VO2-based Oscillatory Ising Machine: The Role of External Temperature on Performance

Corentin Delacour, Stefania Carapezzi, Gabriele Boschetto, Madeleine Abernot, Thierry Gil, Aida Todri-Sanial

VO_2-based Oscillatory Ising Machine: The Role of External Temperature on Performance

Corentin Delacour, Stefania Carapezzi, Gabriele Boschetto, Madeleine Abernot, Thierry Gil, Aida Todri-Sanial

***Abstract*—The rich dynamics of analog oscillators can be controlled to solve Non-deterministic Polynomial-time hard problems by mapping the Ising model to a network of coupled oscillators. Low-power analog oscillators are good candidates to efficiently implement Ising spins thanks to their parallel computing capability. Such as, oscillators based on vanadium dioxide material (VO_2) are often considered as they allow a compact and low-power hardware implementation operating at room temperature. However, as both electrical and thermal variables characterize VO_2 devices, it is essential to understand the impact of temperature on OIM performances before implementing such hardware at large scale. In this work, we study an OIM with oscillating spins based on crossbar VO_2 devices and coupled by capacitors. Via TCAD and circuit simulations, we investigate how the external temperature impacts the performances of the VO_2-OIM when solving NP-hard MAX-CUT problems. We show that a 10-spins OIM can undergo an 75% accuracy variation due to a 5% external temperature variation only. Our electro-thermal study suggests that VO_2 thermal properties should be considered when designing VO_2-OIMs and eventually could be harnessed for thermal annealing.**

***Index Terms*—Oscillatory Neural Network, Oscillatory Ising Machine, NP-hard problems, Coupled Oscillators**

I. Introduction

Recently, new optimisation machines based on non-silicon physical systems such as quantum annealing machines have been developed. These machines have gained a lot of interest as they solve combinatorial NP-hard problems by searching the state of the Ising spins, which minimizes the Ising Hamiltonian [1]. There are various NP-hard combinatorial optimization problems, which can be formulated and mapped into recurrent neural networks such as Hopfield neural networks [2], or to the Ising model thanks to Lucas' seminal work [3].

In this work, we report on an Oscillatory Ising Machine (OIM) based on oscillators with programmable interactions where the phase differences among oscillators represent the Ising spins [4], [5]. In the recent years, a variety of Ising machine implementations have been implemented such as D-Wave [6], a quantum annealing machine, Coherent Ising Machine [7] based on laser network, purely digital CMOS Ising machine by [8], [9], and memristive Ising machines [10].

In previous researches, VO_2-based Ising machine [11], [12] has been investigated to show the feasibility, computation speed and accuracy on solving MAX-CUT problems. Unlike previous works, we study the impact of the external temperature which is raised by frequent switching of VO_2 devices, on the dynamics and solution accuracy of the OIM. Given that the switching mechanism of VO_2 devices is based on Joule heating and heat dissipation on the surrounding is prevalent, this triggers to inquire if external temperature variations have a role on the Ising machine computation accuracy and performance.

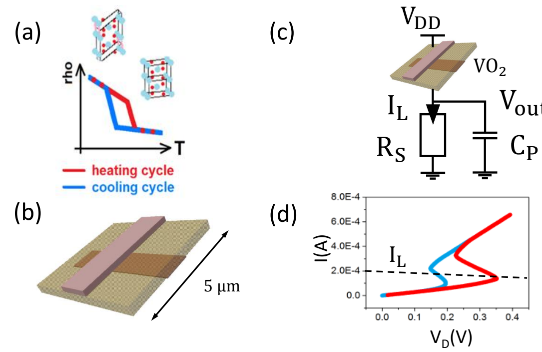
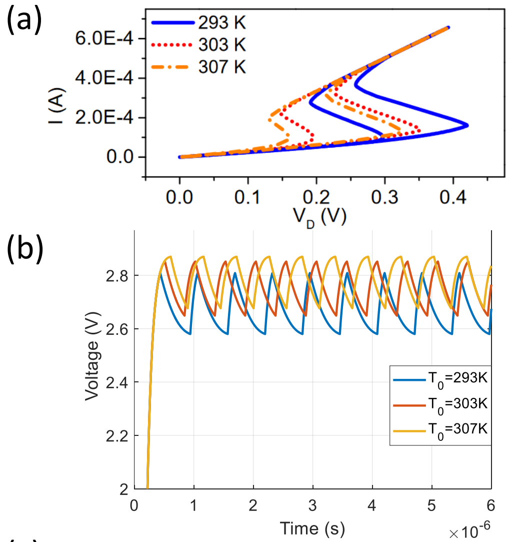


Fig. 1. (a) Scheme of VO_2 resistivity ρ vs temperature T. VO_2 switches between high/low resistivity states depending on being an insulator/metal, respectively. The threshold temperatures for insulator-to-metal and metal-to-insulator transitions are different, giving rise to hysteresis in ρ vs T. A dedicated TCAD model has been implemented [17], [18] to simulate VO_2 resistive switch. (b) Geometry of VO_2 CB device. The VO_2 layer is a square of 5-μm side, and thickness of 80 nm. The contact width is 250 nm. Further details can be found in [18]. (c) VO_2 oscillator circuit. (d) Simulated current vs voltage across VO_2 device for T_0 = 303 K.

II. VO_2-oscillator

VO_2 is a Transition Metal Oxide. The external temperature drives a change in its physical behavior, from being insulator to metal [13] above a threshold temperature of about 340 K (Fig.1a). This volatile resistive switching is also activated by self-heating [14] when bias is applied or current is injected across VO_2. This allows to build a compact and scalable oscillator when the VO_2 device is inserted into a RC circuit [15]. Electrical oscillations will follow the charging/discharging of the external capacitor as a consequence of the device being ON/OFF. We simulate the VO_2 device through the Silvaco TCAD Victory Device tool [16]. The device architecture is so called crossbar (Fig.1b): a squared VO_2 layer is inserted between a top and bottom electrodes. The VO_2 is treated as a conductor. The behavior of VO_2 resistivity with temperature is modeled through a dedicated TCAD approach [17], [18] which has been benchmarked and validated against experimental data.

We bias the VO_2 device with a resistor in series R_S such that the load line I_L lie in the VO_2 negative differential region to obtain oscillation (Fig.1d). We choose the following circuit parameters: V_{DD} = 3 V, R_S = 15 kΩ, and C_P = 150 pF which lead to oscillation frequencies in the MHz range (Fig.2c). We perform electro-thermal simulations at different external temperatures T_0 set as a thermal boundary condition at the



(c)

T_0 (K)	293	303	307
f_{osc} (MHz)	1.59	1.96	1.81
V_{pp} (mV)	220	202	190

Fig. 2. a)VO_2 $I-V$ characteristic for three different external temperatures. b) Oscillating output voltages for V_{DD} = 3 V, R_S = 15 kΩ, and C_P = 150 pF c) Corresponding oscillating frequencies and amplitudes.

bottom surface of the VO_2 layer. Fig. 2a shows the simulated current I vs voltage V characteristics for T_0 of 293, 303 and 307 K. We observe that the $I-V$ curve depends on the external temperature T_0 and therefore impacts the oscillating waveform shape (Fig.2b).

III. VO_2-based Oscillatory Ising Machine

A. Oscillatory Ising Machine to solve MAX-CUT problems

The Ising model was initially derived to describe domain formation in ferromagnetic materials [1]. It consists of a set of N binary variables called *spins* $s_i = \pm 1$ interacting via real coupling coefficients J_{ij}. Without external fields applied to the spins, the Ising Hamiltonian H is expressed as [3]:

$$H = -\sum_{i,j} J_{ij} s_i s_j \tag{1}$$

An Ising machine is a physical system that can minimize H. Such hardware realization is of interest as various NP-hard combinatorial optimization problems can be mapped to H [3]. Solving those problems consist in reaching the ground state of H given by an optimal spin configuration.

The MAX-CUT problem is one example of NP-hard problem that can be mapped to the Ising Hamiltonian H and has direct applications in communication networks or VLSI routing [9]. Given an undirected graph, the objective is to find two subsets of vertices W and X, such that the weighted sum of edges between W and X is maximum. When vertices

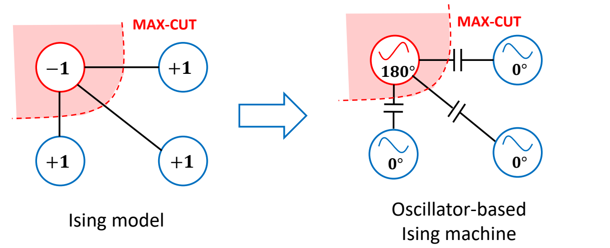


Fig. 3. The Ising model can be mapped to network of coupled oscillators. Spins are binary phases $\Phi_i \in \{0°, 180°\}$. To solve MAX-CUT problem, coupling capacitors implement graph edges.

are mapped to Ising spins s_i and edges to coefficients J_{ij}, H becomes [11]:

$$H = -\sum_{i,j} J_{ij} + 2 \sum_{i \in W, j \in X} J_{ij} \tag{2}$$

For negative coefficients J_{ij}, minimizing H consists in maximizing the weighted sum of edges between W and X, i.e. finding the maximum cut of the graph.

Recently, it has been shown that networks of coupled oscillators can implement Ising machine and solve MAX-CUT problems by encoding spins $s_i = \pm 1$ into binary phases $\Phi_i \in \{0°, 180°\}$ [9], [11], [4]. Coupling elements such as capacitors induce Φ_{ij} = 180° and encode negative interaction coefficients J_{ij} [11]. As VO_2 devices are sensitive to the external temperature, we expect the thermal environment to impact MAX-CUT solutions when using network of VO_2 oscillators.

B. Simulation set-up of the VO_2-oscillator-based Ising machine to solve MAX-CUT problems

We apply our hybrid TCAD-Matlab simulation flow [18] to assess the impact of the external temperature on a VO_2-OIM. We solve numerically the system dynamics on Matlab with Euler's method. At every time step, our circuit solver updates the oscillator state using the VO_2-IV stored in a look-up table.

The binary phase of a single oscillator (with respect to a reference) emulates a spin and coupling capacitors implement graph edges to solve MAX-CUT problems (Fig.3a). We set coupling capacitors as $C_C \sim C_P/N$, and employ Sub Harmonic Injection Locking (SHIL) at twice the oscillator's natural frequency as it ensures binary phase locking [11], [4]. In our work, we investigate SHIL via V_{DD} but injection onto the output port is also possible using additional capacitors [11]. Similarly to the work carried out in [11], we linearly ramp up the SHIL amplitude to let the OIM compute before binarizing phases. After 25 oscillation cycles, the SHIL amplitude reaches $2\% V_{DD}$ chosen as a trade off between escaping local minima and not overdriving the oscillators. Finally, we measure oscillators' phases to extract the two subsets W and X defining the cut size.

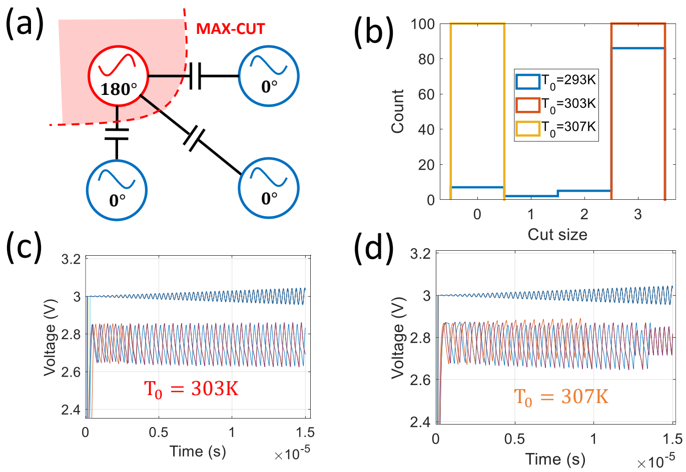


Fig. 4. a) Graph with 4 vertices and 3 edges mapped to an oscillator-based Ising machine. b) Histogram of MAX-CUT solutions for $T_0 \in \{293, 303, 307\}$K. c) and d) Oscillators' output waveforms and supply voltage for cases at T_0=303K and 307K, respectively. At T_0=307K, the SHIL amplitude becomes too large with respect to the oscillation amplitude and finally overdrive the oscillators that all converge in phase.

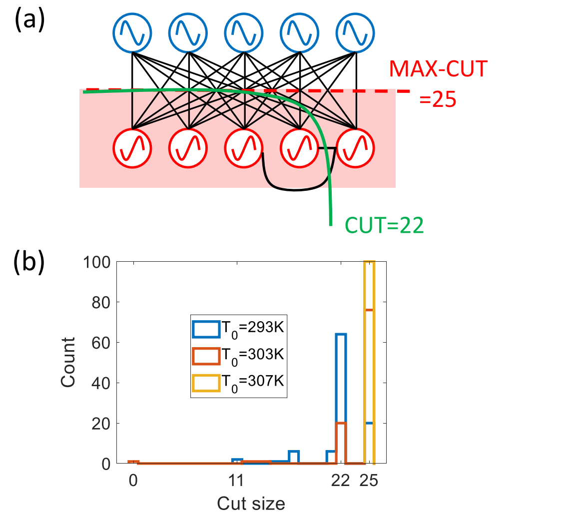


Fig. 5. a) Graph with 10 vertices and 27 edges mapped to an oscillator-based Ising machine. b) Histogram of MAX-CUT solutions for $T_0 \in \{293, 303, 307\}$K.

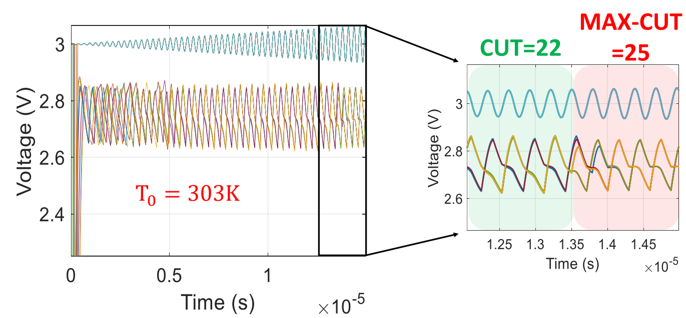


Fig. 6. Oscillating waveforms showing an example at T_0=303K. Oscillators find the optimal solution only when SHIL amplitude is sufficiently large to escape the local energy minimum corresponding to the cut of size 22.

IV. Effect of External Temperature on a VO_2-Based Oscillatory Ising Machine

To study the effect of external temperature on MAX-CUT solutions, we keep the same simulation set-up and we run 100 simulations for each temperature point T_0 with random initial phases between 0° and 180°. Our nominal configuration corresponds to an external temperature T_0 = 303K, thus we set the SHIL frequency at twice the oscillator's natural frequency measured at T_0 = 303K ($f_{SHIL} = 2f_{osc}$ = 3.9 MHz).

A. Effect of external temperature on solving 4-nodes MAX-CUT problems

Fig.4 presents a simple 4-nodes MAX-CUT problem solved by 4 coupled VO_2 oscillators. When T_0=293K and 303K, oscillators find the MAX-CUT=3 with 80% and 100 % accuracy, respectively. However, it reaches 0 % at T_0 = 307K (Fig.4b). We believe the operation fails at T_0 = 307K as the oscillation amplitude is smaller at this temperature point (Fig.2d). SHIL becomes too "strong" and overdrives oscillators that finally all converge in phase (Fig.4d). For robustness, the SHIL amplitude should not be too large with respect to the oscillation amplitude.

B. Effect of external temperature on solving a 10-nodes MAX-CUT problem

We scale up the OIM to solve a 10-nodes MAX-CUT problem as shown in Fig.5a. The graph is built in such a way that there is a non-optimal cut of size 22 corresponding to a spurious energy minimum (cut highlighted in green plain line); whereas the MAX-CUT solution is 25. In this example, we observe that increasing the external temperature leads to higher accuracy: 20%, 75% and 100% for T_0=293K, 303K and 307K respectively (Fig.5b).

As in the 4-oscillators case, one might think this is due to the relative strength of SHIL with respect to the oscillation amplitude that decreases with T_0 (Fig.2c). Fig.6 shows a case at T_0=303K where oscillators find the optimal solution only when SHIL is sufficiently strong. Until then, oscillators' phases are stuck in a local minimum corresponding to the non-optimal cut of size 22. We repeated simulations using various SHIL amplitudes to further assess the impact of SHIL strength on MAX-CUT solutions (Fig.7a). We notice that a SHIL amplitude around 30% of the oscillation peak-to-peak amplitude (V_{SHIL}/V_{pp} = 30%) is sufficiently large to find the MAX-CUT solution (with at least 25% of accuracy) for $T_0 \in \{293, 303, 307\}$K. However, when the SHIL amplitude is too large and reaches a threshold, overdriven oscillators converge in phase and fail to find any solution (cut size=0). Surprisingly, this upper threshold depends on the external temperature as oscillators at lower temperature can tolerate larger relative SHIL amplitudes (Fig.7a). As we can see on Fig.7b, this discrepancy is not related to oscillators' frequency variations as accuracies do not vary significantly with SHIL

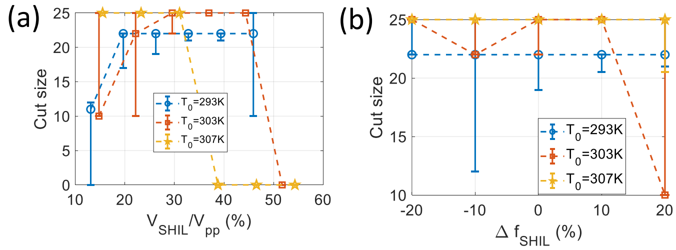


Fig. 7. a) Impact of SHIL amplitude on MAX-CUT solutions for different external temperatures (data points are quartiles Q1, Q2 and Q3). A good trade-off appears for SHIL amplitudes around 30% of peak-to-peak oscillating amplitude. b) Impact of SHIL frequency variations on MAX-CUT solutions for V_{SHIL}/V_{pp} = 30%. The Ising machine is robust to frequency variations as solutions do not vary significantly.

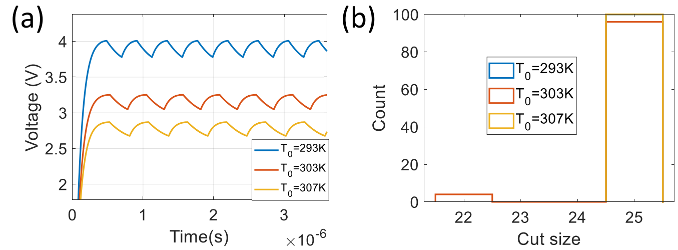


Fig. 8. a) Oscillating waveform with V_{DD}=4.2V, 3.4V and 3V for T_0=293K, 303K and 307K, respectively. The rising time is approximately equal to the falling time for all temperature points. b) Histogram of MAX-CUT solutions obtained with new V_{DD} operating points and for V_{SHIL}/V_{pp} around 25%. The performances of the oscillator-based Ising machine increase when the oscillating waveform is more symmetric.

frequencies.

Instead, we believe that the shape of the oscillating waveform that changes with temperature T_0 (Fig. 2a) has a major impact on MAX-CUT solutions. The waveform shape becomes more symmetric (rising time≈falling time) when increasing the external temperature (Fig. 2a), which could explain the higher accuracy obtained at higher temperature (Fig.5b) and corroborates results from [12]. To obtain a rising time equal to the falling time, we increased V_{DD} for lower external temperatures as V_{DD}=4.2V, 3.4V and 3V for T_0=293K, 303K and 307K, respectively (Fig.8a). By shaping the oscillation waveform, the accuracy of the oscillator-based Ising machine significantly increases and exceeds 96 % for any external temperature (Fig.8b); whereas it was initially 25% at T_0=293K (Fig.5b).

Conclusion

In this work, we simulated VO_2-based Oscillator Ising Machines using an hybrid TCAD-circuit simulation framework that encapsulates both electrical and thermal VO_2 properties. Our study highlights the strong dependency between temperature and VO_2-OIM performances. Specifically, we simulated a 10-spins VO_2-OIM to assess its accuracy sensitivity with respect to external temperature. We showed that a 5% external temperature variation induces an 75% accuracy variation due to changes in the oscillation rising and falling times. The OIM performs better when the oscillator rising and falling times are matched and therefore needs a robust biasing scheme. Our study suggests that VO_2 thermal properties must be considered when designing VO_2-OIMs. Finally, we believe such temperature sensitivity could be harnessed for designing annealing schemes using the temperature built-up at VO_2 device level.

References

[1] E. Ising, Beitrag zur Theorie des Ferromagnetismus, Z. Physik 31, 253–258, 1925.
[2] J. Hopfield and D. Tank, Neural Computation of Decisions in Optimization Problems. Biological cybernetics, 52. 141-52, 1985.
[3] A. Lucas, Ising formulations of many NP problems, Frontiers in Physics, vol. 2, 2014.
[4] T. Wang, L. Wu, P. Nobel and J. Roychowdhury. Solving combinatorial optimisation problems using oscillator based Ising machines. Natural Computing. 20. 1-20, 2021.
[5] M. K. Bashar, A. Mallick and N. Shukla, Experimental Investigation of the Dynamics of Coupled Oscillators as Ising Machines, in IEEE Access, vol. 9, pp. 148184-148190, 2021.
[6] "D-Wave systems". https://www.dwavesys.com/.
[7] T. Honjo, T. Sonobe, K. Inaba, T. Inagaki, T. Ikuta, Y. Yamada, T. Kazama, K. Enbutsu, T. Umeki, R. Kasahara, K. Kawarabayashi and H. Takesue, 100,000-spin coherent Ising machine, Science advances, 2021.
[8] M. Yamaoka, C. Yoshimura, M. Hayashi, T. Okuyama, H. Aoki and H. Mizuno, A 20k-Spin Ising Chip to Solve Combinatorial Optimization Problems With CMOS Annealing, in IEEE Journal of Solid-State Circuits, vol. 51, no. 1, pp. 303-309, 2016.
[9] I. Ahmed, P. -W. Chiu, W. Moy and C. H. Kim, A Probabilistic Compute Fabric Based on Coupled Ring Oscillators for Solving Combinatorial Optimization Problems, in IEEE Journal of Solid-State Circuits, 2021.
[10] Z. Fahimi, M.R. Mahmoodi, H. Nili, et al. Combinatorial optimization by weight annealing in memristive hopfield networks. Sci Rep 11, 16383, 2021.
[11] S. Dutta, A. Khanna, J. Gomez, K. Ni, Z. Toroczkai and S. Datta, Experimental Demonstration of Phase Transition Nano-Oscillator Based Ising Machine, 2019 IEEE International Electron Devices Meeting (IEDM), pp. 37.8.1-37.8.4, 2019.
[12] S. Dutta, A. Khanna, A.S. Assoa, et al. An Ising Hamiltonian solver based on coupled stochastic phase-transition nano-oscillators. Nat Electron 4, 502–512, 2021.
[13] K. Liu, S. Lee, S. Yang, O. Delaire, and J. Wu, Recent progresses on physics and applications of vanadium dioxide, Mater. Today, vol. 21, pp. 875–896, 2018.
[14] A. Zimmers, L. Aigouy, M. Mortier, A. Sharoni, S. Wang, K. G. West, et al., Role of Thermal Heating on the Voltage Induced Insulator-Metal Transition in VO_2, Phys. Rev. Lett., vol. 110, p. 056601, 2013.
[15] E. Corti, B. Gotsmann, K. Moselund, A. M. Ionescu, J. Robertson, and S. Karg, Scaled resistively-coupled VO_2 oscillators for neuromorphic computing, Solid-State Electron., vol. 168, p. 107729, 2020.
[16] "Victory Device User Manual", *version 1.19.1.C*, Silvaco Inc.
[17] S. Carapezzi, C. Delacour, G. Boschetto, E. Corti, M. Abernot, A. Nejim, et al., Multi-Scale Modeling and Simulation Flow for Oscillatory Neural Networks for Edge Computing, IEEE NEWCAS 2021.
[18] S. Carapezzi, G. Boschetto, C. Delacour, E. Corti, A. Plews, A. Nejim, et al., Advanced Design Methods From Materials and Devices to Circuits for Brain-Inspired Oscillatory Neural Networks for Edge Computing, in IEEE J. Emerg. Sel. Top. Circuits Syst., vol. 11, no. 4, pp. 586–596, 2021.

Acknowledgment

This work is supported by the European Union's Horizon 2020 research and innovation program, EU H2020 NEURONN (www.neuronn.eu) project under Grant No. 871501.