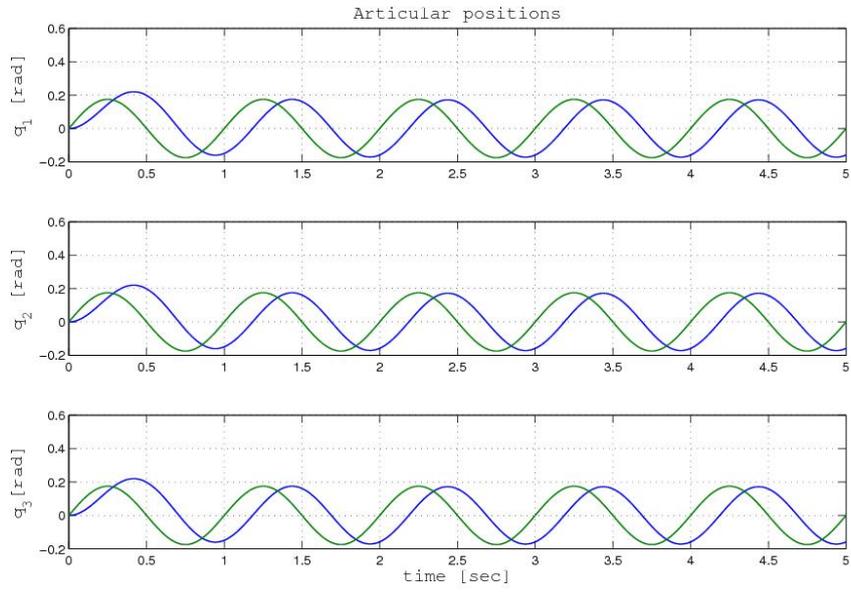
From Figure 5, we remark that the system output follows the referential one with an overshoot  $D = 0.035$  and a delay time  $t_d = 0.14$  s, this characteristic presents the most limitation of the MRAC approach in trajectory tracking control.

In order to reduce the delay time between the trajectory of the SCARA robot and the reference trajectory (29), let us use the fractional Model Reference defined above.

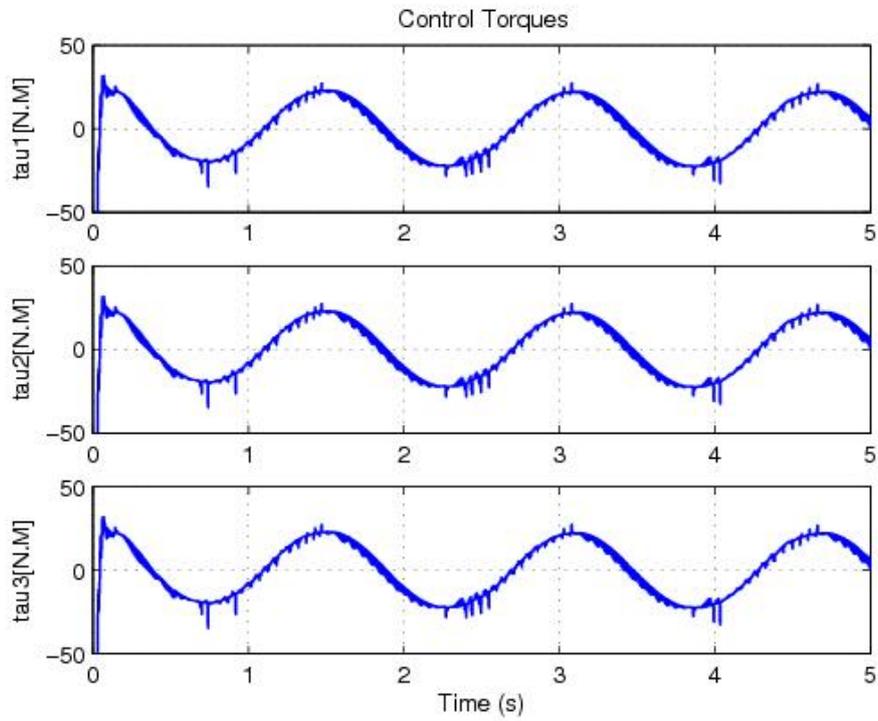
The output response of the robot and the corresponding torques using the FMRAC controller for  $\alpha = 0.7$  are shown in Figure 7 and Figure 8 respectively.

From Figure 7, we remark that the output of the system follows the referential one with very small oscillations comparatively to the integer order case and a reduced delay time of  $t_d = 0.0293$  s with an overshoot  $D = 0.019$ , thus, the system achieved a better asymptotic tracking using a fractional order reference model of order  $\alpha = 0.7$ .

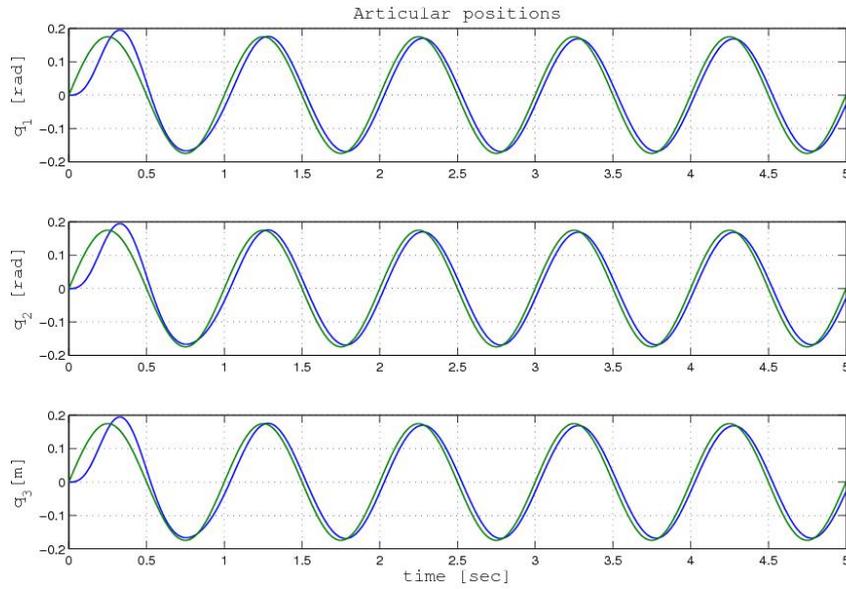
**Figure 5** Articular reference trajectory and robot trajectory using the classical MRAC approach (see online version for colours)



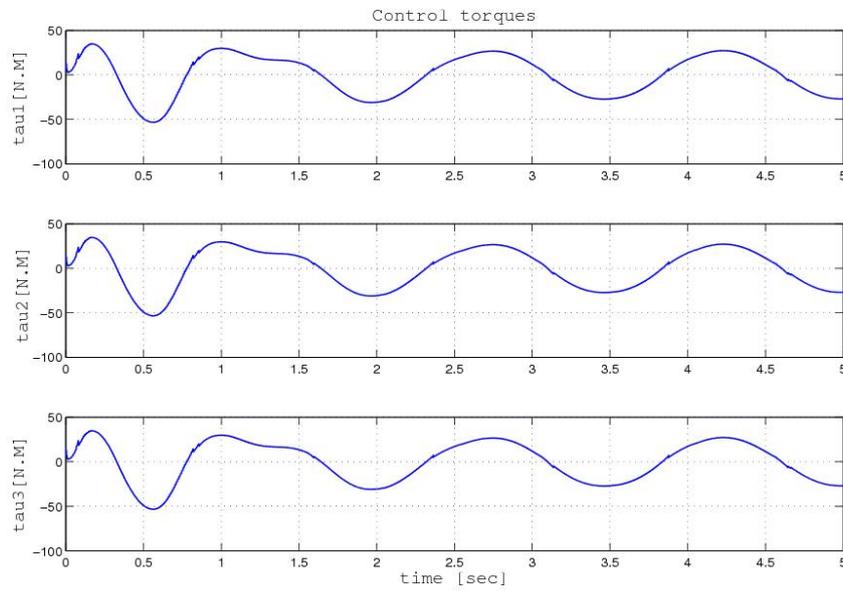
**Figure 6** Control torques for the MRAC controller (see online version for colours)



**Figure 7** Articular reference trajectory and robot trajectory using the proposed FMRAC approach with  $\alpha = 0.7$  (see online version for colours)



**Figure 8** Control torques for the FMRAC controller with  $\alpha = 0.7$  (see online version for colours)



In order to search for the 'optimal' value of the fractional order  $\alpha$  we measured the time delay and the overshoot of the system reference position tracking response for different values of  $\alpha \in [0, 1]$ . The comparative simulation results presented in Table 2, show that the system behaviour is improved for the majority of the fractional order values comparatively to the classical ( $\alpha = 1$ ) MRAC response, and the minimal criteria are obtained for  $\alpha = 0.3$ , as we obtain  $t_d = 0.0052$  s and  $D = 0.012$ .

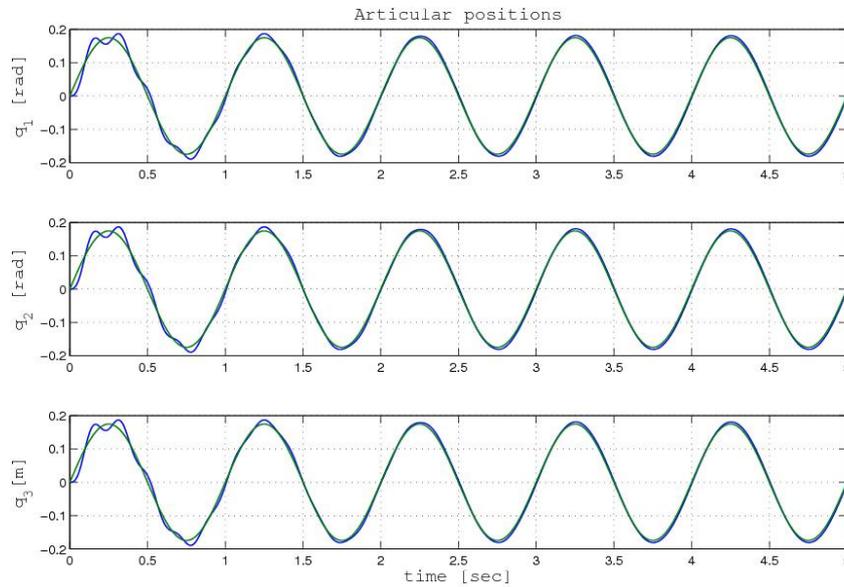
**Table 2** Comparative performance versus the model's fractional order

$\alpha$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$t_d$	0.028	0.022	0.0052	0.021	0.027	0.0128	0.0293	0.0055	0.0277	0.14
$D$	0.019	0.032	0.0120	0.030	0.019	0.0550	0.019	0.0118	0.0310	0.035

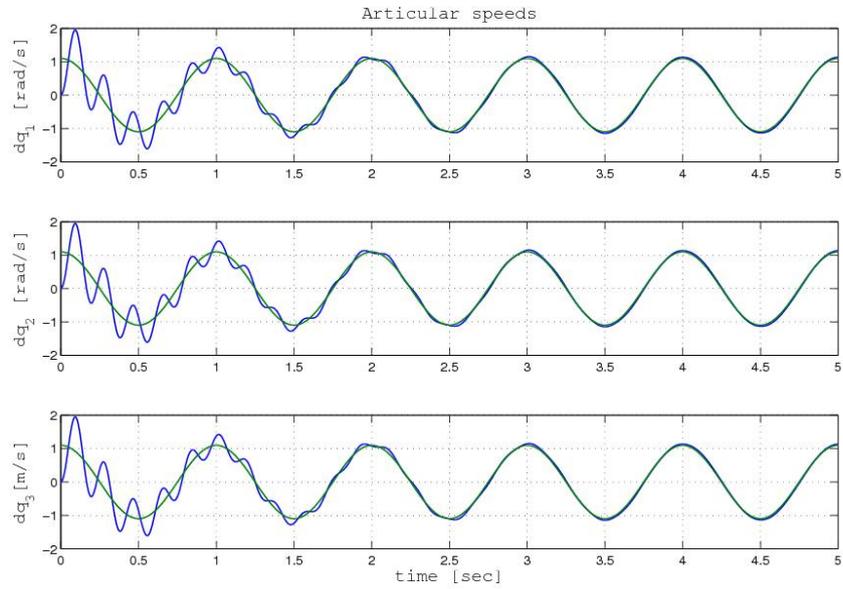
The SCARA robot response for the best fractional order reference model ( $\alpha = 0.3$ ), is presented in Figure 10 for the trajectory position tracking, Figure 9 for the articular speeds and Figure 11 for the input control torques.

The present study for minimising the lag between the adaptive system output and the time varying referential trajectory can be considered as a stability analysis of the robot system under FMRAC control as stabilisation problems can sometimes refer to the weak aims (Sragovich, 2006). The introduction of a fractional order model reference has indeed improved the stability and convergence of the position errors.

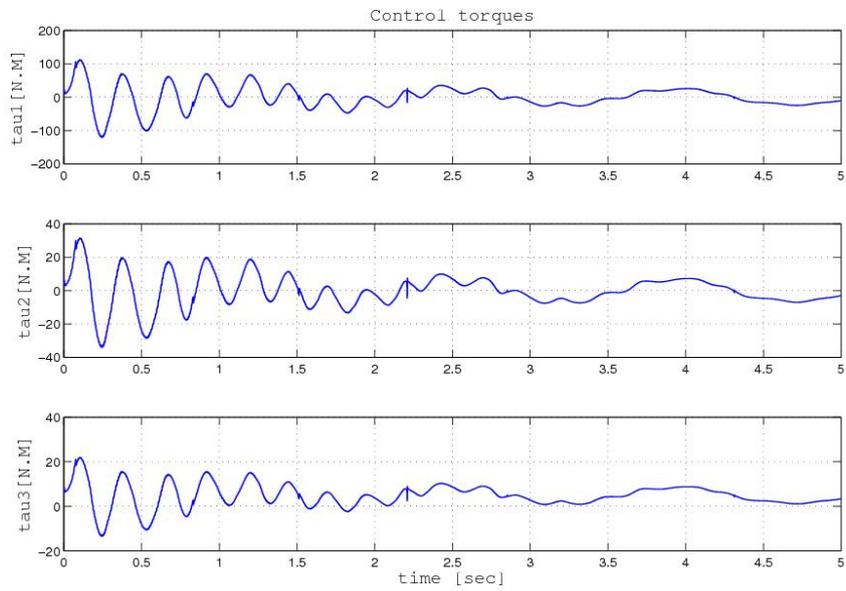
**Figure 9** Best robot trajectory tracking using the proposed FMRAC approach with  $\alpha = 0.3$  (see online version for colours)



**Figure 10** Articular speeds using the proposed FMRAC approach with  $\alpha = 0.3$  (see online version for colours)



**Figure 11** Control torques for the FMRAC controller with  $\alpha = 0.3$  (see online version for colours)



## 6 Conclusions

In this paper, the problem tackled was the time lag observed when a system tracks a varying trajectory under a MRAC. The proposed solution consists in introducing the fractional reference model in the MRAC strategy in order to eliminate the delay time and achieve a better asymptotic tracking. The idea was to get benefit from the high performance quality of fractional order systems confirmed in many precedent research works. The simulations results on a SCARA type robot manipulator show the best performances obtained by using the proposed FMRAC approach.

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