# Efficient Leak Resistant Modular Exponentiation in RNS 

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## Efficient Leak Resistant Modular Exponentiation in RNS

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## Outline

(1) Cryptography

- RSA cryptosystem
- Power analysis
- Montgomery multiplication in RNS
(2) Randomized modular exponentiation in RNS
- Randomized Montgomery multiplication
- Proposed approach
- Level of randomization
(3) Conclusion


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## RSA encryption (Rivest, Shamir and Adleman)

Bob chooses $p$ and $q$ two large prime numbers and computes $N=p q$. He generates $E$ and $D$ two integers such that $E D=1(\bmod (p-1)(q-1))$.

- Public Key: N, D.
- Private Key: $E, p, q$.
- Alice encrypts a message $m$ by: $c=m^{D} \bmod N$.
- Bob decrypts $c$ by doing: $c^{E}=m^{E D} \bmod N=m$.

An algorithm for modular exponentiation : Right-to-left Square-and-multiply

```
Require: A modulus N, an
    integer }X\in[0,N[\mathrm{ and an
    exponent
    E=(e\ell-1},\ldots,\mp@subsup{e}{0}{}\mp@subsup{)}{2}{
Ensure: R=X 會 (mod}N
    1: R\leftarrow1
    2: }Z\leftarrow
    3: for i from 0 to \ell-1
    do
        if }\mp@subsup{e}{i}{}=1\mathrm{ then
        R\leftarrowR\timesZ (mod N)
        end if
        L\leftarrowZZ'(mod}N
        end for
    9: return R
```

6: end if
7: $\quad Z \leftarrow Z^{2}(\bmod N)$
8: end for
9: return $R$

$$
\begin{gathered}
X^{E}=X^{\sum_{i=0}^{\ell-1} e_{i} 2^{i}} \\
X^{E}=X^{e_{\ell-1} 2^{\ell-1}} \times \cdots \times X^{e_{1} 2^{1}} \times X^{e_{0} 2^{0}}
\end{gathered}
$$

## Simple power analysis

$E=\left(e_{\ell}, \ldots, e_{0}\right)_{2}$ and $X \in[0, N[$

$\uparrow$
Square-and-multiply
$R \leftarrow 1$
$Z \leftarrow X$
for $i=0$ to $\ell-1$ do
if $e_{i}=1$ then
$R \leftarrow R \cdot Z \bmod N$
endif
$Z \leftarrow Z^{2} \bmod N$
endfor
return $(R)$

## Simple power analysis

$E=\left(e_{\ell}, \ldots, e_{0}\right)_{2}$ and $X \in[0, N[$


| Square-and-multiply-always |
| :--- |
| $R_{0} \leftarrow 1$ |
| $R_{1} \leftarrow 1$ |
| $Z \leftarrow X$ |
| for $i=0$ to $\ell-1$ do |
| if $e_{i}=0$ then |
| $R_{0} \leftarrow R_{0} \cdot Z \bmod N$ |
| else |
| $R_{1} \leftarrow R_{1} \cdot Z \bmod N$ <br> endif <br> endfor <br> $Z \leftarrow Z^{2} \bmod N$ <br> return $\left(R_{1}\right)$ <br>  <br>  |




## Differential power analysis

## Differential power analysis

$$
\begin{aligned}
& r_{1} \quad r_{2} \quad r_{3} \quad r_{4}<r_{1}^{\prime}
\end{aligned}
$$

Differential power analysis
 loop 1 loop 2 loop 3 loop 4 loop 5 $e_{1}=1 \quad e_{2}=0 \quad e_{3}=1 \quad e_{4}=0 \quad e_{5}=? ?$

$$
r_{1} \quad r_{2} \quad r_{3} \quad r_{4}<_{1>r_{5}^{\prime}}^{0>r_{5}}
$$






## Differential power analysis

$$
\begin{aligned}
& e_{1}=1 \quad e_{2}=0 \quad e_{3}=1 \quad e_{4}=0 \quad e_{5}=? ? \\
& r_{1} \\
& r_{4} \sum_{1>r_{5}^{\prime}}^{0} r_{5}
\end{aligned}
$$


 correct guess

## Differentials:



## Differential power analysis

$$
\begin{aligned}
& e_{1}=1 \quad e_{2}=0 \quad e_{3}=1 \quad e_{4}=0 \quad e_{5}=? ? \\
& r_{1}
\end{aligned}
$$


 correct guess

Differentials:
wrong guess


Counter-measure: Randomization of the exponent and data.

## Montgomery multiplication

Basic modular multiplication. For $X, Y \in[0, N[$
(1) Product. $Z \leftarrow X \times Y$
(2) Reduction. $Q \leftarrow\lfloor Z / N\rfloor$ and $R \leftarrow Z-Q \times N$

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## Montgomery Multiplication

Require: $X, Y \in[0, N[$ and

$$
A=2^{n}>N
$$

Ensure: $R=X \times Y \times A^{-1}(\bmod N)$
1: $Z \leftarrow X \times Y$
2: $Q \leftarrow N^{-1} \times Z(\bmod A)$
3: $R \leftarrow(Z-Q \times N) / A$

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Montgomery representation.
(1) $\tilde{X}=X A \bmod N$ provides
(2) $\operatorname{MontMul}(\widetilde{X}, \widetilde{Y})=(X A) \times(Y A) \times A^{-1} \bmod N=X Y A \bmod N$

Montgomery multiplication in residue number system

- Let $\mathcal{A}=\left\{a_{1}, \ldots, a_{t}\right\}$ be a set $t$ co-prime integers.


## Montgomery multiplication in residue number system

- Let $\mathcal{A}=\left\{a_{1}, \ldots, a_{t}\right\}$ be a set $t$ co-prime integers.
- An integer $X$ such that $0 \leq X<A=\prod_{i=1}^{t} a_{i}$ is represented by

$$
[X]_{\mathcal{A}}=\left(x_{1}=X \quad \bmod a_{1}, \ldots, x_{t}=X \quad \bmod a_{t}\right)
$$

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- The Chinese remainder theorem tell us that for op $\in\{+, \times\}$

$$
[X]_{\mathcal{A}} \text { op }[Y]_{\mathcal{A}}=\left(\left[x_{1} \text { op } y_{1}\right]_{a_{1}}, \ldots,\left[x_{t} \text { op } y_{t}\right]_{a_{t}}\right) \Leftrightarrow X \text { op } Y \bmod A
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$$

## Montgomery Multiplication in RNS

Require: $X, Y$ in $\mathcal{A} \cup \mathcal{B}$
Ensure: $X Y A^{-1} \bmod N$ in $\mathcal{A} \cup \mathcal{B}$
1: $[Q]_{\mathcal{A}} \leftarrow\left[X Y N^{-1}\right]_{\mathcal{A}}$
3: $[Z]_{\mathcal{B}} \leftarrow\left[(X Y-Q N) A^{-1}\right]_{\mathcal{B}}$
5: return $\left(Z_{\mathcal{A} \cup \mathcal{B}}\right)$

## Montgomery multiplication in residue number system

- Let $\mathcal{A}=\left\{a_{1}, \ldots, a_{t}\right\}$ be a set $t$ co-prime integers.
- An integer $X$ such that $0 \leq X<A=\prod_{i=1}^{t} a_{i}$ is represented by

$$
[X]_{\mathcal{A}}=\left(x_{1}=X \quad \bmod a_{1}, \ldots, x_{t}=X \quad \bmod a_{t}\right)
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$$
[X]_{\mathcal{A}} \text { op }[Y]_{\mathcal{A}}=\left(\left[x_{1} \text { op } y_{1}\right]_{a_{1}}, \ldots,\left[x_{t} \text { op } y_{t}\right]_{a_{t}}\right) \Leftrightarrow X \text { op } Y \bmod A
$$

## Montgomery Multiplication in RNS

Require: $X, Y$ in $\mathcal{A} \cup \mathcal{B}$
Ensure: $X Y A^{-1} \bmod N$ in $\mathcal{A} \cup \mathcal{B}$
1: $[Q]_{\mathcal{A}} \leftarrow\left[X Y N^{-1}\right]_{\mathcal{A}}$
2: $[Q]_{\mathcal{B}} \leftarrow B E_{\mathcal{A} \rightarrow \mathcal{B}}\left([Q]_{\mathcal{A}}\right)$
3: $[Z]_{\mathcal{B}} \leftarrow\left[(X Y-Q N) A^{-1}\right]_{\mathcal{B}}$
4: $[Z]_{\mathcal{A}} \leftarrow B E_{\mathcal{B} \rightarrow \mathcal{A}}\left([Z]_{\mathcal{B}}\right)$
5: return $\left(Z_{\mathcal{A} \cup \mathcal{B}}\right)$

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## Randomization in RNS (LRA CHES 2004)

We have

$$
\widetilde{X}_{\text {old }}=\left[X A_{\text {old }}\right]_{\mathcal{A}_{\text {old }} \cup \mathcal{B}_{\text {old }}}
$$

we permute the basis elements $\mathcal{A}_{\text {old }} \cup \mathcal{B}_{\text {old }} \rightarrow \mathcal{A}_{\text {new }} \cup \mathcal{B}_{\text {new }}$

this leads to a new representation of $X$

$$
\widetilde{X}_{\text {new }}=\left[X A_{\text {new }}\right]_{\mathcal{A}_{\text {new }} \cup \mathcal{B}_{\text {new }}}
$$

## Cost

Two Montgomery multiplications :
$X A_{\text {old }} \bmod N \rightarrow X A_{\text {old }} A_{\text {new }} \bmod N \rightarrow X A_{\text {new }} \bmod N$.

## Randomized square-and-multiply-always

- Input: $N, X \in\left[0, N\left[, E=\left(e_{\ell-1}, \ldots, e_{0}\right)_{2}\right.\right.$ and $\mathcal{M}=\left\{m_{1}, \ldots, m_{2 t}\right\}$.
- Output: $X^{E} \bmod N$


## Square-and-mult-always

```
A,\mathcal{B}\leftarrowr random split }\mathcal{M
\widetilde{Z}}\leftarrow[[\widetilde{X}\mp@subsup{]}{\mathcal{A}\cup\mathcal{B}}{}
\mp@subsup{R}{0}{}}\leftarrow[[\tilde{1}\mp@subsup{]}{\mathcal{A\cup\mathcal{B}}}{},\mp@subsup{\widetilde{R}}{1}{}\leftarrow[[\mp@subsup{]}{\mathcal{A\cupB}}{
for i from 0 to }\ell-1\mathrm{ do
    \widetilde{R}}\mp@subsup{\widetilde{\mp@subsup{e}{i}{}}}{}{\leftarrowMM_RNS(\widetilde{R}
    Z}\leftarrowMM_RNS(\widetilde{Z},\widetilde{Z},\mathcal{A},\mathcal{B}
end for
return }\mp@subsup{\widetilde{R}}{1}{
```


## Randomized square-and-multiply-always

- Input: $N, X \in\left[0, N\left[, E=\left(e_{\ell-1}, \ldots, e_{0}\right)_{2}\right.\right.$ and $\mathcal{M}=\left\{m_{1}, \ldots, m_{2 t}\right\}$.
- Output: $X^{E} \bmod N$


## Randomized <br> Square-and-mult-always

```
\mathcal{A},\mathcal{B}\leftarrow[\widetilde{~}
Z}\leftarrow[\widetilde{X}\mp@subsup{]}{\mathcal{A\cupB}}{}
\mp@subsup{R}{0}{}}\leftarrow[\widetilde{1}\mp@subsup{]}{\mathcal{A\cupB}}{},\mp@subsup{\widetilde{R}}{1}{}\leftarrow[\widetilde{1}\mp@subsup{]}{\mathcal{A}\cup\mathcal{B}}{
for i from 0 to }\ell-
    \mp@subsup{\widetilde{R}}{\mp@subsup{e}{i}{}}{}\leftarrowMM_RNS(\widetilde{R}
    Z}\leftarrowMM_RNS(\tilde{Z},\tilde{Z},\mathcal{A},\mathcal{B}
    Randomise(}\mp@subsup{\mathcal{A}}{\mathrm{ old }}{},\mp@subsup{\mathcal{B}}{\mathrm{ old }}{},\mathcal{A},\mathcal{B}
    Z}\leftarrowU\operatorname{Update( }(\tilde{Z},\mp@subsup{\mathcal{A}}{\mathrm{ old }}{},\mp@subsup{\mathcal{B}}{\mathrm{ old }}{\prime},\mathcal{A},\mathcal{B}
    \widetilde{R}
    \widetilde{R}
end for
return }\mp@subsup{\widetilde{R}}{1}{
```


## Randomized square-and-multiply-always

- Input: $N, X \in\left[0, N\left[, E=\left(e_{\ell-1}, \ldots, e_{0}\right)_{2}\right.\right.$ and $\mathcal{M}=\left\{m_{1}, \ldots, m_{2 t}\right\}$.
- Output: $X^{E} \bmod N$

```
Randomized
Square-and-mult-always
```

```
\mathcal{A},\mathcal{B}\leftarrow~
```

\mathcal{A},\mathcal{B}\leftarrow~
Z}\leftarrow[\widetilde{X}\mp@subsup{]}{\mathcal{A\cupB}}{}
Z}\leftarrow[\widetilde{X}\mp@subsup{]}{\mathcal{A\cupB}}{}
\mp@subsup{R}{0}{}}\leftarrow[\widetilde{1}\mp@subsup{]}{\mathcal{A\cup\mathcal{B}}}{},\mp@subsup{\widetilde{R}}{1}{}\leftarrow[\widetilde{1}\mp@subsup{]}{\mathcal{A}\cup\mathcal{B}}{
\mp@subsup{R}{0}{}}\leftarrow[\widetilde{1}\mp@subsup{]}{\mathcal{A\cup\mathcal{B}}}{},\mp@subsup{\widetilde{R}}{1}{}\leftarrow[\widetilde{1}\mp@subsup{]}{\mathcal{A}\cup\mathcal{B}}{
for i from 0 to }\ell-1\mathrm{ do
for i from 0 to }\ell-1\mathrm{ do
\mp@subsup{\widetilde{R}}{\mp@subsup{e}{i}{}}{}\leftarrowMM_RNS(\widetilde{R}
\mp@subsup{\widetilde{R}}{\mp@subsup{e}{i}{}}{}\leftarrowMM_RNS(\widetilde{R}
Z}\leftarrowMM_RNS(\tilde{Z},\tilde{Z},\mathcal{A},\mathcal{B}
Z}\leftarrowMM_RNS(\tilde{Z},\tilde{Z},\mathcal{A},\mathcal{B}
Randomise ( }\mp@subsup{\mathcal{A}}{\mathrm{ old }}{},\mathcal{B
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Z}\leftarrow\mathrm{ Update( }\tilde{Z},\mp@subsup{\mathcal{A}}{\mathrm{ old }}{\mathrm{ , }

```
    Z}\leftarrow\mathrm{ Update( }\tilde{Z},\mp@subsup{\mathcal{A}}{\mathrm{ old }}{\mathrm{ , }
```




```
    \mp@subsup{R}{1}{}\longleftarrowU\mathrm{ pdate (}\mp@subsup{\widetilde{R}}{1}{},\mp@subsup{\mathcal{A}}{\mathrm{ old }}{},\mp@subsup{\mathcal{B}}{\mathrm{ old }}{},\mathcal{A},\mathcal{B})
```

    \mp@subsup{R}{1}{}\longleftarrowU\mathrm{ pdate (}\mp@subsup{\widetilde{R}}{1}{},\mp@subsup{\mathcal{A}}{\mathrm{ old }}{},\mp@subsup{\mathcal{B}}{\mathrm{ old }}{},\mathcal{A},\mathcal{B})
    end for
end for
return }\mp@subsup{\widetilde{R}}{1}{

```
return }\mp@subsup{\widetilde{R}}{1}{
```


## Randomized square-and-multiply-always

- Input: $N, X \in\left[0, N\left[, E=\left(e_{\ell-1}, \ldots, e_{0}\right)_{2}\right.\right.$ and $\mathcal{M}=\left\{m_{1}, \ldots, m_{2 t}\right\}$.
- Output: $X^{E} \bmod N$


## Randomized <br> Square-and-mult-always

$\mathcal{A}, \mathcal{B} \leftarrow{ }^{\text {random split }} \mathcal{M}$
$\underset{\widetilde{R}}{\mathcal{R}} \leftarrow[\widetilde{X}]_{\mathcal{A} \cup \mathcal{B}}$,
$\widetilde{R}_{0} \leftarrow[\widetilde{1}]_{\mathcal{A} \cup \mathcal{B}}, \widetilde{R}_{1} \leftarrow[\widetilde{1}]_{\mathcal{A} \cup \mathcal{B}}$
for $i$ from 0 to $\ell-1$ do
$\widetilde{R}_{e_{i}} \leftarrow M M \_\operatorname{RNS}\left(\widetilde{R}_{e_{i}}, \widetilde{Z}, \mathcal{A}, \mathcal{B}\right)$
$\widetilde{Z} \leftarrow M M \_\operatorname{RNS}(\tilde{Z}, \tilde{Z}, \mathcal{A}, \mathcal{B})$
Randomise $\left(\mathcal{A}_{\text {old }}, \mathcal{B}_{\text {old }}, \mathcal{A}, \mathcal{B}\right)$
$\widetilde{Z} \leftarrow \operatorname{Update}\left(\widetilde{Z}, \mathcal{A}_{\text {old }}, \mathcal{B}_{\text {old }}, \mathcal{A}, \mathcal{B}\right)$
$\widetilde{R}_{0} \cup U$ pdate $\left(\widetilde{R}_{0}, \mathcal{A}_{\text {old }}, \mathcal{B}_{\text {old }}, \mathcal{A}, \mathcal{B}\right)$ $\widetilde{R}_{1} \longleftarrow \operatorname{Update}\left(\widetilde{R}_{1}, \mathcal{A}_{\text {old }}, \mathcal{B}_{\text {old }}, \mathcal{A}, \mathcal{B}\right)$ end for return $\widetilde{R}_{1}$

## Proposed

$$
\begin{aligned}
& \mathcal{A}, \mathcal{B} \leftarrow{ }^{\text {random split }} \mathcal{M} \\
& \underset{\underset{R}{Z}}{\widetilde{R}} \leftarrow[\widetilde{X}]_{\mathcal{A} \cup \mathcal{B}}, \\
& \widetilde{R}_{0} \leftarrow[\widetilde{1}]_{\mathcal{A} \cup \mathcal{B}}, \widetilde{R}_{1} \leftarrow[\widetilde{1}]_{\mathcal{A} \cup \mathcal{B}} \\
& \text { for } i \text { from } 0 \text { to } \ell-1 \text { do } \\
& \mathcal{A}_{e_{i}}^{\prime}, \mathcal{B}_{e_{i}}^{\prime} \leftarrow{ }^{\text {random split }} \mathcal{M} \\
& \widetilde{R}_{e_{i}} \leftarrow \operatorname{MM} \_\operatorname{RNS}\left(\widetilde{R}_{e_{i}}, \widetilde{Z}, \mathcal{A}_{e_{i}}^{\prime}, \mathcal{B}_{e_{i}}^{\prime}\right) \\
& \tilde{Z} \leftarrow \operatorname{MM} \_\operatorname{RNS}(\tilde{Z}, \tilde{Z}, \mathcal{A}, \mathcal{B}) \\
& \text { Randomise }\left(\mathcal{A}_{\text {old }}, \mathcal{B}_{\text {old }}, \mathcal{A}, \mathcal{B}\right) \\
& \widetilde{Z} \leftarrow \operatorname{Update}\left(\widetilde{Z}, \mathcal{A}_{\text {old }}, \mathcal{B}_{\text {old }}, \mathcal{A}, \mathcal{B}\right) \\
& \text { end for } \\
& \text { return } \widetilde{R}_{1}
\end{aligned}
$$

## Example

For $E=7=(111)_{2}$ and $\mathcal{M}=\left\{m_{1}, m_{2}, m_{3}, m_{4}\right\}$

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For $E=7=(111)_{2}$ and $\mathcal{M}=\left\{m_{1}, m_{2}, m_{3}, m_{4}\right\}$

- Initialization: $\mathcal{A}=\left\{m_{1}, m_{2}\right\}, \mathcal{B}=\left\{m_{3}, m_{4}\right\}$ leads to

$$
\begin{aligned}
R_{1} & =m_{1} m_{2} \quad \bmod N \\
Z & =X m_{1} m_{2} \quad \bmod N
\end{aligned}
$$

## Example

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$$

- Loop 1: $\mathcal{A}_{1}=\left\{m_{2}, m_{4}\right\}, \mathcal{B}_{1}=\left\{m_{1}, m_{3}\right\}$ we get

$$
R_{1}=\left(m_{1} m_{2}\right) \times \underbrace{\left(X m_{1} m_{2}\right)}_{Z} \times \underbrace{\left(m_{2}^{-1} m_{4}^{-1}\right)}_{\text {Mont. factor }}=X m_{1}^{2} m_{2} m_{4}^{-1}
$$

## Example

For $E=7=(111)_{2}$ and $\mathcal{M}=\left\{m_{1}, m_{2}, m_{3}, m_{4}\right\}$

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$$

$$
\mathcal{A}=\left\{m_{1}, m_{3}\right\}, \mathcal{B}=\left\{m_{2}, m_{4}\right\} \text { leads to }
$$

$$
Z=X^{2} m_{1} m_{3}
$$

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$$

$\mathcal{A}=\left\{m_{1}, m_{3}\right\}, \mathcal{B}=\left\{m_{2}, m_{4}\right\}$ leads to

$$
Z=X^{2} m_{1} m_{3}
$$

- Loop 2: $\mathcal{A}_{1}=\left\{m_{1}, m_{4}\right\}, \mathcal{B}_{1}=\left\{m_{2}, m_{3}\right\}$ we get

$$
R_{1}=X m_{1}^{2} m_{2} m_{4}^{-1} \times\left(X^{2} m_{1} m_{3}\right) \times\left(m_{1}^{-1} m_{4}^{-1}\right)=X^{3} m_{1}^{2} m_{2} m_{3} m_{4}^{-2}
$$

## Example

For $E=7=(111)_{2}$ and $\mathcal{M}=\left\{m_{1}, m_{2}, m_{3}, m_{4}\right\}$

- Initialization: $\mathcal{A}=\left\{m_{1}, m_{2}\right\}, \mathcal{B}=\left\{m_{3}, m_{4}\right\}$ leads to

$$
\begin{aligned}
R_{1} & =m_{1} m_{2} \quad \bmod N \\
Z & =X m_{1} m_{2} \quad \bmod N
\end{aligned}
$$

- Loop 1: $\mathcal{A}_{1}=\left\{m_{2}, m_{4}\right\}, \mathcal{B}_{1}=\left\{m_{1}, m_{3}\right\}$ we get

$$
R_{1}=\left(m_{1} m_{2}\right) \times \underbrace{\left(X m_{1} m_{2}\right)}_{Z} \times \underbrace{\left(m_{2}^{-1} m_{4}^{-1}\right)}_{\text {Mont. factor }}=X m_{1}^{2} m_{2} m_{4}^{-1}
$$

$\mathcal{A}=\left\{m_{1}, m_{3}\right\}, \mathcal{B}=\left\{m_{2}, m_{4}\right\}$ leads to

$$
Z=X^{2} m_{1} m_{3}
$$

- Loop 2: $\mathcal{A}_{1}=\left\{m_{1}, m_{4}\right\}, \mathcal{B}_{1}=\left\{m_{2}, m_{3}\right\}$ we get

$$
R_{1}=X m_{1}^{2} m_{2} m_{4}^{-1} \times\left(X^{2} m_{1} m_{3}\right) \times\left(m_{1}^{-1} m_{4}^{-1}\right)=X^{3} m_{1}^{2} m_{2} m_{3} m_{4}^{-2}
$$

- Etc.


## Random evolution of the mask

After $i$ loop iterations we have

$$
\widetilde{R}_{1}^{(i)}=X^{\sum_{j=0}^{i-1} e_{j} 2^{j}} \times \prod_{j=0}^{2 t} m_{j}^{\gamma_{j}^{(i)}} \bmod N
$$

and each $\gamma_{j}^{(i)}$ evolves randomly as

$$
\gamma_{j}^{(i+1)}=\gamma_{j}^{(i)}+\delta_{j}^{(i)} \text { with } \delta_{j}^{(i)} \in\{-1,0,1\} \text { and }\left\{\begin{array}{c}
\mathbb{P}\left(\delta_{j}^{(i)}=1\right)=1 / 8 \\
\mathbb{P}\left(\delta_{j}^{(i)}=-1\right)=1 / 8 \\
\mathbb{P}\left(\delta_{j}^{(i)}=0\right)=3 / 4
\end{array}\right.
$$



## Removing the final mask

Problem: at the end we have to remove the final mask $\prod_{j=1}^{2 t} m_{j}^{\gamma_{j}^{(\ell)}}$ from

$$
\tilde{X}=X^{E} \cdot \prod_{j=1}^{2 t} m_{j}^{\gamma_{j}^{(\ell)}} \bmod N .
$$

Strategy: we force $\gamma_{j}^{(\ell)}$ to be equal 0 as follows

- During the first half of the iterations each $\gamma_{j}^{(i)}$ evolves freely.
- During the second half we constrain each $\left|\gamma_{j}^{(i)}\right|$ to decrease toward 0 .



## Level of randomization

- The probabilities of the mask exponents satisfy

$$
\begin{aligned}
& \mathbb{P}\left(\gamma_{j}^{(i)}=d\right)=\sum_{k=d}^{d+\lfloor(i-d) / 2\rfloor}\binom{i}{k}\binom{i-k}{k-d}\left(\frac{1}{8}\right)^{2 k-d}\left(\frac{3}{4}\right)^{i-2 k+d} \\
& \mathbb{P}\left(\Gamma^{(i)}=\Gamma\right) \leq \prod_{j=1}^{t} \mathbb{P}\left(\gamma_{j}^{(i)}=\gamma_{j}\right) \leq \prod_{j=1}^{t} \mathbb{P}\left(\gamma_{j}^{(i)}=0\right)
\end{aligned}
$$

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\end{aligned}
$$

- Comparison: for a 2048-bit RSA modulus and $t=32$ :
- CHES 04:
$\star$ Montgomery-ladder,
* 4MM_RNS per randomization,
$\star$ all masks are controled.
- Proposed:
« right-left square-and-multiply-always,
$\star$ 2MM_RNS per randomization
$\star$ the masks for $R_{0}$ and $R_{1}$ are not controled.

| Approach | loop 1 | loop 5 | loop 10 | loop 50 | loop 100 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CHES 04 | $4.17 \cdot 10^{-38}$ | $4.17 \cdot 10^{-38}$ | $4.17 \cdot 10^{-38}$ | $4.17 \cdot 10^{-38}$ | $4.17 \cdot 10^{-38}$ |
| Proposed | $10^{-8}$ | $5 \cdot 10^{-28}$ | $1.7 \cdot 10^{-38}$ | $2.69 \cdot 10^{-61}$ | $5.75 \cdot 10^{-71}$ |

## Outline

(1) Cryptography

- RSA cryptosystem
- Power analysis
- Montgomery multiplication in RNS
(2) Randomized modular exponentiation in RNS
- Randomized Montgomery multiplication
- Proposed approach
- Level of randomization
(3) Conclusion


## Conclusion

Secure embedded implementation of RSA:

- Randomized modular exponentiation
- But leak resistant arithmetic (CHES 04) is costly: 4 MM_RNS per randomization


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Secure embedded implementation of RSA:

- Randomized modular exponentiation
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We proposed:
- To apply LRA to right-to-left exponentiation.
- Avoid some correction of Montgomery Factor.
- This decreases the computational cost: 2 MM_RNS per randomization.
- Increases the level of randomization after a small number of loop.


## Conclusion

Secure embedded implementation of RSA:

- Randomized modular exponentiation
- But leak resistant arithmetic (CHES 04) is costly: 4 MM_RNS per randomization
We proposed:
- To apply LRA to right-to-left exponentiation.
- Avoid some correction of Montgomery Factor.
- This decreases the computational cost: 2 MM_RNS per randomization.
- Increases the level of randomization after a small number of loop.

Perspectives:

- A better estimation of the level of randomization.
- Is it a good counter-measure against horizontal power analysis ?


## Thank you for your attention!

