Efficient Leak Resistant Modular Exponentiation in RNS

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Outline

1. Cryptography
   - RSA cryptosystem
   - Power analysis
   - Montgomery multiplication in RNS
2. Randomized modular exponentiation in RNS
   - Randomized Montgomery multiplication
   - Proposed approach
   - Level of randomization
3. Conclusion

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RSA encryption (Rivest, Shamir and Adleman)

Bob chooses p and q two large prime numbers and computes $N = pq$. He generates E and D two integers such that $ED = 1 \pmod{(p-1)(q-1)}$.

- Public Key: N, D.
- Private Key: E, p, q.
- Alice encrypts a message m by: $c = m^D \mod N$.
- Bob decrypts c by doing: $c^E = m^{ED} \mod N = m$.

An algorithm for modular exponentiation : Right-to-left Square-and-multiply

Require: A modulus N, an integer $X \in [0, N[$ and an exponent $E = (e_{\ell-1}, \ldots, e_0)_2$
Ensure: $R = X^E \pmod{N}$

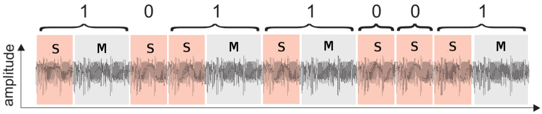
$R \leftarrow 1$
$Z \leftarrow X$
for i **from** 0 **to** $\ell - 1$ **do**
    if $e_i = 1$ **then**
        $R \leftarrow R \times Z \pmod{N}$
    end if
    $Z \leftarrow Z^2 \pmod{N}$
end for
return R

$$X^E = X^{\sum_{i=0}^{\ell-1} e_i 2^i}$$

$$X^E = X^{e_{\ell-1}2^{\ell-1}} \times \cdots \times X^{e_1 2^1} \times X^{e_0 2^0}$$

Simple power analysis

$E = (e_\ell, \ldots, e_0)_2$ and $X \in [0, N[$



Square-and-multiply

$R \leftarrow 1$
$Z \leftarrow X$
for $i = 0$ **to** $\ell - 1$ **do**
  if $e_i = 1$ **then**
    $R \leftarrow R \cdot Z \mod N$
  endif
  $Z \leftarrow Z^2 \mod N$
endfor
return(R)

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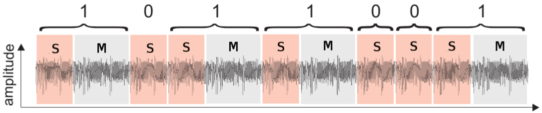

Square-and-multiply

$R \leftarrow 1$
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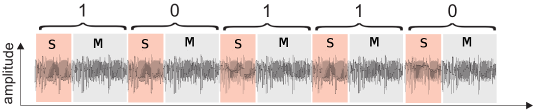
for $i = 0$ **to** $\ell - 1$ **do**
  if $e_i = 1$ **then**
    $R \leftarrow R \cdot Z \mod N$
  endif
  $Z \leftarrow Z^2 \mod N$
endfor
return(R)

Square-and-multiply-always

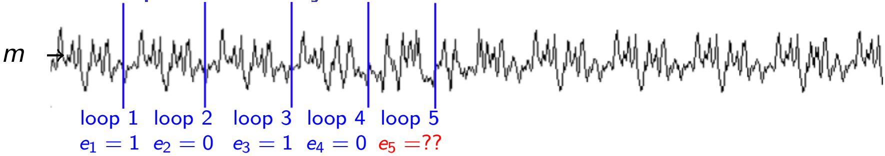
$R_0 \leftarrow 1$
$R_1 \leftarrow 1$
$Z \leftarrow X$
for $i = 0$ **to** $\ell - 1$ **do**
  if $e_i = 0$ **then**
    $R_0 \leftarrow R_0 \cdot Z \mod N$
  else
    $R_1 \leftarrow R_1 \cdot Z \mod N$
  endif
endfor
  $Z \leftarrow Z^2 \mod N$
return(R_1)

Montgomery-ladder

$R \leftarrow 1$
$R' \leftarrow X$
for $i = \ell$ **to** 1 **do**
  if $k_i = 1$ **then**
    $R \leftarrow R \cdot R' \mod N$
    $R' \leftarrow R'^2 \mod N$
  else
    $R' \leftarrow R \cdot R' \mod N$
    $R \leftarrow R^2$
  endif
endfor
return(R)



Differential power analysis



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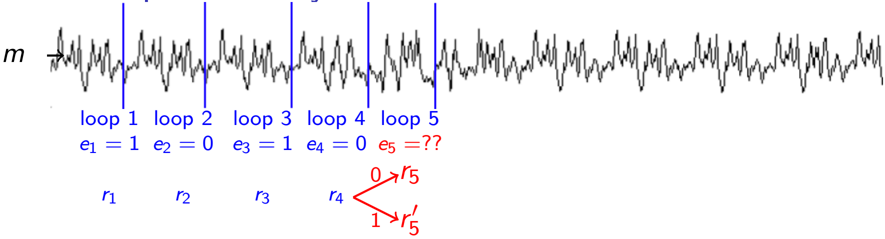

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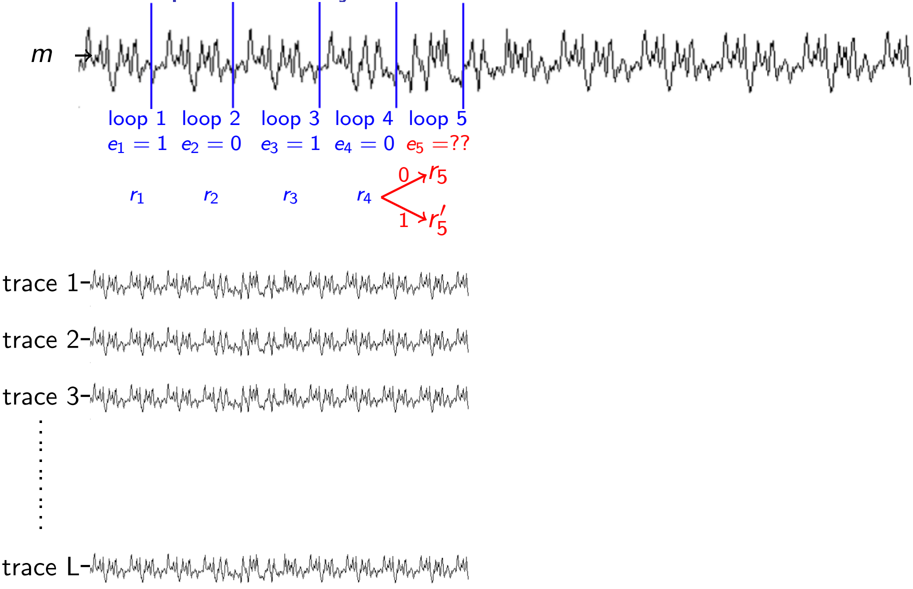

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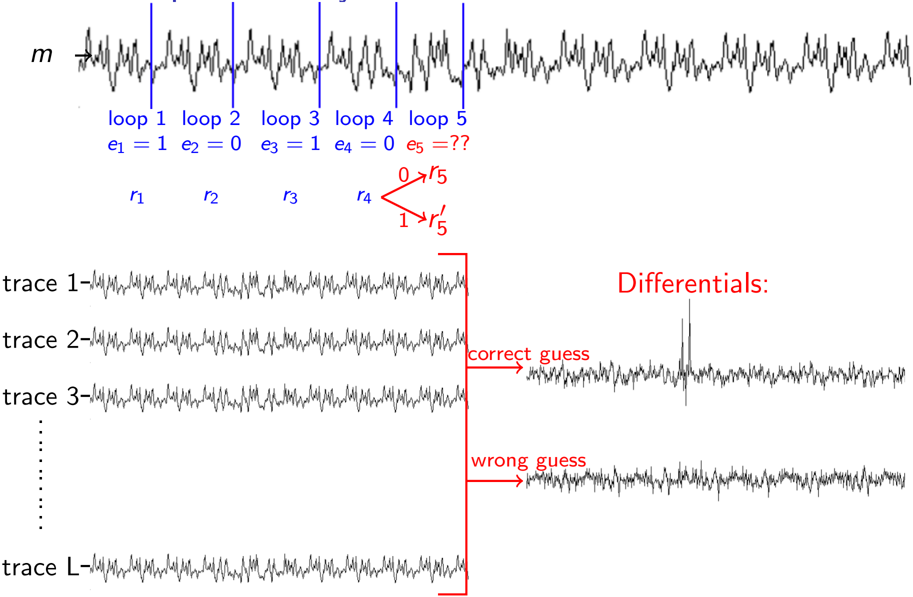

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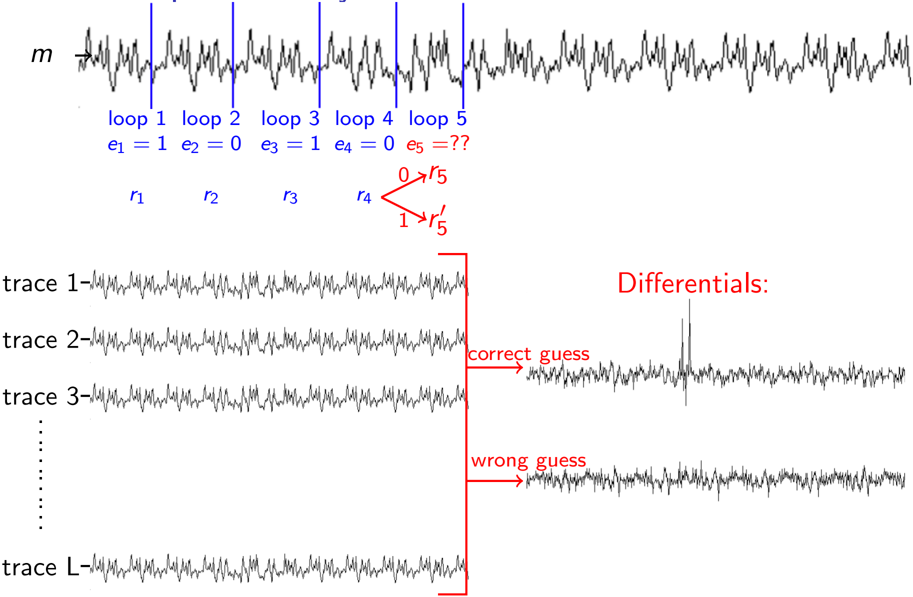

Counter-measure: Randomization of the exponent and data.

Montgomery multiplication

Basic modular multiplication. For $X, Y \in [0, N[$

1. Product. $Z \leftarrow X \times Y$
2. Reduction. $Q \leftarrow \lfloor Z/N \rfloor$ and $R \leftarrow Z - Q \times N$

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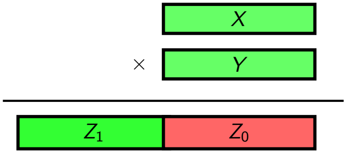
Montgomery Multiplication

Require: $X, Y \in [0, N[$ and
$A = 2^n > N$
Ensure: $R = X \times Y \times A^{-1} \pmod N$
1: $Z \leftarrow X \times Y$
2: $Q \leftarrow N^{-1} \times Z \pmod A$
3: $R \leftarrow (Z - Q \times N)/A$



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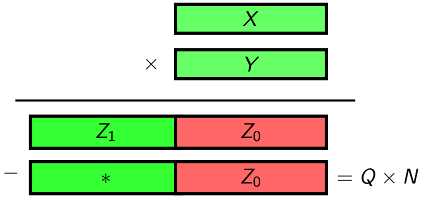

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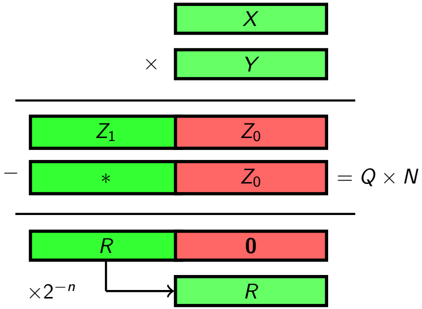

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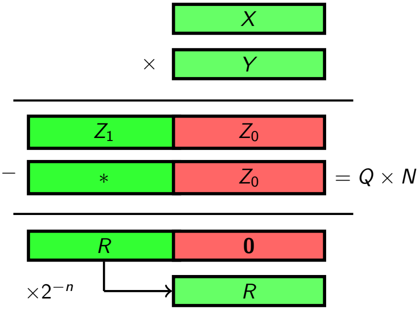

Montgomery representation.

1. $\widetilde{X} = XA \mod N$ provides
2. $MontMul(\widetilde{X}, \widetilde{Y}) = (XA) \times (YA) \times A^{-1} \mod N = XYA \mod N$

Montgomery multiplication in residue number system

- Let $\mathcal{A} = \{a_1, \ldots, a_t\}$ be a set t co-prime integers.

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- The Chinese remainder theorem tell us that for op $\in \{+, \times\}$

$$[X]_{\mathcal{A}} \text{ op } [Y]_{\mathcal{A}} = ([x_1 \text{ op } y_1]_{a_1}, \ldots, [x_t \text{ op } y_t]_{a_t}) \Leftrightarrow X \text{ op } Y \mod A$$

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Montgomery Multiplication in RNS

Require: X, Y in $\mathcal{A} \cup \mathcal{B}$
Ensure: $XYA^{-1} \mod N$ in $\mathcal{A} \cup \mathcal{B}$

1: $[Q]_{\mathcal{A}} \leftarrow [XYN^{-1}]_{\mathcal{A}}$

3: $[Z]_{\mathcal{B}} \leftarrow [(XY - QN)A^{-1}]_{\mathcal{B}}$

5: **return** $(Z_{\mathcal{A} \cup \mathcal{B}})$

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2: $[Q]_{\mathcal{B}} \leftarrow BE_{\mathcal{A} \to \mathcal{B}}([Q]_{\mathcal{A}})$
3: $[Z]_{\mathcal{B}} \leftarrow [(XY - QN)A^{-1}]_{\mathcal{B}}$
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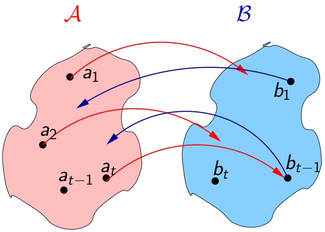
Randomization in RNS (LRA CHES 2004)

We have

$$\widetilde{X}_{old} = [XA_{old}]_{\mathcal{A}_{old} \cup \mathcal{B}_{old}}$$

we permute the basis elements $\mathcal{A}_{old} \cup \mathcal{B}_{old} \to \mathcal{A}_{new} \cup \mathcal{B}_{new}$



this leads to a new representation of X

$$\widetilde{X}_{new} = [XA_{new}]_{\mathcal{A}_{new} \cup \mathcal{B}_{new}}$$

Cost

Two Montgomery multiplications :

$$XA_{old} \mod N \to XA_{old}A_{new} \mod N \to XA_{new} \mod N.$$

Randomized square-and-multiply-always

- Input: N, $X \in [0, N[$, $E = (e_{\ell-1}, \ldots, e_0)_2$ and $\mathcal{M} = \{m_1, \ldots, m_{2t}\}$.
- Output: $X^E \mod N$

Square-and-mult-always

$\mathcal{A}, \mathcal{B} \xleftarrow{\text{random split}} \mathcal{M}$
$\widetilde{Z} \leftarrow [\widetilde{X}]_{\mathcal{A}\cup\mathcal{B}}$,
$\widetilde{R}_0 \leftarrow [\widetilde{1}]_{\mathcal{A}\cup\mathcal{B}}$, $\widetilde{R}_1 \leftarrow [\widetilde{1}]_{\mathcal{A}\cup\mathcal{B}}$
for i **from** 0 **to** $\ell - 1$ **do**
    $\widetilde{R}_{e_i} \leftarrow \mathrm{MM\_RNS}(\widetilde{R}_{e_i}, \widetilde{Z}, \mathcal{A}, \mathcal{B})$
    $\widetilde{Z} \leftarrow \mathrm{MM\_RNS}(\widetilde{Z}, \widetilde{Z}, \mathcal{A}, \mathcal{B})$
end for
return $\widetilde{R}_1$

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  $\widetilde{Z} \leftarrow \text{MM\_RNS}(\widetilde{Z}, \widetilde{Z}, \mathcal{A}, \mathcal{B})$
  Randomise($\mathcal{A}_{old}, \mathcal{B}_{old}, \mathcal{A}, \mathcal{B}$)
  $\widetilde{Z} \leftarrow$ Update($\widetilde{Z}, \mathcal{A}_{old}, \mathcal{B}_{old}, \mathcal{A}, \mathcal{B}$)
  $\widetilde{R}_0 \leftarrow$ Update($\widetilde{R}_0, \mathcal{A}_{old}, \mathcal{B}_{old}, \mathcal{A}, \mathcal{B}$)
  $\widetilde{R}_1 \leftarrow$ Update($\widetilde{R}_1, \mathcal{A}_{old}, \mathcal{B}_{old}, \mathcal{A}, \mathcal{B}$)
end for
return $\widetilde{R}_1$

Randomized square-and-multiply-always

- Input: N, $X \in [0, N[$, $E = (e_{\ell-1}, \ldots, e_0)_2$ and $\mathcal{M} = \{m_1, \ldots, m_{2t}\}$.
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$\mathcal{A}, \mathcal{B} \xleftarrow{\text{random split}} \mathcal{M}$
$\widetilde{Z} \leftarrow [\widetilde{X}]_{\mathcal{A}\cup\mathcal{B}}$,
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for i **from** 0 **to** $\ell - 1$ **do**
    $\widetilde{R}_{e_i} \leftarrow \text{MM\_RNS}(\widetilde{R}_{e_i}, \widetilde{Z}, \mathcal{A}, \mathcal{B})$
    $\widetilde{Z} \leftarrow \text{MM\_RNS}(\widetilde{Z}, \widetilde{Z}, \mathcal{A}, \mathcal{B})$
    Randomise$(\mathcal{A}_{old}, \mathcal{B}_{old}, \mathcal{A}, \mathcal{B})$
    $\widetilde{Z} \leftarrow \text{Update}(\widetilde{Z}, \mathcal{A}_{old}, \mathcal{B}_{old}, \mathcal{A}, \mathcal{B})$
    $\cancel{\widetilde{R}_0 \leftarrow \text{Update}(\widetilde{R}_0, \mathcal{A}_{old}, \mathcal{B}_{old}, \mathcal{A}, \mathcal{B})}$
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- Input: N, $X \in [0, N[$, $E = (e_{\ell-1}, \ldots, e_0)_2$ and $\mathcal{M} = \{m_1, \ldots, m_{2t}\}$.
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  $\text{Randomise}(\mathcal{A}_{old}, \mathcal{B}_{old}, \mathcal{A}, \mathcal{B})$
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end for
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Proposed

$\mathcal{A}, \mathcal{B} \xleftarrow{\text{random split}} \mathcal{M}$
$\widetilde{Z} \leftarrow [\widetilde{X}]_{\mathcal{A}\cup\mathcal{B}}$,
$\widetilde{R}_0 \leftarrow [\widetilde{1}]_{\mathcal{A}\cup\mathcal{B}}$, $\widetilde{R}_1 \leftarrow [\widetilde{1}]_{\mathcal{A}\cup\mathcal{B}}$
for i **from** 0 **to** $\ell - 1$ **do**
  $\mathcal{A}'_{e_i}, \mathcal{B}'_{e_i} \xleftarrow{\text{random split}} \mathcal{M}$
  $\widetilde{R}_{e_i} \leftarrow \text{MM\_RNS}(\widetilde{R}_{e_i}, \widetilde{Z}, \mathcal{A}'_{e_i}, \mathcal{B}'_{e_i})$
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Example

For $E = 7 = (111)_2$ and $\mathcal{M} = \{m_1, m_2, m_3, m_4\}$

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- Initialization: $\mathcal{A} = \{m_1, m_2\}, \mathcal{B} = \{m_3, m_4\}$ leads to

$$\begin{aligned} R_1 &= m_1 m_2 \mod N \\ Z &= X m_1 m_2 \mod N \end{aligned}$$

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- Loop 1: $\mathcal{A}_1 = \{m_2, m_4\}, \mathcal{B}_1 = \{m_1, m_3\}$ we get

$$R_1 = (m_1 m_2) \times \underbrace{(X m_1 m_2)}_{Z} \times \underbrace{(m_2^{-1} m_4^{-1})}_{\text{Mont. factor}} = X m_1^2 m_2 m_4^{-1}$$

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$\mathcal{A} = \{m_1, m_3\}, \mathcal{B} = \{m_2, m_4\}$ leads to

$$Z = X^2 m_1 m_3$$

- Loop 2: $\mathcal{A}_1 = \{m_1, m_4\}, \mathcal{B}_1 = \{m_2, m_3\}$ we get

$$R_1 = X m_1^2 m_2 m_4^{-1} \times (X^2 m_1 m_3) \times (m_1^{-1} m_4^{-1}) = X^3 m_1^2 m_2 m_3 m_4^{-2}$$

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  $\mathcal{A} = \{m_1, m_3\}, \mathcal{B} = \{m_2, m_4\}$ leads to

$$Z = X^2 m_1 m_3$$

- Loop 2: $\mathcal{A}_1 = \{m_1, m_4\}, \mathcal{B}_1 = \{m_2, m_3\}$ we get

$$R_1 = X m_1^2 m_2 m_4^{-1} \times (X^2 m_1 m_3) \times (m_1^{-1} m_4^{-1}) = X^3 m_1^2 m_2 m_3 m_4^{-2}$$

- Etc.

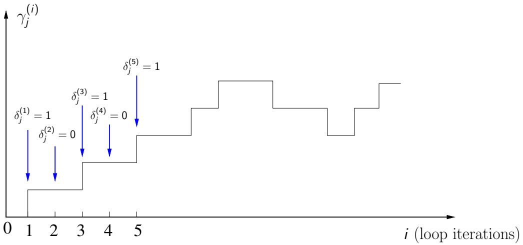
Random evolution of the mask

After i loop iterations we have

$$\widetilde{R}_1^{(i)} = X^{\sum_{j=0}^{i-1} e_j 2^j} \times \prod_{j=0}^{2t} m_j^{\gamma_j^{(i)}} \mod N$$

and each $\gamma_j^{(i)}$ evolves randomly as

$$\gamma_j^{(i+1)} = \gamma_j^{(i)} + \delta_j^{(i)} \text{ with } \delta_j^{(i)} \in \{-1, 0, 1\} \text{ and } \begin{cases} \mathbb{P}(\delta_j^{(i)} = 1) = 1/8, \\ \mathbb{P}(\delta_j^{(i)} = -1) = 1/8, \\ \mathbb{P}(\delta_j^{(i)} = 0) = 3/4. \end{cases}$$



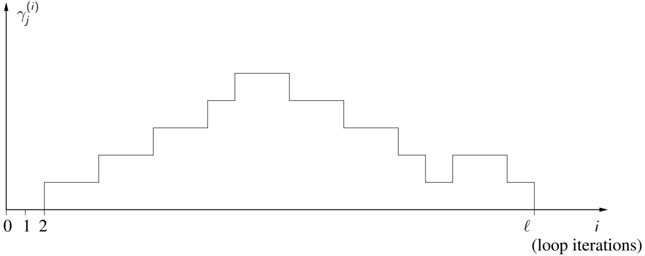
Removing the final mask

Problem: at the end we have to remove the final mask $\prod_{j=1}^{2t} m_j^{\gamma_j^{(\ell)}}$ from

$$\widetilde{X} = X^E \cdot \prod_{j=1}^{2t} m_j^{\gamma_j^{(\ell)}} \mod N.$$

Strategy: we force $\gamma_j^{(\ell)}$ to be equal 0 as follows

- During the first half of the iterations each $\gamma_j^{(i)}$ evolves freely.
- During the second half we constrain each $|\gamma_j^{(i)}|$ to decrease toward 0.



Level of randomization

- The probabilities of the mask exponents satisfy

$$\begin{aligned} \mathbb{P}(\gamma_j^{(i)} = d) &= \sum_{k=d}^{d+\lfloor (i-d)/2 \rfloor} \binom{i}{k} \binom{i-k}{k-d} \left(\tfrac{1}{8}\right)^{2k-d} \left(\tfrac{3}{4}\right)^{i-2k+d} \\ \mathbb{P}(\Gamma^{(i)} = \Gamma) &\leq \prod_{j=1}^{t} \mathbb{P}(\gamma_j^{(i)} = \gamma_j) \leq \prod_{j=1}^{t} \mathbb{P}(\gamma_j^{(i)} = 0) \end{aligned}$$

Level of randomization

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$$\begin{array}{rcl} \mathbb{P}(\gamma_j^{(i)} = d) & = & \sum_{k=d}^{d+\lfloor (i-d)/2 \rfloor} \binom{i}{k} \binom{i-k}{k-d} \left(\frac{1}{8}\right)^{2k-d} \left(\frac{3}{4}\right)^{i-2k+d} \\ \mathbb{P}(\Gamma^{(i)} = \Gamma) & \leq & \prod_{j=1}^{t} \mathbb{P}(\gamma_j^{(i)} = \gamma_j) \leq \prod_{j=1}^{t} \mathbb{P}(\gamma_j^{(i)} = 0) \end{array}$$

- Comparison: for a 2048-bit RSA modulus and $t = 32$:
  - CHES 04:
    - Montgomery-ladder,
    - 4MM_RNS per randomization,
    - all masks are controled.
  - Proposed:
    - right-left square-and-multiply-always,
    - 2MM_RNS per randomization
    - the masks for R_0 and R_1 are not controled.

Approach	loop 1	loop 5	loop 10	loop 50	loop 100
CHES 04	$4.17 \cdot 10^{-38}$	$4.17 \cdot 10^{-38}$	$4.17 \cdot 10^{-38}$	$4.17 \cdot 10^{-38}$	$4.17 \cdot 10^{-38}$
Proposed	10^{-8}	$5 \cdot 10^{-28}$	$1.7 \cdot 10^{-38}$	$2.69 \cdot 10^{-61}$	$5.75 \cdot 10^{-71}$

Outline

1. Cryptography
   - RSA cryptosystem
   - Power analysis
   - Montgomery multiplication in RNS
2. Randomized modular exponentiation in RNS
   - Randomized Montgomery multiplication
   - Proposed approach
   - Level of randomization
3. Conclusion

Conclusion

Secure embedded implementation of RSA:

- Randomized modular exponentiation
- But leak resistant arithmetic (CHES 04) is costly: 4 MM_RNS per randomization

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We proposed:

- To apply LRA to right-to-left exponentiation.
- Avoid some correction of Montgomery Factor.
- This decreases the computational cost: 2 MM_RNS per randomization.
- Increases the level of randomization after a small number of loop.

Conclusion

Secure embedded implementation of RSA:

- Randomized modular exponentiation
- But leak resistant arithmetic (CHES 04) is costly: 4 MM_RNS per randomization

We proposed:

- To apply LRA to right-to-left exponentiation.
- Avoid some correction of Montgomery Factor.
- This decreases the computational cost: 2 MM_RNS per randomization.
- Increases the level of randomization after a small number of loop.

Perspectives:

- A better estimation of the level of randomization.
- Is it a good counter-measure against horizontal power analysis ?

Thank you for your attention!