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A Fuzzy Sliding Mode Controller for Reducing Torques Applied to a Rehabilitation Robot

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Abstract – Exoskeletons are the most efficient devices used to help children, suffering from several diseases, to walk compared to the use of wheelchairs. In rehabilitation, relearning to walk is very important. In fact, repetitive tasks improve human locomotory performances. This paper aims to deal with the control problem of a two degrees of freedom lower limb exoskeleton. During the control process, high torques are considered as a major constraint. Thus, a fuzzy sliding mode controller is implemented. Moreover, stability analysis are presented using Lyapunov theory.

Keywords – Fuzzy Sliding Mode, Lyapunov theory, Exoskeletons, Rehabilitation.

I. INTRODUCTION

Stroke, spinal cord injury, traumatic brain injury or cerebral palsy are neurological disorders that can affect limbs [1]. In this purpose, several rehabilitation techniques exist all over the world. Nowadays, exoskeletons are considered as the most efficient technique used to improve the movement ability for elderly persons and handicaps [2]. In fact, they can allow patients to move and to do challenging tasks. Besides, they provide longer training duration with huge number of repetitions comparing to therapists who can get tired while supporting patients with severe motor deficits [3]. Studies have shown that repeating a specific task improves the patient functional walking ability by stimulating the brain plasticity [4]. Furthermore, it helps to reduce patients’ anxiety and improve self confidence. Among the new technologies that provide training procedures is Lokomat [5]. It guarantees repetitive training sessions through a predefined gait cycle in a safe environment. In fact, individuals did not feel the fear of falling, rather than, they feel motivated and able to concentrate in order to achieve the gait training goals. In this work, we are interested in the rehabilitation of kids suffering from cerebral palsy. This disease represents the common physical disability of childhood. It causes a reduction of the movement capacity, walking asymmetries and influences kids’ motor skills. Moreover, it causes pain, fatigue and musculoskeletal dysfunctions which provide, generally, slow gait speed with disturbed motor control. Statistics prove that 10000 babies are affected by this disease each year worldwide [6]. So, it is needed to find a solution for this population. There is not a specific cure but several treatments are used such as standard and robotic rehabilitation [7]. Among the standard rehabilitation, we can cite, doman and delcato, rood, bobath, vojta-european method, and conductive education therapies [8]. Comparing to the standard rehabilitation, lower limb exoskeletons seem to be more efficient. In fact, they can encourage kids to make longer therapeutic sessions and help physiotherapists to reduce their hard work. Recently, studies focus on the most important part which is the control of the exoskeletons using the adequate controllers [9]. Hence, in the present paper, we propose to control a two degrees of freedom lower limb exoskeleton at the hip and knee joints with a good tracking [6], [7], [10], [11]. The choice of the suitable and the robust controller law is fundamental [12], [13], [14]. In this paper, we propose a robust sliding mode controller [10].

The design of sliding surfaces is one of the factors for improving performances. To achieve robustness, various methods have been suggested [15], [16]. For example, Slotine and Sastry [17] have proposed a time varying sliding surface to remove the reaching phase by imposing a constraint that initial errors be zero in tracking control. Choi et al. [18] have introduced a piecewise continuous moving sliding mode, yet this still includes a reaching phase. Ha et al. [19] have proposed a continuously moving sliding mode obtained by fuzzy tuning. The sliding surface can rotate or shift in the phase space in order to enhance the tracking behavior. The fuzzy tuning considered only the error in the phase plane as the input of the fuzzy system. Medhaffar et al. [20] have replaced fixed sliding surfaces by continuously moving sliding mode surfaces. Moreover, the proposed fuzzy tuning considers the error and the error velocity in the phase plane, as inputs of the fuzzy system, in order to improve control performances.

From the simulation results, we notice that torques are too high, so the idea is to implement a fuzzy sliding mode controller [21]. Then, each position error
is quantified into five fuzzy subsystems, in order to control the parameters of the sliding mode controller. These parameters take high values for small values of the error, and they take small values for high values of the error. The rest of the paper is organized as follows. The second section introduces the dynamic model of the lower limb exoskeleton. The third section describes the proposed control solutions. The fourth section presents the simulation results. Finally, the fifth and last section presents the conclusion.

II. DYNAMIC MODEL OF SYSTEM

Exoskeletons are considered as complex systems. Hence, a dynamic model is established using Lagrange formulation. The proposed model includes the exoskeleton and the kind legs. [22]

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \]  \hspace{1cm} (1)

with:

\[ M_{11} = a_1 + a_2 \cos q_2 \]
\[ M_{12} = M_{21} = a_3 + a_4 \cos q_2 \]
\[ M_{22} = a_5 \]
\[ C_{11} = a_6 \sin q_2 \]
\[ C_{12} = a_6 \sin q_2 \]
\[ C_{21} = \frac{1}{2}a_6 \sin q_2 \]
\[ C_{22} = 0 \]
\[ G_1 = a_7 \sin q_1 - a_8 \sin(q_1 + q_2) \]
\[ G_2 = a_9 \sin(q_1 + q_2) \]

where:
\[ q, \dot{q}, \ddot{q} \in \mathbb{R}^2 \] represent the position, velocity and acceleration vectors, respectively,
\[ M(q) \in \mathbb{R}^2 \] is the inertia matrix
\[ C(q, \dot{q}) \in \mathbb{R}^2 \] represents the Coriolis and centrifugal torques,
\[ G(q) \in \mathbb{R}^2 \] denotes the gravity vector,
\[ \tau \in \mathbb{R}^2 \] denotes the vector of applied torques by actuators,
\[ a_i \] are positive model parameters which depend on thigh and shank masses, lengths, position of the center of mass, moments of inertia and the gravity acceleration.

III. FUZZY SLIDING MODE CONTROL

In this section, a sliding mode controller is proposed [10]. Define the sliding function as follows:

\[ s = \dot{e} + \lambda e \]  \hspace{1cm} (2)

where; \( e = q - q_d \) is the tracking error, \( \dot{e} = \dot{q} - \dot{q}_d \) is the velocity tracking error.

It is required to drive the system to the sliding surface. Hence, we choose \( \tau \) such that \( \dot{s} = -K \text{ sign}(s) \)

\[ \dot{s} = M^{-1}(\tau - C\dot{q} - G) - \ddot{q}_d + \lambda \dot{e} = -K \text{ sign}(s) \]  \hspace{1cm} (3)

We obtain:

\[ \tau = C\dot{q} + G(q, \ddot{q}_d - \lambda \dot{e} - K \text{ sign}(s)) \]  \hspace{1cm} (4)

However, due to the large values of the error, especially at the beginning, torques become high. Hence, we suggest to implement a fuzzy sliding mode, which consists to apply small values of parameters \( K \) and \( \lambda \) for high values of the error, and high values of parameters \( K \) and \( \lambda \) for small values of the error. Then, each position error \( e_i (i=1,2) \) is quantified into five fuzzy subsystems as illustrated in Fig. 1 [23].

- \text{NB}: Negative Big
- \text{NS}: Negative Small
- \text{EZ}: Equal Zero
- \text{PS}: Positive Small
- \text{PB}: Positive Big

Two fuzzy systems are designed as supervisors of the sliding mode controllers. Their outputs are the parameters: \( \lambda \) and \( K \). In this study, we have chosen triangular membership functions as shown in Fig. 1, regarding their simple expressions, their sum is equal to the unity, and their derivatives are also simple.

![Fig. 1. Membership functions of the fuzzy supervisor](image)

The fuzzy rules are presented in table I. In this case, the choice of the fuzzy systems yields to write:

\[ \sum_j \mu_{ij} = 1 \]  \hspace{1cm} (5)

<table>
<thead>
<tr>
<th>( e_i )</th>
<th>NB</th>
<th>NS</th>
<th>EZ</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{ij} )</td>
<td>( \mu_{i1} )</td>
<td>( \mu_{i2} )</td>
<td>( \mu_{i3} )</td>
<td>( \mu_{i4} )</td>
<td>( \mu_{i5} )</td>
</tr>
<tr>
<td>( K_i )</td>
<td>( K_{i1} )</td>
<td>( K_{i2} )</td>
<td>( K_{i3} )</td>
<td>( K_{i4} )</td>
<td>( K_{i5} )</td>
</tr>
</tbody>
</table>

The outputs of the fuzzy system \( i \) \((i = 1,2)\) can be expressed as:

\[ \lambda_i = \frac{1}{\sum_k \mu_{ik}} \sum_j \mu_{ij} \lambda_{ij} \]  \hspace{1cm} (6)

\[ K_i = \frac{1}{\sum_k \mu_{ik}} \sum_j \mu_{ij} K_{ij} \]  \hspace{1cm} (7)
The membership functions are expressed as described in table II. Their differentials with respect to $e_i$ are expressed in table III.

Based on the new expression of $\lambda$ which depends on the time, the expression of $\dot{s}$ becomes as follows:

$$\dot{s} = M^{-1}(\tau - C\dot{q} - G) - \dot{q}_d + \lambda \dot{e} + \dot{\lambda} e$$  \hspace{1cm} (8)

with:

$$\dot{\lambda}_i = \sum_j \mu_{ij} \dot{\lambda}_{ij} = \sum_j \frac{d\mu_{ij}}{de_j} \lambda_{ij} \dot{e}_j$$  \hspace{1cm} (9)

The control law is expressed by:

$$\tau = \tau_{eq} + \Delta \tau$$  \hspace{1cm} (10)

where $\tau_{eq}$ is the equivalent control which is the required control which ensures $\dot{s} = 0$. However, the term $\Delta \tau$ is responsible for the robustness of the control law. Then:

$$\tau_{eq} = C\dot{q} + G + M \left[ \dot{q}_d - \lambda \dot{e} + \dot{\lambda} e \right]$$  \hspace{1cm} (11)

and:

$$\Delta \tau = -M \lambda (e) \text{ sign}(s)$$  \hspace{1cm} (12)

Thus, the control law can be expressed as:

$$\tau = C\dot{q} + G + M \left[ \dot{q}_d - \lambda \dot{e} + \dot{\lambda} e - K(e) \text{ sign}(s) \right]$$  \hspace{1cm} (13)

Consequently, based on the new expression of $\lambda$ which depends on the time, the derivative of the sliding function $s$ becomes as follows:

$$\dot{s} = -K(e) \text{ sign}(s)$$  \hspace{1cm} (14)

In figures 2 and 3, evolutions of $\lambda_i(e_i)$ and $K_i(e_i)$ are presented. Observing these figures, it is clear that $\lambda_i(e_i)$ and $K_i(e_i)$ are positive functions.

The Lyapunov function associated to the system is:

$$V_1 = \frac{1}{2} s^T s$$  \hspace{1cm} (15)

Its differential with respect to time gives:

$$\dot{V}_1 = -s^T K(e) \text{ sign}(s) = -\sum_i K_i(e_i) |s_i| < 0$$  \hspace{1cm} (16)

where $K_i(e_i)$ is a positive function (see figures 2 and 3).

Equation (16) guarantees the fact that the state system converges to the sliding surface $s = 0$ and remains on it. Now, we should show that if the state system is on the sliding surface, the error converges to zero. For this reason, let’s consider a second Lyapunov function as:

$$V_2 = \frac{1}{2} e^T e$$  \hspace{1cm} (17)

Its differential with respect to time is expressed as:

$$\dot{V}_2 = -e^T \lambda(e) e = -\sum_i \lambda_i(e_i) e_i^2 < 0$$  \hspace{1cm} (18)

since $\lambda_i(e_i)$ is a positive function (see figures 2 and 3).

In figures 2 and 3, evolutions of sliding surfaces in planes $(e_i, \dot{e}_i)$ are presented. In this case, sliding surfaces are not straight lines but curves.

IV. Simulation Results

In this section, a comparison between sliding mode and fuzzy sliding mode controllers is presented.

### Table II

<table>
<thead>
<tr>
<th>$e_i \in$</th>
<th>$(-\infty, -e_{IM})$</th>
<th>$[-e_{IM}, -e_{IM}]$</th>
<th>$[-e_{IM}, 0]$</th>
<th>$[0, e_{IM}]$</th>
<th>$[e_{IM}, e_{IM}]$</th>
<th>$[e_{IM}, +\infty]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{i1}$</td>
<td>1 \hspace{1cm} $e_{IM} - e_{IM}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_{i2}$</td>
<td>0 \hspace{1cm} $e_{IM} - e_{IM}$</td>
<td>$-e_{IM}$ \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_{i3}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} $e_{IM} - e_{IM}$</td>
<td>$-e_{IM}$ \hspace{1cm} $-e_{IM}$ \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0</td>
<td></td>
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</tr>
<tr>
<td>$\mu_{i4}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} $e_{IM}$</td>
<td>$e_{IM}$ \hspace{1cm} $e_{IM} - e_{IM}$ \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0</td>
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<tr>
<td>$\mu_{i5}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} $e_{IM}$</td>
<td>$e_{IM}$ \hspace{1cm} $e_{IM} - e_{IM}$ \hspace{1cm} 1 \hspace{1cm} 0 \hspace{1cm} 0</td>
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</tbody>
</table>

### Table III

<table>
<thead>
<tr>
<th>$e_i \in$</th>
<th>$(-\infty, -e_{IM})$</th>
<th>$[-e_{IM}, -e_{IM}]$</th>
<th>$[-e_{IM}, 0]$</th>
<th>$[0, e_{IM}]$</th>
<th>$[e_{IM}, e_{IM}]$</th>
<th>$[e_{IM}, +\infty]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d\mu_{i1}}{de_i}$</td>
<td>0 \hspace{1cm} $-\frac{1}{e_{IM} - e_{IM}}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0</td>
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<tr>
<td>$\frac{d\mu_{i2}}{de_i}$</td>
<td>0 \hspace{1cm} $-\frac{1}{e_{IM} - e_{IM}}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0</td>
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<tr>
<td>$\frac{d\mu_{i3}}{de_i}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} $-\frac{1}{e_{IM} - e_{IM}}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0</td>
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<tr>
<td>$\frac{d\mu_{i4}}{de_i}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} $-\frac{1}{e_{IM} - e_{IM}}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0</td>
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</tr>
<tr>
<td>$\frac{d\mu_{i5}}{de_i}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} $-\frac{1}{e_{IM} - e_{IM}}$</td>
<td>0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0</td>
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</table>
Fig. 2. The evolutions of: (a) membership functions \( \mu_{1j} \), (b) \( \lambda_1(e_1) \), (c) \( K_1(e_1) \), and (d) the sliding surface \( s_1(e_1, \dot{e}_1) = 0 \) in the plane \((e_1, \dot{e}_1)\).  

Fig. 3. The evolutions of: (a) membership functions \( \mu_{2j} \), (b) \( \lambda_2(e_2) \), (c) \( K_2(e_2) \), and (d) the sliding surface \( s_2(e_2, \dot{e}_2) = 0 \) in the plane \((e_2, \dot{e}_2)\).
Figures 4 and 5 represent the evolution of positions, speeds, applied torques and the evolution of the position errors of hip and knee joints, respectively, using sliding mode controllers. From these figures, it is shown that at the beginning the errors are high which cause the raised values of the torques. For this reason, a fuzzy sliding mode controller is proposed. Figures 6 and 7 show the evolution of positions, speeds, applied torques and position errors using the proposed controller. In order to test the robustness of the proposed controller, we have applied +50\% variations on masses and +25\% variations on lengths, representing 13 aged years aged kids.

Figures 8 and 9 represent results given by sliding mode controllers, and figures 10 and 11 represent results given by fuzzy sliding mode controllers. It is obvious that fuzzy sliding mode controllers give better results illustrated by smaller applied torques.
This paper concerns the control of lower limb exoskeletons used for the rehabilitation of children suffering from cerebral palsy. In this context, two controllers have been proposed. First, a sliding mode controller has been implemented. It has been found that the system follows the desired trajectories. However, the applied torques are two high and exceed the actuator limitations. For this reason, we have proposed to design fuzzy systems as supervisors, in order to limit the applied torques when the tracking errors are high. Especially at the beginning. The stability of the closed loop system has been ensured using the Lyapunov theory. Simulation results show that the proposed controller is effective. Moreover, it has been shown that the fuzzy sliding mode controllers are robust against parametric variations such as masses and lengths of kid’s legs.

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