

**LIRMM**

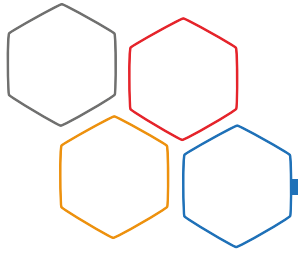
# Complexity Results in Optimistic/Pessimistic Preference Reasoning

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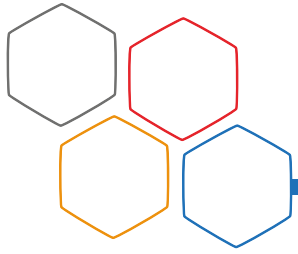
tellmeplus 





# Summary

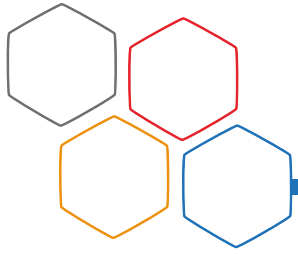
- Background in preference reasoning
  - Semantics
  - Main problems
- Contributions
  - Complexity study of main problems in preference reasoning
  - Knowledge compilation : improving through pre-processing



# How express a preference ?

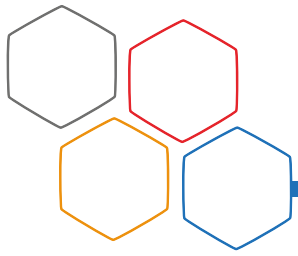
Suppose the preference « I prefer salad to tomato »

- Quantitative preference
  - “ I like a salad with weight 0.7 and tomato with weight 0.3 ”
- Qualitative preference
  - “ I prefer salad rather than tomato ”



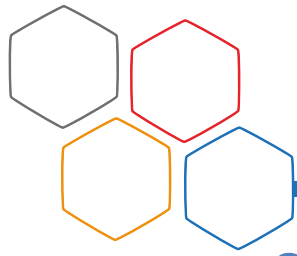
# Main problems

- Undominated
  - “ Does it exist an outcome which is the most preferred? ”
- Dominance
  - “ Given two outcomes  $\omega, \omega'$  , is  $\omega$  strictly preferred as  $\omega'$ ? ”
- Consistency
  - “ Is the network consistent ? ” (absence of dominance cycle)



# Formalisms - Complexity

	Dominance	Consistency
CP-nets[1]	PSPACE-complete[4]	PSPACE-complete[4]
Conditionnal Logic[2]	?	?
Prototypical preference logic[3]	PSPACE-complete[3]	PSPACE-complete[3]



# Semantics — How to interpret a preference?

Suppose the preference « I prefer salad to tomato »

## 1:Optimistic

[Pearl, 1990]

at least one salad meal is strictly preferred to all tomato meals

## 2:Strong (called strict)

[Boutilier, 1994]

all salad meals are strictly preferred to all tomato meals regardless main dish / wine

## 3:Ceteris paribus

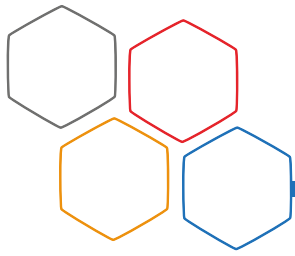
[Hansson, 1996]

Salad meals are strictly preferred to tomato meals all other things being equal

## 4:Pessimistic



[Benferhat et al., 2002]


at least one tomato meal is less preferred to all salad meals





# Notations

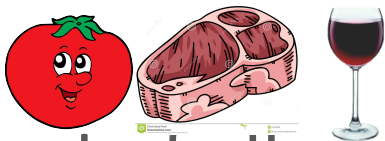
Guideline example  
 Choosing a meal according to

Starter = {   }

Main dish = {   }

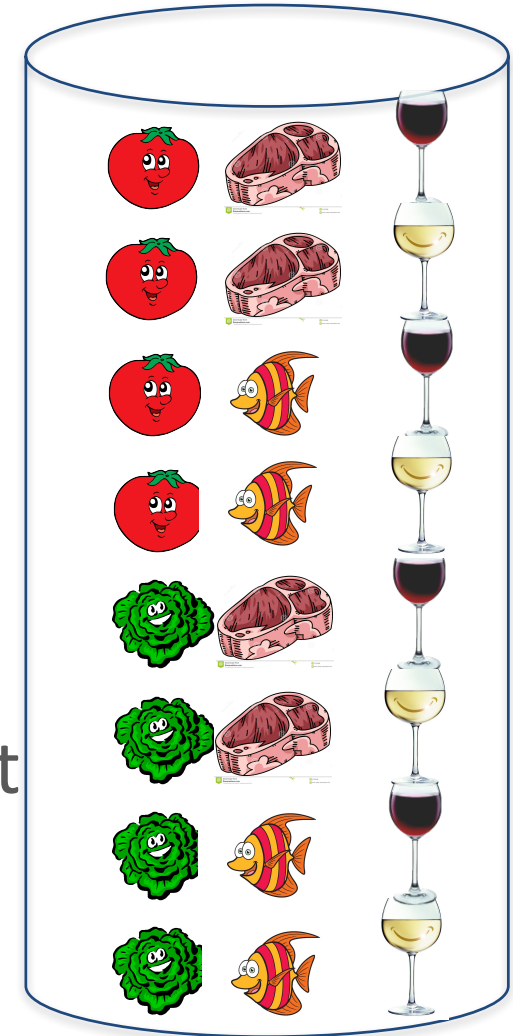
Wine = {   }

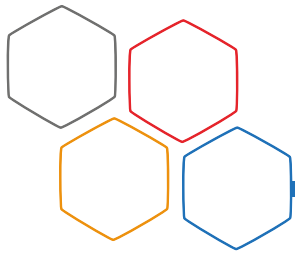
- An outcome  $\omega$  is a complete assignment



- The set of all outcomes  $\Omega$

$\Omega$





# Framework – Conditionnal logic formalism

$$p_1 = \text{broccoli} > \text{tomato}$$

$$p_2 = \text{steak} : \text{red wine} > \text{white wine}$$

$$p_3 = \text{fish} : \text{white wine} > \text{red wine}$$

Semantic = Optimistic

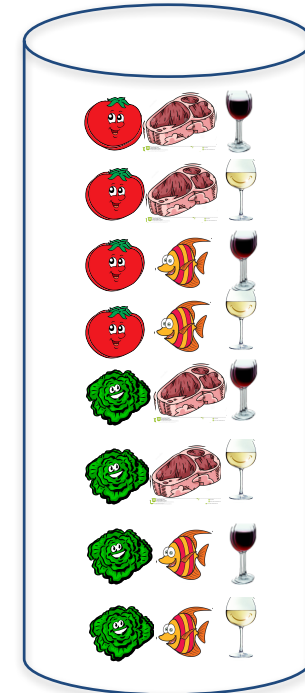
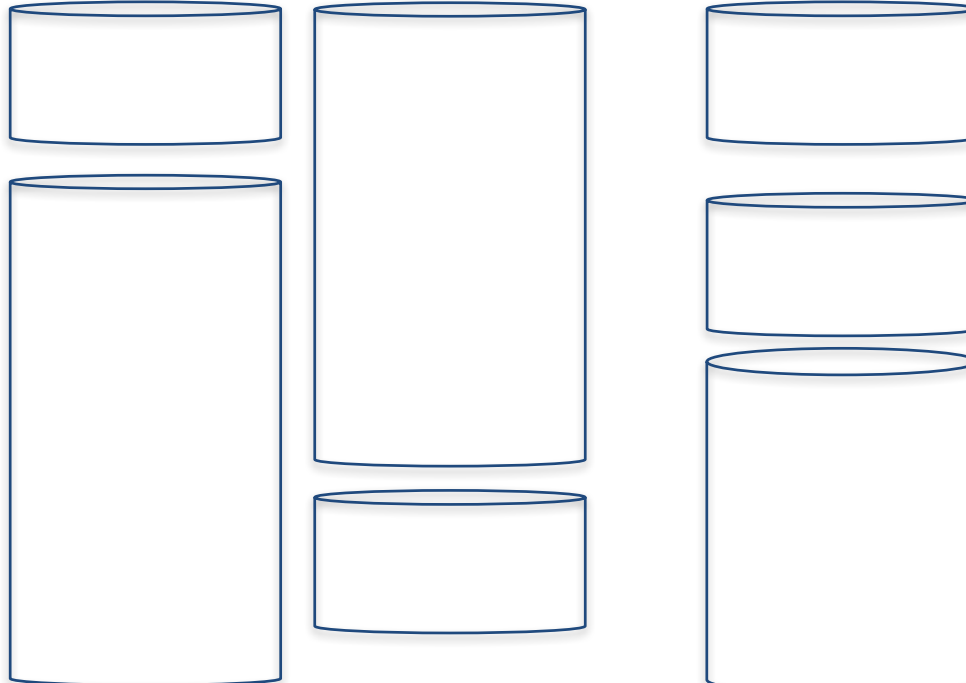
Pessimistic

Optimistic Strong= $p_1$

Layer<sub>0</sub>

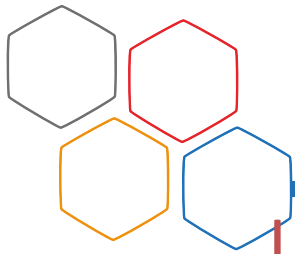
Layer<sub>1</sub>

Layer<sub>2</sub>



$\Omega$





# State of the art - Conditionnal logic formalism

Layers are explicitly built :

Algorithm Exponential in space

- Undominated

Choose an outcome in Layer<sub>0</sub>

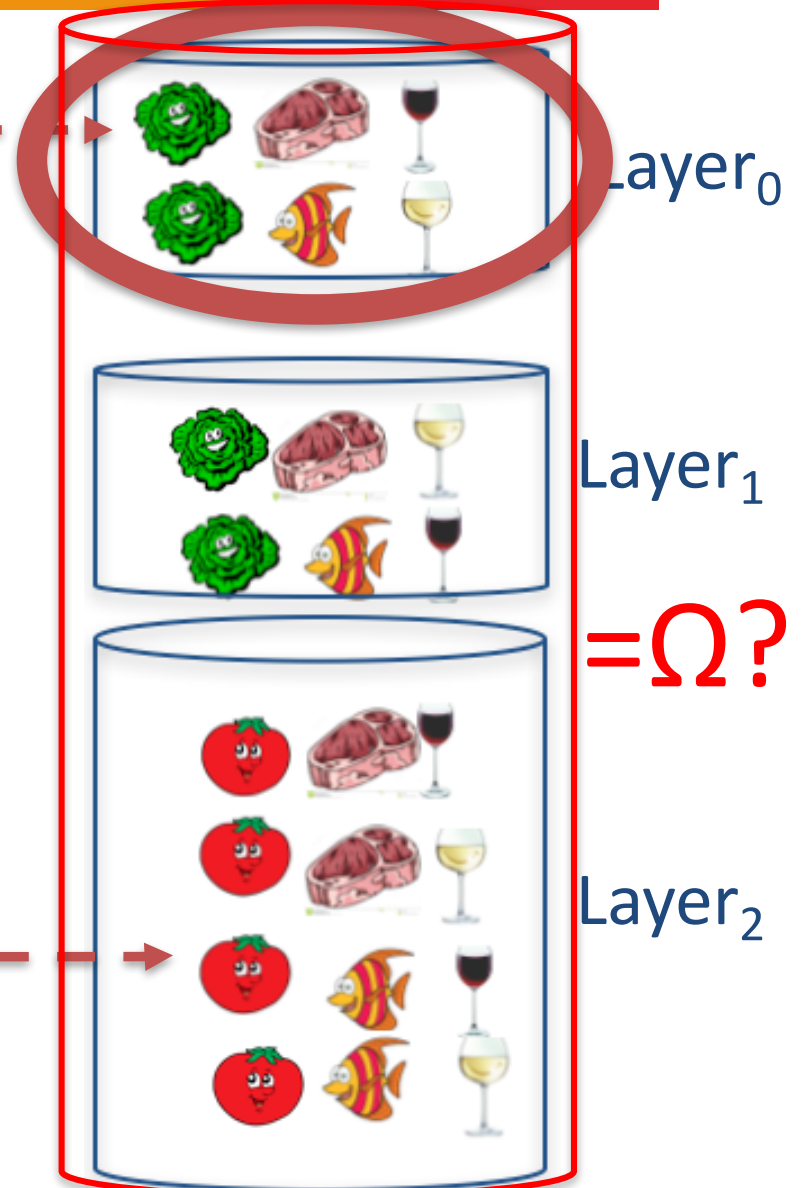
- Dominance ( )

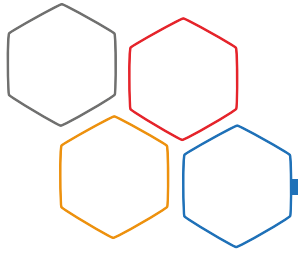
Layer < Layer

Yes

- Consistency

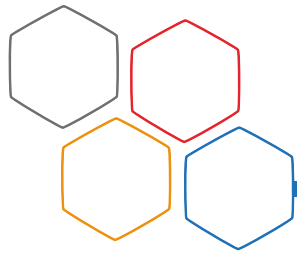
Check that Layer<sub>0</sub> U ... U Layer<sub>n</sub> = Ω





# Contributions - Complexity map

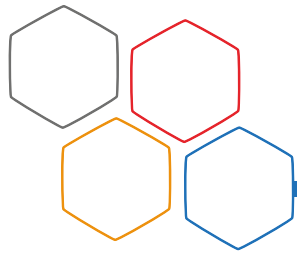
Problem	Strong Preferences	Optimistic	Pessimistic
Undominated	All		
	Not All		
Consistency	All		
	Not All		
Dominance	All		
	Not All		



# Contributions - Complexity map

Problem	Strong Preferences	Optimistic	Pessimistic
Undominated	All	NP-complete	Polynomial
	Not All	NP-complete	NP-complete
Consistency	All		
	Not All		
Dominance	All		
	Not All		

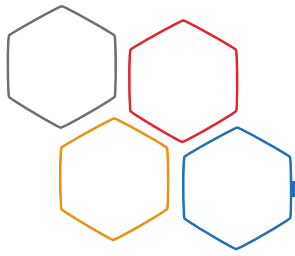
- Complexity depends on the semantic



# Contributions - Complexity map

Problem	Strong Preferences	Optimistic	Pessimistic
Undominated	All	NP-complete	Polynomial
	Not All	NP-complete	NP-complete
Consistency	All	Polynomial	Polynomial
	Not All	NP-complete	NP-complete
Dominance	All		
	Not All		

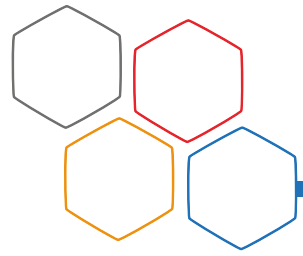
- Complexity depends if all preferences are strong



# Contributions - Complexity map

Problem	Strong Preferences	Optimistic	Pessimistic
Undominated	All	NP-complete	Polynomial
	Not All	NP-complete	NP-complete
Consistency	All	Polynomial	Polynomial
	Not All	NP-complete	NP-complete
Dominance	All	Polynomial	Polynomial
	Not All	DP-complete	DP-complete

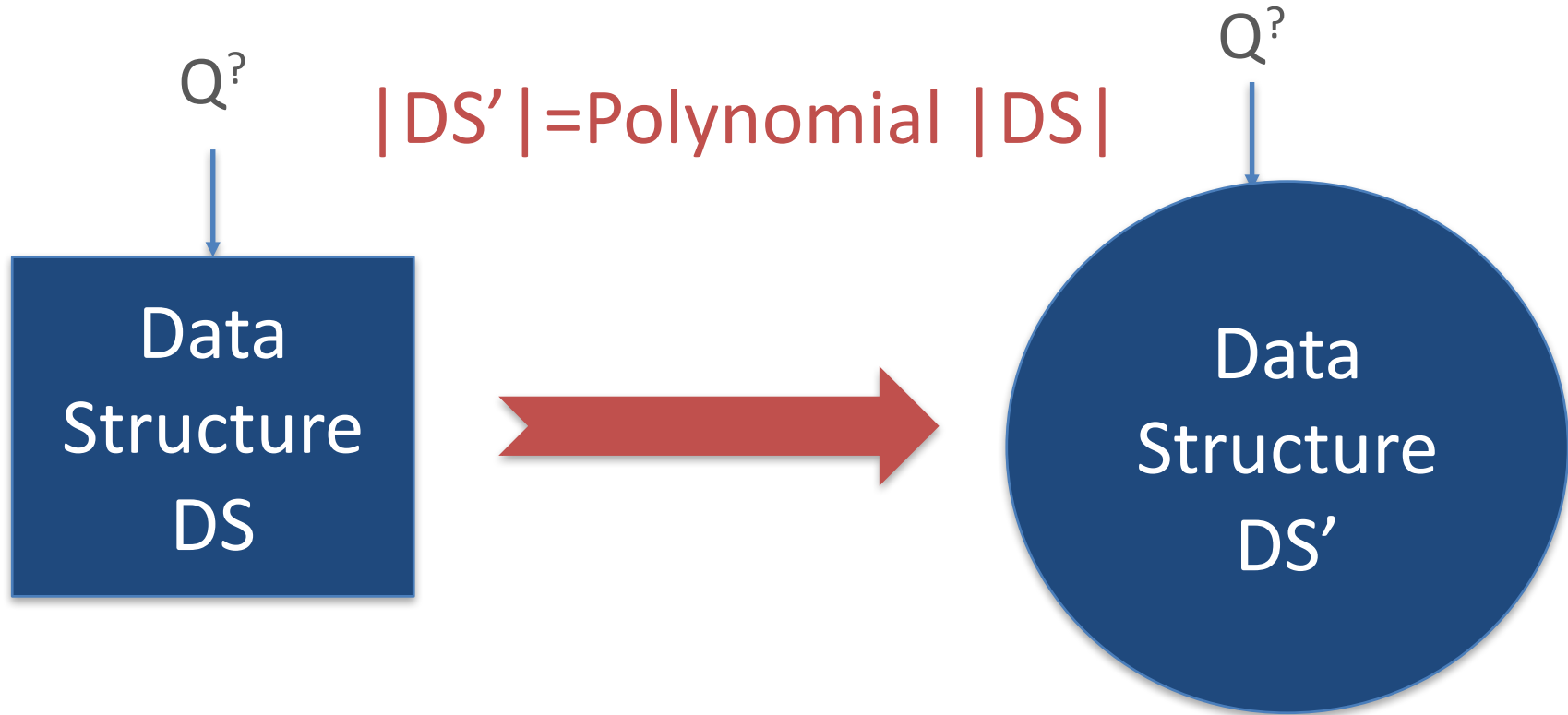
- Dominance is the only problem called several times
  - Many different pairs  $(\omega, \omega')$
  - Unfortunately DP-complete



# Knowledge compilation- Background

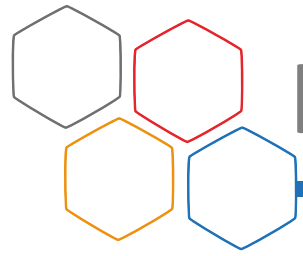
Take as much time as needed

$$|DS'| = \text{Polynomial } |DS|$$



Complexity of  $Q^?$  : Hard

Complexity of  $Q^?$  : Easier



# Knowledge compilation- Dominance

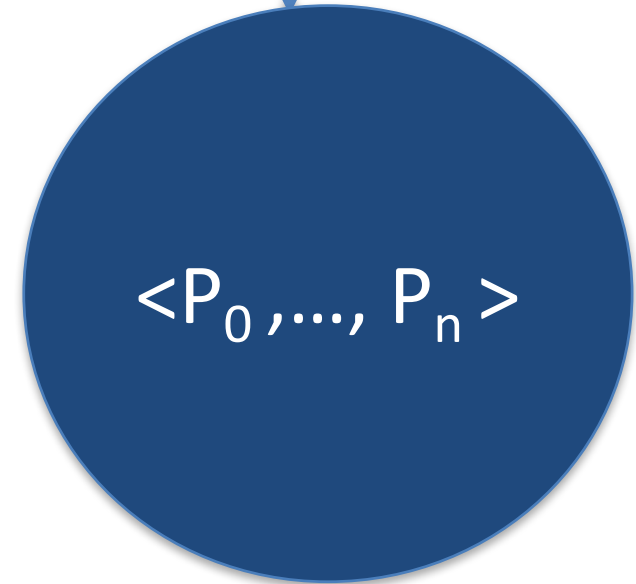
$$\omega >? \omega'$$



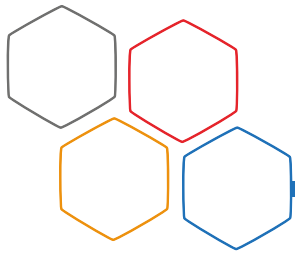
DP-complete



$$\omega >? \omega'$$

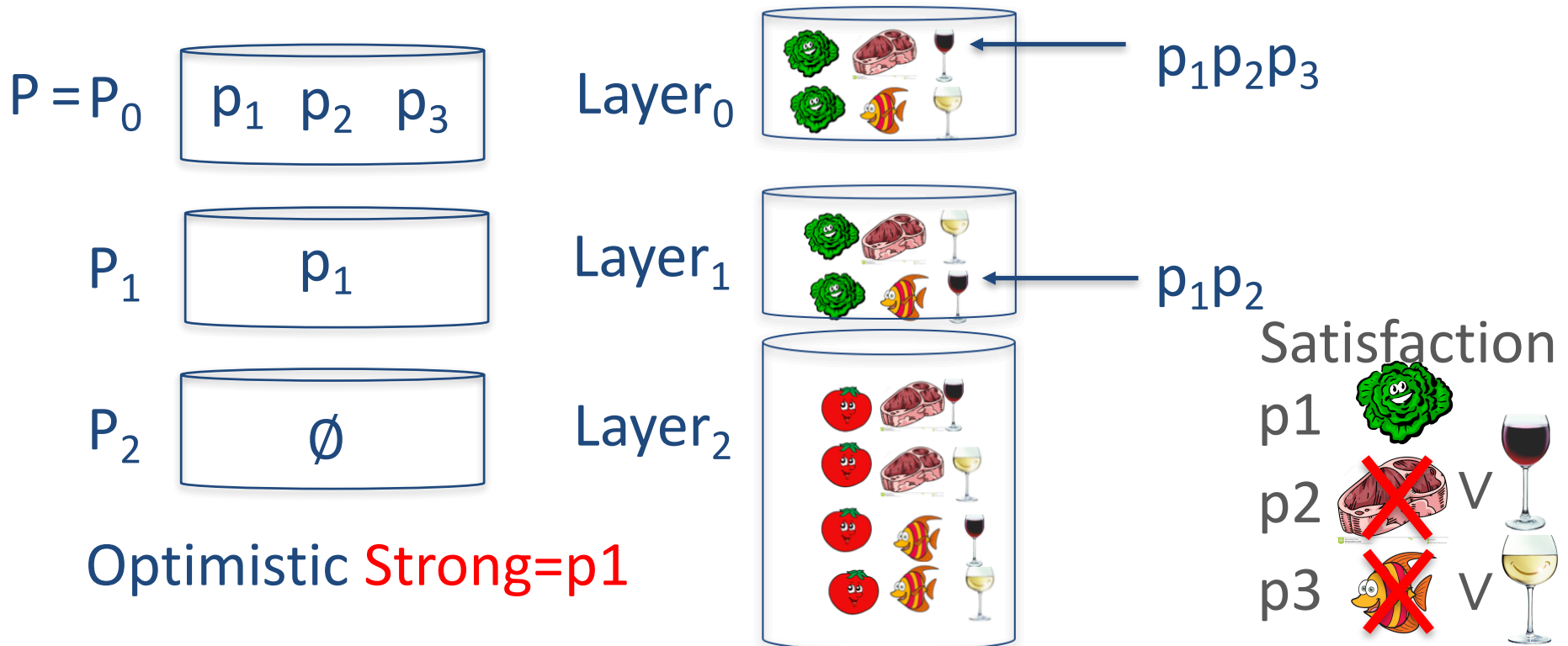


Polynomial



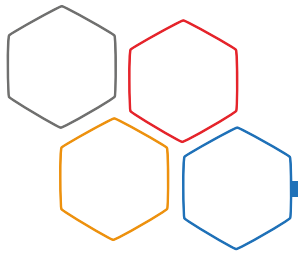
# Compiled preferences - Equivalence

- Outcomes in Layer<sub>i</sub> are those which :
  - satisfy all preferences in  $P_i$
  - violate at least one preference in  $P_{i-1} \setminus P_i$



Optimistic Strong= $p_1$





# Compiled preferences - Complexity

$$p_1 = \text{broccoli} > \text{tomato}$$

$$p_2 = \text{salmon} : \text{red wine} > \text{white wine}$$

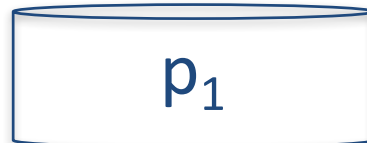
$$p_3 = \text{fish} : \text{white wine} > \text{red wine}$$

Optimistic **Strong=p1**

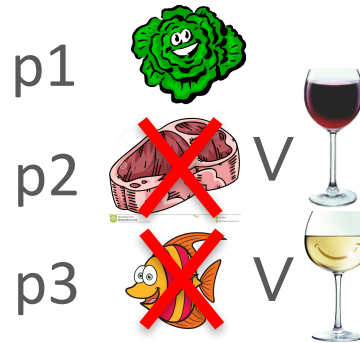
$$P = P_0$$



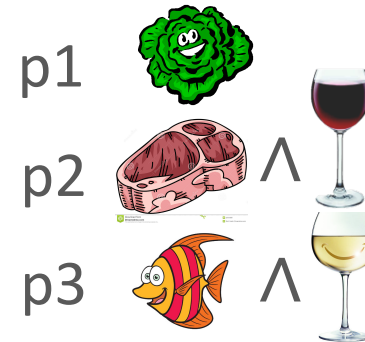
$$P_1$$



Satisfaction

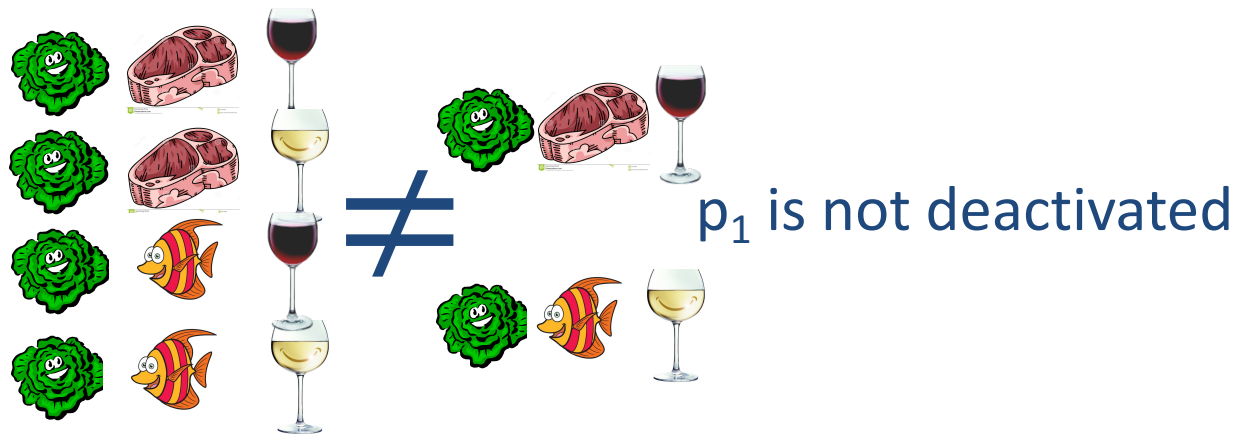


Full Satisfaction

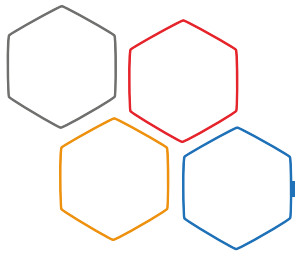


Deactivation of  $p_1$

$$\{\text{fully satisfaction } p_1\} = \{\text{satisfaction } p_1 \text{ and } p_2 \text{ and } p_3\}$$



The deactivation of a strong preference is Polynomial



# Compiled preferences - Complexity

$$p_1 = \text{broccoli} > \text{tomato}$$

$$p_2 = \text{salmon} : \text{red wine} > \text{white wine}$$

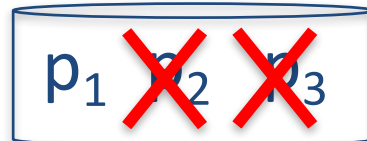
$$p_3 = \text{fish} : \text{white wine} > \text{red wine}$$

Optimistic **Strong=p1**

$$P = P_0$$



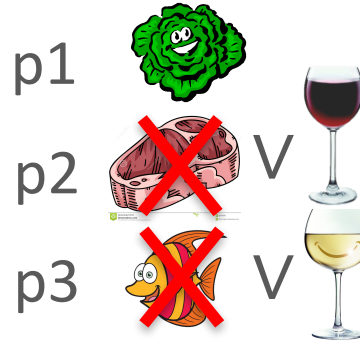
$P_1$



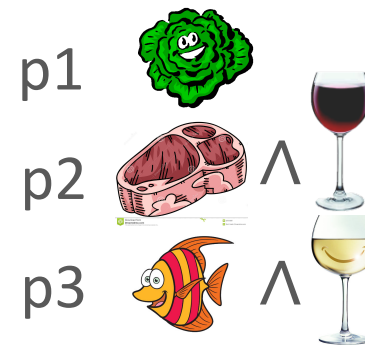
$P_2$



Satisfaction

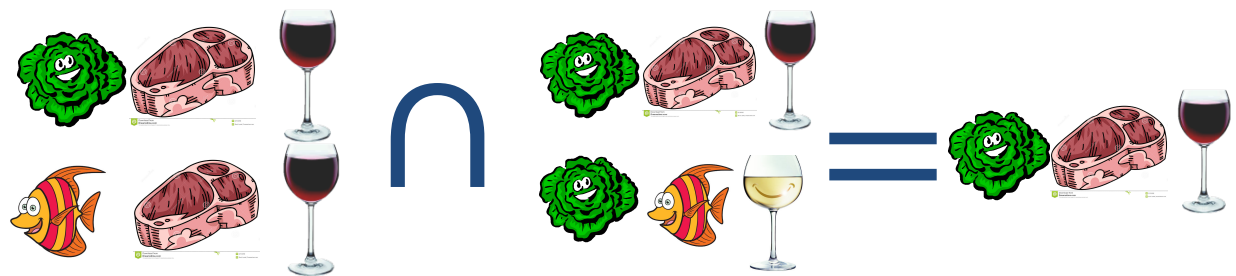


Full Satisfaction



Deactivation of  $p_2$

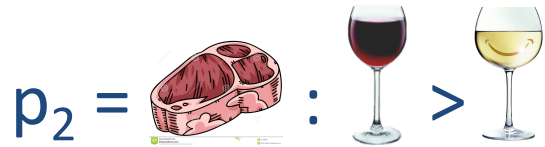
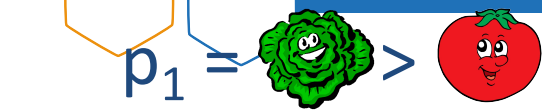
$$\{\text{fully sat } p_2\} \cap \{\text{sat } p_1 \text{ and } p_2 \text{ and } p_3\} \neq \emptyset$$



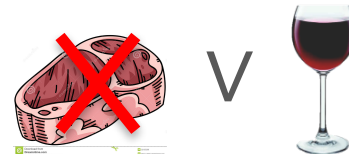
$p_2$  is deactivated

The deactivation of a preference is NP-complete

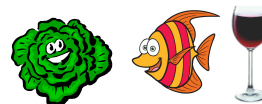
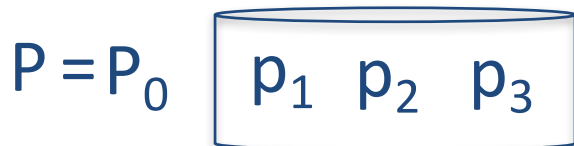
# Contributions: Dominance( $\langle P_0, \dots, P_n \rangle$ ) is Polynomial\*



Optimistic **Strong**= $p_1$



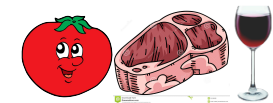
Satisfaction



$>?$



$>?$



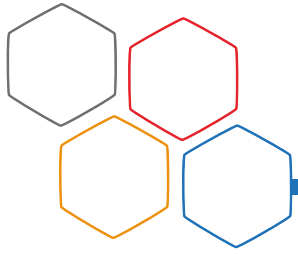
No



Yes

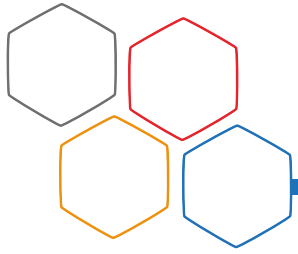


\*Linear with an index encoding **Layer 1 = Layer 1 < Layer 2**



# Conclusion

- Complexity study of main problems in preference reasoning in an existing framework
  - Depends on the semantic  $S$
  - Depends on the set of strong preferences  $Str$
- Dominance is compilable to polynomial time



Thank you for your  
attention