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Abstract: This paper deals with the development of a new time-varying adaptive feedforward sliding mode control (SMC). The proposed controller is designed based on the original SMC by adding a feedforward term, which can be adaptive, in order to estimate the dynamic parameters. The main contribution of this study is a time-varying adaptation gain, which is proposed to efficiently identify these parameters. The objective is then to outperform the original SMC scheme. The proposed controller has been designed and applied to parallel kinematic manipulators (PKMs). It is validated through numerical simulation for different operating conditions on the VELOCE PKM. The obtained simulation results show the efficiency of the proposed controller in terms of tracking performances and robustness, with improvements up to 55% with respect to another adaptive controller (Adaptive Feedforward PD: AFFPD).

Keywords: Parallel Kinematic Manipulators, Sliding Mode Control, Adaptive Control, Time-Varying Gains, Feedforward, Dynamic Model.

1. INTRODUCTION

Since the beginning of technological and industrial development, several researchers have been interested in robotic systems, and especially their control design. Accordingly, control theory has experienced a real harvest of progress, dealing with a wide variety of industrial systems and robots.

Among industrial robots, parallel kinematic manipulators are used for various industrial applications. VELOCE PKM is a fully actuated example of these manipulators. Their control aroused the interest of many researchers within robotic community (Taghirad, 2013), (Staicu, 2019), (Kelly et al., 2006). The early control techniques applied to these manipulators were empirical, based on measurements in real time and without the need of mathematical models. These controllers are commonly called Non-model-based and may give acceptable results in nominal operating conditions. However, in the presence of external disturbances or mainly uncertainties, these controllers may lose their performances and can not guarantee the closed-loop system stability. [Such as PD and PID control (Cheng et al., 2003), (Kelly, 1997)]. In order to improve their control performances, nonlinear PD and PID controllers have been proposed (Shang et al., 2012), (Kelly and Carelli, 1996). Besides, some robust methods were introduced without the consideration of the dynamic model in the control design. They can ensure the stability against uncertainties and external disturbances, such as RISE feedback control (Hassan et al., 2020), (Saied et al., 2019a). Furthermore, some researchers developed adaptive control schemes, able to adapt to the system variations without using its dynamic model, such as Model Reference Adaptive Control (MRAC), $L_1$ Adaptive Control, and the neural network control (Bennehar et al., 2015a), (Zhao et al., 2006), (Escorcia-Hernandez et al., 2019). These controllers require a good tuning of theirs parameters, which may be complex and time consuming.

With the development of new identification and control tools, dynamic models were introduced in control design, aiming at improving the control performances. Some researchers proposed to use the dynamic model in the design of the control scheme, such as model predictive control (MPC) (Kouki et al., 2020), (Santos et al., 2021), and the sliding mode control (Jafarinasab et al., 2011), (Kim et al., 1998). However, in presence of constraints or in the case of nonlinear systems, predictive control needs an important computing time especially for complex prediction model to give the best performances possible. The sliding mode control is able to guarantee the robustness against uncertainties and external disturbances, but its main drawback is the chattering.

The dynamic model can also be used as an extended term of non-model-based control schemes, by introducing a feedforward term, which can be nominal or adaptive (Saied et al., 2019b). (Bennehar et al., 2014), (Bennehar et al., 2016), (Natal et al., 2012), (Bennehar et al., 2018), (Lamaury et al., 2013), (Bennehar et al., 2015b). Adding this term to robust control laws can lead to significantly improved control performances.
As the parameters estimation and the feedforward term are closely related to the adaptation gain, it influences directly the control performances. In the literature this gain is taken constant, which may effect the stability for high gain values and speed of convergence for the low ones. In this paper, a time-varying adaptation gain is proposed, to avoid the constant adaptation gain limitations and to enhance the parameters estimation as it inherits the nonlinear gains advantages. This solution will be integrated in the adaptation law of an adaptive feedforward sliding mode controller as an extension of the original SMC (Jafariyasab et al., 2011). The proposed control scheme has been validated in numerical simulation on a 4-DoF parallel kinematic manipulator (VELOCE).

The paper is organized as follows. Section II introduces a structure description and modeling of VELOCE PKM. The proposed control solution is detailed in section III. The obtained simulation results are presented and discussed in section IV. Section V concludes this paper.

2. DESCRIPTION AND MODELING OF VELOCE PKM

In this section, the mechanical structure of VELOCE PKM is described, followed by a presentation of its dynamic model.

2.1 Description

VELOCE (cf. Fig. 1) is a Delta-like parallel robot with an additional kinematic chain and one additional rotational degree-of-freedom. It is a non-redundant parallel manipulator, designed for pick-and-place applications. It consists of four identical kinematic chains, composed of an actuator, a rear-arm and a forearm. Each forearm includes two parallel rods connected from one extremity to the rear-arm and from the other extremity to the traveling plate through spherical passive joints (Bennehar et al., 2017).

2.2 Dynamic Model

The dynamic model is obtained by analyzing VELOCE dynamics in both the joint space and the traveling plate space separately, then summing up the two obtained equations together. Nevertheless, to simplify the dynamic model some assumptions are considered:

- **Assumption 1**: The forearm inertia is neglected in view of its small mass. Additionally, its mass is split-up into two parts: one included in the rear arm and the other one in the traveling plate.
- **Assumption 2**: The dry and viscous frictions in the passive and active joints are neglected, due to the joints design which minimizes the friction effects.

The resulting force acting on the traveling plate is composed of (i) the gravity force, (ii) the inertial forces and (iii) the force of the payload as follows:

\[
F_p = M_p \ddot{X} - (M_p + M_{load}) G \tag{1}
\]

where \(M_p\) is the mass matrix of the traveling plate, \(M_{load}\) is the payload mass matrix, \(\dot{X}\) is the Cartesian space acceleration vector and \(G = [0 \ 0 \ q \ 0]^T\) represents the gravity vector with \(q\) is the gravity acceleration. This resulting force is converted into torque contributions using the Jacobian matrix \(J (\dot{X} = J \dot{q})\) (Bennehar et al., 2017) as follows:

\[
\Gamma_p = J^T (M_p \ddot{X} - (M_p + M_{load}) G) \tag{2}
\]

On the joints side, the gravity and the inertia of arms are expressed as joint torques. Additionally, the torque vector resulting from the gravity effect on the arms is given by:

\[
\Gamma_{\alpha_a} = J^T (m_{ra} r_{g} + m_{fa} l) g \cos(q) \tag{3}
\]

where \(m_{ra}\) and \(m_{fa}\) are the rear-arm and forearm masses respectively, \(l\) is the rear-arm length and \(r_{g}\) is the distance between the rotation axis and the rear-arm center of mass. Then, the total joint torques vector is obtained by summing up the contributions of all torques as follows:

\[
I_{\alpha} \ddot{q} + \Gamma_p + \Gamma_{\alpha_a} = \Gamma \tag{4}
\]

where \(I_{\alpha}\) is the inertia matrix of arms and \(\ddot{q}\) is the joint acceleration vector. Finally, using the Jacobian matrix \(J\) the inverse dynamic model can be written as follows:

\[
M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = \Gamma \tag{5}
\]

where \(M(q) \in \mathbb{R}^{4 \times 4}\) is the inertia matrix of the robot, \(C(q, \dot{q}) \in \mathbb{R}^{4 \times 4}\) is the Coriolis and centrifugal force matrix, and \(G(q) \in \mathbb{R}^4\) is the gravitational forces vector. This inverse dynamic model of the robot is linear with respect to the system dynamic parameters. This property is very important for model-based adaptive controllers and especially when an adaptive feedforward term is used. Accordingly, The VELOCE PKM inverse dynamic model can be rewritten as follows:

\[
Y(q, \dot{q}, \ddot{q}) \Theta(t) = \Gamma \tag{6}
\]

where \(Y(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{4 \times p}\) is the regressor, \(\Theta(t) \in \mathbb{R}^p\) is the dynamic parameters vector and \(p\) is the number
of dynamic parameters (Saied et al., 2019b). The main dynamic parameters of VELOCE PKM are summarized in Table 1.

Table 1. The main dynamic parameters of VELOCE PKM.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traveling platform mass (kg)</td>
<td>0.999</td>
</tr>
<tr>
<td>Actuator inertia (kg.m^2)</td>
<td>0.0011</td>
</tr>
<tr>
<td>Rear-arm mass (kg)</td>
<td>0.541</td>
</tr>
<tr>
<td>Rear-arm length (m)</td>
<td>0.2</td>
</tr>
<tr>
<td>Forearm mass (kg)</td>
<td>0.08</td>
</tr>
<tr>
<td>Forearm length (m)</td>
<td>0.53</td>
</tr>
</tbody>
</table>

3. PROPOSED CONTROL SOLUTIONS

In this section, the proposed control solution for the VELOCE PKM is detailed.

3.1 Motivation

The sliding mode control (SMC) is a nonlinear controller which targets the asymptotic convergence of the state variables in the presence of unknown disturbances and uncertainties. On the phase portrait, this controller drives the state variables to a sliding surface. Furthermore, it keeps them on the sliding surface even in the presence of bounded disturbances (Jafarinassab et al., 2011). The sliding surface can be defined as a combined state error, such as:

\[ s = \dot{e} + \lambda e \]  
\[ e = q_d - q \]

where \( q_d \) is the desired joint positions and \( \lambda \) is a positive constant. Then, the control law of the sliding mode controller is given by:

\[ \Gamma = M(q)(\ddot{q}_d + \lambda \dot{e}) + \dot{C}(q, \dot{q})\dot{q} + G(q) \]  
\[ \dot{\Theta} = \dot{\Theta} + \Gamma_{SMC} + \dot{\Gamma}_{FF} \]  
\[ \dot{\Theta} = \dot{\Theta} + \dot{\Theta} + \ddot{q}_d \Theta(t) \]

where \( \Theta(t) \) is an estimate of the unknown parameters vector \( \Theta(t) \). Note that the desired trajectories are used instead of the actual ones. Consequently, the feedforward term can be computed and stored off-line which can reduce the computation burden if needed. Additionally, the controller can better reproduce the nonlinear dynamics of the system, allowing to compensate its effects thanks to the dynamic parameters estimation. The vector \( \dot{\Theta}(t) \) is adjusted to identify the unknown parameters while keeping the stability in closed loop. Consequently, the adaptation law is given by:

\[ \dot{\Theta} = \dot{\Theta} + \ddot{q}_d \Theta(t) \]
where \( \Xi \in \mathbb{R}^p \) is a positive definite diagonal matrix known as the adaptation gain matrix and \( p \) is the number of dynamic parameters. In our case, \( \Xi \) is a nonlinear gain matrix. The idea is to use low adaptation gain values when the estimated parameters are far from the nominal parameters, in order to avoid the overshoots in their estimation. Otherwise, high adaptation gain values can be used to ensure a rapid convergence. The control scheme block diagram is shown in Fig. 3.

As the parameter estimation errors are unknown, the adaptation gain matrix varies according to the position and velocity joint errors. This choice is motivated by the direct relationship between the parameter estimation errors and the tracking errors. Thus, a nonlinear time-varying adaptation gain matrix according to the sliding surface vector norm \( n_s = ||s|| = \sqrt{s^T s} \) is proposed. The non-linear gain is similar to the ones proposed in (Shang et al., 2009) which satisfy our requirements. This gain is given as follows:

\[
\Xi(n_s) = \begin{cases} 
\Xi_0 \delta^{-1} & \text{if } n_s < \delta \\
\Xi_0 n_s^{-1} & \text{if } n_s \geq \delta 
\end{cases}
\]

(14)

where \( \Xi_0, \epsilon \) and \( \delta \) are positive control design parameters to be tuned to get the desired control performances.

4. SIMULATION RESULTS

VELOCE PKM is equipped with four motors, with a maximum torque of 127 N.m and a maximum speed of 550 rpm. A numerical simulator of the robot is developed in the Matlab/Simulink environment. The controllers are implemented in discrete-time with fixed-step solver whose sampling time is equal to 0.1 ms. The desired trajectory illustrated in Fig. 4 was designed to be singularity-free within the work-space of the robot. This trajectory is divided into two parts: the first one is a sequence of point-to-point motions with a time period of \( T = 0.5s \) for each motion (A-B, B-C-D-E-B "two times", B-C-D-E-F, F-G-H-F "two times"). The second part includes two rotations of the traveling plate with an angle \( \alpha \), the period of each rotation is \( T_\alpha = 0.5s \). In this study, a robustness scenario with the consideration of masses and inertia uncertainties as well as a payload change is proposed.

The platform and payload masses as well as the arm mass and inertia are considered unknown, and need to be estimated online. From equation (12) the dynamic parameters vector is defined as \( \hat{\Theta} = [M_{tp} \ M_{utp} \ I_a \ m_a \ m_{load}] \),

where \( M_{tp} \) is the total mass of the traveling plate, \( M_{utp} \) is the upper traveling plate mass, \( I_a \) and \( m_a \) are the arm moment of inertia and mass respectively, and \( m_{load} \) is the payload mass.

The proposed evaluation criteria are based on the calculation of the root mean square error of translation \( RMS_t \), of rotation \( RMS_r \) and of joint angular positions \( RMS_q \) as follows:

\[
RMS_t = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (e_x^2(i) + e_y^2(i) + e_z^2(i))}
\]

(15)

\[
RMS_r = \sqrt{\frac{1}{N} \sum_{i=1}^{N} e_x^2(i)}
\]

(16)

\[
RMS_q = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \sum_{j=1}^{4} e_{\theta_j}(i) \right) ^2}
\]

(17)

where \( e_x, e_y, e_z \) and \( e_\theta \) are the Cartesian tracking errors, \( e_q \) are the joint tracking errors and \( N \) is the number of samples.

VELOCE PKM is a Delta-like parallel robot designed to perform pick-and-place tasks. For such applications, the payload mass is often subject to variations. The robustness test aims at evaluating the performances and stability of the proposed controller against these uncertainties. Indeed, uncertainties of 50\% \( [\hat{\Theta}(t)]_\text{New} = \hat{\Theta}(t) + \Delta \hat{\Theta}(t), \Delta \hat{\Theta}(t) = 0.5\Theta(t) \) as well as a payload of 500g are considered. The proposed controller is compared along the reference trajectory with another adaptive controller (Adaptive Feedforward PD: AFFPD) proposed in (Saied et al., 2019b). The tuning of the control design parameters for the both controllers is based on trial and error technique. The obtained results for this tuning are summarized in Table 2.

The obtained Cartesian tracking errors for the robustness scenario are shown in Fig. 5. To better view, the plots are zoomed within the interval \([0,2]\) s. As illustrated in this figure, the adaptive controllers keep the same perfor-
Fig. 5. Evolution of the Cartesian tracking errors.

Fig. 6. Evolution of the control input torques.

Fig. 7. Evolution of the parameters estimation.

In this paper, a new time-varying adaptive feedforward sliding mode controller is proposed and validated in numerical simulation with the AFFPD controller on VELOCE PKM. The obtained results show the superiority of the proposed controller in terms of tracking performances, robustness and parameters estimation. As future works, the proposed contribution will be validated in real time experimentation. The adaptation law will be enhanced, with a more complete dynamic model of VELOCE PKM including, as example the friction dynamics.

### Table 2. Summary of the control design parameters

<table>
<thead>
<tr>
<th>AFFPD</th>
<th>TVAFFSMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_p = 8000 )</td>
<td>( K = \text{diag}(100, 100) )</td>
</tr>
<tr>
<td>( K_d = 100 )</td>
<td>( K_1 = 90 )</td>
</tr>
<tr>
<td>( K = \text{diag}(100, 100, 2.5, 2.2) )</td>
<td>( K_2 = 5.25 )</td>
</tr>
<tr>
<td>( E_0 = \text{diag}(5000, 5000, 80, 80, 80) )</td>
<td>( \epsilon = 0.8 )</td>
</tr>
<tr>
<td>( \delta = 0.0002 )</td>
<td>( \delta = \frac{1}{0.0002} )</td>
</tr>
</tbody>
</table>

### Table 3. Control performances evaluation for both controllers.

<table>
<thead>
<tr>
<th>Controller</th>
<th>( \text{RMS}_{x}(\text{cm}) )</th>
<th>( \text{RMS}_{y}(\text{deg}) )</th>
<th>( \text{RMS}_{r}(\text{deg}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFFPD</td>
<td>( 2.0252 \times 10^{-4} )</td>
<td>( 2.2911 \times 10^{-4} )</td>
<td>( 4.3995 \times 10^{-4} )</td>
</tr>
<tr>
<td>TVAFFSMC</td>
<td>( 8.9912 \times 10^{-5} )</td>
<td>( 9.0991 \times 10^{-5} )</td>
<td>( 1.9528 \times 10^{-4} )</td>
</tr>
<tr>
<td>Imp./AFFPD</td>
<td>55.61%</td>
<td>60.32%</td>
<td>55.36%</td>
</tr>
</tbody>
</table>

### 5. CONCLUSION AND FUTURE WORK

The proposed controller shows better results with improvements up to 60% with respect to AFFPD, thanks to the time-varying adaptation gain which varies according to the sliding surface vector norm \( n_u \). It allows fast convergence while guaranteeing stability (i.e. smaller overshoots). The tracking control performances evaluation as well as the percentages of improvements are summarized in Table 3.
Fig. 8. Evolution of the sliding surface vector norm and the Time-Varying Adaptation Gain.

REFERENCES


